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SIMULATION OF ELECTRON SPIN DEPOLARISATION WITH THE COMPUTER CODE SITROS

by

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Abstract

A new tracking program "SITROS" which enables simulation of polarising and depolarising effects in electron-positron storage rings is presented. The effectiveness of the simulation is illustrated for the storage ring PETRA.

1. Introduction

Electrons or positrons circulating in a storage ring can become polarised during emission of synchrotron radiation by the Sokolov-Ternov effect /1/. The maximum possible degree of polarisation is 92.4 %.
Experimental studies on polarisation at PETRA /2/ have led to the achievement of a maximum measured degree of polarisation in PETRA of about 80 %. This degree of polarisation could only be achieved when the uncompensated solenoidal detector fields were switched off and after a careful optimization of the vertical closed orbit.
The failure to reach the theoretical maximum degree of polarisation is caused by various depolarising effects. The first computer program capable calculating depolarising effects in electron-positron storage rings was the program "SLIM" developed by A. Chao /1/. SLIM calculates only the linear coupling between spin rotation and particle oscillations and is therefore restricted to the calculation of depolarising resonances of first order. The influence of higher order effects on polarisation driven by nonlinear forces (sextupoles, chromaticity, beam-beam force) is not taken into account. Since experimental work /2/ has indicated that nonlinear effects are important it has become necessary to develop a new program.
In constrast to SLIM which calculates the polarisation by a perturbation method, this new program is based on a tracking algorithm. Trajectories and spin vectors of a number of representative particles are traced over many revolutions and both linear and nonlinear effects and quantum emission are taken into account. The program is called SITROS.

2. Transformation Algorithm for a Section of the Ring

2.1 Orbital Motion

The first task of SITROS is the calculation of the closed orbit. In an ideal machine the closed orbit goes through the center of the quadrupoles. In a real machine the closed orbit deviates from the ideal owing to magnet errors and the nonuniform distribution of cavities. All other trajectories will oscillate around this closed orbit. Thus the closed orbit defines the center of the bunch and can be taken as the average trajectory.
The motion of an electron is described by a 6 dimensional vector $\vec{X} = (x, x', z, z', s, \delta)$, where x, z, s are the horizontal, vertical and longitudinal displacements of the particle relative to the closed orbit particle and $\delta = \Delta E/E$ is the relative energy deviation.
In linear approximation the space coordinates are transformed from a point s_o to an other point s_1 by a constant matrix M:

$$\vec{X}(s_1) = M \cdot \vec{X}(s_o)$$

To include 2nd order effects the matrix must be extended:

$$\vec{X}(s_1) = (M_{lin} + M_{x'} + M_z + M_{z'} + M_s + M\delta) \cdot \vec{X}(s_o)$$

where the additional matrices depend on the vector $\vec{X}(s_o)$. The matrix $M\delta$ describes the chromaticity effects and the effects of sextupoles are contained in M_x, $M_{x'}$, M_z, $M_{z'}$.

Defining a vector $\tilde{\vec{X}} = (x, x', ..., s, \delta, x^2, xx', ..., s, \delta, \delta^2)$, this transformation can be rewritten as:

$$\vec{X}(s_1) = \tilde{M} \cdot \tilde{\vec{X}}(s_o)$$

where $\tilde{M}$ is a 6x27 matrix with constant coefficients.

SITROS finds these matrices by calculating 72 different trajectories in the vicinity of the closed orbit and then making a quadratic interpolation table of these trajectories. The 72 trajectories are themselves generated by a tracking algorithm based on the methods used in the PETROS /9/ program for finding the closed orbit. The PETROS routines are in the present case extended to caculate synchrotron oscillations and the damping of the transverse and longitudinal motion.

In contrast to SLIM in which the optical elements are described as thin lenses, SITROS takes the thickness of the lenses into consideration. Thus the Q-values calculated by SITROS are rather close to the measured Q-values in PETRA. As in PETROS, it is possible to introduce different kinds of magnet errors (e.g. misalignment of quadrupoles) into the optical lattice.

2.2 Spin Motion

The spin direction of a particle is described by a vector $\vec{S} = (sx, sz, ss)$. The rotation of the spin of a particle moving in an electromagnetic field is described classically by the well known BMT-equation /6/:

$$\frac{d\vec{S}}{dt} = \vec{S} \times (\vec{\omega}_e + \vec{\omega}_b + \vec{\omega}_r)$$

where $\vec{\omega}_e$ describes the rotation by electric fields:

$$\vec{\omega}_e = - \frac{e}{m\gamma} (\gamma a + 1)\, \vec{\beta} \times \vec{E}$$

$\vec{\omega}_b$ describes the rotation by magnetic fields:

$$\vec{\omega}_b = \frac{e}{m\gamma} [\gamma a \, (\vec{B} - \frac{\gamma}{+1} (\vec{\beta}\cdot\vec{B})\cdot\vec{\beta} \,) + \vec{B}\,]$$

As usual in optics calculations the reference frame rotates with the curvature of the reference orbit. The radius of this curvature can be described by a fictitious magnetic field:

$$\frac{1}{r} = \frac{e}{m\gamma_o} \vec{B}_r$$

In this rotating frame the electron spin is rotating backwards and is described by the term $\vec{\omega}_r$:

$$\vec{\omega}r = - \frac{e}{m\gamma_o} \vec{B}_r$$

where m is the electron mass, γ_o is the Lorentz factor at the nominal energy $\gamma_o = E_o/m$ and $\gamma = \gamma_o(1+\delta)$. $\vec{\beta}$ is the relativistic velocity of the electron. $\vec{\beta} = \vec{v}/c$ · $\vec{\beta}$ can be approximated for small x', z' by (x', z', 1). The strength of the electromagnetic fields varies with the position (x, z, s) of the particle.

As a means of saving computer time, in the SITROS program dipole magnets and other optical elements are treated in different ways:

In a dipole magnet, the field is constant over the magnet and independent of the particle position. However, a small constant solenoidal component of the field due to a slope of the closed orbit must be included in $\vec{\omega}r$. The BMT-equation can be easily solved for a constant vector $\vec{\omega} = \vec{\omega}_b + \vec{\omega}_r$; the solution is the rotation of the spin by an angle $|\vec{\omega}|$ around the axis given by the direction of $\vec{\omega}$.

In quadrupoles, higher multipoles and cavities the solenoidal fields can be neglected. For a purely transverse field and no curvature of the reference orbit the BMT-equation can be solved as:

$$\varphi d\varphi = (\gamma a + 1)\, d\alpha$$

where φ is the rotation angle of the spin vector and α is the change of the direction of the particle. This formula can then be applied for a whole quadrupole or cavity, because small rotations are commutative.

The rotations in successive magnets are combined into a single rotation vector $\vec{\Omega} = (\Omega x, \Omega z, \Omega s)$ /8/. This vector depends on the electromagnetic fields along the particle trajectory and can be written in the following way:

$$\vec{\Omega} = \vec{\Omega}_{c.o} + \vec{\Omega}(\vec{\tilde{X}})$$

where $\vec{\Omega}_{c.o.}$ is the rotation vector on the closed orbit. The rotation vector $\vec{\Omega}_{c.o.}$ for a whole revolution in an ideal flat machine is given by:

$$|\vec{\Omega}_{c.o.}(s_o + C, s_o)| = 2\pi \gamma a$$

The quantity $\nu \equiv \gamma a$ is called the spin tune.
In SITROS, the quantities $|\Omega|$, Ωx, Ωs are calculated in a similar way to the components of $\vec{X}(s_1)$. If the spin rotation angle is not too close to a multiple of 2π, these quantities can be described in a good approximation by first and second order terms. By combining both spin and particle motion a 9x28 matrix can be defined:

$$\begin{pmatrix} x \\ x' \\ z \\ z' \\ s \\ |\Omega| \\ \Omega x \\ \Omega s \end{pmatrix}_{(s_1)} = M(9x28) \cdot \begin{pmatrix} \\ 1 \\ \tilde{\vec{X}} \\ \\ \end{pmatrix}_{(s_0)}$$

3. Photo Emission

An electron loses energy stochastically by the emission of photons. On average about 11 photons per electron are emitted in a PETRA magnet at 16.5 Gev /7/.
Each photon emission excites synchrotron oscillations which are in general coupled to betatron oscillations. The stochastic character of the emission leads to a random phase and amplitude distribution of the oscillations, and together with the damping defines the beam size /10/. Coupling of the spin motion to these oscillations will result in depolarisation.
Since the M(9x28) matrices already contain terms describing the average (smooth) radiative energy loss, a correct description of real particle motion would involve inclusion of additional positive or negative random energy changes, $\Delta\delta$, in each bending magnet. However, in order to save computing time the random energy jump in the SITROS program is only made in two bending magnets (PETRA has 228 bending magnets). This has the advantage that the matrix M(9x28) can be calculated for a whole section containing many bending and quadrupole magnets, so that the computing time for the spin tracking is strongly reduced.

The strength of these random jumps is chosen in such a way that the horizontal emittance of the beam is the same as in the PETROS program. The emittance ε is proportional to the excitation strength $\langle\Delta\delta\rangle$.

The beam size is calculated by using the well known optical parameters α, β and γ. The emittance of a single particle is given by:

$$\varepsilon_x = \beta x'^2 + 2 \alpha x x' + \gamma x^2$$

The emittance of the whole beam is given by the rms-value of these single particle emittances.
Since particles with different energies differ much less in their trajectories than in their spin behaviour, it is a good approximation to calculate the polarisation at different energy points by using the same space parameters at each energy so that in this version of SITROS 100 different energy points can be calculated in one job. This saves a factor of two in computing time.

4. Evaluation of the Degree of Polarisation

The SITROS program, as described so far, calculates the depolarisation rate of a perfectly polarised beam. The program starts with all particles on the closed orbit and with the spin vectors pointing in the direction of the socalled n-axis /4/ which is the spin rotation axis on the closed orbit for one revolution. For particles remaining on the closed orbit this spin direction will be preserved.
Since the particles experience random energy kicks they will leave the closed orbit and the spins are kicked away from the direction of the n-axis by the quadrupole and sextupole fields.
The average deviation of the spin from the n-axis is zero far away from the depolarising resonances but accumulate to a non zero value near resonances.
We define a polarisation vector:

$$\vec{P} = \frac{1}{k} \sum_i \vec{S}_i \cdot \vec{n}$$

where k is the number of tracked particles and $\vec{S}i$ is the spin vector of the ith particle. The length P of the vector $\vec{P}$ will decrease with the number of revolutions. In SITROS, it is assumed that the function P(t) can be described by the exponential function:

$$P(t) = Po \quad \exp(-\,t/\tau_d)$$

Taking two samples of this function at t_1 and t_2 the depolarisation time τ_d is:

$$\frac{1}{\tau_d} = \ln \quad \frac{P(t_1)/P(t_2)}{t_2 - t_1}$$

τ_d converges after a few betatron damping times to the final value and the calculation can then be stopped. Fig. 1 shows a typical dependence of the polarisation P_∞ on the number of calculated revolutions.

The build-up time τ_p of the polarisation is given by:

$$\tau_p = 98 \cdot \frac{R^3 \; \langle R \rangle}{E^5 \; R}$$

where E is the beam energy in GeV, R is the magnet radius in meters and <R> is the average radius of the ring in meters.

From the values of τ_p and τ_d the equilibrium degree of polarisation can be calculated using /5/:

$$P_\infty = 92.4\ \% \ \frac{\tau_d}{\tau_p + \tau_d}$$

5. <u>Present Version of SITROS</u>

In the present version of SITROS 20 electrons are tracked around the ring. The ring is devided into four sections. The break points are two bending magnets in which photon emissions are simulated and two interaction points. At these points the beam-beam force or time dependent fields can be included.

SITROS runs in three steps:

1. generation of the transfer matrix
2. tracking of the particles
3. graphic display of the results.

On the IBM 168 the first step consumes approximately 10 minutesof cpu time and 600 Kbyte. In the second step for 600 revolutions it consumes 16 minutes and 1 Mbyte. At the end of the job the particle coordinates are dumped into a file. The tracking can then be continued in a subsequent job.

6. <u>Simulation Results for PETRA</u>

The first SITROS runs were made with a PETRA "M15" optics. The parameters of this optics evaluated with PETROS are:

Q-values: Qx = 23.207 Qz = 25.317 Qs = 0.0925

ß-function in the IP: ßx* = 2.5 m ßz* = .15 m

Maximum dispersion in the arc: 1.800 m

Emittance ε_x: $14 \cdot 10^{-8}$ rad m

Bunch length σ_s: 1 cm

Relative energy spread σ_e/E $1 \cdot 10^{-3}$

Fig. 2 shows the measured degree of polarisation in PETRA versus the spin tune in the range of 37.00 to 38.00. This corresponds to an energy range from 16.30 to 16.74 GeV. The vertical closed orbit was optimized in this measurement (see also Fig. 5).

The plateau region of high polarisation is narrowed down by strong depolarising resonances. These strong resonances are not predicted by SLIM and so we expect that by including nonlinear resonances in SITROS we will achieve a better description of the reality. Indeed the SITROS results (Fig. 3, 5, 7) do exhibit strong nonlinear resonances and thus accounts for loss of polarisation.

Fig. 3 shows the degree of polarisation versus the spin tune for a M15 optics with a vertical closed orbit shape generated by a random distribution of kicks in the vertical correction coils. The rms amplitude of the closed orbit is 1.5 mm. The decrease of polarisation from the theoretical value of 92.4 % can be explained by the occurrence of various resonances. In general these resonances occur, when the condition

$$\nu + i\ Qx + j\ Qz + k\ Qs = m$$

is fulfilled, where i, j, k, m are integer. The resonances are expected to be strongest when the integers are small. The resonances $\nu \pm Qx = m$ and $\nu \pm Qs = m$ are most strong. Other strong resonances are $\nu \pm 2(Qx+Qs) = m$.

The resonances with $j \neq 0$ (Qz resonances) do not appear strongly in this calculations. Due to the fourfold symmetry, the driving terms of these resonances cancel each onther in the different parts of the ring.

For comparison Fig. 4 shows the corresponding results of SLIM. The shape of the curves in Fig. 3 and 4 are roughly the same but SLIM can only detect the linear resonances $\nu \pm Qx = m$ and $\nu \pm Qs = m$. Due to the thin lens approximation of SLIM the Q-values are slightly different to thos in Fig. 3.
It has been shown /7/ that the strength of depolarising resonances can be reduced by a special orbit correction scheme. The closed orbit is adjusted, so that harmonic components near the spin tune vanish.
Fig. 5 shows the polarisation of an optics where, with special symmetric kicks, the vertical closed orbit is distorted so that only the 4th harmonic component exists. Thus the closed orbit is perfectly corrected with respect to the spin in the range between $\nu = 37$ to 38. The main resonances are seen in both the corrected and uncorrected optics, but in the optimized optics the resonances are smaller and the polarisation between them is higher. However, the correction scheme does not cure all problems as the corresponding SLIM calculation would suggest (Fig. 6).
Only two resonances can be seen in the SLIM result: $\nu - Qx = 14$ and $\nu + Qs = 38$. The resonances $\nu + Qx = 51$ and $\nu - Qs = 37$ are compensated by the symmetry of the machine.
In Fig. 3 and 5 the vertical kicks are generated with the correction coils. So the same optics can be calculated with SITROS and SLIM. In Fig. 7 the machine distortions are generated with the quadrupoles.
The following randomly distributed magnet displacement and current errors are used:

horizontal displacement	0.05 mm
vertical displacement	0.055 mm
magnet current error	0.1 %

These imperfections generate an asymmetric machine. Therefore Qz resonances appear in Fig. 7.

7. Depolariser

It is well known that polarisation can be destroyed by application of a transverse oscillating field with a frequency given by:

$$f_{dep} = \widetilde{\gamma a} \cdot f_{rev}$$

where $\widetilde{\gamma a}$ is the fractional part of the spin tune, f_{rev} is the revolution frequency of the beam and f_{dep} is the frequency of the depolariser field.
This depolarising effect has already been demonstrated at PETRA /3/ and other machines /11/. It can also be simulated by introducing a time dependent kicker into the SITROS program. SLIM only calculates the static behaviour of the polarisation and can therefore not describe time dependent phenomena.
Fig. 8 shows the polarisation as a function of the frequency f_{dep} for a SITROS calculation in which, as in the experiment, a field of 0.1 Γ m was used. In order to avoid a change of the program, the kicker was implemented in the interaction point in this simulation. As the vertical beta function in this point is much smaller (15 cm) than in the position used in the experiment (30 m) the width of the measured and simulated resonance differ.
Fig. 9 shows the curve measured in PETRA. Due to the smaller accuracy of the energy calibration by the magnet currents this resonance occurred at the lower frequency.
Since the depolarisation time is much longer than the synchrotron oscillation time the resonance is smaller ($\Delta E/E = 1 \cdot 10^{-5}$) than the calculated energy spread of the beam is $1 \cdot 10^{-3}$.

8. Summary

This report introduces a new simulation program for calculating the degree of polarisation in electron positron storage rings and presents the first results achieved with the program. This program differs from the well known program SLIM in that it is a tracking program which allows computation of nonlinear and nonstationary effects. Nevertheless, it is a common problem of all tracking programs that for exact calculations unlimited computing time is necessary. This time is not available and therefore some simplifications and restrictions must be used. This report shows that despite these approximations and the classical treatment SITROS is a useful tool for the understanding of depolarising effects.

9. Acknowledgement

I wish to thank Prof. Dr. G.-A. Voss, Dr. R. Rossmanith, Dr. R. Schmidt, Dr. H. Mais, Dr. R. Barber and Dr. F. Willeke for helpful discussions and for encouraging me in this work.

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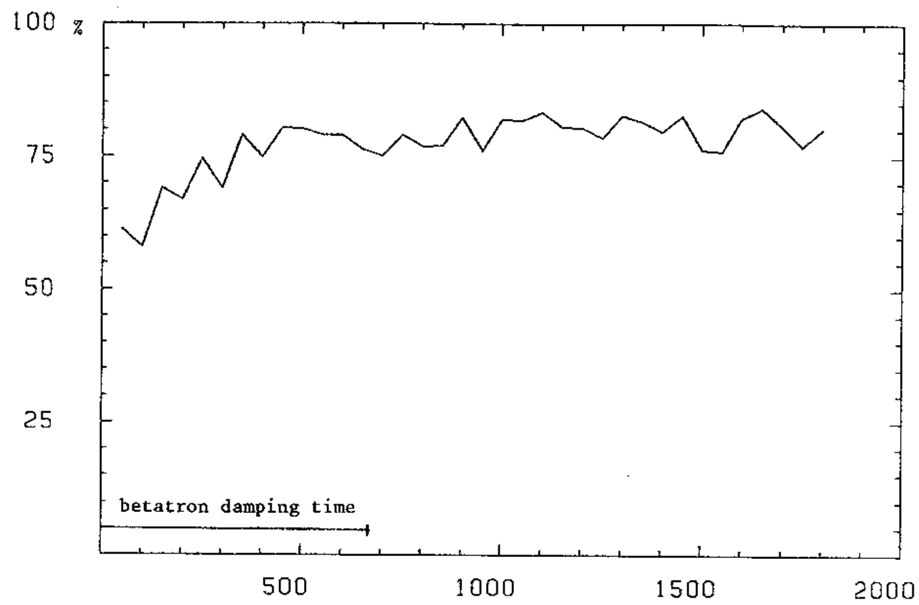

Fig. 1 : Dependence of the extrapolated polarisation as a function of the revolutions calculated with SITROS. The optics of Fig. 3 is displayed at a spin tune of ν = 37.49

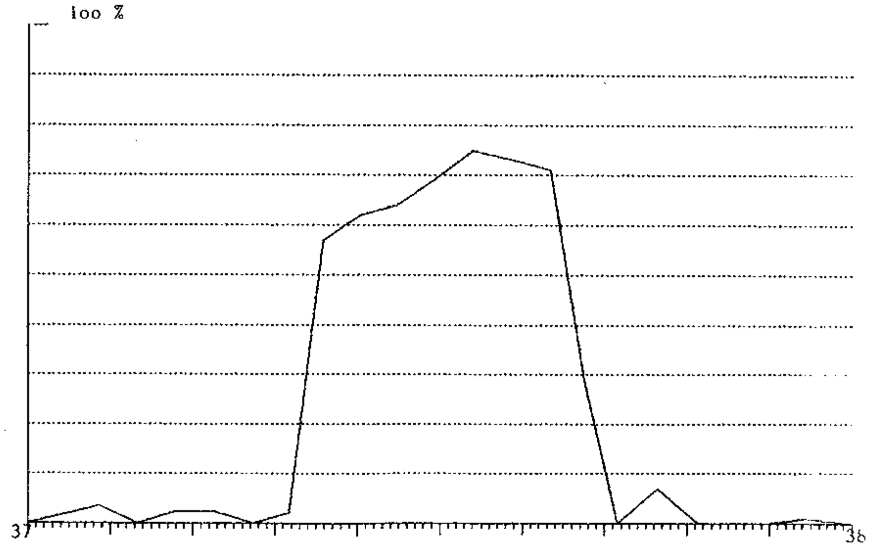

Fig. 2 : Polarisation versus spin tune measured in the storage ring PETRA.

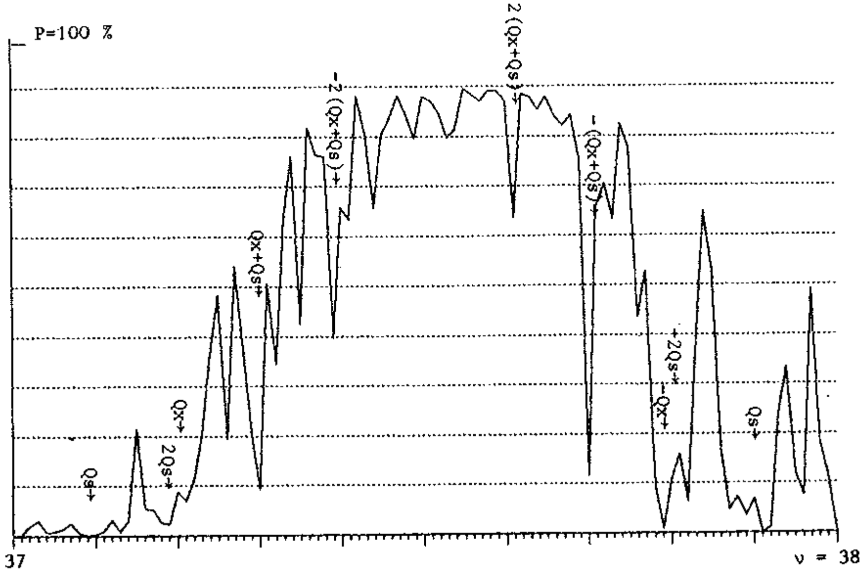

Fig. 3 : Polarisation versus spin tune of an optics with normal distributed vertical kicks calculated with SITROS.

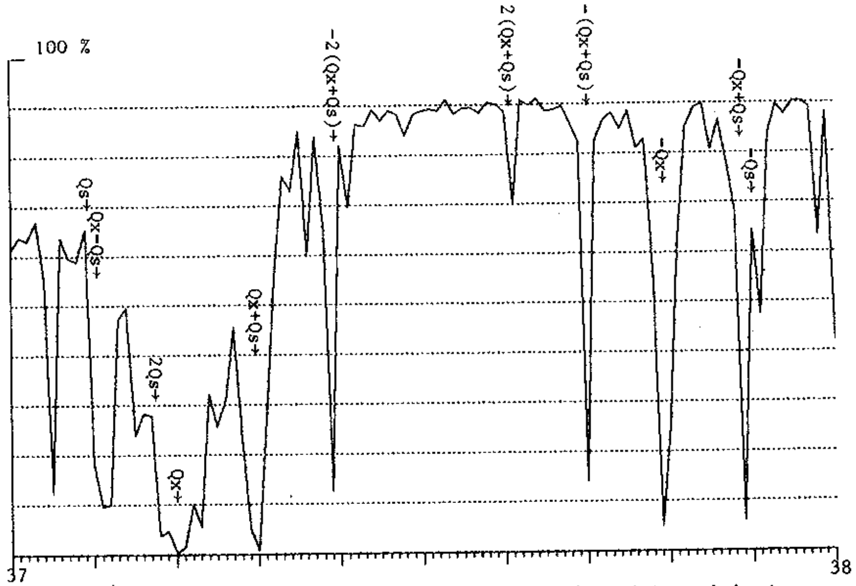

Fig. 5 : Polarisation versus spin tune of an optics with optimized closed orbit calculated with SITROS.

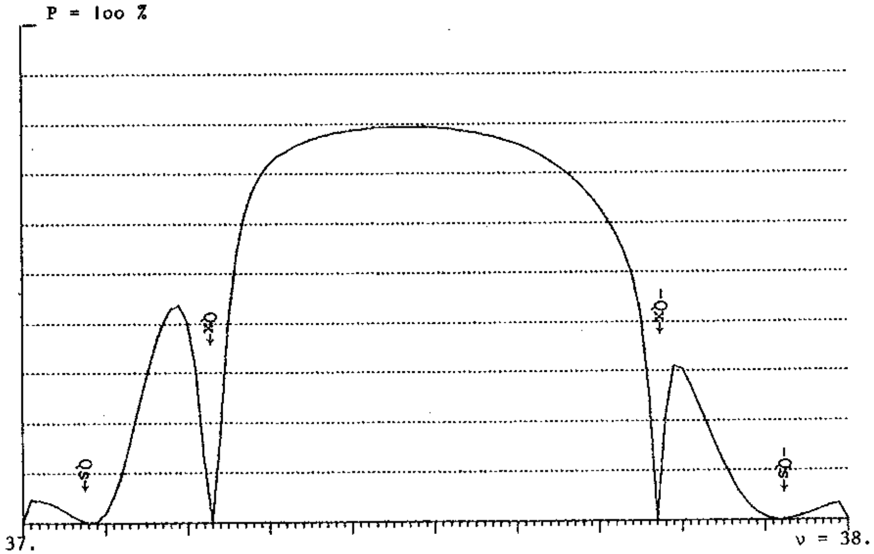

Fig. 4 : Polarisation versus spin tune of the optics of Fig.1 calculated with SLIM.

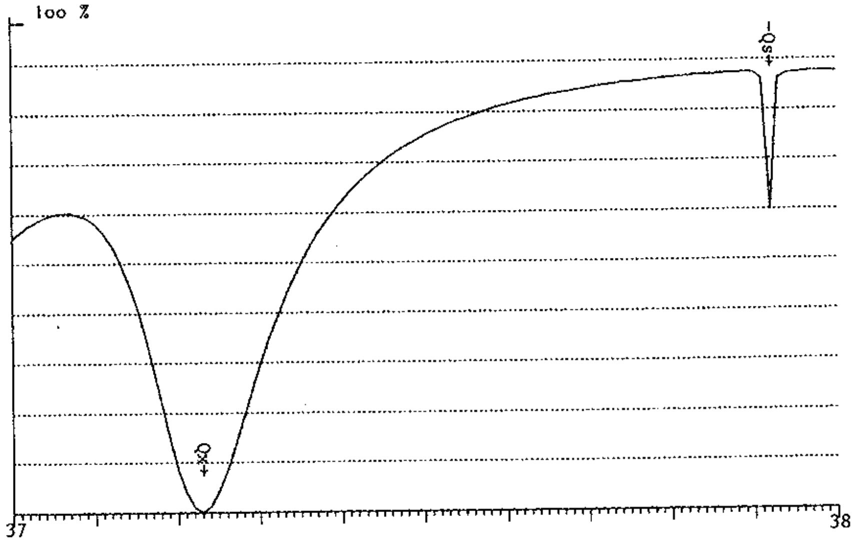

Fig. 6 : Polarisation versus spin tune of the optics of Fig. 3 calculated with SLIM.

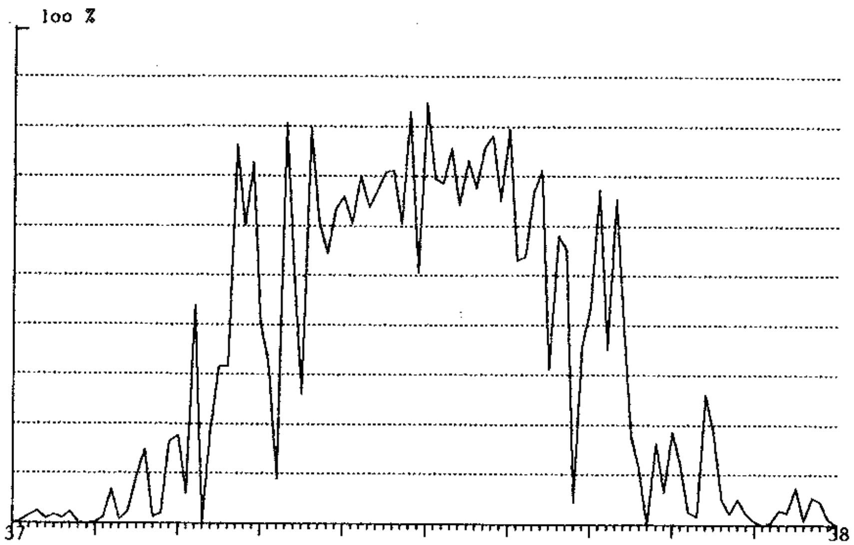

Fig. 7 : Polarisation versus spin tune of an optics with random quadrupole displacement and current errors calculated with SITROS.

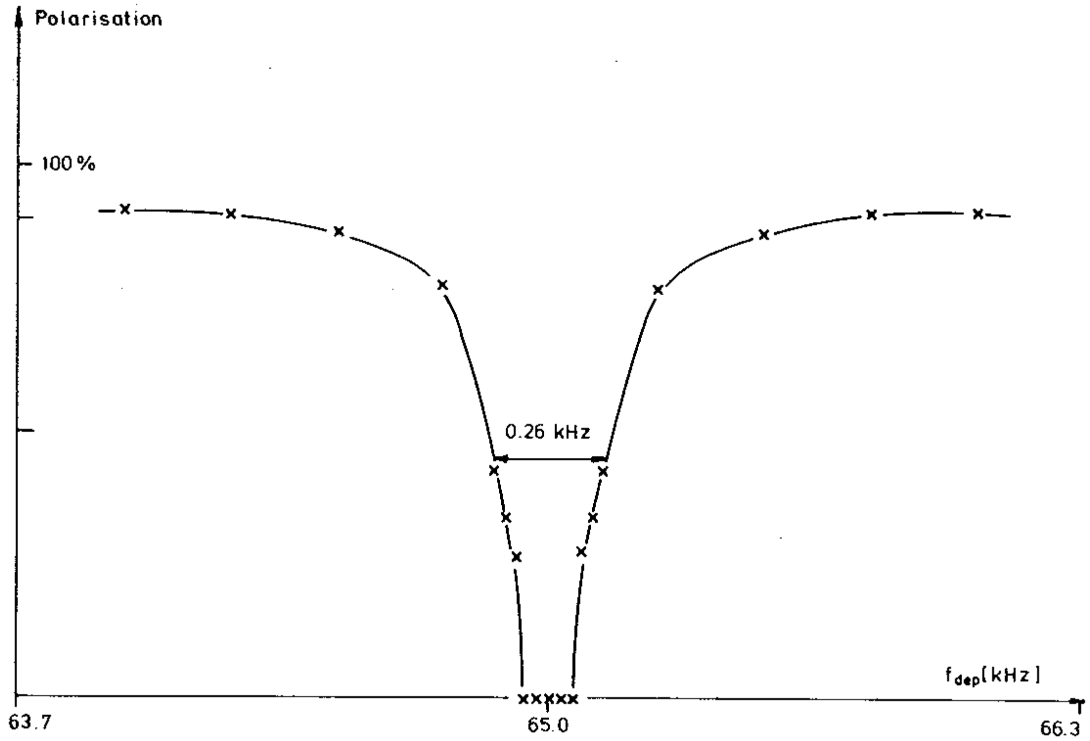

Fig. 8 : Polarisation versus depolariser frequency calculated with SITROS

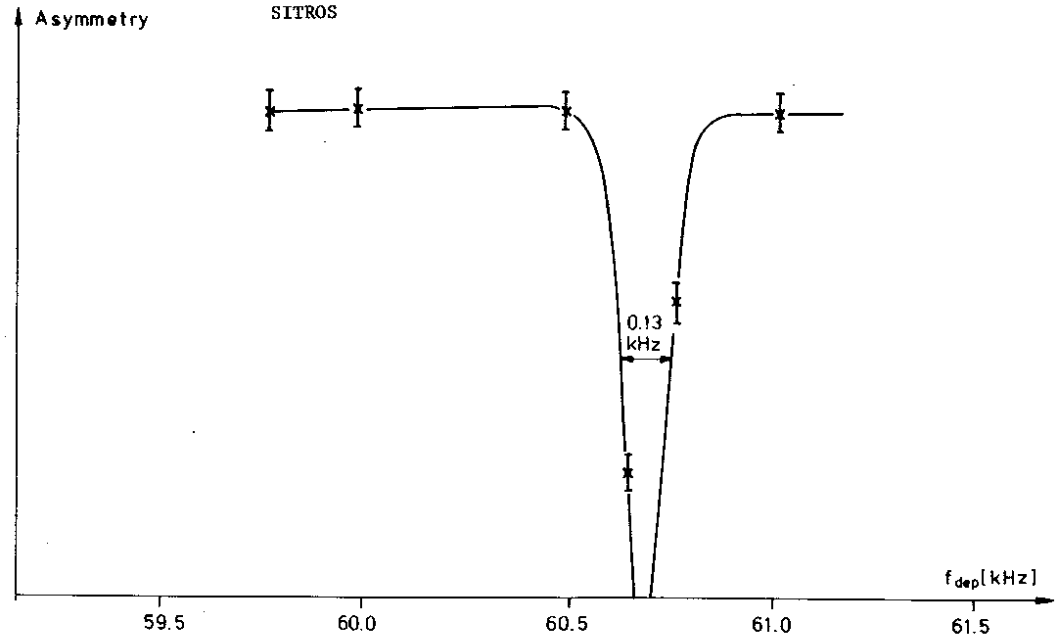

Fig. 9 : Polarisation versus depolariser frequency measured in PETRA