DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 82-040
July 1982

FERMIONS IN LATTICE QUANTUM CHROMODYNAMICS

by

I.Montvay

II. Institut für Theoretische Physik der Universität Hamburg

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 82-040
July 1982

Fermions in Lattice Quantum Chromodynamics[+]

I. Montvay
II. Institut für Theoretische Physik der Universität Hamburg[++]

Abstract: The basic concepts and recent progress in calculation of the hadron spectrum in lattice QCD are reviewed.

I. Introduction: lattice actions.

The formulation of quantum gauge field theories on a space-time lattice [1] allows for an approximate numerical computation of non-perturbative features like masses, spontaneous symmetry breaking, phase transition temperatures etc.. The first exiting results in lattice gauge theories were obtained in pure gauge field systems without matter fields [2-4]. In order to calculate hadron properties in quantum chromodynamics, however, the inclusion of quark fields is necessary. The quarks, as every fermion, are described on the lattice by anticommuting Grassmann-variables and this causes new, non-trivial problems for the numerical evaluation. The purpose of this lecture is to introduce the basic concepts and tools for lattice fermions and to overview some of the recent Monte Carlo results in lattice QCD. The review will be incomplete in many respects. I shall concentrate mainly on the mass spectrum leaving out many important questions such as fermion condensates etc..

\+ Lecture presented at the Workshop on Quark Matter Formation and Heavy Ion Collisions, Bielefeld, 10 - 14 May 1982

++ Supported by Bundesministerium für Forschung und Technologie, Bonn, Germany

The euclidean lattice action for the pure $SU(N_c)$ gauge field (without fermions) can be written like

$$S_g = g^{-2} \sum_{\Box} \left(N_c - \mathrm{Tr}(U_{\Box}) \right) , \tag{1.1}$$

where the sum runs over the plaquettes $\Box$ on the lattice and $U_{\Box}$ is the $SU(N_c)$ group element belonging to the plaquette $\Box$. (It is a product of the four link variables belonging to the boundary of the plaquette: $U_{\Box} = U_4 U_3 U_2 U_1$; $U_i \in SU(N_c)$.) The bare coupling constant $g = g(a)$ is a function of the lattice spacing a. For small a the behaviour of the function g(a) is dictated by the renormalization group Callan-Symanzik β-function. For $a \to 0$ and $g \to 0$ we have:

$$a \Lambda_{latt} = \left(\beta_0 g^2 \right)^{-\beta_1/(2\beta_0^2)} \exp\left\{ -\frac{1}{2\beta_0 g^2} + \sigma(g^2) \right\} ; \tag{1.2}$$

$$\beta_0 = \frac{1}{(4\pi)^2} \left(\frac{11 N_c}{3} - \frac{2 N_f}{3} \right) ; \quad \beta_1 = \frac{1}{(4\pi)^4} \left[\frac{34 N_c^2}{3} - \frac{10}{3} N_c N_f - \frac{(N_c^2-1) N_f}{N_c} \right] .$$

Here N_f denotes the number of (massless) quark flavours. The lattice scale parameter Λ_{latt} is proportional to the Λ-parameter of other renormalization schemes [5]. The factor of proportionality depends on the form of the lattice action choosen.

The relevance of numerical lattice calculations for the continuum theory relies on the observation of the renormalization group behaviour of the results in the small lattice spacing (hence small g) region. As a matter of fact, present day computers do not allow to go to very small couplings, but one of the pleasant surprises is, that the renormalization group behaviour (1.2) sets in for the available, not very small (or even intermediate) couplings.

The first difficulty on the way of including fermions in the lattice action is the so called "fermion doubling", which can already be seen in the free case. Substituting the continuum derivates $\partial_\mu \psi$ "naively" by lattice derivates $a^{-1}(\psi_{x+\hat{\mu}} - \psi_x)$, the free fermion euclidean action with mass m would be

$$S_f = \sum_x \left\{ m a^4 \tilde{\psi}_x \psi_x - \frac{a^3}{2} \left(\tilde{\psi}_{x+\hat{\mu}} \gamma_\mu \psi_x - \tilde{\psi}_x \gamma_\mu \psi_{x+\hat{\mu}} \right) \right\} . \qquad (1.3)$$

Here, and in the following, x denotes lattice points and $\hat{\mu}$ is a lattice unite vector in the direction $\mu = 1,2,3,4$. As it can be easily seen, the fermion propagator corresponding to (1.3) has poles not only at $ap_\mu \cong 0$ (for $a \to 0$) but also at $ap_\mu \cong \pm \pi$. The appearence of these superfluous states is not an accident. It has deep geometrical and topological grounds and is, in fact, a serious problem for putting fermion fields on the lattice. One way of elimivating the unwanted states was given by Wilson [1,6], who changed the free fermion action to

$$S_f = \sum_x a^3 \left\{ (4+am) \tilde{\psi}_x \psi_x - \frac{1}{2} \sum_\mu \tilde{\psi}_{x+\hat{\mu}} (1+\gamma_\mu) \psi_x \right\} . \qquad (1.4)$$

Here the summation $\sum_\mu$ is ment for both positive and negative directions ($\gamma_{-\mu} = -\gamma_\mu$). For finite lattice spacing a the extra states are still there, but for $a \to 0$ they get a mass proportional to a^{-1}, hence they are decoupled. The price to pay for the elimination of fermion doubling is the loss of chiral symmetry. Even for zero mass fermions the chiral symmetry can only be restored in the continuum limit.

Taking the interaction with gauge fields into account, the full Wilson-action for lattice QCD is, in a convenient normalization:

$$S = S_g + S_f = g^{-2} \sum_\square (N_c - \mathrm{Tr}(U_\square)) \; + \qquad (1.5)$$
$$+ \sum_x \left\{ \tilde{\psi}_x \psi_x - K \sum_\mu \tilde{\psi}_{x+\hat{\mu}} \, U[x+\hat{\mu}, x] (1+\gamma_\mu) \psi_x \right\} .$$

The dependence of the "hopping parameter" K on the bare quark mass m is given here by

$$K = (8 + 2ma)^{-1} , \qquad (1.6)$$

An important property of the action (1.5) is that it is bilinear in the fermion fields:

$$S_f = \sum_{x,y} \tilde{\psi}_x Q_{xy} \psi_y \; ; \qquad Q \equiv 1 - KM \; ;$$
$$M \equiv \sum_{x,\mu} M_{x,\mu} = \sum_{x,\mu} (1+\gamma_\mu) U[x+\hat{\mu}, x] . \qquad (1.7)$$

The matrix $M_{x,\mu}$ is associated to the "hopping" from the point x into the neighbouring point $x+\hat{\mu}$. After performing the integral over the fermion fields we obtain e.g. for the partition function:

$$Z \equiv \int dU \, d\psi \, d\tilde{\psi} \, e^{-S} = \int dU \, e^{-S_g} \det Q \equiv \qquad (1.8)$$
$$\equiv \langle \det Q \rangle .$$

This is the expectation value of the "fermion determinant" in the pure gauge field configuration governed by the gauge field action S_g. The expectation values of physical quantities in the theory with fermions can, therefore, be determined in the pure gauge theory. For instance, the expectation value of the fermion bilinear $\sum_x \tilde{\psi}_x \Gamma \psi_x$ is:

$$\langle \det(1-KM) \cdot \mathrm{Tr}[(1-KM)^{-1} \Gamma] \rangle \Big/ \langle \det(1-KM) \rangle . \qquad (1.9)$$

In fact, the interesting expectation values (like hadron propagators) have a structure similar to (1.9). Therefore, the task is to evaluate the quark determinant det(1-KM) and the matrix elements of the quark propagator $(1-KM)^{-1}$.

In the continuum limit the hopping parameter K goes to a rather small value 1/8, therefore it seems legitimate to try an expansion in it:

$$(1-KM)^{-1} = \sum_{k=0}^{\infty} K^k M^k ,$$
$$S_{eff} \equiv -\ln \det(1-KM) = \sum_{k=1}^{\infty} \frac{K^k}{k} \mathrm{Tr}(M^k) . \qquad (1.10)$$

This "hopping parameter expansion" is an important tool for studying fermions in lattice gauge field theory.

Let us mention also some other ways of putting fermions on the lattice. The "naive" action (1.3) describes 16 identical fermions. This number can be reduced to 4 by putting the different Dirac-components of the field to different lattice sites [7-9]. This "Susskind-fermion" was originally formulated in the Hamitonian formalism with continuous time [7], when the degeneracy of fermion states is only 2. The Susskind fermion action can be written in the form [8]

$$S_f = \sum_x \left\{ am \tilde{\chi}_x \chi_x - \frac{1}{2} \sum_\mu \eta_{\mu,x} \tilde{\chi}_{x+\hat{\mu}} \, \mathcal{U}[x+\hat{\mu},x] \, \chi_x \right\} , \qquad (1.11)$$

where $\eta_{-\mu,x} = -\eta_{\mu,x}$ and at the lattice point $x=(x_1 x_2 x_3 x_4)$

$$\eta_{1,x} = 1 \ , \quad \eta_{2,x} = (-1)^{x_1} \ , \quad \eta_{3,x} = (-1)^{x_1+x_2} \ , \quad \eta_{4,x} = (-1)^{x_1+x_2+x_3} \ . \qquad (1.12)$$

χ is a single component field, hence the different spin components reside on different sites and the four "flavours" described by the χ field are also represented on the lattice non-locally. It can be shown, that the "naive" action (1.3) es equivalent to a direct sum of 4 actions like (1.11). The advantage of the Susskind-fermion action is that it preserves a discrete part of chiral symmetry on the lattice. This is of importance e.g. for the study of spontaneous chiral symmetry breaking.

There are several reasons to believe that it is impossible to further reduce the number of fermions described by a lattice action. One argument relies on triangle anomaly considerations (see later, in chapter III.). Another explanation [10-12] is based on the introduction of the analogue of differential forms on the lattice. The continuum Dirac-equation formulated in terms of differential forms by Kähler [13] describes four degenerate fermions. Putting it on the lattice with the help of the concepts of algebraic topology, one obtains the four fermions described by the Susskind-action.

In order to extend the class of calculable models to include parity violating interactions (like the Glashow-Weinberg-Salam theory of electroweak interactions) it would be necessary to describe chiral fermions on the lattice. This is, however, impossible without giving up some of the principles like translation invariance, hermiticity and locality (in a weak sense, asuring only that the Fourier-transform to momentum space exists) [14,15]. It follows, for instance, from the Poincaré-Hopf theorem on the four-dimensional torus that the number of L-handed and R-handed components must always be equal.

II. Recent results on the hadron spectrum.

The hopping parameter expansion (1.10) of the quark determinant and quark propagator can be used to build up the hopping parameter expansion of hadron propagators. The numerical determination of the expansion coefficients then requires the evaluation of Dirac traces and of the colour traces in some gauge field configuration generated by pure gauge field Monte Carlo [16]. The propagator poles in momentum space, corresponding to the hadrons, imply poles in the complex hopping parameter plane. These poles can be localized by Padé approximants build from the hopping parameter expansion coefficients [17]. At present only relatively few coefficients are known. The extrapolation of the K^{10} order series gives a good description of the mesons but (from K^{12} series) the proton (and other baryon) masses come out too high (e.g. $m_p \cong 17oo$ MeV) and the proton-delta splitting is too small [17]. In general these features resemble the strong coupling expansion results. This shows that the K^{12} series are too short to penetrate the weak coupling ("continuum") region. It remains to be seen whether higher order calculations (24^{th} order and higher [18]) help.

The parameters appearing in the hadron spectrum calculations are g, $K_u = K_d$, K_s and K_c (for u,d,s,c quark flavours, forgetting for the moment heavier flavours). These can be fixed, for instance, by the m_ρ, m_π, m_K and m_D masses and then the other masses are uniquely prodicted. The behaviour of π- and ρ-meson masses as a function of hopping parameter (for fixed g^{-2}) is illustrated by Figure 1. The physical values of K_u, K_s and K_c as a function of g^{-2} are shown in

Figure 2. The physical value of K_u is rather close to the critical value $K = K_{cr}$ where the pion mass vanishes. At $K = K_{cr}$ the hopping parameter expansion does not converge (there is presumably at $K = K_{cr}$ a singularity in the complex K-plane [19]). This implies that a direct evaluation of the Green's functions at the physical value of K_u is a very delicate (and perhaps even dangerous) procedure. It is better to calculate far away from K_{cr} and obtain the physical values by some analytic extrapolation in K.

The quark determinant or the effective action S_{eff} in (1.10) describes the effect of closed internal quark loops in the Green's functions. The phenomenological success of the Zwei-rule, $^1/N_c$ expansion etc. shows that in many cases the closed quark loops are not very important. This is supported by available numerical evidence, showing that e.g. at $K = {}^1/8$ ($m = 0$) the effective action S_{eff} contributes in the average only about 5% in the Metropolis updating of link variables. This implies that the so called "quenched approximation", when S_{eff} is neglected (i.e. the quark determinant is replaced by 1), can be a good first approximation for many problems in QCD. In fact, recent calculations using the quenched approximation [20-23] gave surprisingly good results for the hadron spectrum.

In the quenched approximation the task is to calculate matrix element of the euclidean quark propagator $(1-KM)^{-1}$. For instance, the meson masses are extracted from the large x_4 dependence $\exp(-x_4 m)$ of the four quark Green's functions ($\underset{\sim}{x} \equiv x_1 x_2 x_3$):

$$\begin{aligned} &\langle 0 | \sum_{\underset{\sim}{x}} \tilde{\psi}_{\underset{\sim}{x} x_4} \Gamma \psi_{\underset{\sim}{x} x_4} \; \tilde{\psi}_{\underset{\sim}{y} 0} \Gamma \psi_{\underset{\sim}{y} 0} | 0 \rangle \\ &\Rightarrow \mathrm{Tr} \left\{ \Gamma (1-KM)^{-1} \Gamma (1-KM)^{-1} \right\} . \end{aligned} \tag{2.1}$$

Here Γ is some Dirac-matrix ($\Gamma = \gamma_5$ for the pion, $\Gamma = \gamma_{1,2,3}$ for the ρ-meson etc.) and the sum over the space points $\sum_{\underset{\sim}{x}}$ is done in order to project out zero three-momentum intermediate states. The calculation of $(1-KM)^{-1}$ is usually done by some variant of the Gauss-Seidel iteration method (see e.g. [24]). If the matrix element $(f, (1-KM)^{-1} i)$ is needed then for the vector $g \equiv (1-KM)^{-1} i$ the iterative equation is

$$\begin{aligned} &g_n = i + KM g_{n-1} \qquad (n \geq 1) , \\ &g = \lim_{n \to \infty} g_n \quad . \end{aligned} \tag{2.2}$$

Using the previously updated elements of g_{n-1} in the calculation of g_n is equivalent to the iteration

$$g_n = i + KM_\Delta g_n + KM_\nabla g_{n-1} \quad , \tag{2.3}$$

where in the decomposition $M = M_\Delta + M_\nabla$ the part M_Δ has non-zero elements only below the main diagonal (and M_∇ is zero there). It is also possible to introduce in (2.3) some relaxation parameter $\omega > 0$:

$$g_n = (1-\omega) g_{n-1} + \omega \left[i + KM_\Delta g_n + KM_\nabla g_{n-1} \right] . \tag{2.4}$$

These equations allow for a sufficiently fast computation of the required matrix elements, if the hopping parameter is not much larger than $K = 0.125$. The required physical value of K is, however, larger. Therefore, either a lot of iterations (up to 500) are necessary, or there is need for an extrapolation in K.

A crutial and up to now not very much studied question in the spectrum calculations is the effect of the finite lattice size on the results. Most calculations were done on rather small lattices (a recent typical example is with SU(3) colour $5^3 \times 10$ [23]). On a small lattice the inclusion of the through-going Wilson-lines closed by the periodic boundary conditions might in principle be very dangerous for smaller values of the bare coupling. As a matter of fact, the calculations show up to a factor 2-3 differences in the hopping parameter expansion coefficients if calculated with periodic, antiperiodic or free boundary conditions on the same 6^4 lattice. The reason of the big difference can already be seen by comparing the average of Dirac-traces calculated in an infinite space [25] with the average on a periodic lattice (including the lines closed by the boundary condition). In the hopping parameter expansion method the Dirac-traces can in principle be calculated without boundaries (in a "infinite space") [17], but the colour traces have to be evaluated always on a finite lattice.

The inclusion of the quark determinant in lattice QCD calculations seems difficult, but there are important new developments also in this direction. The exact calculation of det(1-KM) is very, very hard even by using powerful vector-processors (see e.g. [26]). In fact, by using the Metropolis algorithm, only the change of the effective action is needed:

$$\Delta S_{eff} = -\ln\det \frac{1-KM(U')}{1-KM(U)} = -\ln\det\left\{1 + K\left[M(U)-M(U')\right]\left(1-KM(U)\right)^{-1}\right\} . \qquad (2.5)$$

Since the gauge configurations U' and U differ only on one link, the last form requires the evaluation of only a 16 x 16 subdeterminant, once the corresponding matrix elements of $(1-KM(U))^{-1}$ are known. A promising way for the calculation of $(1-KM)^{-1}$ is to use a variant of the von Neumann-Ulam stochastic method [27]. We shall hear more about this in the talk of Julius Kuti at this conference.

III. Current amplitudes on the lattice.

An important piece of low - and high-Q^2 QCD phenomenology refers to the amplitudes of electromagnetic and weak currents. The study of currents on the lattice received up to now relatively little attention, partly because the calculation of masses is simpler and partly because the introduction of currents involves some new, non-trivial problems (see, for instance, [28,29]). In what follows we shall only consider the case of the Wilson-action. The conventional continuum normalization of the fermion fields is to take $\psi'_x = \sqrt{2Ka^{-3}}\,\psi_x$ instead of ψ_x in (1.5). The corretly normalized conserved vector current on the lattice, is therefore [28]:

$$V_j(x)_\mu = Ka^{-3}\left\{\tilde{\psi}_{x+\hat{\mu}}\, U[x+\hat{\mu},x]\frac{\lambda_j}{2}(1+\gamma_\mu)\psi_x - \tilde{\psi}_x\, U[x,x+\hat{\mu}]\frac{\lambda_j}{2}(1-\gamma_\mu)\psi_{x+\hat{\mu}}\right\} . \qquad (3.1)$$

(λ_j denotes, as usual, the flavour Gell-Mann matrices.) This form is a lattice analogue of the point-split definition of the continuum current operator, satisfying for flavour independent hopping parameters the lattice conservation law:

$$\nabla_\mu V_j(x-\hat{\mu})_\mu \equiv \sum_{\mu>0}\frac{1}{a}\left[V_j(x)_\mu - V_j(x-\hat{\mu})_\mu\right] = 0 . \qquad (3.2)$$

This conservation law implies the validity of lattice Ward-identities for the amplitudes containing V_j. As a consequence, $V_j(x)_\mu$ is not renormalized by the colour interactions [28], therfore its matrix elements have an immediate physical significance.

The situation for the axialvector current

$$A_j(x)_\mu = Ka^{-3}\left\{\tilde{\psi}_{x+\hat{\mu}}\, U[x+\hat{\mu},x]\frac{\lambda_j}{2}\gamma_5\gamma_\mu\psi_x + h.c.\right\} \qquad (3.3)$$

is less transparent. The Wilson-action explicitly breaks the chiral symmetry, therefore the axial currents are not conserved and hence they are renormalized.

In the case of a chiral invariant action there is the problem of fermion degeneracy ("doubling"). It is enlightening to consider the question of triangle anomaly associated to the axialvector currents. We can say, that in the lattice regularization scheme either the axial triangle anomaly is cancelled by extra fermions or it is introduced by the explicit chiral symmetry breaking. (Note that 2 fermions would be enough to cancel the anomaly, therefore the actual degeneracy of 4 is not needed for this purpose.)

The renormalization of the axialvector current is a difficulty for any non-perturbative calculation of axial current amplitudes. One possible way of fixing the normalization would be to calculate and fix the known value of the VVA anomaly, althougn in the praxis this is presumably rather cumbersome.

Measuring by Monte Carlo the two electromagnetic current amplitude

$$\langle 0|\sum_{\underset{\sim}{x}} J_m(\underset{\sim}{x},x_4)\, J_n(\underset{\sim}{y},0)|0\rangle \qquad (m,n=1,2,3) \qquad (3.4)$$

it is possible to determine the spectral function measured in e^+e^- annihilation. Assuming the dominance (for large x_4) of a vector meson pole gives in (3.4) the behaviour $m_V^3/(2f_V^2)\exp(-x_4 m_V)$. This allows to extract the mass m_V and the current-vectormeson junction parameter f_V. A preliminary result [30] on a 6^4 lattice at $4g^{-2} = 2.2$ with SU(2) colour gives, however, a factor 4-5 too small f_ρ and $f_{J\psi}$. The ratio $f_{J\psi}/f_\rho \cong 2.5$ and the masses come out well. Possible reasons for this failure are: the lattice could be too small and/or the separation from higher excited vector meson states could be very incomplete.

As a short resumê: a reliable calculation of low energy hadronic properties in QCD with lattice Monte Carlo seems feasible. There are already many encouraging results and progress is fast, but the unsolved problems are not yet completely negligible.

References

[1] K.G. Wilson, Phys. Rev. D10, 2445 (1974)

[2] M. Creutz, L. Jacobs, C. Rebbi, Phys. Rev. Lett. 42, 1390 (1979); Phys. Rev. D20, 1915 (1979)

[3] M. Creutz, Phys. Rev. Lett. 43, 553 (1979); Phys. Rev. D21, 2308 (1980)

[4] K.G. Wilson, (in Recent Developments in Gauge Theories (Cargêse 1979), ed. G. 't Hooft et al., Plenum Press, New York 1980

[5] A. Hasenfratz, P. Hasenfratz, Phys. Lett. 93B, 165 (1980); Nucl. Phys. B193, 210 (1981)

[6] K.G. Wilson, in New Phenomena in Subnuclear Physics (Erice 1975), ed. A. Zichichi, Plenum Press, New York 1977

[7] L. Susskind, Phys. Rev. D16, 3031 (1977)

[8] N. Kawamoto, J. Smit , Nucl. Phys. B192, 100 (1981)

[9] H. Sharatchandra, H.J. Thun, P. Weisz, Nucl. Phys. B192, 205 (1981)

[10] P. Becher, Phys. Lett. 104B, 221 (1981)

[11] J.M. Rabin, Yale University preprint YTP81-29 (1981)

[12] P. Becher, H. Joos, DESY preprint 82-031 (1982)

[13] E. Kähler, Rendiconti di Mat. (Roma) Ser. V. 21, 425 (1962)

[14] H.B. Nielson, M. Ninomiya, Nucl. Phys. B185, 20 (1981); Phys. Lett. 105B, 219 (1981); Nucl. Phys. B193, 173 (1981)

[15] L.H. Karsten, Phys. Lett. 104B, 315 (1981)

[16] C.B. Lang, H. Nicolai, Nucl. Phys. B200, 135 (1982)

[17] A. Hasenfratz, P. Hasenfratz, Z. Kunszt, C.B. Lang, Phys. Lett. 110B, 289 (1982); CERN preprint TH. 3313 (1982)

[18] P. Hasenfratz, I. Montvay, in preparation.

[19] N. Kawamoto, Nucl. Phys. B190, 616 (1981)

[20] H. Hamber, G. Parisi, Phys. Rev. Lett. 47,1972 (1981); Brokkhaven preprint BNL 31322 (1982)

[21] E. Marinari, G. Parisi, C. Rebbi, Phys. Rev. Lett. 47, 1795 (1981)
H. Hamber, E. Marinari, G. Parisi, C. Rebbi, Phys. Lett. 108B, 314 (1982)

[22] D. Weingarten, Phys. Lett. 109B, 57 (1982)

[23] F. Fucito, G. Martinelli, C. Omero, G. Parisi, R. Petronzio, F. Rapuano, CERN preprint TH. 3288 (1982)

[24] R.S. Varga, Matrix Iterative Analysis, Prentice-Hall. Inc. (1965)

[25] B. Berg, A. Billoire, D. Forster, CERN preprint TH. 3214 (1982)

[26] I.O. Stamatescu, MPI Munich prerint 82/81 (1982)

[27] J. Kuti, Santa Barbara preprint NSF-ITP-81-151 (1981)

[28] L.H. Karsten, J. Smit, Nucl. Phys. B183, 103 (1981)

[29] P.H. Ginsparg, K.G. Wilson, Cornell preprint CLNS-81/520 (1981)

[30] A. Ali, I. Montvay, unpublished

Figure Captions

Fig. 1: The dependence of $(am_\pi)^2$ and $(am_\rho)^2$ on the hopping parameter K_u at $g^{-2} = 0.925$. (Ref. [17]).

Fig. 2: The physical values of the hopping parameters $K_{u,s,c}$ according to Ref. [17].

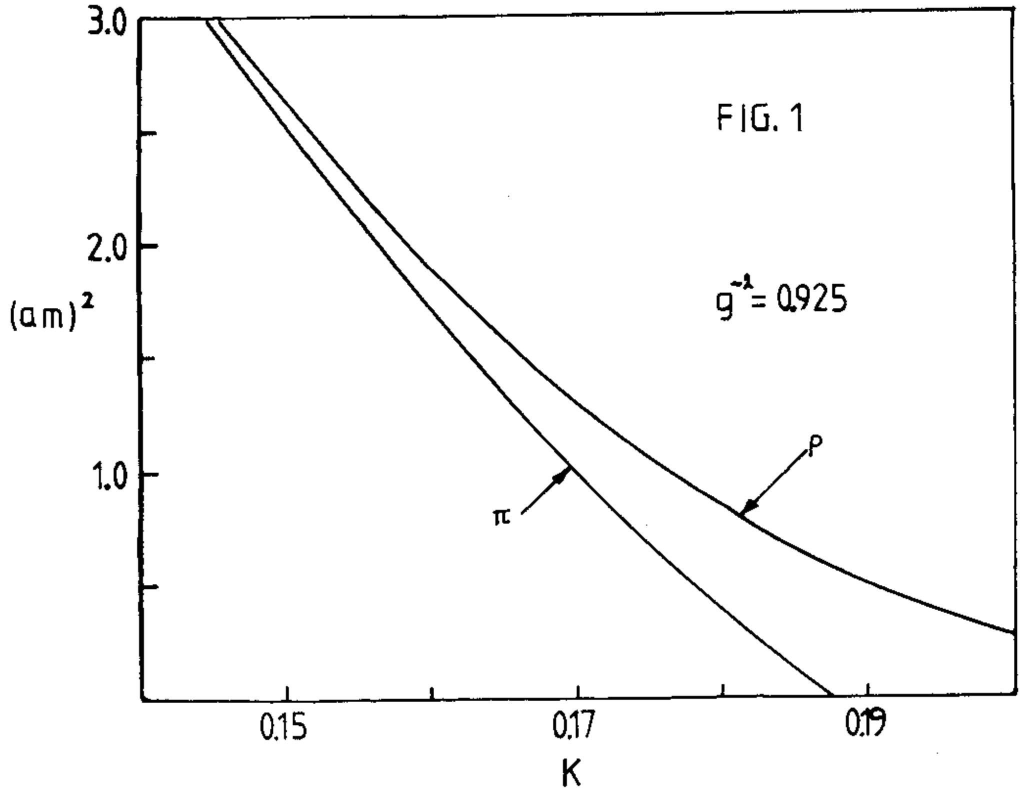
FIG. 1
3.0
2.0
1.0
(am)²
g⁻² = 0.925
ρ
π
0.15
0.17
0.19
K

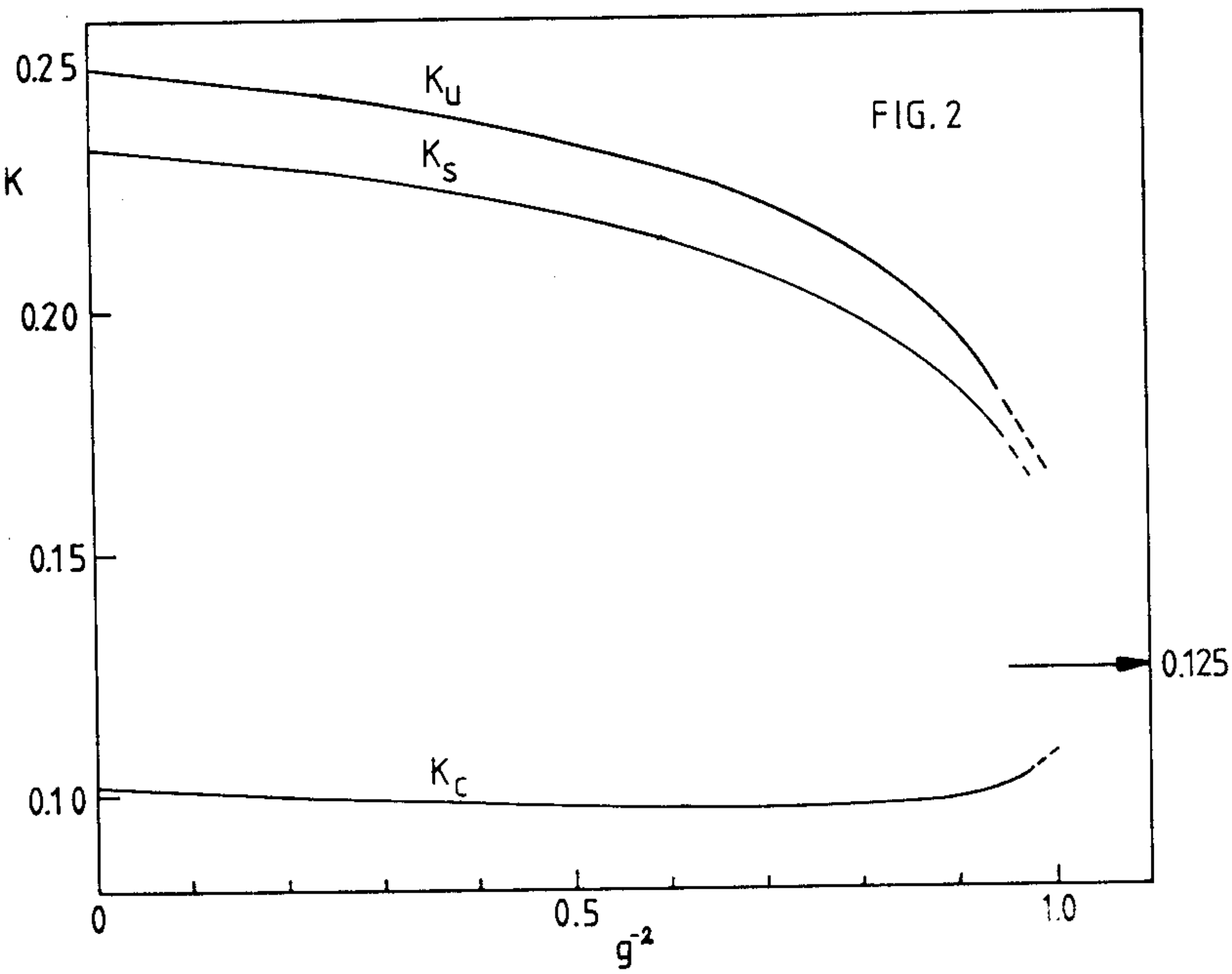
FIG. 2
0.25
0.20
0.15
0.10
K
K_u
K_s
K_c
0.125
0
0.5
1.0
g⁻²