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COMPENSATION OF THE DEPOLARIZING EFFECTS OF SOLENOIDS IN ELECTRON-POSITRON STORAGE RINGS

by

D.P. Barber

Laboratory for Nuclear Science

Massachusetts Institute of Technology

Cambridge, Massachusetts

J. Kewisch, H. Mais, G. Ripken, R. Rossmanith and Rüdiger Schmidt

Deutsches Elektronen-Synchrotron DESY, Hamburg

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Abstract

We show how solenoids lead to loss of vertical polarization in electron-positron storage rings and propose a general prescription for compensating depolarizing effects with the aid of additional opposite polarity "antisolenoids" which need not necessarly be placed next to the main solenoid but may be placed in positions where x' of the particle trajectory is similar to the x' at the interaction point. We also discuss the implications for the achievement of longitudinal polarization.

Introduction

Following recent successes in obtaining large reproducible vertical polarization at PETRA /1/ and other storage rings /2/, the next step in PETRA should be to use the polarization as a tool in high energy physics experiments /3/. Most experiments utilize solenoidal magnetic fields for particle momentum measurement and as well as introducing some betatron coupling into the beam optics, these fields tilt the electron or positron spins away from the vertical and thus cause a serious loss of polarization. As will be seen below, the simplest way to cancel both the coupling and spin effects is to place opposite polarity "antisolenoids" immediately to either side of the solenoid so that the total disturbance felt by the particles is zero (Fig. 1). However, with the commonly used low-beta optics where achievement of high luminosity requires that the interaction region (I.R.) quadrupoles are as close to the interaction point (I.P.) as possible, there is frequently no free space beyond the ends of the experimental solenoid and other means of compensation are then needed.

In this article we describe the principal optical requirements for a compensation scheme based on antisolenoids placed among the low-beta quadrupoles which does not jeopardize luminosity. We illustrate the effectiveness of the scheme using an optics chosen to satisfy the compensation conditions and then discuss the implications for longitudinal polarization.

The influence of uncompensated solenoids on beam polarization

1) Spin behaviour in a storage ring

In this subsection we briefly recall some well known facts about spin behaviour. A fuller description can be found in Reference 4.

Electron beams in storage rings acquire vertical polarization by the Sokolov-Ternov mechanism /5/. Ensembles of spins travelling in a magnetic field precess according to the B.M.T. /6/ equation:

$$\frac{d\vec{S}}{ds} = \frac{e\,\vec{S}}{mc\gamma} \wedge \left[(1 + a)\,\vec{B}_{/\!/} + (1 + a\,\gamma)\,\vec{B}_{\perp}\right] \qquad (1)$$

where

$\vec{S}$ = total vector spin

s = distance around the ring

e = electric charge

m = electron mass

c = velocity of light

γ = Lorentz factor

$\vec{B}_{/\!/}$ = magnetic field parallel to electron motion

$\vec{B}_{\perp}$ = magnetic field perpendicular to electron motion

a = $(g-2)/2 = 0.00116$ and g = electron spin g factor.

For horizontal motion in a purely vertical field, eqn. (1) implies that $\Delta\Psi$, the spin precession angle around the vertical axis is

$$\Delta\Psi = (a\gamma)\,\Delta\theta \qquad (2)$$

where $\Delta\theta$ is the change of direction of motion of the particle on travelling around the ring. The quantity $a\gamma = \nu$ is called the spin tune and depends on energy. (For example at 16 GeV $a\gamma = 36.31$).

For motion parallel to the longitudinal field of a solenoid eqn.(1) implies that $\vec{S}$ precesses around the beam direction by an angle

$$\Delta\varphi = \frac{(1+a)e}{mc\gamma} \int B_{/\!/s}\, ds \qquad (3a)$$

In the case where $B_{/\!/s}$ is constant in s, we may write

$$\Delta\varphi = (1+a)\,k\Delta s \qquad (3b)$$

where $$k = \frac{eB_{/\!/s}}{mc\gamma} \qquad (3c)$$

and Δs is the solenoid length.

For a typical solenoid at PETRA $\Delta\varphi = 32$ mrad at 16 GeV.

Consider an electron travelling on the closed orbit. It is straightforward to demonstrate that at every point along the closed orbit, a unit vector $\underline{\hat{n}}$ for the spin direction can be found which, when transformed once around the closed orbit according to eqn.(1) reproduces the original vector $\underline{\hat{n}}$. The $\underline{\hat{n}}$ vector is periodic and at each point on the ring represents the only direction along which a net polarization can point; all spin components orthogonal to $\underline{\hat{n}}$ will, on one circuit around the ring, precess around $\underline{\hat{n}}$ by an angle given by the spin tune and orthogonal components will quickly average to zero leaving only the components along $\underline{\hat{n}}$.

If the ring is perfectly flat and contains only dipole, quadrupole and sextupole magnets, the beam will have essentially zero vertical thickness but a small horizontal width. On the closed orbit, the particles only experience vertical fields and $\underline{\hat{n}}$ is vertical everywhere. In practice the particles execute horizontal betatron oscillations around the closed orbit but since $\underline{\hat{n}}$ is parallel to the field there is (eqn.(1)) no precession away from the vertical and the beam maintains the maximum polarization, viz. 92 %.

Real storage rings are never perfectly flat and the beam then experiences the horizontal fields in the quadrupole, sextupole and vertical correction magnets with the result that $\underline{\hat{n}}$ is no longer exactly vertical everywhere and the beam also acquires some vertical thickness. Now, in executing orbital oscillations the spins experience additional small torques which depend on particle coordinates and the beam may become depolarized. The depolarization effects are strongest when the following resonance conditions are satisfied

$$\nu = a\gamma = k_0 \pm Q_I\,, \quad I = x, y, s, \qquad k_0 = \text{integer} \qquad (4)$$

i.e. when the spin precession rate matches the frequency Q of the disturbance caused by orbital oscillations.

ii) Spin behaviour in a storage ring with a solenoid

In this subsection we restrict discussion to perfectly flat storage rings where, if there were no solenoid, $\underline{\hat{n}}$ would be vertical.

Consider an ensemble of particles with vertical net spin entering a solenoid on the closed orbit. We recall from eqn.(3) that on leaving the solenoid, the spin will be tilted by an angle $\Delta\varphi$ around the direction of motion. The spin will then precess in the vertical fields in the ring and it becomes clear that in the presence of a solenoid the $\underline{\hat{n}}$ axis cannot be vertical at any point in the ring or at any energy. Since $\underline{\hat{n}}$ is not vertical in the arcs, orbital oscillations again lead to smearing of spin directions and loss of polarization. This is confirmed by a calculation for PETRA using the program SLIM /7/ with a solenoid which gives $\Delta\varphi$ = 32 mrad at 16 GeV (see fig. 2). This loss of polarization has been confirmed experimentally /8/.

Solenoid compensation

i) The preferred solution

From the above discussion it is clear that high polarization is the presence of solenoids can only be achieved if some means is found to ensure that $\underline{\hat{n}}$ is nevertheless vertical in the arcs. The obvious way is to place antisolenoids on either side of the main solenoid as in fig. 1 so that $\int B_{\parallel}\, ds$ across the interaction region is zero and $\Delta\varphi = 0$.

ii) Alternative solution

As we have mentioned previously there is frequently no space available for the above solenoid solution. An alternative is to place antisolenoids further away from the I.P. among the low-beta quadrupoles (Fig. 3). Again, by ensuring $\int B_{\parallel s}\, ds = 0$ on the closed orbit $\underline{\hat{n}}$ will be vertical in the arcs. However, owing to the fact that they couple vertical and horizontal betatron motions, solenoids increase the beam size and can cause a serious fall in luminosity. This is especially true for antisolenoids placed among the quadrupoles. Furthermore, since $\underline{\hat{n}}$ is no longer vertical in the I.R. quadrupoles, betatron oscillations in the quadrupoles are also expected to lead to depolarization.

Finally, the antisolenoids and solenoids are now optically separated by the quadrupoles and as we shall see below this can lead to further depolarization effects.

Similar phenomena also occur in another scheme which has been suggested recently /9/ where, instead of using antisolenoids the rotation in the solenoids is compensated by vertical beam bumps in the arcs.

Remedies

In this section we examine the influence of solenoids on orbital oscillations and on spin motion for particles not on the closed orbit. We assume that the closed orbit coincides with the axis of the solenoids.

i) Influence on orbital motion:

In Fig. 4 we sketch an idealized representation of the magnetic field lines near the axis of a solenoid. Clearly, in computing the optical properties, not only the longitudinal field but also the radial end fields must be included /10/. The total effect of both fields on the beam is that of a rotation in the transverse plane by an angle $k\Delta s/2$ around the beam direction (Fig.5a) and a weak focussing in both transverse directions.

Quadrupoles aligned in the normal manner in the x-y plane would couple horizontal and vertical motion and reduce the luminosity. However, the coupling may be completely removed and the luminosity restored by rotating the quadrupoles between the solenoid and antisolenoids (RQ_1--RQ_n in Fig. 3) by angles $k\Delta s/2$ /11/. It should be noted that quadrupole groups on opposite sides of the I.P. must be rotated in opposite senses.

ii) Influence on spin

From eqn. (3..) we see that on the closed orbit the $\underline{\hat{n}}$ axis is rotated in a solenoid by an angle $\approx k\Delta s$ which is twice the angle of rotation of the beam (see Fig. 5a).

For convenience we now describe all motion within the rotated frame of reference that may be associated with the twist of the beam generated by the solenoids. In this frame /10/ the quasi-horizontal and quasi-vertical betatron motions are decoupled. We also work in the approximation that in these frames the particle trajectories are straight lines in the solenoids. Since we have no coupling, it is only necessary in the following to consider quasi-horizontal motion.

Consider a particle entering the solenoid at an angle $dx/ds = x'$ (Fig. 5b). The particle leaves the solenoid with the same angle and a change in transverse position $x'\Delta s$ and in the process has experienced additional spin precession due to its transverse motion in the longitudinal field and its passage across the radial end fields (eqn. (1)).

In the approximation that the radial end fields vary only linearly with distance from the solenoid axis /10/ and noting that the end fields at the two ends act in opposite senses, the total spin precession around the radial direction is (eqn. (1) and (3c)) /12/:

$$\delta\theta_1 = (1+a\gamma)\ x'\ k\Delta s/2 \qquad (5)$$

Thus off closed orbit particles experience forward/backward tilting of spin direction due to radial fields by an amount depending only on the transverse direction of the particle and the field integral of the solenoid but not on the transverse position.

Particles traversing a solenoid with non-zero x' so that they no longer travel parallel to the longitudinal field also experience a forward/backward tilt of the spins by angle /6,12/

$$\delta\theta_2 = a\gamma\ x'\ k\Delta s \qquad (6)$$

in addition to the closed orbit rotation $k\Delta s\ (1+a)$. $\delta\theta_2$ depends only on x' and the field integral.

It is now clear how $\delta\theta_1$ and $\delta\theta_2$ should be cancelled; each half-solenoid at the I.P. should be accompanied by an antisolenoid in a position among the quadrupoles where individual electrons have the same x' as they have at the I.P. This then implies that individual electrons have the same x' in both an-

tisolenoids. It is also interesting to note that $\delta\theta_1$ and $\delta\theta_2$ (eqns. 5,6) are of the same order of size.

The condition on x' is trivially satisfied if the antisolenoids are adjacent to the solenoids as in Fig. 1 but is less easy to deal with if the antisolenoids are further out as in Fig. 3. Nevertheless, the condition on x' required for cancelling additional solenoid precession is exactly the condition needed to cancel the additional precession in the intervening quadrupoles. As seen from eqn. 2, since a particle would not change the direction of its quasi-horizontal motion on travelling between the antisolenoids and I.P. it also has no net additional precession around the internal vertical axes of the rotated quadrupoles /13/.

Thus we see that if it can be arranged that for an individual particle, x' in the antisolenoids and in the solenoid at the I.P. is the same, all three kinds of precession are cancelled and since $\underline{\hat{n}}$ is vertical in the arcs, depolarizing resonances should be strongly suppressed.

A suitable optics

Since development of an optics satisfying the condition on x' is not a straightforward matter we have instead chosen to evaluate the principle using an optics in which the condition on x' is only approximately fulfilled.

Consider an arrangement as in Fig. 6 where the antisolenoids are placed at a position where the horizontal betatron phase advance between the I.P. and the antisolenoid is $\pi/2$, $\alpha = -\beta'/2$ is negative and β is large. In general we may write:

$$x \sim \sqrt{\beta}\cos\left(\int\frac{ds}{\beta} + \delta\right) \qquad \text{where } \delta \text{ is an arbitrary phase} \qquad (7a)$$

and
$$x' \sim \frac{1}{\sqrt{\beta}}\left(-\alpha\cos\left(\int\frac{ds}{\beta} + \delta\right) - \sin\left(\int\frac{ds}{\beta} + \delta\right)\right) \qquad (7b)$$

At the I.P.
$$x'^{*} \sim -\frac{1}{\sqrt{\beta^{*}}}\sin(\delta) \qquad (7c)$$

(since $\alpha = 0$ and we put $\int\frac{ds}{\beta} = 0$)

At the antisolenoid

$$x' \sim \frac{1}{\sqrt{\beta}}\left(-\alpha\cos\left(\delta + \frac{\pi}{2}\right) - \sin\left(\delta + \frac{\pi}{2}\right)\right) \qquad (7d)$$
$$= \frac{1}{\sqrt{\beta}}(\alpha\sin\delta - \cos\delta)$$

If we now impose the conditions:

$$-\alpha = \sqrt{\frac{\beta}{\beta^*}} \qquad (7e)$$

we find

$$x'^* - x' \sim \frac{\cos(\delta)}{\sqrt{\beta}} \qquad (7f)$$

Since β is large (or equivalently if $-\alpha = \sqrt{\beta/\beta^*}$ is large), we then have approximate equality of x'^* and x'. As we shall see in the next section even this approximate equality can be sufficient to ensure that depolarizing effects are very small although exact equality would be preferred.

Evaluation

Fig. 7 shows the horizontal beta function of a modified PETRA optics which approximately satisfies the above optical conditions, has $\beta^* \approx 1$ metre and which we used in a test of the cancellation mechanism. The phase advance between the I.P. and the antisolenoid is 1.44 radians, $\alpha\sqrt{\beta^*/\beta} \approx -1$ at the antisolenoid and $\beta \approx 250$ m. We also indicate an alternative, "unfavoured", position for the antisolenoid at a position between RQ1 and RQ2 where $\alpha\sqrt{\beta^*/\beta} \approx -3{\cdot}5$ and $\beta \approx 120$ so that $x' \approx 3{\cdot}5\, x'^*$ over most of the range of δ.

In order to test this arrangement we used the program SLIM /7/ with a modification to include rotated quadrupoles and ran it in a mode in which the solenoid fields were allowed to scale with beam energy so that precession angle and optical tune were independent of energy. No imperfections were introduced so as not to obscure the solenoid effects. The success of the compensation scheme is clearly apparent in Fig. 8 where polarizations are plotted as a function of beam energy for the antisolenoids in the favoured position (Fig. 8a) and in the alternative unfavoured position (Fig. 8b). As can be seen, correct positioning of the antisolenoids leads to an almost complete cancellation of depolarizing effects even though the condition on x' is only approximately fulfilled, while in the other position the polarization is rendered unuseable due to strongly depolarizing horizontal betatron resonances. In Figures 9, 10, 11 we show similar curves for the cases where additional precession due only to the longitudinal fields, the radial solenoid fields and the intervening quadrupoles is turned on. It is clear that all three contributions to depolarization are of similar importance.

It is also evident that the curves of Fig. 10 and 11 and of Figs. 8 and 9 are essentially identical. This may be understood by noting that the additional precession due to quasi-horizontal motion in the quadrupoles is in first order equal and opposite to that due to the radial fields /14/. These fields then give identical depolarizations. However, when they are both activated together they cancel. Then, when all three sources of depolarization are activated as in Fig. 8, only the effect of the longitudinal fields remains and the curves of Fig. 8 are expected to be similar to the curves of Fig. 9. Finally therefore, it is only the depolarizing effect of the longitudinal solenoid fields that needs to be compensated although as we have seen, compensation of the longitudinal fields with the condition on x' will automatically lead to separate compensation of the radial and quadrupole fields.

Conclusion

We have shown that in schemes which utilize antisolenoids to compensate for the depolarizing effects of solenoids the necessary condition for success is that individual electrons have the same direction of quasi-horizontal transverse motion both in the solenoids and the antisolenoids and we have demonstrated the effectiveness of an optics which allows the condition to be approximately satisfied.

It is clear that is is unwise to neglect the effects of betatron motion in the solenoids and that compensation of quadrupole effects alone or only arranging for $\underline{\hat{n}}$ to be vertical in the arcs is not sufficient. Indeed since all damaging solenoid effects depend on x' it is likely that solenoids can only be compensated by antisolenoids and in no other way.

Finally we would like to point out that in future storage rings where it is hoped that longitudinal polarization will be available, the solenoid fields will still have a strong depolarizing effect even though in this case the $\underline{\hat{n}}$ axis will be parallel to the longitudinal field.

Acknowledgements

We wish to thank Prof. G.-A. Voss for his encouraging us to carry out this investigation.

Figure Captions

Fig. 1: Preferred arrangement of solenoids and antisolenoids where each half-solenoid with field integral $\int B_{\parallel s} ds = f$ is compensated by an antisolenoid with field integral $\int B_{\parallel a} ds = -f$. The coordinate system is also indicated.

Fig. 2: Polarization prediction for PETRA with one uncompensated solenoid having $\Delta\varphi = 32$ mrad at 16 GeV.

Fig. 3: Alternative solenoid-antisolenoid arrangement with rotated quadrupole groups RQ_1---RQ_k---RQ_n. The antisolenoids are positioned between RQ_n and Q_1. The field integrals cancel as in Fig. 1.

Fig. 4: a) View from above of field lines near the axis of a solenoid showing the radial end fields.
b) End view of radial field lines of a solenoid.
c) Plot of longitudinal field versus distance along axis illustrating the "top hat" shape.

Fig. 5: a) Sketch depicting a view along the beam of the transverse beam cross-section on entering an antisolenoid from the arc. The beam distribution is "horizontal" on entry and the $\underline{n}$ axis is vertical. On exit the beam has rotated by an angle $k\Delta s/2$ and $\underline{n}$ by an angle $k\Delta s$.
b) Schematic of a straighttrack traversing a solenoid. The change in transverse position is $x_1 + x_2 = x'\Delta s$.

Fig. 6: Sketch of a horizontal beta-function of the kind needed to satisfy the conditions on x' and showing the corresponding positions of the optical elements. The phase shift between the I.P. and the antisolenoid is $\pi/2$. The broken line is to indicate that apart from the conditions on β_x at the I.P. and antisolenoid, the β_x shape is otherwise arbitrary.

Fig. 7: β_x function used in the evaluation of the compensation scheme. The positions of the quadrupoles are shown together with the "favoured" antisolenoid positions between RQ2 and Q3 and the unfavoured position between RQ1 and RQ2.

Fig. 8: Polarizations plotted as a function of energy.
a) with the antisolenoids in the favoured position.
b) with the antisolenoids in the unfavoured position.

Fig. 9: Polarizations plotted as a function of energy but with only additional precession due to the longitudinal fields turned on
a) with the antisolenoids in the favoured position.
b) with the antisolenoids in the unfavoured position.

Fig. 10: Polarizations plotted as a function of energy but with only additional precession due to solenoid radial end fields turned on.
a) with the antisolenoids in the favoured position.
b) with the antisolenoids in the unfavoured position.

Fig. 11: Polarization plotted as a function of energy but with only additional precession due to the rotated quadrupoles turned on.
a) with the antisolenoids in the favoured position.
b) with the antisolenoids in the unfavoured position.

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/13/ This result is a special case of the spin orbit decoupling conditions of A. Chao and K. Yokoya, Internal Report KEK-81-7, July 1981, (unpublished)

/14/ This may be demonstrated by multiplying together the appropriate 8x8 transport matrices used in the SLIM program.

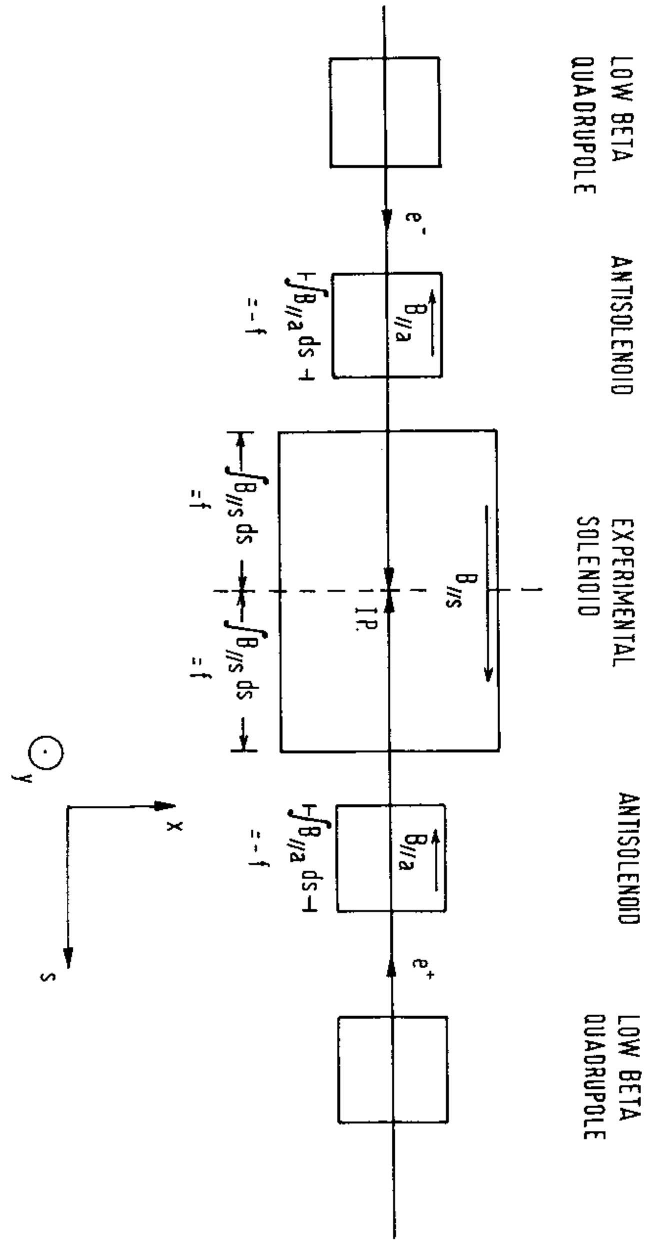

Fig. 1

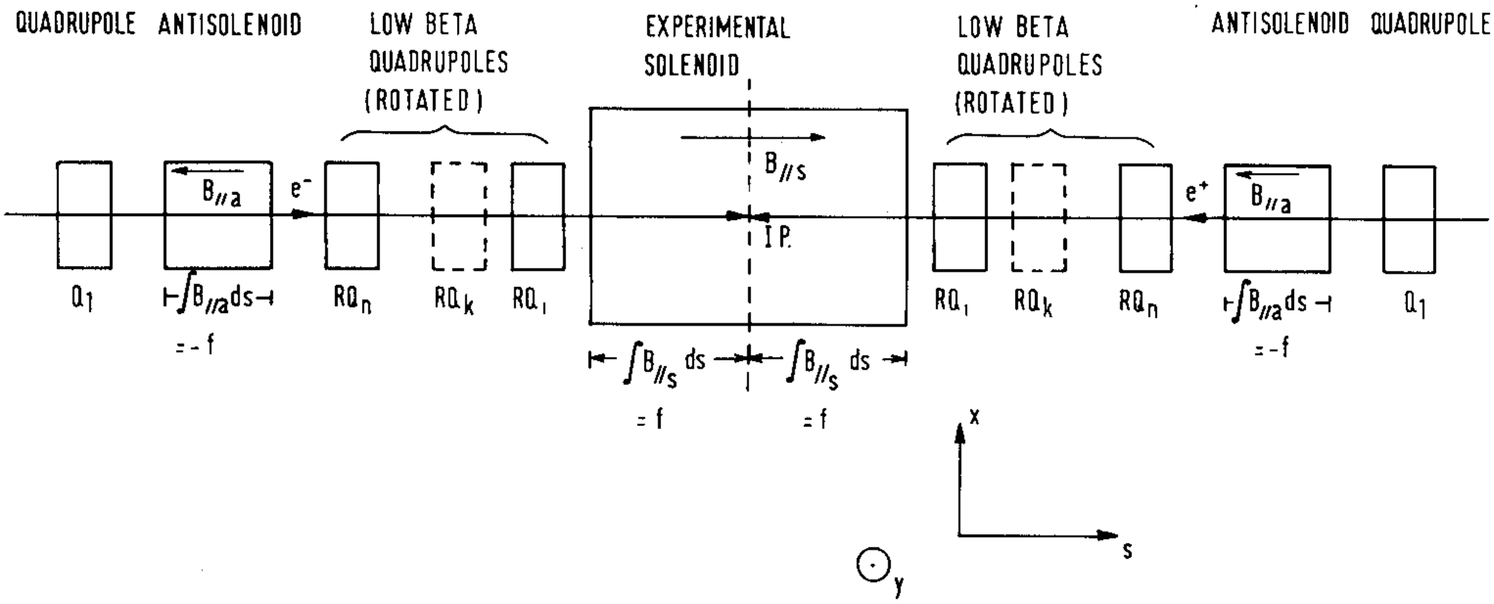
QUADRUPOLE
ANTISOLENOID
LOW BETA QUADRUPOLES (ROTATED)
EXPERIMENTAL SOLENOID
LOW BETA QUADRUPOLES (ROTATED)
ANTISOLENOID
QUADRUPOLE
$B_{//a}$
e^-
$B_{//s}$
I.P.
e^+
$B_{//a}$
Q_1
$\int B_{//a} ds = -f$
RQ_n
RQ_k
RQ_1
$\int B_{//s} ds = f$
$\int B_{//s} ds = f$
RQ_1
RQ_k
RQ_n
$\int B_{//a} ds = -f$
Q_1
x
s
y

Fig. 3

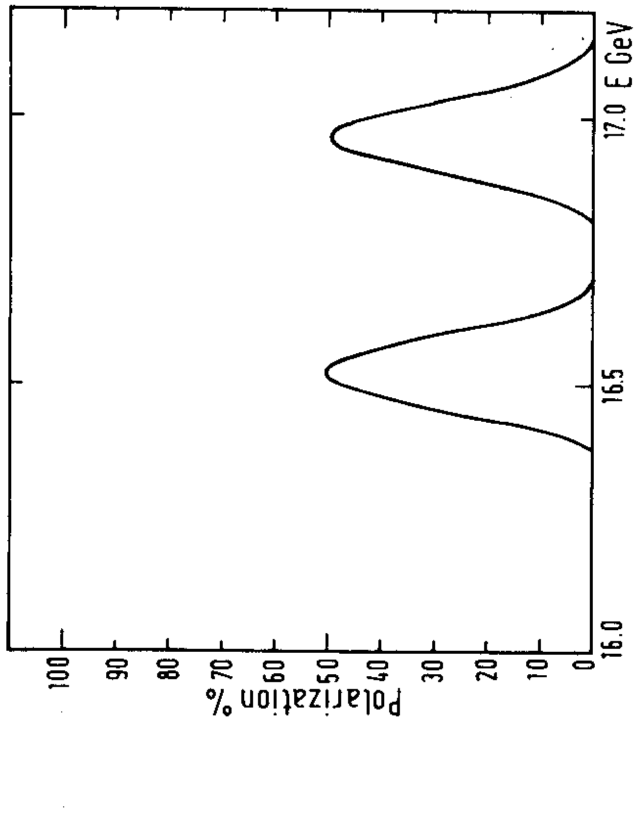
Polarization %
100
90
80
70
60
50
40
30
20
10
0
16.0
16.5
17.0
E GeV

Fig.2

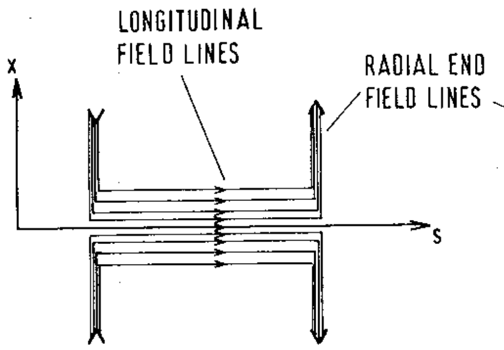

Fig. 4a

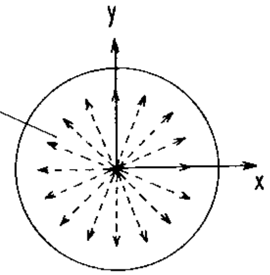

Fig. 4b

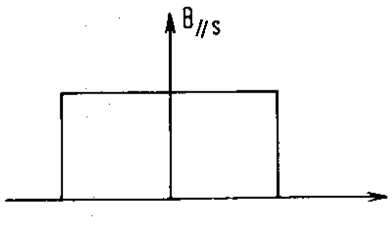

Fig. 4c

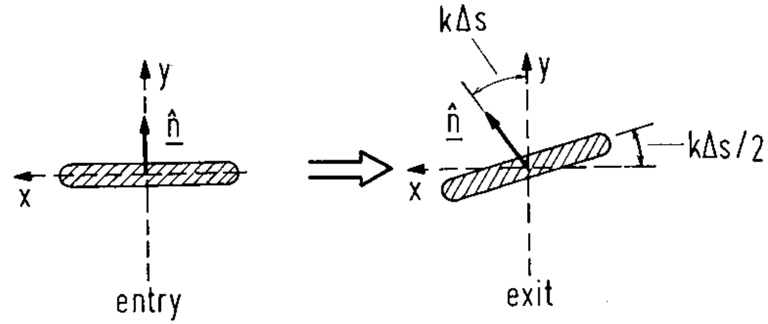

Fig.5a

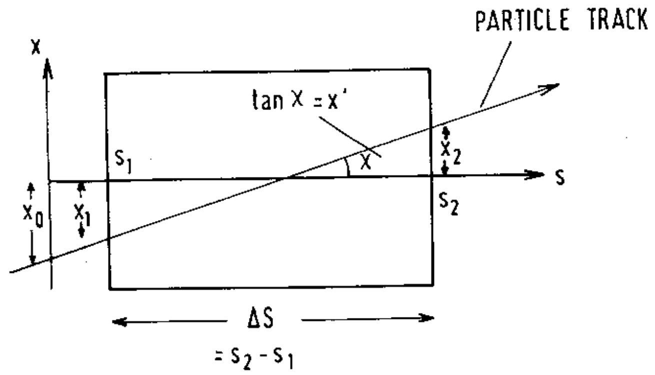

Fig. 5b

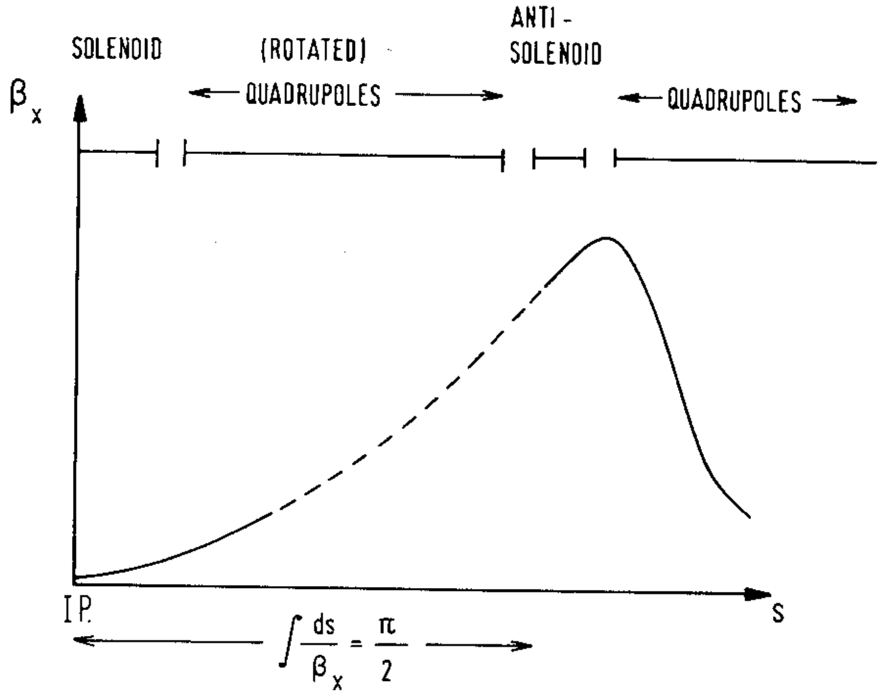
SOLENOID
(ROTATED)
QUADRUPOLES
ANTI-
SOLENOID
QUADRUPOLES
β_x
I.P.
s
$\int \frac{ds}{\beta_x} = \frac{\pi}{2}$

Fig. 6

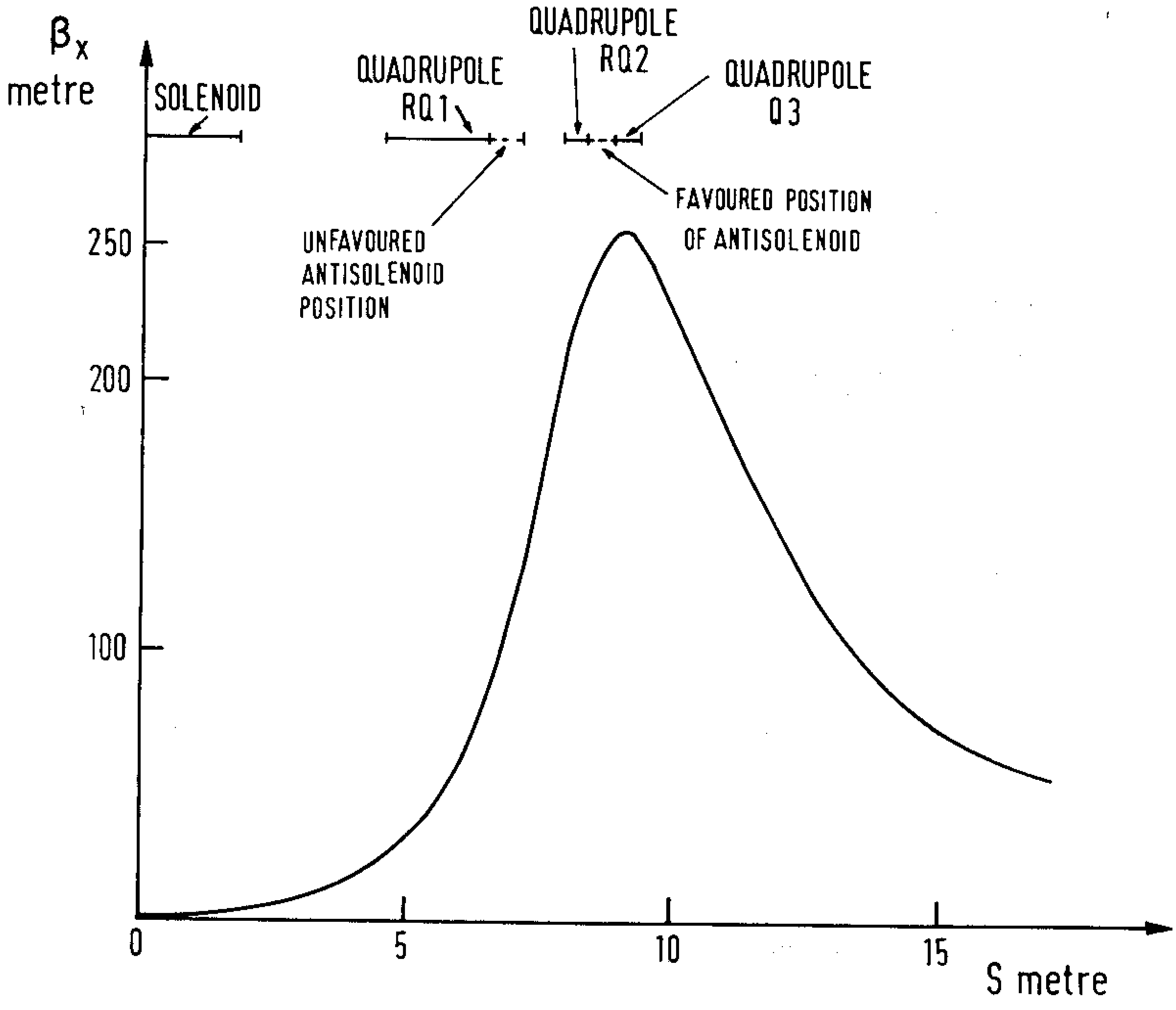
β_x
metre
SOLENOID
QUADRUPOLE
RQ1
QUADRUPOLE
RQ2
QUADRUPOLE
Q3
UNFAVOURED
ANTISOLENOID
POSITION
FAVOURED POSITION
OF ANTISOLENOID
250
200
100
0
5
10
15
S metre

Fig. 7

favoured antisolenoid position

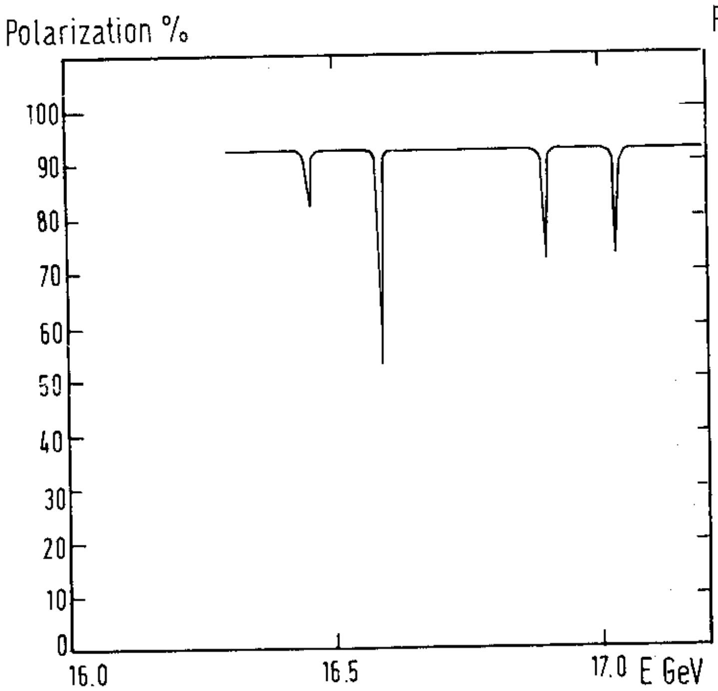

Fig. 8a

unfavoured antisolenoid position

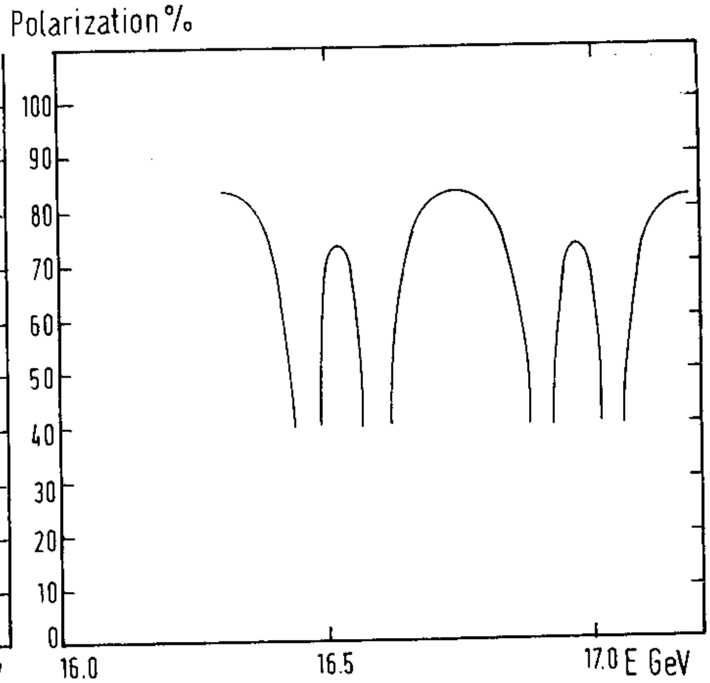

Fig. 8b

favoured antisolenoid position

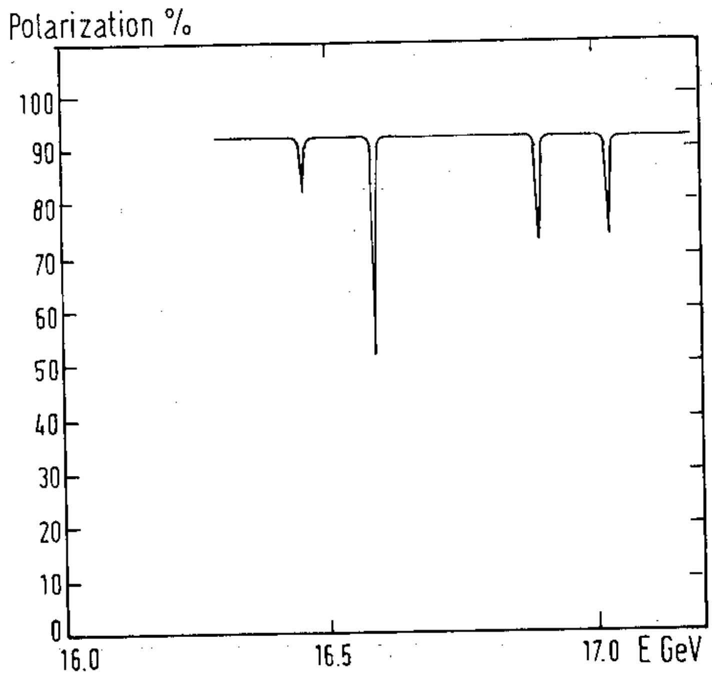

Fig. 9a

unfavoured antisolenoid position

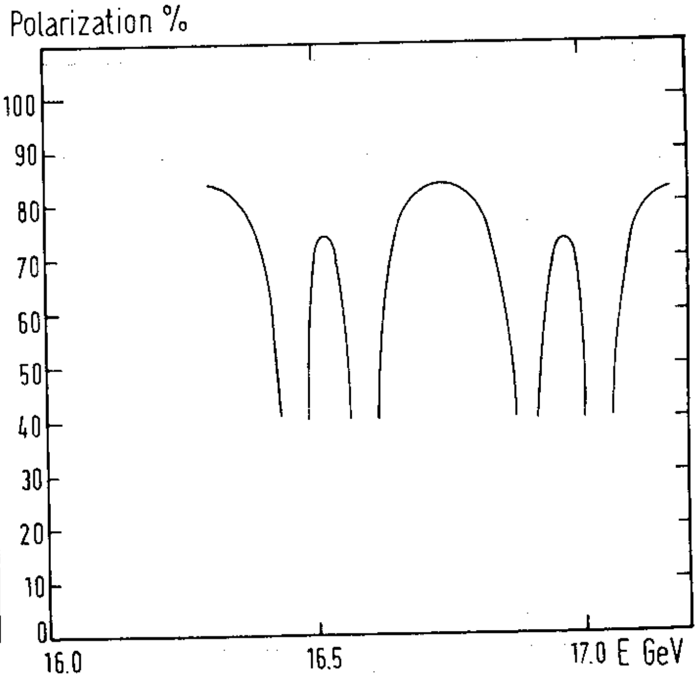

Fig. 9b

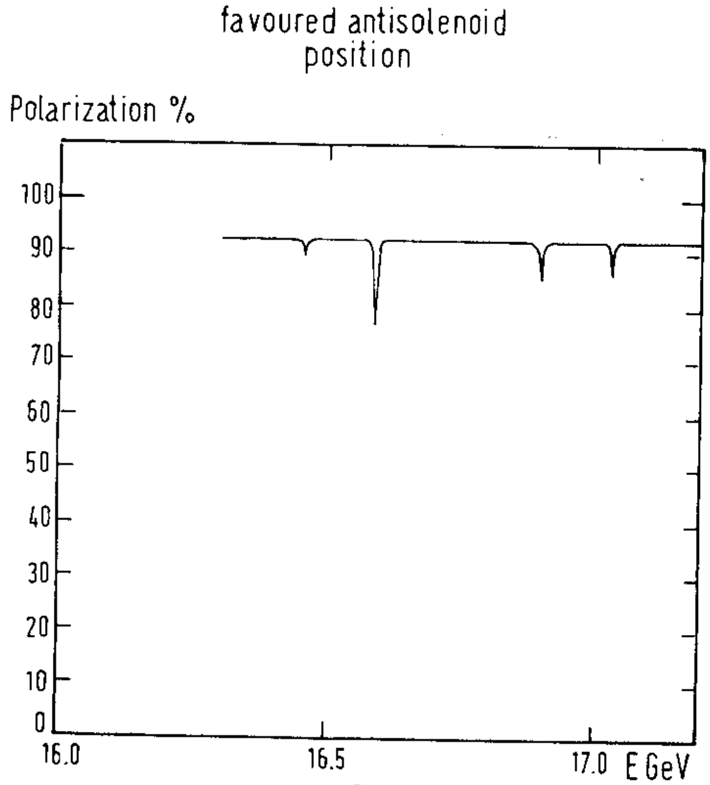

Fig. 10a

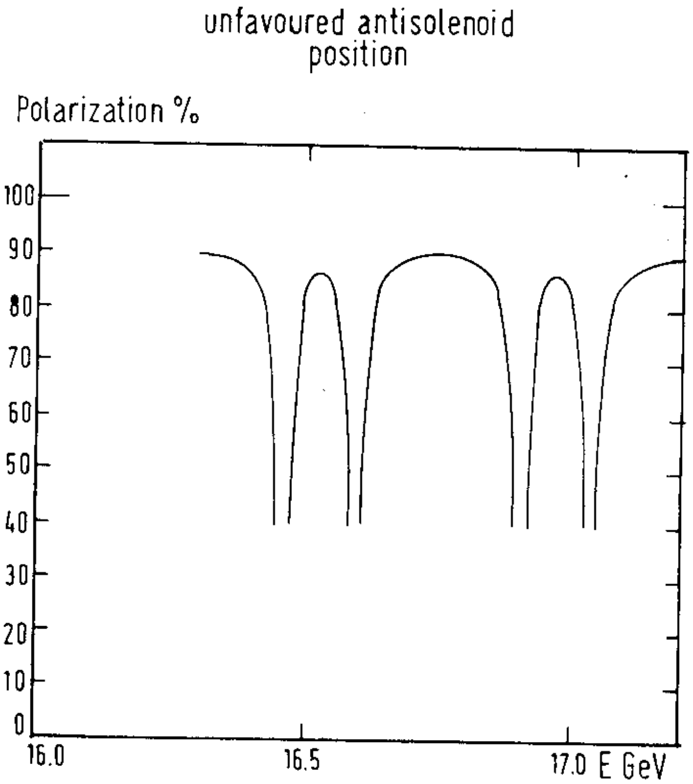

Fig. 10 b

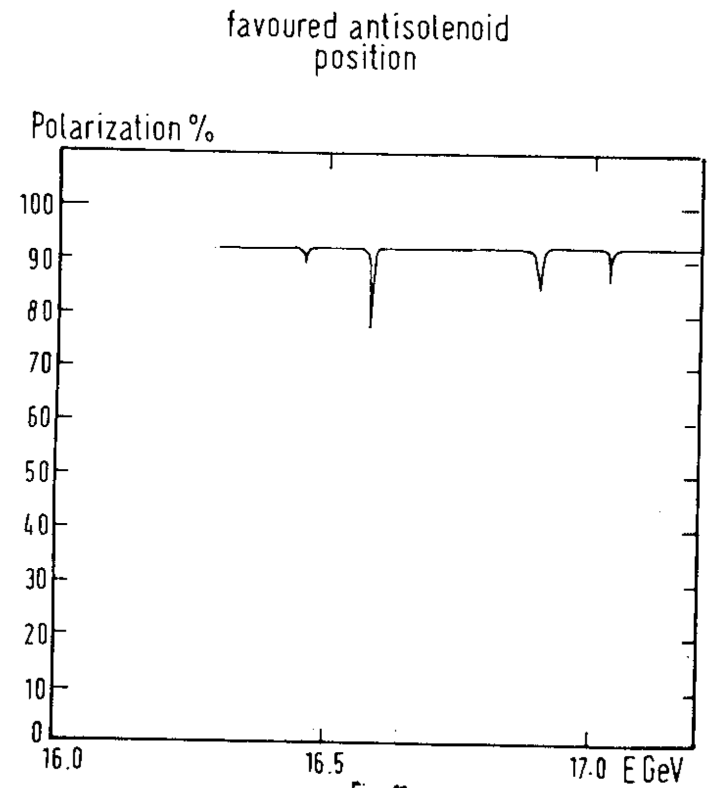

Fig. 11a

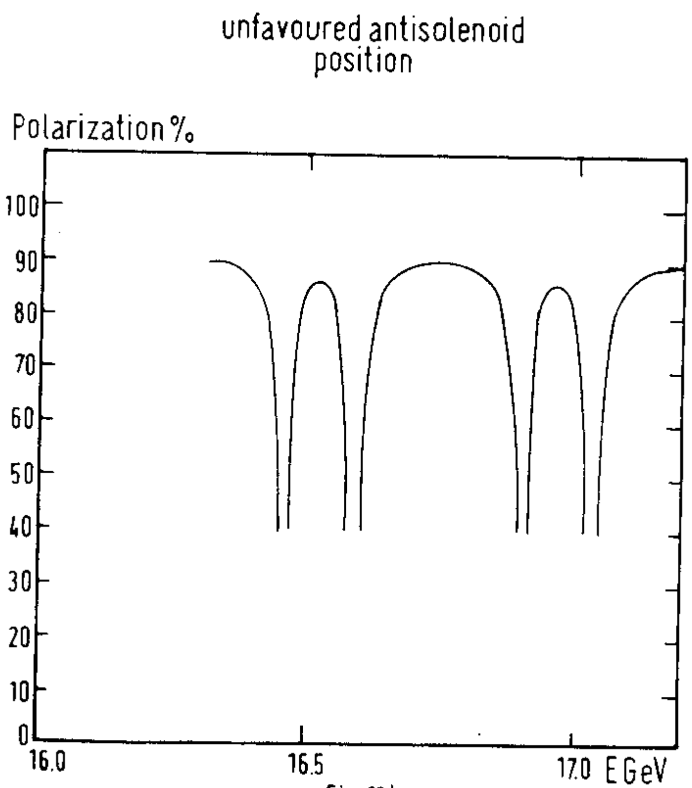

Fig. 11b