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ON THE QUANTITATIVE PREDICTION OF BUNCH LENGTHENING IN HIGH ENERGY ELECTRON STORAGE RINGS

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On the quantitative prediction of bunch lengthening in high energy electron storage rings

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Abstract

The longitudinal current dependent electromagnetic interaction between a bunch of charged particles and accelerator components can be described by a Green's Function in time domain or by an impedance in frequency domain. The aim of this paper is to describe a procedure which yields an approximate Green's Function for cylindrically symmetric objects. Once this Green's Function is quantitatively known the equation of motion for the particles can be solved easily by a turn-by-turn tracking code on a computer. Thus it is possible to predict the bunch length and width as a function of charge per bunch for future accelerators and storage rings based on pure geometrical data of the accelerator components. Results are presented for PETRA and LEP. A comparison between measurements at PETRA and computations shows an excellent agreement.

1. Introduction

This work was stimulated by papers of Renieri[1)], Bane, Satoh and Wilson[2)], who described turn-by-turn tracking codes for the computation of the charge density in longitudinal phase space.

The electromagnetic interaction between the charged particles and all accelerator components has been modelled by an RC-network[1)] and by the Fourier Transform of the assumed SPEAR-impedance[2)].

In this paper a method is described which enables a quantitative a priori computation without using models or measurements for the impedance.

The effective impedance of PETRA is known from computation and measurements[3,4)]. Roughly at least 2/3 of the PETRA-impedance are due to the accelerating cavities. Thus the interaction between the bunch and all accelerator components is dominated by the cavities.

The electromagnetic interaction between a bunch of charged particles and its environment can be described by a Green's Function (δ-function wake potential). This potential gives the energy loss of a particle behind a leading particle after the passage of the entire structure as a function of their distance. Such δ-function wake potentials cannot be calculated exactly for realistic structures[12)].

An approximate Green's Function can be computed by means of the computer program BCI[5)] which solves Maxwell's equations in the domain including the presence of free moving charges. Instead of a leading point charge a leading short Gaussian bunch is used. It has been verified that the error introduced by this approximation is small.

The "single passage" limit in the beam-cavity interaction which neglects memory effects between successive passages is nearly reached in PETRA and will be reached in larger storage rings with room temperature cavities as well. Since memory effects can always be cured by lowering the Q-value of the resonant modes, these effects do not really limit the performance of a ring. Thus we restrict our considerations to the more important "single passage" limit.

The approximate Green's Function for PETRA has been used in a turn-by-turn tracking code including:

1) distributed rf-stations
2) quantum excitation and damping
3) higher harmonic rf-system with automatic phasing, mismatched phase and amplitude
4) rf-noise
5) continous radiation energy loss between rf-stations

The code can be used interactively and thus enables the simulation of realistic injection procedures and acceleration of an accumulated beam to higher energies.

A comparison of computed results with measurements at PETRA is quite limited since the current in short bunches is limited by the transverse PETRA instability[6]. A situation with a high bunch lengthening cannot be reached. The few data which have been taken during recent machine studies*) show (within the accuracy of the measurement) an excellent agreement with the computer simulations.

Results for PETRA at different energies and LEP are presented.

*) The measurements have been performed by the PETRA storage ring group

2. The Green's Function for the PETRA cavities

Since the electromagnetic interaction between the bunch of particles and the accelerator is assumed to take place only in the accelerating cavities, we need to know the wake potential inside the bunch whenever the particles pass an rf-station.

The easiest way is to run each time the BCI-program which needs at least 30 seconds computer time. A computer simulation over a few damping times needs of the order of a few hundred turns at least. Thus such a simulation is impracticable.

A much faster way is to try to compute a Green's Function for the wake potential and then to sum over all contributions of the single particles. This brings the computer time per turn down to the order of milliseconds.

Since the particle motion is determined by the wake potential inside the finite bunch it is not of particular importance whether the Green's Function itself is accurate. The only important question is how accurate the Green's Function generates the potential inside the finite bunch. Since it is not possible to reconstruct a δ-function wake potential from potentials in finite bunches[7], the computer code BCI has been used with a short Gaussian bunch approximating the leading point charge.

For the 500 MHz PETRA cavity[8] a mesh with 55.000 grid points has been used with a step size 1/8 cm. The shortest bunch length which is permissible in BCI is determined by convergence criteria[5] and computational effort. A reasonable limit is

$$\sigma_{r.m.s.} \geq \text{structure length}/100$$

This limit corresponds to a total computation cost of 2 Mbyte * 1 hour on an IBM 370/168. For accelerating cavities this limit can also be expressed in degrees of the accelerating voltage as $\sigma \geq 3° rf$. For the PETRA cavity $\sigma = 5$ mm has been chosen as lower limit.

For such big meshes together with short bunches higher order parasitic effects due to the numerical approximations in BCI become significant. In analogy to Ref. 9 the potentials can be improved significantly by subtracting the erroneous contribution of a beam pipe of the same length as the original structure. This correction term has to be computed in a second run with BCI.

The wake potential behind the middle of the short Gaussian bunch ($\sigma_{r.m.s.} = 5$ mm) is now considered to be a Green's Function. Fig. 1 shows the approximate δ-function wake potentials for the future PETRA cavity[10], the present PETRA cavity[8] and the higher harmonic 1 GHz cavity in PETRA[10].

The wake potential $w_\lambda(x)$ inside a bunch of finite length with line charge density $\lambda(x)$ can be computed from the Green's Function (δ-function wake potential) $w_\delta(x)$ as:

$$w_\lambda(x) = \frac{1}{Q} \int_x^\infty \lambda(y)\, w_\delta(y-x)\, dy \qquad (1)$$

The potential $w_\lambda(x)$ (together with RF- and other machine parameters) finally determines the particle motion. The accuracy of the approximate Green's Functions can now be checked by evaluating the integral (1) for a set of line charge densities with different r.m.s. length. For the same bunches the potentials can be computed directly by the BCI-program, which gives correct results. A comparison of both results yields a measure for the accuracy of the Green's Functions. Fig. 2 gives for three different Gaussian bunches the correct (straight line) and the approximated potential (broken line) from Equ. (1). Both potentials start off from zero on the left where the early (leading) particles are sitting.

The maximum of the difference between the potentials goes down with increasing bunch length as shown in Fig. 2d. Since a wrong wake potential cannot introduce significant errors where no particles are, a more realistic measure is probably the difference between the two potentials weighted with the normalized particle density. This weighted error as a function of bunch length is also given in Fig. 2d.

The conclusion from Fig.2 is that the approximate Green's functions are valid for bunches with r.m.s. length above 5 mm.

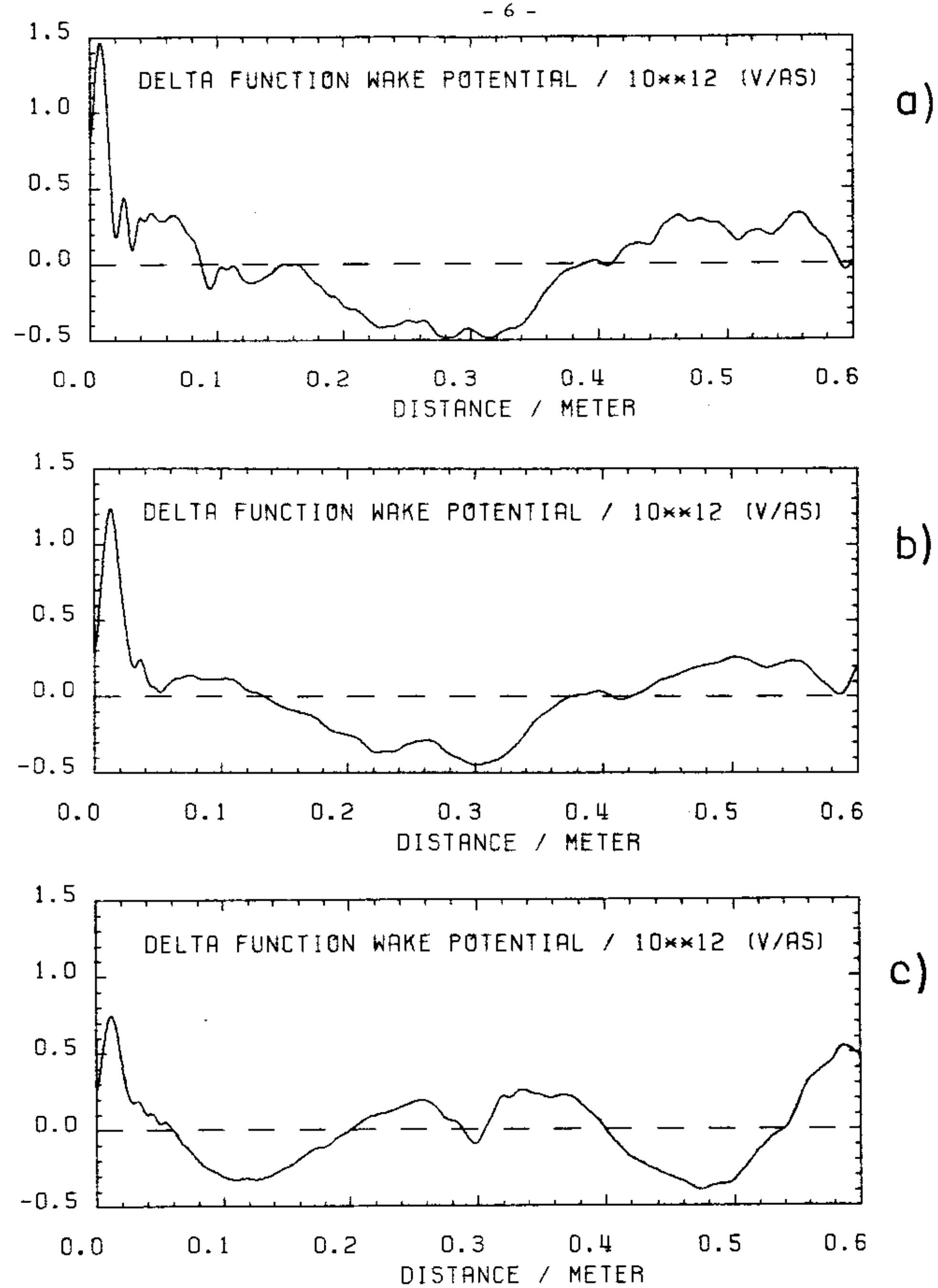


Fig.1: The approximate Green's Functions for
a) the future PETRA cavity (500 MHz)
b) the present PETRA cavity (500 MHz)
c) the higher harmonic PETRA cavity (1 GHz)
(all normalized to a single cell)

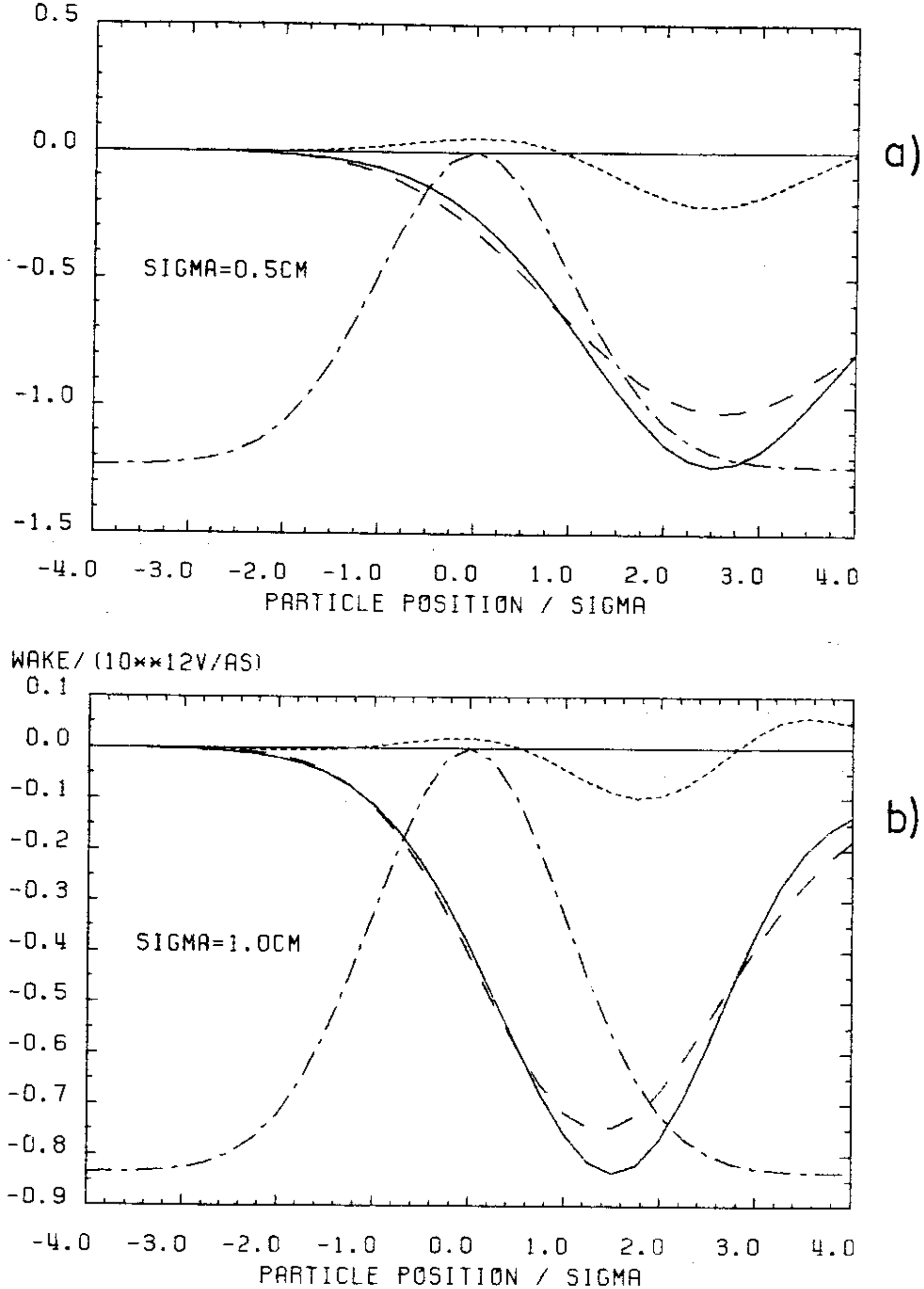
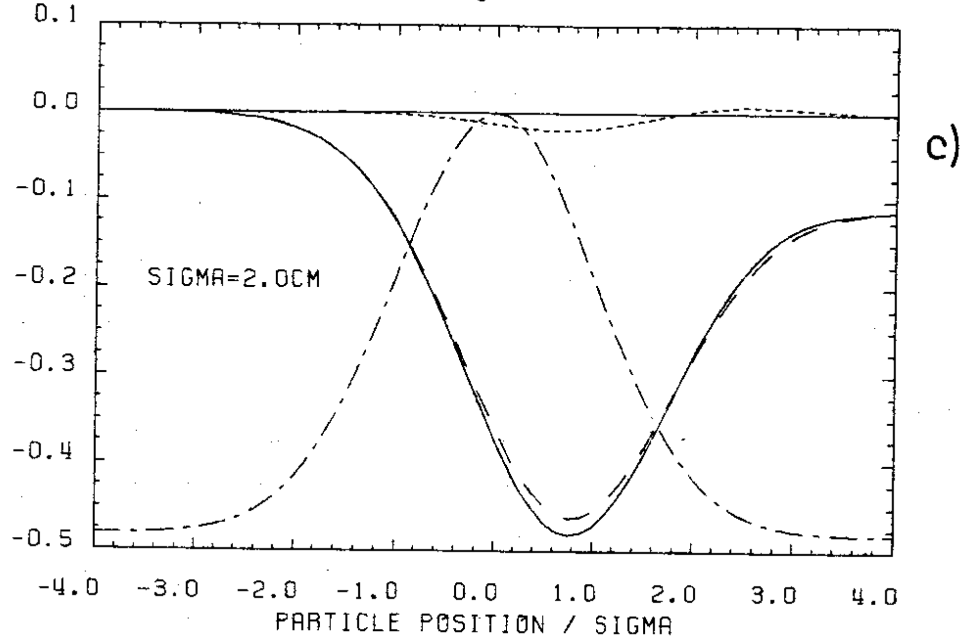


Fig. 2a, b, c:
- ——— correct wake potential (directly by BCI)
- — — approximate potential (generated by Green's Function)
- ----- error of the approximate potential
- -·-·- particle density in the bunch (front left)



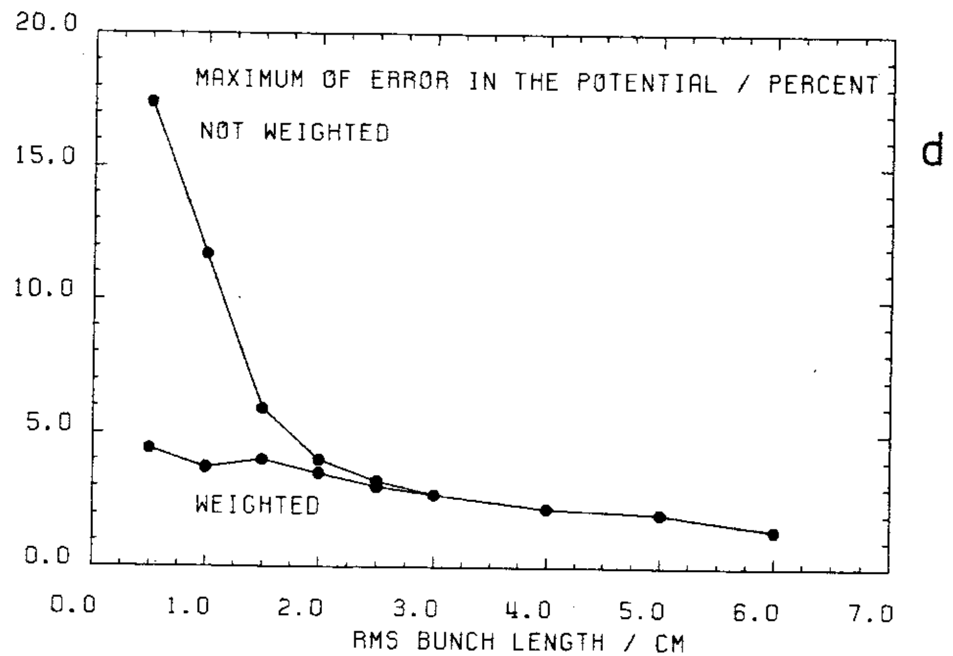


Fig. 2d: Maximum error of the approximate wake potential generated by the "quasi"-Green's Function as a function of bunch length (weight = particle density)

3. The tracking equations

A ring can be modeled (for the longitudinal particle motion) by point like rf-stations and arcs in between. Assuming a superperiodicity given by the (N_{RF}) rf-stations, the transport equations for a particle over one period are known to be:

$$\tau^{n+1} = \tau^n + \frac{\alpha T_o}{E_o N_{RF}} \cdot \varepsilon^n - \frac{\alpha T_o U_o}{2 N_{RF}^2 E_o} \quad (2a)$$

$$\varepsilon^{n+1} = \varepsilon^n + \frac{e \hat{V}_o}{N_{RF}} \sin(\omega_o \tau^{n+1}) + \frac{e \hat{V}_1}{N_{RF}} \sin(\omega_1 \tau^{n+1} + \varphi_1) - e w(\tau^{n+1}) - \frac{U_o}{N_{RF}} - \frac{2 T_o}{N_{RF} T} \varepsilon^n + 2 \sqrt{\frac{T_o}{N_{RF} T_\varepsilon}} \cdot \sigma_{o\varepsilon} \cdot R \quad (2b)$$

With

$\hat{V}_o$, $\hat{V}_1$	total circumferential voltage of first and second RF system
τ	particle position in time with origin at zero of main RF
ε	particle energy deviation from nominal energy E_o
U_o	synchrotron radiation loss per turn
N_{RF}, N_P	number of rf-stations, number of "super particles "
$\sigma_{o\varepsilon}$	zero current r.m.s. energy spread
T_ε	damping time
T_o	revolution time
α	momentum compaction
R	random munber, $\langle R \rangle = 0$, $\langle R^2 \rangle = 1$
$w(\tau)$	wake potential at τ

Memory effects due to resonant modes in cavities are not included. The third term on the right hand sight of equ. 2a takes care of the continous radiation energy loss between two successive rf-stations. Due to this term a synchronous particle with amplitude zero does not exist.
The wake potential at position τ can be computed by summing all contributions of the particles in front of τ as[2]:

$$w(\tau) = \frac{Q}{N_P N_{RF}} \sum_{\substack{i=1.N_P \\ \tau_i > \tau}} w_\delta (\tau_i - \tau) \quad (3)$$

An evaluation of this sum needs $\sim N_P^2$ operations.
A second way is to solve approximately equ. (1). First all super particles are put into bins of width Δ. With $n(\tau_i)$ as the number of particles in bin i the wake potential becomes

$$w(\tau_j) = \frac{Q}{N_P N_{RF}} \sum_{\substack{i=1.N_{bin} \\ \tau_i > \tau_j}} n(\tau_i) \, w_\delta(\tau_i - \tau_j) \quad (4)$$

An evaluation of this sum needs only $\sim N_P$ operations plus an additional term $\sim N_{bin}^2$ for filling the bins. For $N_P > N_{bin}$ a significant gain in computation time can be achieved.

4) The computer program

With increasing charge/bunch the particle distribution becomes more and more sensitive to changes in the parameters. The maximum current can be reached only by slow injection. In order to have a computer simulation as realistic as possible the tracking equations have been coded in an interactive computer program. The code enables simulation of injection, dynamic change of machine parameters like rf-power as well as acceleration of an accumulated beam to higher energies.
In addition to equ. (2) some options are available like rf-noise, automatic regulation of the rf-parameter of the higher harmonic system including dynamic beam loading and mismatch of phase and amplitude of the second rf-system.
The number of super particles determines the statistical fluctuation and should not be much less than 1000. The bin width should be smaller than the r.m.s. length of the bunch. A general formula for the computation time cannot be given. For machines like PETRA and LEP in which the injection energy is not the top energy, a complete simulation of an injection procedure may need the order of hours of cpu-time.

At top energy the simulation is much faster and takes only a few minutes for ten damping times.

5. Results

5.1 PETRA

In order to check the computer simulation code described in the previous sections it has been tried during recent machine studies to achieve a high bunch lengthening. The following machine parameters have been chosen (optic = MI 100):

$E_o = 7$ GeV , $\hat{V}_o = 34$ MV ,

$\alpha = 0.00272$, $\sigma_{os} = 0.5$ cm ,

$f_s = 11.6/11.7$kHz , $\sigma_{o\varepsilon} = 3$ MeV ,

60 five-cell cavities installed

At currents above 4 mA (single bunch) the bunch length could not be measured due to transverse instabilities*) .

Fig. 3a shows the measured and computed bunch length versus single bunch current and an excellent agreement between both. The computed energy spread is shown in Fig. 36. Bunch length and width increase simultaneously.

A comparison with earlier measurements has not been done since those data have been taken with fewer cavities installed.

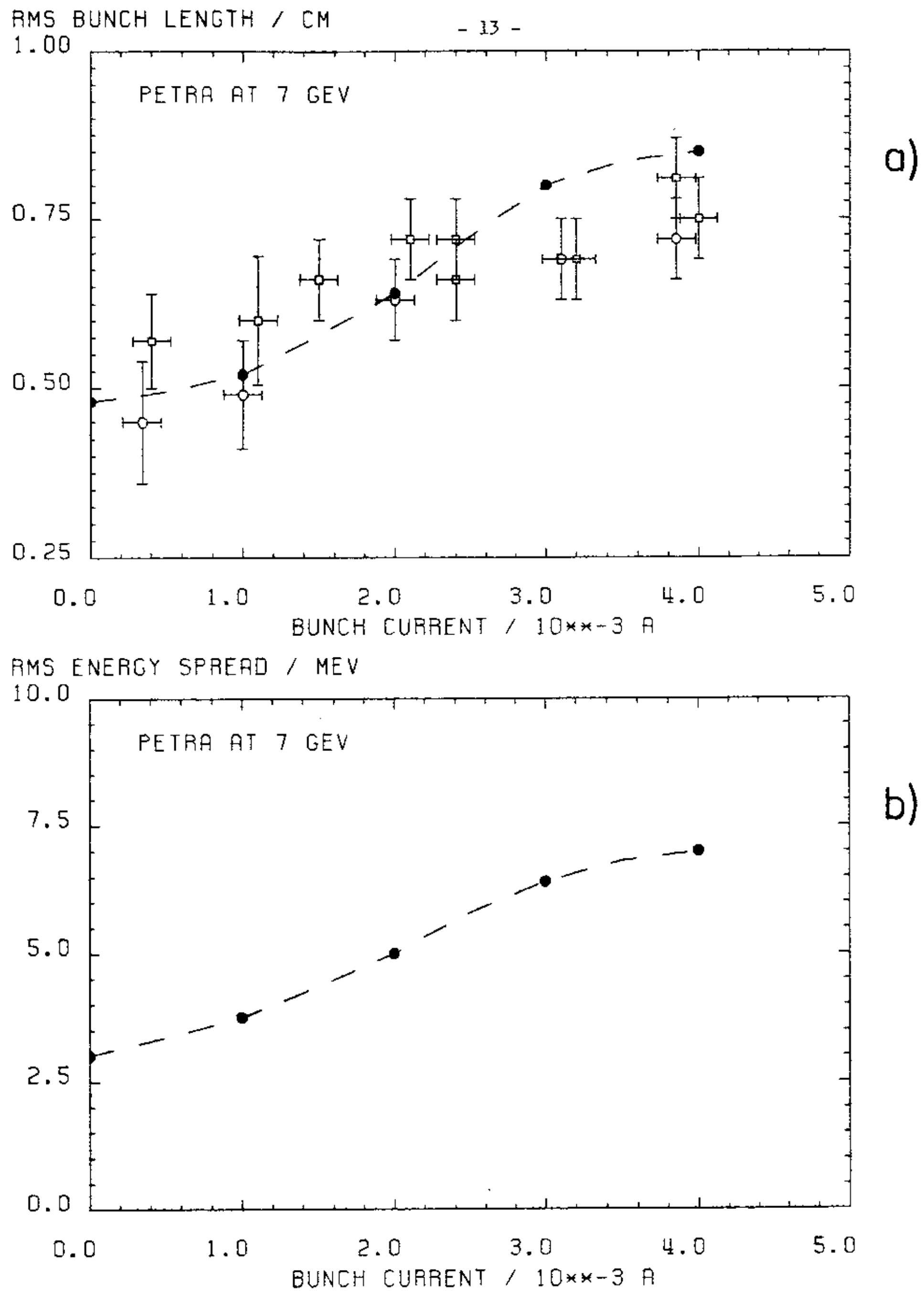
*) For physic runs the single bunch current at injection (7 GeV) can be up to 20 mA when the bunch is prelengthened by changing the damping partition J_ε , but for these parameters (long bunches) no bunch lengthening is observed.

Fig. 4 shows results of the computer simulation at 18 GeV. It can be seen that significant collective effects due to the longitudinal wake potential occur but for currents much higher than the currents which can be accelerated to this energy in PETRA ($I_b \leq 10$ mA).

Near the threshold current ($I_{th} = 30$ mA) at 18 GeV strong dipole and quadrupole oscillations are found. The dipole moment of motion can be made visible by plotting the mean values of energy deviation ε and phase position φ ($\varphi = \omega\tau$).

Fig. 5 shows the phase space of $< \varepsilon(t) >$ and $< \varphi(t) >$ for a few hundred turns (each point gives mean energy and phase after one turn). At low current ($I_b = 5$ mA) the bunch center is seen to stay constant in time at nominal energy and at a stable phase angle of $\varphi_s = 143.5°$. Near the threshold current ($I_b = 30$ mA) the bunch center moves on an ellipse in the phase space $\{<\varepsilon>, <\varphi>\}$ with time. Thus a dipole moment is found with an amplitude of about 21 MeV in energy and 11° in phase (1° $\hat{=}$0.17 cm $\hat{=}$5.6 ps).

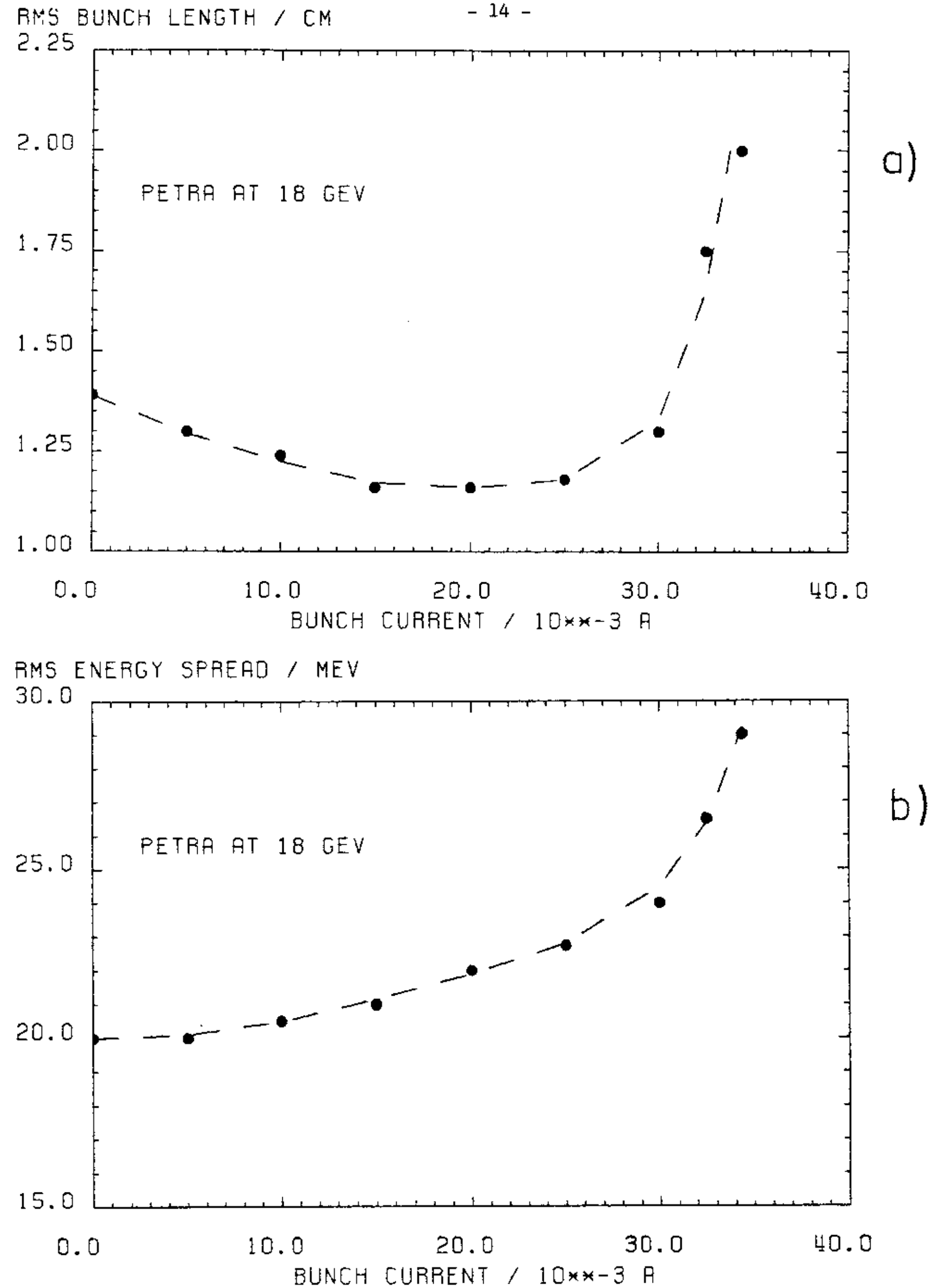
Quadrupole oscillations can be detected by displaying the r.m.s. bunch energy spread and length as a function of time as shown in Fig. 6 (each point gives the r.m.s. energy spread and r.m.s. bunch length in degree after one turn). At low current bunch length and spread stay constant with time at values of 20 MeV and 8° ($\hat{=}$1.3 cm). Above the threshold current ($I_b = 30$ mA) length and width oscillate alternatively with an amplitude of roughly 5 MeV in energy spread and 3-4° degree in phase ($\hat{=}$0.6 cm in bunch length).



<u>Fig. 3:</u> computed and measured bunch length/width for PETRA at 7 GeV (60 cavities)

-- computations

measurements, f_s = 11.7/11.6 kMz



<u>Fig. 4:</u> bunch length and width versus single bunch current in PETRA at 18 GeV (60 cavities, α = .003, $\sigma_{o\epsilon}$ = 20 MeV, $\hat{V}_o$ = 90 MV, $\sigma_{o\epsilon}$ = 1.39 cm, f_s = 11.44 kHz)

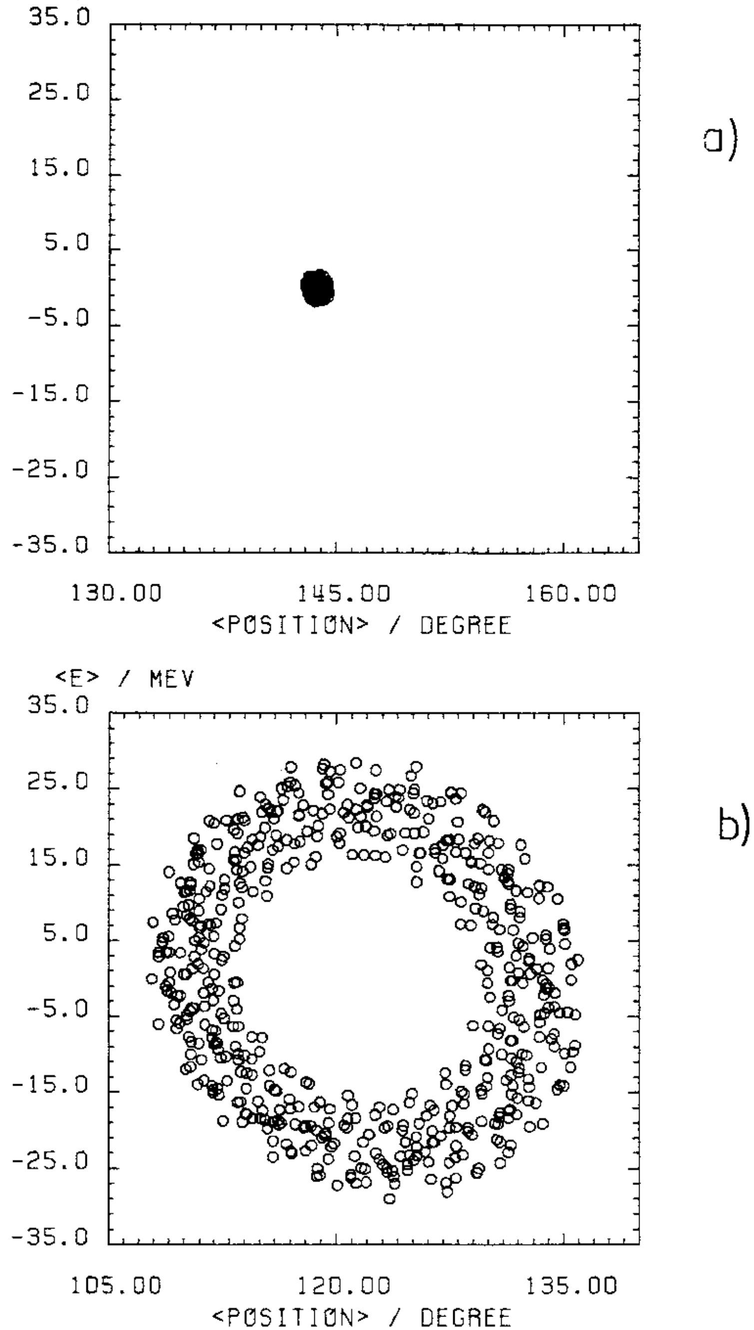


Fig. 5: Phase space of mean values $<\varepsilon>$ and $<\varphi>$. Each point gives the mean energy and phase (position) after one turn for a) I_b = 5 mA, b) I_b = 30 mA. (PETRA, 18 GeV, 60 cavities)

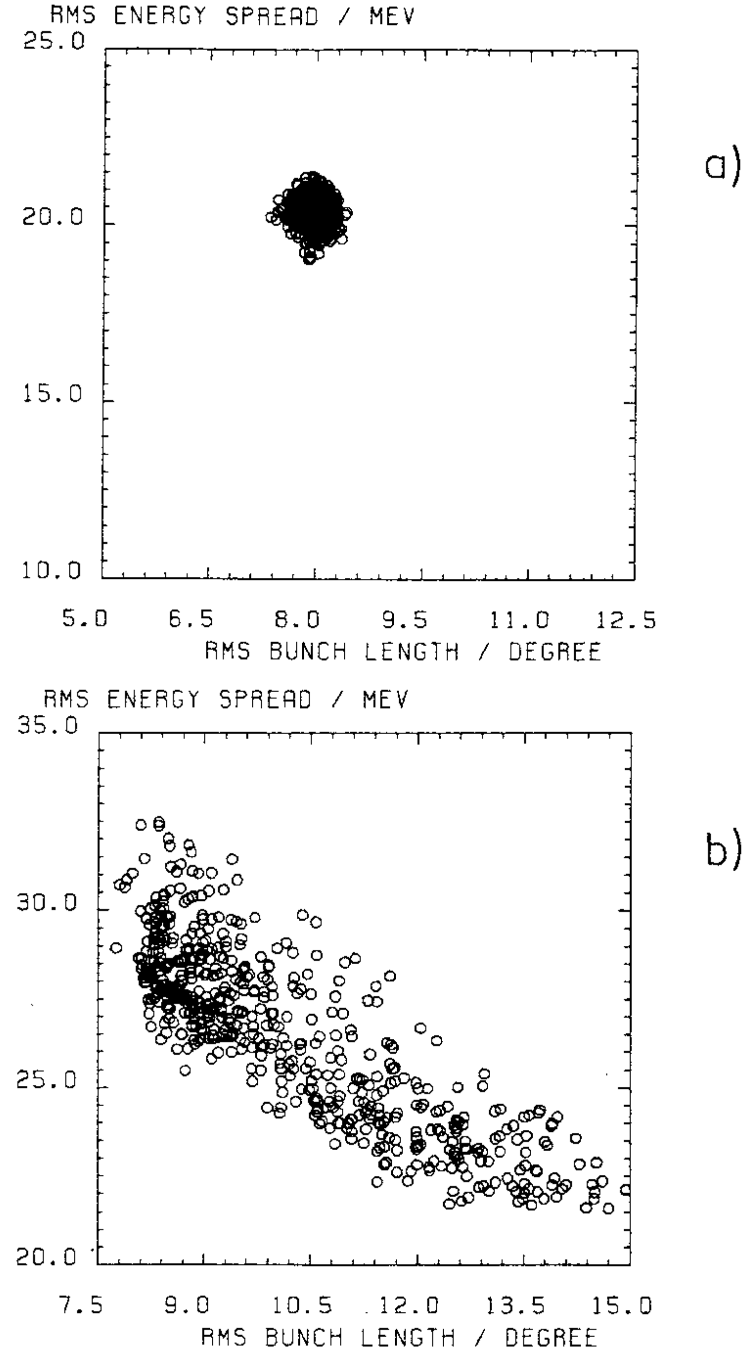


Fig. 6: Phase space of $\varepsilon_{r.m.s.}$ and $\varphi_{r.m.s.}$ Each point gives $\varepsilon_{r.m.s}$ $\varphi_{r.m.s.}$ (bunch width and length) after one turn for a) I_b = 5 mA, b) I_b = 30 mA. (PETRA 18 GeV, 60 cavities)

5.2 LEP

In order to get some results for LEP[13] without computing the δ-function wake potential for the LEP cavity, the Green's Function for the future PETRA cavity was used (beam hole diameter 9 cm). The number of single cavity cells was scaled to the same active length as foreseen for the 353 MHz system. The following machine parameters[11] were used (no wiggler magnet, no higher harmonic rf-system):

	Phase I		Phase II
E_o	18 GeV	50 GeV	85 GeV
$\hat{V}_o$	90 MV	302 MV	2050 MV
Q_{so}	.07	.067	.126
σ_{os}	.5 cm	.94 cm	.84 cm
$\sigma_{o\epsilon}$	5 MeV	39 MeV	111 MeV
N_{cell} (500 MHz)	910	910	5440
L_{RF}	273 m	273 m	1632 m
$\alpha = 1.93\ 10^{-4}$, $h = 31316$, $f_{rf} = 353$ MHz			

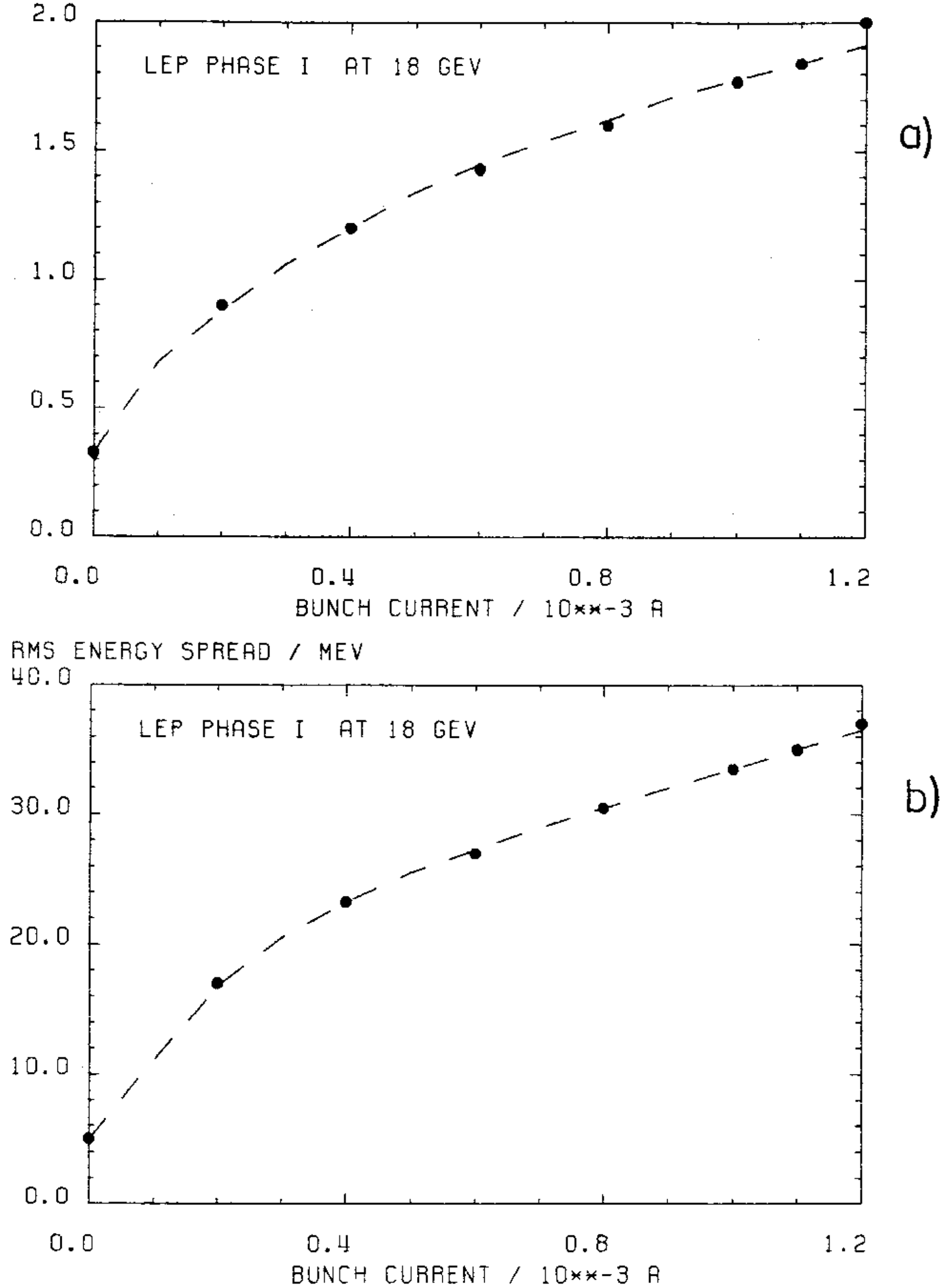
Fig. 7 and 8 show the bunch length and energy spread versus single bunch current in LEP-Phase I at injection energy and at 50 GeV. All results are qualitatively very similar to the PETRA results (Fig. 3 and 4).

At 50 GeV and $I_b = 1.6$ mA an incoherent instability is found similar to the overshoot mechanism[13]. Such instabilities have been observed in DORIS[14] and PETRA. Fig. 9 shows the bunch length as a function of time. In Fig. 9b growing dipole and quadrupole oscillations can be observed during the blow up.

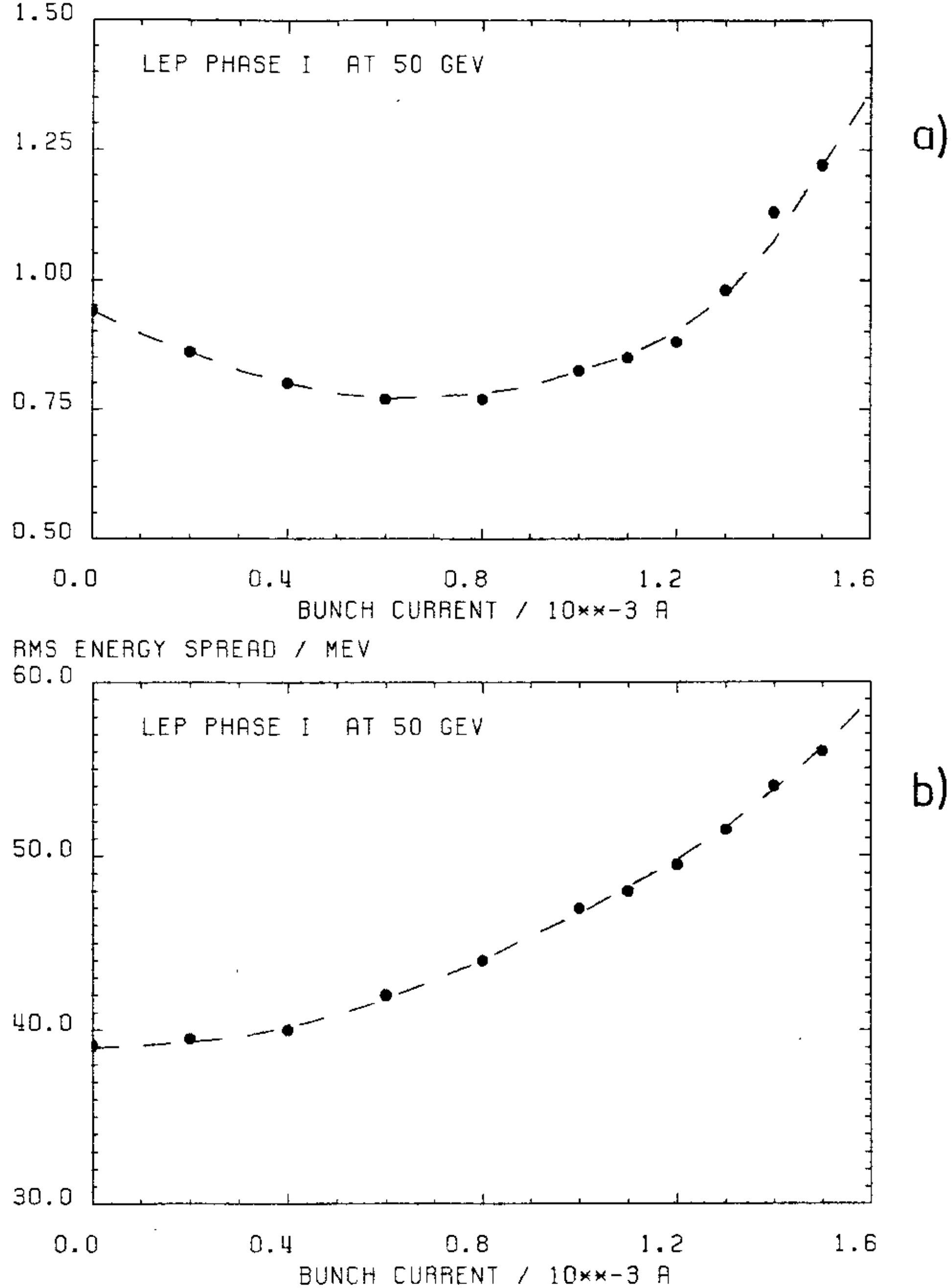
For LEP-Phase II with full rf-system the previous results are still valid after scaling the bunch current with the inverse number of cavities. At top energy (85 GeV) bunch length and spread behave very similar to 50 GeV (at six times lower current), see Fig. 10.

A comparison of theses results with simulations using the Green's Function for the present PETRA cavity did show quantitatively very different results which cannot be explained by simple scaling of the effective impedance.

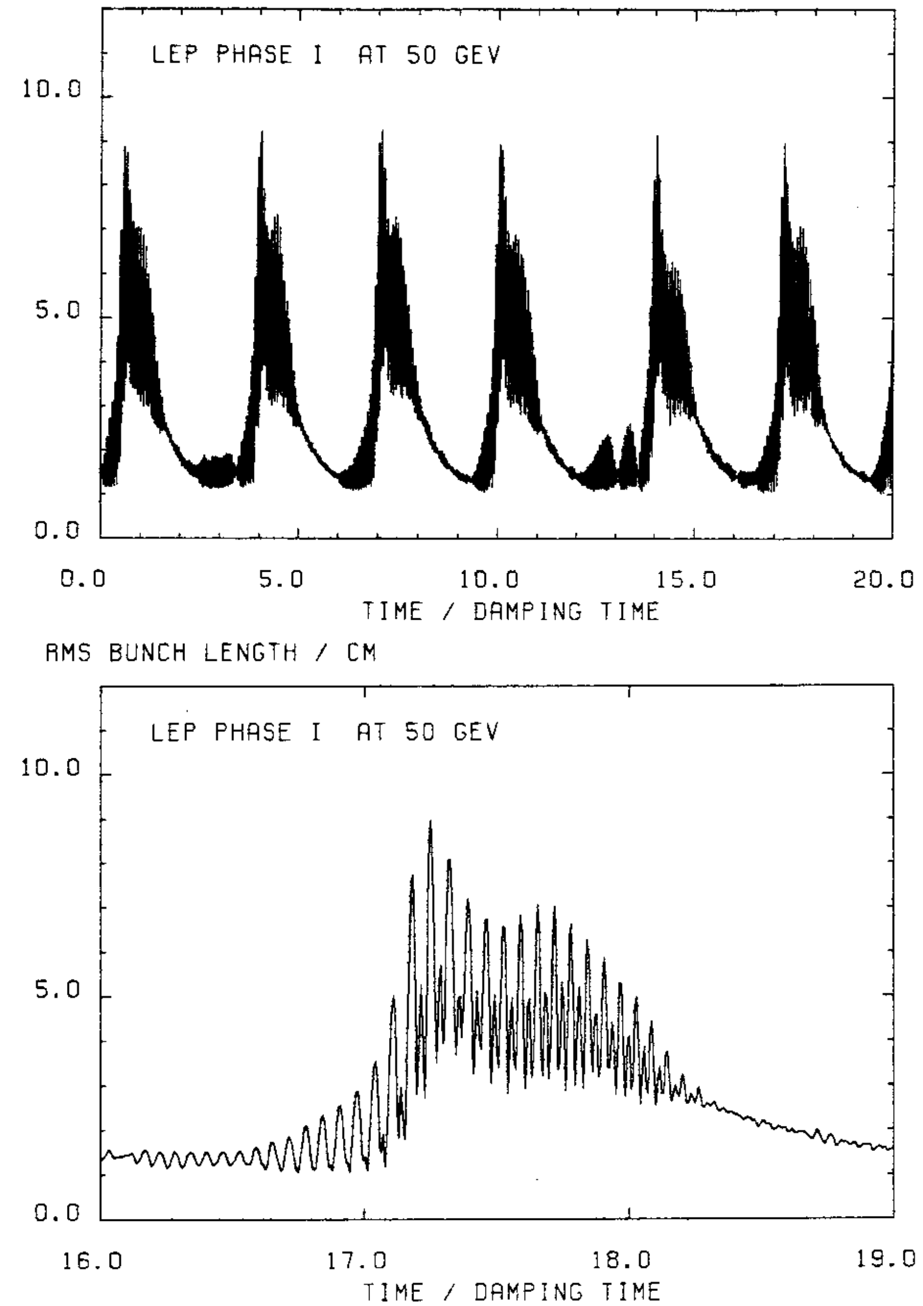
The effective impedances of the new and old PETRA cavities are 3.3 MΩ and 2.3 MΩ at σ = 1 cm in PETRA. A linear scaling of the threshold current yields $I_{th} = 1.3$ mA for the future cavity and $I_{th} = 1.9$ mA for the present one. The simulation shows instead of 1.9 mA a threshold current of more than 3 mA. The behaviour of bunch length and spread obviously depends sensitively on the particular shape of δ-function wake potential and thus on the cavity shape.



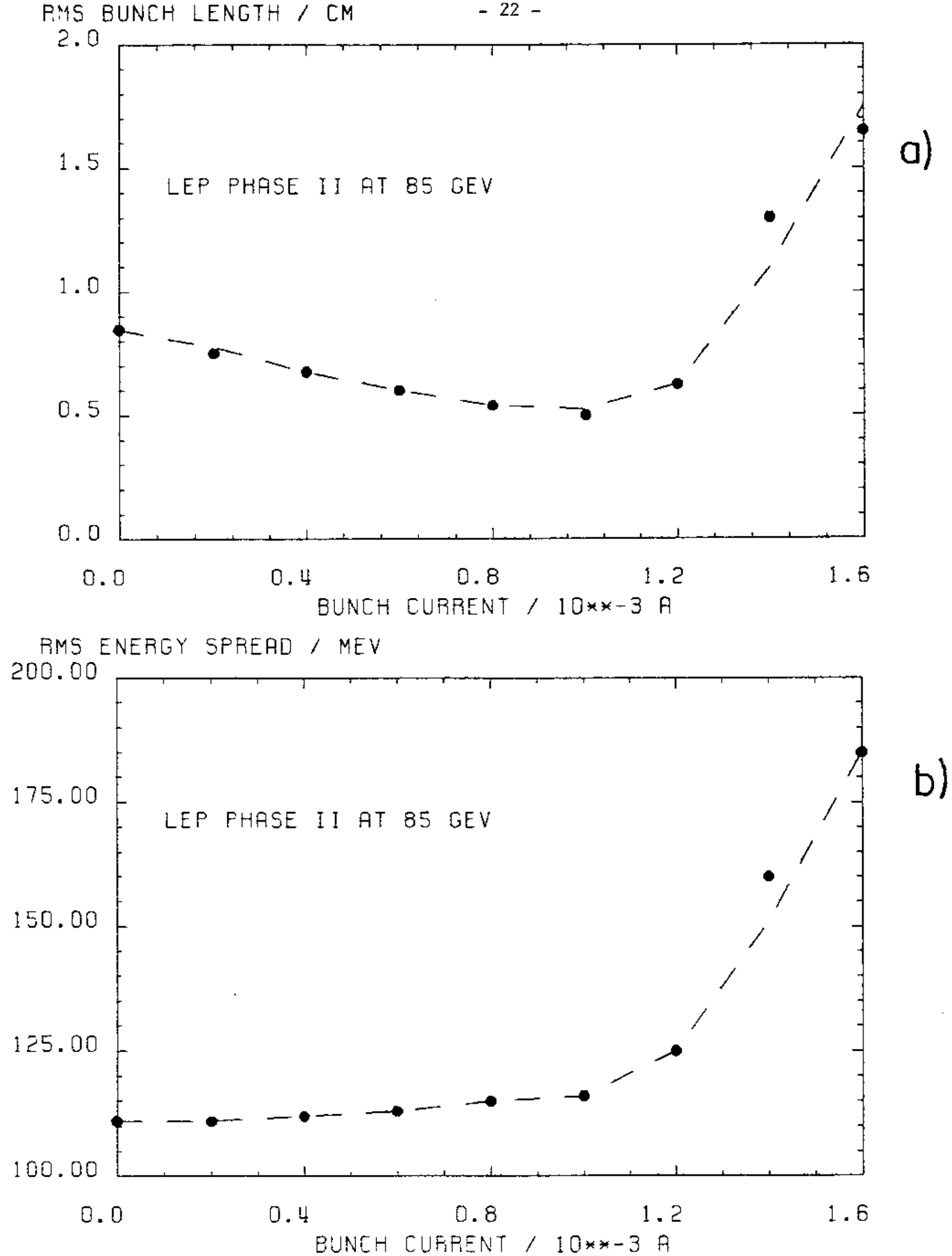
<u>Fig. 7:</u> bunch length and width versus single bunch current in LEP-phase I (1/6 rf) at injection energy



<u>Fig. 8:</u> bunch length and width versus single bunch current in LEP-phase I (1/6 rf) at top energy



<u>Fig. 9:</u> bunch length as a function of time is LEP-phase I at 50 GeV I_b = 1.6 mA.



<u>Fig. 10:</u> bunch length and width versus single bunch current in LEP-phase II (full rf) at 85 GeV.

6. Conclusions

The procedure described enables a quantitative prediction of bunch length and energy spread as a function of single bunch current. Necessary input data are some basic machine parameters and the geometrical shape of the most numerous structures contributing to the impedance (i.e. rf cavities in high energy storage rings).
Results of the computer simulations agree with measurements at PETRA.
All simulations show bunch shortening at top energy for a wide range of current in PETRA and LEP. This result may have an impact on accelerator design since the bunch length at top energy determines the rf-voltage needed (or the top energy for a given rf-structure).
The detailed form of the Green's Function seems to be important. Simple scaling between different rf-cavities by means of the effective impedance do not hold.

7. Literature

1) A. Renieri — Laboratori Nazionali di Frascati del CNEN, Report No. LNF-75/11 R, February 1976

2) P.B. Wilson, K.L.F. Bane & Kohtaro Satoh — IEEE Transactions on Nuclear Science, Vol NS-28, No. 3, June 1981

3) T. Weiland — DESY M-81-04

4) T. Weiland — DESY M-81-23

5) T. Weiland — Proceedings of XI International Conference on High Energy Accelerators, Geneva 1980 and CERN/ISR-TH/80-07, 45, 46

6) R.D. Kohaupt — DESY 80-22

7) H. Sands — private communication

8) H. Gerke et al. — DESY PET-77-08

9) T. Weiland — CERN, Lep-note 236

10) H. Gerke — private communication

11) A. Hutton, J. Jowett, E. Keil — private communication

12) T. Weiland, B. Zotter — Particle Accelerators 1981 Vol. 11, pp. 143-151 and references in it

13) The LEP Study Group — CERN/ISR-LEP/79-33

14) R. A. Dory, — Ph. D. Thesis, MURA Report 64 Madison, Wiscousin 1962

15) R. D. Kohaupt — DESY 77/66

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