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EXOTIC INFRARED REPRESENTATIONS OF INTERACTING SYSTEMS

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Abstract: It is shown that the algebra of observables in a local quantum field theory has representations of type II and III with positive energy, if the theory includes massless particles which form an asymptotically complete set of scattering states below some mass threshold. So the exotic infrared representations, which have recently been exhibited in free field theory [1] do also appear in interacting models.

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1. Introduction

In quantum field theory one normally considers representations of the algebra of all local observables which are irreducible (superselection sectors). It is known that the vacuum representation is always irreducible [2] and that the particle representations in massive theories can be naturally decomposed into irreducible representations [3] (i.e. they are of type I according to the standard classification [4]). Therefore one might be led to the conclusion that in quantum field theory all representations with positive energy are type I.

In this note, however, we show that in quantum field theoretic models containing massless particles there appear infrared (positive energy) representations of the algebra of local observables which are of type II and III, respectively. This means that in such models there exists an abundance of states with finite energy which cannot be decomposed into pure states in any natural manner; so in particular there is no superposition principle for these states.

The existence of this new type of infrared representations was recently established for the field theory of a free, massless Majorana respectively Dirac Fermion [1], and it is clear that with similar techniques one can construct such representations also for free massless Bosons [5]. We will extend here these results to a much wider class of models, including quantum electrodynamics.

The basic idea in our approach is to consider the states leading to exotic infrared representations in free field theory as incoming respectively outgoing collision states in the interacting case. These collision states induce representations of the algebra of local observables which turn out to be of the same type as in the free field case, provided the massless particles in the model form a complete set of scattering states below some mass threshold [6]. So by a suitable choice of the collision states one can generate positive energy representations of the algebra of observables of any type.

It has long been known that the positive energy representations are quite abundant in the presence of massless particles due to the numerous possibilities of forming infrared clouds [7]. On the other hand the detailed structure of infrared clouds is only of little interest in particle physics, and it seems therefore appropriate to select a managable subset of representations from which the relevant physical information can be extracted.

Such a selection criterion has been proposed in [6] and used to initiate a systematic study of the physical state space of quantum electrodynamics. (For a different approach see [8,9] and [10].) According to this criterion one is led to restrict attention to positive energy representations π of the algebra of observables $\mathfrak{A}$ which are closely related to the vacuum representation π_o in the following sense. Representations describing, for example, states of zero total charge have to be equivalent to the vacuum representation on the subalgebras $\mathfrak{A}(V_+), \mathfrak{A}(V_-)$ of observables in the forward and backward lightcone of Minkowski space [6,11],

$$\begin{aligned} \pi \restriction \mathfrak{A}(V_+) &\simeq \pi_o \restriction \mathfrak{A}(V_+) \\ \pi \restriction \mathfrak{A}(V_-) &\simeq \pi_o \restriction \mathfrak{A}(V_-). \end{aligned} \qquad (1.1)$$

It is noteworthy that the representations constructed here fulfill only one of the conditions in (1.1). One may therefore hope that the full condition is sufficiently strong in order to exclude all undesired infrared representations, in particular those which are not of type I.

2. Assumptions and Generalities

Our basic assumptions are essentially the same as in [6] in particular concerning the formulation of asymptotic completeness of the massless particles below the threshold of massive particles. For the convenience of the reader we recall here the relevant facts.

The setting used for our analysis is standard [12]: we assume that the algebra of observables $\mathfrak{A}$ is given as a concrete C*-algebra on the vacuum Hilbert space $\mathcal{H}_o$ and that it is generated by a net $\mathcal{O} \longrightarrow \mathfrak{A}(\mathcal{O})$ of von Neumann algebras which are attached to the bounded regions $\mathcal{O}$ of Minkowski space. We need not specify here the usual structural assumptions such as locality, Poincaré covariance, spectrum condition and uniqueness of the vacuum $\Omega \in \mathcal{H}_o$. In addition to these fundamental properties we assume that there are massless particles in the model.

In the simplest case, where these particles carry the charge quantum numbers of the vacuum, one can construct on $\mathcal{H}_o$ outgoing (free) fields of these particles which generate a net $\mathcal{O} \rightarrow \mathfrak{A}^{out}(\mathcal{O})$ of asymptotic algebras [13]. The relation between these asymptotic algebras and the original observables can be summarized in the inclusions [1)]

$$\mathfrak{A}^{out}(V_+)^- \subset \mathfrak{A}(V_+)^- \subset \mathfrak{A}^{out}(V_-)' \qquad (2.1)$$

which follow from the very construction of the outgoing fields and Huygens' principle [13].

We denote by E the orthogonal projection onto the Fock-space of the outgoing collision states of massless particles,

$$E \cdot \mathcal{H}_o = \overline{\mathfrak{A}^{out}\Omega} = \mathcal{H}_o^{out}, \qquad (2.2)$$

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1) If $\mathcal{R}$ is an unbounded region of Minkowski space, we set

$$\mathfrak{A}(\mathcal{R}) = \overset{C^*}{\bigcup_{\mathcal{O} \subset \mathcal{R}}} \mathfrak{A}(\mathcal{O}),$$

and similarly for the asymptotic algebras. $\mathcal{C}^-$ and $\mathcal{C}'$ denote the weak closure and commutant, respectively, of any subalgebra $\mathcal{C} \subset \mathfrak{B}(\mathcal{H}_o)$.

where $\mathfrak{A}^{out} = \mathfrak{A}^{out}(\mathbb{R}^4)$. It is clear from this relation that $E \in \mathfrak{A}^{out\,\prime}$ and that $U(L)\, E\, U(L)^{-1} = E$ for all Poincaré transformations L . Moreover, it has been shown in [14,15] that for the algebra $\mathfrak{A}^{out}$ of free massless fields timelike duality holds on $\mathcal{H}_o^{out}$ in the sense of the following relation

$$(\mathfrak{A}^{out}(V_-) \upharpoonright \mathcal{H}_o^{out})' = \mathfrak{A}^{out}(V_+)^- \upharpoonright \mathcal{H}_o^{out}. \qquad (2.3)$$

So multiplying (2.1) from both sides by E we get

$$E\, \mathfrak{A}(V_+)^-\, E = \mathfrak{A}^{out}(V_+)^-\, E \qquad (2.4)$$

which is the key relation in our construction of collison states.

An important additional piece of information is given by the relation

$$E \in \mathfrak{A}(V_+)^- \qquad (2.5)$$

which can be derived from the assumption that the massless particles form a complete set of scattering states in $\mathcal{H}_o$ below some mass threshold. Namely, if for some $\varepsilon > 0$ $P_{[0,\varepsilon]}\mathcal{H}_o = P_{[0,\varepsilon]}\mathcal{H}_o^{out}$, where P_Δ are the spectral projections of the energy operator, it follows [6, Prop. 4.2] that the second inclusion in relation (2.1) is actually an equality, and consequently $E \in \mathfrak{A}^{out\,\prime} \subset \mathfrak{A}(V_+)^-$. This fact will allow us to control the type of the representations which are induced by the collision states of massless particles.

If there are massless particles in the model carrying a charge one can still establish relation (2.4) and (2.5), if these particles can be generated from the vacuum with the help of local Bose - or Fermi fields. (Note that the latter assumption means that the massless particles do not carry a gauge charge, such as the electric charge.) By an analogous discussion one can show then that relations (2.1) to (2.5) hold if one replaces everywhere the algebra of observables $\mathfrak{A}$ by the field algebra $\mathfrak{F}$ and, in the presence of massless

Fermions, the commutants by twisted (Klein-transformed) commutants [12]. One then obtains relations (2.4) and (2.5) for the observables by taking means of the corresponding relations for the fields over the underlying symmetry group [12]. Here one uses the fact that the observables in $\mathfrak{A}$ are the fixed points in $\mathfrak{F}$ under the global symmetry-transformations and that E commutes with these transformations because it projects onto the space $\overline{\mathfrak{F}^{out}\Omega}$ which is stable under the action of the symmetry group.

Let us remark in conclusion that $\mathfrak{A}(V_+)^-$ is a factor of type III [16] hence, since $E \in \mathfrak{A}(V_+)^-$, there exists an isometry $W \in \mathfrak{A}(V_+)^-$ such that $WW^* = E$ and $W^*W = 1$, i.e. $E \sim 1 \bmod \mathfrak{A}(V_+)^-$. We recall in this context the following elementary fact for future use: if $\mathcal{M} \subset \mathcal{B}(\mathcal{H})$ is a von Neumann algebra and $E \in \mathcal{M}$ an orthogonal projection with $E \sim 1 \bmod \mathcal{M}$, then the reduced von Neumann algebra $\mathcal{M}_E = E\mathcal{M}E \upharpoonright E\mathcal{H}$ is unitarily equivalent to $\mathcal{M}$. For with $W \in \mathcal{M}$ such that $WW^* = E$ and $W^*W = 1$ we have the following inclusions

$$W\mathcal{M}W^* = EW\mathcal{M}W^*E \subset E\mathcal{M}E = \\ = WW^*\mathcal{M}WW^* \subset W\mathcal{M}W^*. \qquad (2.6)$$

3. The Induction Procedure

We are prepared now to explain our induction procedure, which assignes to any suitable state ω^{out} on the algebra $\mathfrak{A}^{out}$ the corresponding collision state ω on the algebra $\mathfrak{A}$ of observables. The states ω^{out} to which our procedure can be applied have to be locally normal relative to the vacuum on the algebras $\mathfrak{A}^{out}(V_+ + a)$, $a \in \mathbb{R}^4$. Note in this context that the vacuum (Fock-) representation π_0^{out} of $\mathfrak{A}^{out}$ is faithful and can be identified with the representation of $\mathfrak{A}^{out}$ on $\mathcal{H}_0^{out}$ given by [13]

$$\pi_0^{out}(A) = A \restriction \mathcal{H}_0^{out} \quad , \quad A \in \mathfrak{A}^{out}. \tag{3.1}$$

In the following we will deal with normal states ω and representations π of certain C*-algebras $\mathcal{C} \subset \mathcal{B}(\mathcal{H}_0)$ and their unique normal extensions to $\mathcal{C}^-$. For the sake of clarity we denote these extensions by $\tilde{\omega}$ and $\tilde{\pi}$, respectively.

Lemma 3.1: Let ω^{out} be a state on $\mathfrak{A}^{out}$ which is normal on $\mathfrak{A}^{out}(V_+ + a)$ relative to π_0^{out} for all $a \in \mathbb{R}^4$. Then there exists a unique state ω on $\mathfrak{A}$ such that

i) ω is normal on each $\mathfrak{A}(V_+ + a)$, $a \in \mathbb{R}^4$ and

$$\tilde{\omega}(A) = \omega^{out}(A) \quad \text{for } A \in \mathfrak{A}^{out}(V_+ + a), \tag{3.2}$$

$\tilde{\omega}$ being the normal extension of ω to $\mathfrak{A}(V_+ + a)^-$. Moreover, $\tilde{\omega}(E) = 1$, where $E \in \mathfrak{A}(V_+)^-$ is the orthogonal projection onto $\mathcal{H}_0^{out}$ (cf. equation (2.5)).

ii) The GNS-representations induced by ω^{out} and ω, respectively, generate isomorphic von Neumann algebras:

$$\pi_\omega^{out}(\mathfrak{A}^{out})^- \approx \pi_\omega(\mathfrak{A})^-.$$

Proof: Let σ denote the normal compression map of $\mathcal{B}(\mathcal{H}_0)$ onto $\mathcal{B}(\mathcal{H}_0^{out})$,

$$\sigma(B) = EBE \restriction \mathcal{H}_0^{out} \quad , \quad B \in \mathcal{B}(\mathcal{H}_0).$$

From equation (2.4) and the fact that E is invariant under arbitrary translations we get for any $a \in \mathbb{R}^4$

$$\sigma(\mathfrak{A}(V_+ + a)) \subset \sigma(\mathfrak{A}(V_+ + a)^-) = \pi_0^{out}(\mathfrak{A}^{out}(V_+ + a)^-),$$

hence we can define a state ω on $\mathfrak{A}$, setting

$$\omega(A) = \tilde{\omega}^{out}(\sigma(A)) \quad \text{for} \quad A \in \mathfrak{A}(V_+ + a). \tag{3.3}$$

By this relation ω is fixed, since $\mathfrak{A}$ is the inductive limit of the net $\bigcup_{a \in \mathbb{R}^4} \mathfrak{A}(V_+ + a)$. In view of the normality properties of ω^{out} and σ it is clear that ω is a normal state on each $\mathfrak{A}(V_+ + a)$, and since $\mathfrak{A}^{out}(V_+ + a) \subset \mathfrak{A}(V_+ + a)^-$ relation (3.2) follows. Also, by (3.3), we have $\tilde{\omega}(E) = 1$. Conversely, the properties i) imply relation (3.3), so ω is unique.

In order to prove the second half of the statement we use the fact that $E \in \mathfrak{A}(V_+)^-$ (cf. relation (2.5)). Since the GNS-representation $\{\pi_\omega, \xi_\omega, \mathcal{H}_\omega\}$ induced by ω is normal on $\mathfrak{A}(V_+ + a)$ (as a consequence of the normality of ω [17, Prop. 6]) there exists then the projection

$$F = \tilde{\pi}_\omega(E),$$

and it is an immediate consequence of the relation $\tilde{\omega}(E) = 1$ that $F\xi_\omega = \xi_\omega$. We define now on $F\mathcal{H}_\omega$ a representation π^{out} of $\mathfrak{A}^{out}$, setting

$$\pi^{out}(A) = \tilde{\pi}_\omega(A \cdot E) \restriction F\mathcal{H}_\omega$$

if $A \in \mathfrak{A}^{out}(V_+ + a)$ for some $a \in \mathbb{R}^4$. This representation is the link between π_ω and π_ω^{out}: first, we have according to (2.4)

$$F\pi_\omega(\mathfrak{A}(V_+ + a)^-)F = \tilde{\pi}_\omega(E\mathfrak{A}(V_+ + a)^- E) = \pi^{out}(\mathfrak{A}^{out}(V_+ + a))^- F$$

and letting a vary we get

$$F\,\pi_\omega(\mathfrak{A})^- F = \pi^{out}(\mathfrak{A}^{out})^- F. \qquad (3.4)$$

Since $F \sim 1 \bmod \pi_\omega(\mathfrak{A}(V_+))^-$, hence a fortiori $F \sim 1 \bmod \pi_\omega(\mathfrak{A})^-$, it follows from the remarks at the end of Section 2 that $\pi_\omega(\mathfrak{A})^-$ and $\pi^{out}(\mathfrak{A}^{out})^-$ are isomorphic von Neumann algebras. Second, we get from (3.2)

$$(\xi_\omega, \pi^{out}(A)\,\xi_\omega) = \omega^{out}(A) \quad \text{for} \quad A \in \mathfrak{A}^{out}$$

and using relation (3.4) and $F\xi_\omega = \xi_\omega$ it is easy to see that ξ_ω is cyclic for π^{out} in $F\,\mathcal{H}_\omega$:

$$\overline{\pi^{out}(\mathfrak{A}^{out})\,\xi_\omega} = F \cdot \overline{\pi_\omega(\mathfrak{A})\,\xi_\omega} = F \cdot \mathcal{H}_\omega .$$

Hence in view of the uniqueness of the GNS-construction we conclude that π^{out} and π^{out}_ω are equivalent representations of $\mathfrak{A}^{out}$, and the statement follows. □

The first part of this lemma shows that for any freely moving configuration of massless particles which can be represented by a state ω^{out} of the above type, there exists a collision state ω which behaves at large positive times like this configuration. From the second part of the lemma it follows then that the representations which are induced by ω^{out} and ω , respectively, are of the same type.

If ω^{out} induces a positive-energy representation of $\mathfrak{A}^{out}$, then one can easily show that ω induces a representation of $\mathfrak{A}$ which is covariant with respect to the translations. However, it is not clear in general whether this representation has also positive energy. For this reason we now consider a more restricted class of states ω^{out} where this result can be established. These states are distinguished by the fact that they can be approximated by sequences of unit vectors $\xi_n \in \mathcal{H}^{out}_o$ such that

i) ξ_n has energy-momentum spectrum contained in a compact set independent from n;

ii) for each $a \in \mathbb{R}^4$

$$\lim_{n\to\infty} \|(\omega_{\xi_n} - \omega^{out}) \upharpoonright \mathfrak{A}^{out}(V_+ + a)\| = 0.$$

It follows from the second property that ω^{out} is normal on $\mathfrak{A}^{out}(V_+ + a)$ relative to π^{out}_o , so we can apply to it the construction in the previous lemma.

<u>Lemma 3.2</u>. Let ω^{out} be a state on $\mathfrak{A}^{out}$ with the properties given above, and let ω be the corresponding state on $\mathfrak{A}$ constructed in Lemma 3.1. Then the GNS-representation π_ω of $\mathfrak{A}$ is covariant for the translation group and fulfills the spectrum condition.

<u>Proof</u>: Since $E\xi_n = \xi_n$, it follows from the definition (3.3) of ω and the normality of ω^{out} on $\mathfrak{A}^{out}(V_+ + a)$ that

$$\|(\omega_{\xi_n} - \omega) \upharpoonright \mathfrak{A}(V_+ + a)\| = \|(\omega_{\xi_n} - \tilde{\omega}^{out}) \upharpoonright \sigma(\mathfrak{A}(V_+ + a))\|$$
$$= \|(\omega_{\xi_n} - \omega^{out}) \upharpoonright \mathfrak{A}^{out}(V_+ + a)\|$$

for each $a \in \mathbb{R}^4$. So in particular

$$\omega = \text{weak} * - \lim_{n\to\infty} \omega_{\xi_n} \upharpoonright \mathfrak{A}. \qquad (3.5)$$

Now let $\mathcal{C}$ be the C*-subalgebra of $\mathfrak{A}$ consisting of all elements with norm-continuous orbit under translations; $\mathcal{C}$ is generated by the operators [18]

$$\int d^4x\, f(x)\, \alpha_x(A) , \quad A \in \mathfrak{A} , \quad f \in L^1(\mathbb{R}^4).$$

By a theorem of Borchers [19] equation (3.5) restricted to $\mathcal{C}$ implies that the GNS-representation $\pi_{\omega \upharpoonright \mathcal{C}}$ induced by $\omega \upharpoonright \mathcal{C}$ is covariant and fulfills the spectrum condition, because of the restricted energy-momentum support of the vectors ξ_n.

Now $\mathfrak{A}(V_+ + a) \cap \mathcal{C}$ is weakly dense in $\mathfrak{A}(V_+ + a)$ for each

$a \in \mathbb{R}^4$, and since π_ω is normal on $\mathfrak{A}(V_+ + a)$ we have

$$\pi_\omega(\mathfrak{A}(V_+ + a) \cap \mathcal{C})^- = \pi_\omega(\mathfrak{A}(V_+ + a))^- . \tag{3.6}$$

Thus, varying a, we get $\pi_\omega(\mathcal{C})^- = \pi_\omega(\mathfrak{A})^-$, hence the GNS-vector ξ_ω is cyclic for $\pi_\omega(\mathcal{C})$, and consequently the representations $\pi_\omega \upharpoonright \mathcal{C}$ and $\pi_{\omega \upharpoonright \mathcal{C}}$ are unitarily equivalent. This shows that $\pi_\omega \upharpoonright \mathcal{C}$ is covariant and fulfills the spectrum condition. But the normality of π_ω on $\mathfrak{A}(V_+ + a)$ and relation (3.6) imply, that the unitary operators inducing the translations on $\pi_\omega \upharpoonright \mathcal{C}$ also induce the translations on π_ω, so the statement follows. □

4. Examples

By the above considerations we have reduced the problem of constructing positive energy representations of $\mathfrak{A}$ of type II and III, respectively, to a problem in free field theory: we must exhibit states ω^{out} with properties specified above inducing representations of $\mathfrak{A}^{out}$ of this type.

In order to keep this discussion brief we treat here only models with one massless Majorana particle carrying no other charge than univalence; in this case we can rely on results in [1,11]. But with some technical modifications our argument extends to the general situation where massless particles of arbitrary helicity are present.

Let $\mathcal{H}_1$ be the single particle space of our massless Fermion, ψ^{out} the corresponding outgoing field operator and $\mathfrak{F}^{out}$ the C*-algebra which it generates; the observables in $\mathfrak{A}^{out}$ are then the fixed points in $\mathfrak{F}^{out}$ under "spatial rotations by 2π". By an obvious modification of the argument in [11, Theorem 2.2] we can choose a sequence of unit vectors $\xi_n \in \mathcal{H}_1$, $n \in \mathbb{N}$ such that

i) the energy momentum spectra of the vectors ξ_n are mutually disjoint and contained in $|p_0| = |\underline{p}| \le \varepsilon_n$, where $\sum_{n=1}^{\infty} \varepsilon_n < \infty$;

ii) with $a^{out}(\xi_n)$ denoting the destruction operator of the outgoing single particle mode ξ_n there is a smooth function g_n with compact support in $V_- - n \cdot e$, e being a fixed positive timelike vector, such that

$$\| \psi^{out}(g_n) - a^{out}(\xi_n) \| \le 2^{-n} \quad , \; n \in \mathbb{N} . \tag{4.1}$$

Now let P be the orthogonal projection in $\mathcal{H}_1$ onto the subspace generated by the vectors ξ_n , $n \in \mathbb{N}$. Identifying $\mathfrak{F}^{out}$ with the CAR-algebra over $\mathcal{H}_1$, we consider then the even (gauge invariant) quasifree state ω_λ^{out} , $0 \le \lambda \le 1$ on $\mathfrak{F}^{out}$ defined by

$$\begin{aligned} &\omega_\lambda^{out}(a^{out}(\varphi_m)^* \cdots a^{out}(\varphi_1)^* \, a^{out}(\varphi_1) \cdots a^{out}(\varphi_n)) \\ &= \delta_{mn} \cdot \det \left[(\varphi_i , \lambda P \, \varphi_k) \right]_{i,k=1,\dots n} \end{aligned} \tag{4.2}$$

with $\varphi_i \in \mathcal{H}_1$. It follows from the discussion in [1,11] that the restriction of ω_λ^{out} to $\mathfrak{A}^{out}$ fulfills the conditions on which Lemma 3.2 is based. So the corresponding collision state ω_λ induces a representation π_{ω_λ} of the algebra of observables $\mathfrak{A}$ which is covariant with respect to translations, fulfills the spectrum condition, is normal on each $\mathfrak{A}(V_+ + a)$, and

$$\pi_{\omega_\lambda}(\mathfrak{A})^- \approx \pi^{out}_{\omega_\lambda}(\mathfrak{A}^{out})^-.$$

It has been shown in [1] that the von Neumann algebra $\pi^{out}_{\omega_\lambda}(\mathfrak{A}^{out})^-$ is the Powers factor of type II_∞ if $\lambda = \frac{1}{2}$ and of type III_μ , $0 < \mu < 1$ if $\lambda = \frac{1}{1+\sqrt{\mu}}$ or $\frac{\sqrt{\mu}}{1+\sqrt{\mu}}$, respectively. Hence the positive energy representations π_{ω_λ} of $\mathfrak{A}$ do also generate these factors! We conjecture that actually every properly infinite and approximately finite dimensional von Neumann algebra with separable predual can be generated by a suitable positive energy representation of $\mathfrak{A}$ (cf. [1 , Theorem 2.4]).

In the presence of massless particles of arbitrary helicity one can construct states ω^{out} inducing positive energy representations of $\mathfrak{A}^{out}$ of type II and III, respectively, along similar lines. If these particles are Bosons it is, however, more complicated to establish the properties of ω^{out} which are needed for our induction procedure, since the asymptotic Bose fields are unbounded operators. Yet, using the fact that these fields behave like bounded operators on states with a finite particle number one can still establish the relevant properties [5]. We omit the details and just state our main result.

<u>Theorem 4.1</u> Let $\mathfrak{A}$ be the algebra of observables in a theory fulfilling the assumptions of Section 2. Then there are representations π of $\mathfrak{A}$ of type II and III, respectively, which are covariant with respect to translations, fulfill the spectrum condition and are normal relative to the vacuum representation on $\mathfrak{A}(V_+ + a)$ for any $a \in \mathbb{R}^4$.

By an analogous discussion based on the incoming massless fields, one can also construct representations π which are normal relative to the vacuum representation on $\mathfrak{A}(V_- + a)$, $a \in \mathbb{R}^4$. It is an

open problem whether there can exist positive energy representations of type II or III which are normal on $\mathfrak{A}(V_+ + a)$ and $\mathfrak{A}(V_- + a)$.

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