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## Abstract

Siberian Snake schemes can be inserted in ring accelerators for making the spin tune almost independent of energy[1-2]. Two such schemes are here suggested which lend particularly well to practical application over a wide energy range. Being composed of horizontal and vertical bending magnets, the proposed snakes are designed to have a small maximum beam excursion in one plane. By applying in this plane a bending correction that varies with energy, they can be operated at fixed geometry in the other plane where most of the bending occurs, thus avoiding complicated magnet motion or excessively large magnet apertures that would otherwise be needed for large energy variations.
The first of the proposed schemes employs a pair of standard-type Siberian Snakes, i.e. of the usual 1st and 2nd kind which rotate the spin about the longitudinal and the transverse horizontal axis, respectively. The second scheme employs a pair of novel-type snakes which rotate the spin about either one of the horizontal axes that are at 45° to the beam direction. In obvious reference to these axes, they are called left-pointed and right-pointed snakes.

## Introduction

If, in a transverse magnetic field, the direction of motion of a relativistic electron is rotated about the field axis by an angle

$$\varphi = \frac{1}{\gamma}\,\frac{e}{m_0 c}\int B_\perp\,ds \qquad (1)$$

its spin direction is rotated about the same axis by an angle

$$\psi = \gamma a\cdot\varphi = a\,\frac{e}{m_0 c}\int B_\perp\,ds \qquad \text{with} \begin{cases}\gamma a = \dfrac{E}{440.65\ \text{MeV}}\\[2ex] a = \dfrac{g-2}{2} = 1.16\cdot 10^{-3}\end{cases} \qquad (2)$$

Thus the spin rotation depends on the field only, independent of energy, while the deflection of the trajectory varies inversely proportional to energy. In a plane ring accelerator or storage ring, for a particle moving on a closed orbit, there exists an equilibrium spin direction that closes upon itself after one revolution. All other spin directions will precess about this equilibrium spin, and the number of precessions per revolution is called the spin tune $\nu$. Particles with energies differing by $\Delta\gamma$ will move on different closed orbits which, by definition, have the same beam deflection $\varphi = 2\pi$ per revolution and accordingly have spin tunes that differ by $\Delta\nu = a\cdot\Delta\gamma$(eq. 2). The natural beam energy spread $\sigma_E$ therefore gives rise to a spread $\sigma_\nu = a\cdot\sigma_E$ in spin tune, and this tune spread, like the beam energy spread, will increase quadratically with beam energy and will eventually cause depolarization of the stored beam.

This is because there exist certain resonant spin tunes, sideband resonances centered about the integer tune values which, when excited by vertical bends or orbit errors, will cause depolarization.
The linear sidebands are given by

$$\nu = n \pm Q_{s,h,v} \qquad (3)$$

where $Q_{h,v}$ are the horizontal and vertical betatron tune, $Q_s$ the synchrotron tune and n an integer. In order to avoid depolarization, the beam tune spread must not straddle these resonances but must be placed between them.

With increasing beam energy, this becomes more difficult and finally impossible due to the increase in beam spread and the decrease in resonance spacing as measured in units of relative energy (Integer $\nu$-values are spaced at equal intervals of 440 MeV). The depolarization enhancement due to large energy spread, including also the effect of higher order sideband resonances, is treated in several papers[3]. Above a certain critical energy it may become prohibitively difficult to maintain a polarized beam in a given flat storage ring. Now Derbenev and Kondratenko have already in 1977 devised an ingenious scheme[1-2] that drastically reduces the spin tune spread, making $\nu \approx \frac{1}{2}$ for all particle energies in the beam. They use the topological trick of rotating the spin by 180° in two opposite symmetry points of the ring such that the spin components transverse to the equilibrium spin direction close upon themselves after two revolutions, independent of particle energy, which means a precession frequency of $\nu = \frac{1}{2}$. The scheme is explained in Fig. 1 where the "Siberian Snake of the 1st kind" rotates the spin by 180° about the longitudinal axis and the (oppositely located) "Siberian Snake of the 2nd kind" rotates the spin by 180° about the transverse horizontal axis. Fig. 1a shows the equilibrium spin direction which is parallel to the bending field in one arc of the ring and anti-parallel to the field in the other. For this reason, there will be no radiative polarization in a symmetric electron storage ring equipped with a "Double Siberian Snake Scheme", but polarization can be reestablished by installing, for example, shorter and stronger bending magnets ("kink magnets") in one of the arcs.

The design of the two types of Siberian Snakes is a formidable problem whenever they are required to work in a large energy range, as e.g. in proton accelerators or $e^+e^-$ storage rings. In practice, such a snake will be an alternating sequence of horizontal and vertical bending magnets, and various such schemes have been proposed that restore the original beam line and can thus, in principle, be inserted into the straight section of an accelerator ring. However, the total amount of bending in the snake should be small, especially in electron rings, and the maximum excursion from the straight line should be small, also. The reason for this is the following: In order to procure the correct spin rotation (Eq. 2), the bending fields in the snake magnets must be kept fixed when changing the energy of the ring. Then, according to Eq. 1, the bending angles will change and the beam <u>geometry</u> in the snake will <u>vary</u>. Large energy variations will mean large variations in beam position which, in turn, require large magnet apertures or a controlled variation of magnet positions.



In the light of the criteria stated above, not many good snakes were known so far. Among them the probably most attractive snake of the 1st kind is shown as an example at the bottom of Fig. 2, where the angles given in braces indicate the spin rotations in the horizontal and vertical plane, respectively[4]. It is not difficult to follow up the motion of each of the 3 spin components with a pencil indicating the direction and find that rotation is indeed 180° about the longitudinal axis. A similarly sleek and simple snake of the 2nd kind was not known; the one shown at the top of Fig. 2 requires a rather elaborate mounting on a pair of rotating girders[5].

The snakes to be proposed in this note follow a different principle for energy variation: The <u>geometry</u> is kept <u>fixed</u> in one coordinate (e.g. the horizontal) and the resulting error in spin rotation is then corrected by a controlled, energy-dependent variation of geometry in the other coordinate (e.g. the vertical). This has the practical advantage that, when changing the beam energy, the beam will change its position only in one coordinate, and even much less than in the previous case, such that magnet positions can be fixed and apertures can be made rather small.
In addition, one of the snake pairs given below differs from previous schemes also in a more fundamental respect: the two snakes do not rotate the spin about the longitudinal and transverse horizontal axis, respectively, but about two orthogonal axes that are at 45° with respect to these. It turns out that they have the same effect on the spin tune, making it $\nu = \frac{1}{2}$ independent of particle energy.

In the snake magnets, the spin generally is not parallel to the field direction. In an electron ring, this will reduce the maximum degree of polarization. Also, any snake composed of transverse magnetic fields contains vertical bending which, in an electron ring, will lead to radiative depolarization if the optic is not carefully adjusted to cancel this effect. The depolarization and the required optical "spin matching" are discussed in the last paragraph.

The pair of standard-type snakes,

i.e. of the 1st and 2nd kind respectively, is shown in Figs. 3 and 4 where the sequence and magnitude of horizontal and vertical spin rotations is indicated. This sequence is the same in both kinds of snakes, except for some outer horizontal bends. Each snake resembles a pair of HERA spin rotators with inverted symmetry. Both snakes have the same vertical geometry with small maximum vertical beam excursion. The horizontal excursion is larger, but the snakes can be operated with a correction in vertical bending that makes it possible to work at fixed horizontal bending geometry (i.e. at variable horizontal spin rotation) in an energy range of, say, ± 40 % centered about the design energy.

Thus assuming that the horizontal bends are ramped with energy (fixed geometry), their spin rotation varies proportional to $(1+\delta)$ where $\delta = \frac{\Delta p}{p}$ is the relative deviation from the design momentum of the snake. Due to the anti-symmetry of horizontal bending, the variation of horizontal spin rotation with $\delta$ will drop out to first order, and if the vertical spin rotation is kept constant by working there at fixed field, i.e. variable vertical geometry, the snake will give the correct overall spin rotation in a $\delta$-range of plusminus a few percent.

For larger $\delta$-values, this will not properly work any more without a correction of vertical bending, as can be seen by following up the projection of the spin motion onto the vertical (z, s)-plane. When entering the snake the spin is vertical, and the first vertical magnet rotates it backward by 45°. The first horizontal magnets, then, rotates it about the vertical axis by an angle $(1+\delta)\cdot 180°$, and the projection of the spin onto the (z, s)-plane will then form an angle $\varphi$ with the s-direction given by

$$\tan\varphi = \frac{1}{\cos\alpha} \quad \text{with } \alpha = \delta \cdot 180°$$

For larger values of $\delta$, $\cos\alpha$ will be appreciably smaller than unity and the projection angle $\varphi$ will be appreciably larger than 45°. Since the following second vertical magnet then rotates the spin forward by exactly 45°, the spin, at the symmetry point, will not exactly end up in the horizontal symmetry plane and, after traversing the second half of the snake, will not coincide with the vertical axis as required.



This suggest for off-energy operation to make all vertical bends slightly stronger such that, at the symmetry point, the spin will be in the horizontal symmetry plane. In order to meet this condition, the second vertical magnet will have to rotate the spin by an angle $\varphi$ with

$$\tan\varphi = \tan(90° - \psi) \cdot \frac{1}{\cos\alpha} \quad ; \quad \alpha = \delta \cdot 180°$$

if $\psi$ is the spin rotation in the first vertical magnet. Demanding both vertical rotations to be of equal size, we have

$$\tan\psi = \tan(90° - \psi) \cdot \frac{1}{\cos\alpha} \qquad \text{or}$$

$$\tan^2\psi = \frac{1}{\cos(\delta \cdot 180°)} \tag{4}$$

Writing $\psi = (1+\varepsilon)\cdot 45°$, we find the vertical magnet excitation factor $(1+\varepsilon)$ as a function of the relative energy deviation $\delta$; it is shown as a dashed curve in Fig. 5.

The vertical excursion h in the center of the snake depends on energy. Without the correction, it would vary as $(1+\delta)^{-1}$. With the correction, it is also proportional to vertical magnet excitation $(1+\varepsilon)$; thus

$$\frac{h}{h_o} = \frac{1+\varepsilon}{1+\delta}$$

This relative beam excursion is shown in Fig. 5 as a solid curve. In a range of $-9\,\% < \delta < +47\,\%$ it varies only by ± 11 %, and if the vertical beam aperture is made big enough to accomodate this rather small variation in vertical beam position, the snakes can be operated in this large energy range without any change in magnet geometry. The range can be extended down to $\delta$-values of about − 40 % by a rather moderate controlled vertical motion of horizontal bending magnet plus vacuum chamber, as indicated in Fig. 3.

As an unwanted side effect, the corrected snake pair, when operated at an energy offset $\bar{\delta}$, produces a spin tune shift

$$\Delta\nu = \pm\frac{1}{2}\delta \qquad \text{per revolution,}$$

where the sign depends on the relative polarity of bending in the snakes and the accelerator ring. This means that the spin tune of the ring including the corrected snake pair does not exactly remain $\nu = 0.5$ but, for a $\delta$-variation between ± 40 %-limits, will linearly shift from $\nu = 0.3$ to $\nu = 0.7$. The tune shift $|\Delta\nu|$ is indicated as a dotted curve in Fig. 5.

Example for standard-type snake scheme

In LEP, for example, the snakes could be given a design energy of 60 GeV. Then, between 55 GeV and 88 GeV they could be operated with fixed magnet geometry, and between ~ 36 GeV and 55 GeV they could be operated with some controlled vertical motion of the horizontal bending magnets.

In polarized proton acceleration one might hope that, at some intermediate energy, the snakes can be turned on adiabatically without loss of polarization, but this complex question calls for separate investigation.

Summary of correction principle

The proposed correction may be viewed as a trick for exactly freezing the horizontal snake geometry, where most of the bending occurs, on account of an added variation in vertical snake geometry. This vertical variation turns out to be quite beneficial at positive energy deviations where it counteracts the usual geometry variation and thus keeps the vertical beam position rather constant, while at negative energy deviations it enhances the usual geometry variation and thus, from a certain point on, necessitates a controlled vertical motion of the horizontal bending magnets.

The pair of novel-type snakes

is shown in Fig. 6, giving the sequence and magnitude of horizontal and vertical spin rotations. Both kinds of snakes are identical, except for an inverted polarity in horizontal bending. They are very compact, and the maximum horizontal beam excursion is very small, smaller that the vertical excursion in the standard-type snakes of Figs. 3 and 4.

The novel feature of these snakes is the fact that they don't rotate the spin about the same axes as the standard-type snakes of the 1st and 2nd kind. Their spin rotation, instead, is about either one of the two horizontal axes that point at 45° to the beam direction, either to the left or to the right. Correspondingly, we call these snakes left-pointed and right-pointed, respectively. When inserting one of each type in opposite symmetry points of the ring, such a snake pair again makes the spin tune $\nu = \frac{1}{2}$ indepedent of particle energy.



These snakes have their maximum beam excursion in vertical direction, where most of the bending occurs, and it would therefore be desirable to operate them at fixed vertical geometry, i.e. at variable vertical spin rotation. It turns out that this is possible by applying a correction scheme quite similar to the one described above for the standard-type snakes. Here, for operation off the design energy, the field in all horizontal bending magnets must be increased by a factor $(1+\varepsilon)$ which is shown as a function of the relative energy deviation $\delta$ in Fig. 5 and numerically agrees with the corresponding factor in the previous scheme. Naturally, then also the variation of relative maximum beam excursion $h/h_0$ is the same as before.

What is not quite the same in this scheme, however, is the spin tune shift $\Delta\nu$ induced by the correction. It is larger than before, especially for positive $\delta$, and reaches the value $|\Delta\nu| = 0.5$ at $\delta = \frac{1}{3}$. What here happens in the corrected snake pair can be viewed as follows: At $\delta = 0$, the rotation axes of the two snakes are orthogonal to one another. With increasing $\delta$, they rotate in the horizontal plane each by an angle $\Delta\nu \cdot 90°$ in opposite direction and at $\delta = \frac{1}{3}$ they coincide, having each rotated by 45°. The machine then contains two identically acting snakes of the 2nd kind and is on an integer spin resonance. For negative $\delta$, the rotation of the axes and the tune shift $|\Delta\nu|$ is smaller, as indicated as a dotted line in Fig. 7. Thus in the range $-40\ \% < \delta < +25\ \%$ we have $|\Delta\nu| < \frac{1}{3}$, i.e. the spin tune shifts between the extreme values of $\nu = 0.17$ and $\nu = 0.83$ which may be acceptable.

Effect of snakes on the polarization in an electron ring

The maximum degree of polarization in an electron ring is given by

$$P = .924 \frac{\oint \frac{1}{|\rho|^3}(\vec{h}\,\vec{n})ds}{\oint \frac{1}{|\rho|^3}(1 + \frac{11}{18}\vec{d}^2(s))ds}$$

where $\vec{n}$ is the equilibrium spin direction and $\vec{h}$ the direction of the magnetic field of strength $\frac{1}{\rho}$. $\vec{d}(s)$ is the "spin orbit coupling vector" which, as a function of the azimut s in the ring, is determined by the linear optics. It consists of 3 additive terms which, at a given point, represent the coupling of the spin to the synchrotron motion and to horizontal and vertical betatron motion, respectively[6-7].

When ignoring this coupling for a moment, it appears from Eq. 5 that all magnets with a scalar product $(\vec{n}\vec{n}) < 1$ will reduce the maximum polarization. If, for a given snake design, the reduction is too large, it can be made smaller by reducing the relative weight of the snake magnets in the ring by making them weaker and longer.

The role of the "spin orbit coupling vector" may, in principle, be seen as follows: If, in any particular bending magnet of the ring, the electron emits a synchrotron radiation quantum, its sudden energy change will excite a synchrotron oscillation of a certain amplitude and phase, and also a horizontal/vertical betatron oscillation if the horizontal/vertical dispersion does not vanish in that magnet. Now, if the "spin orbit coupling vector" is not zero in that magnet, the sudden change in oscillation amplitude will in turn cause a change in spin motion and polarization, and repeated radiation will cause a "spin diffusion" leading to depolarization. The vector $\vec{d}$ contains a resonant factor, and the radiative depolarization will be strongest for spin tunes near the sideband resonances that correspond to the oscillation excited by radiation.

In a flat machine without field errors, the "spin orbit coupling vector" is zero everywhere. However, this is not so any more when introducing snakes with vertical bends into the ring, which make the equilibrium spin direction deviate from the vertical and generate vertical dispersion. In order to make the vector $\vec{d}$ vanish everywhere in that case, the quadrupole focusing strengths in the ring must be adjusted to meet certain "spin matching conditions" in addition to the usual optical matching conditions. It has been shown by examples that this is possible in practical cases[8-9].



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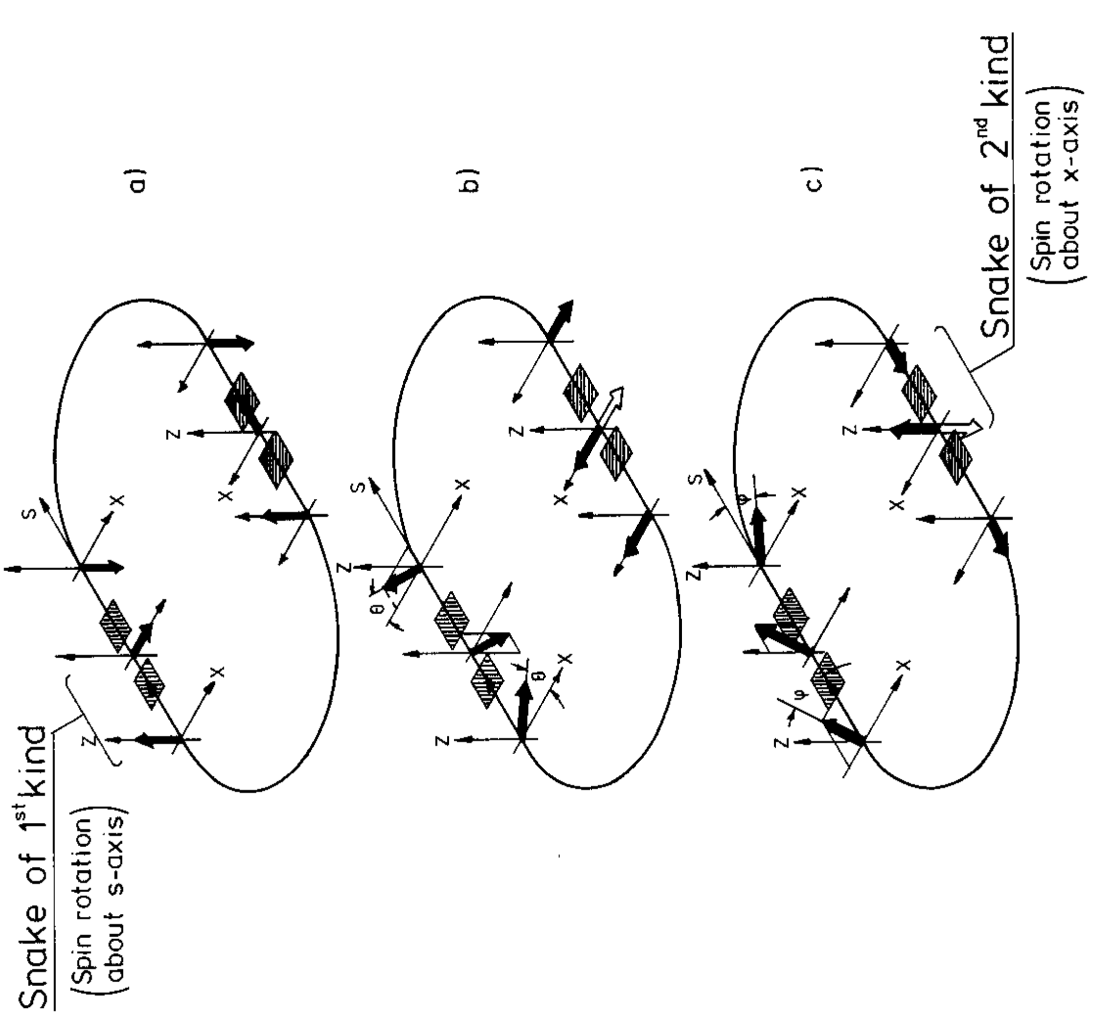


Fig. 1 **Ring with Double Siberian Snake Scheme**

Spin component a) closes after one revolution

Spin components b), c) close after two revolutions

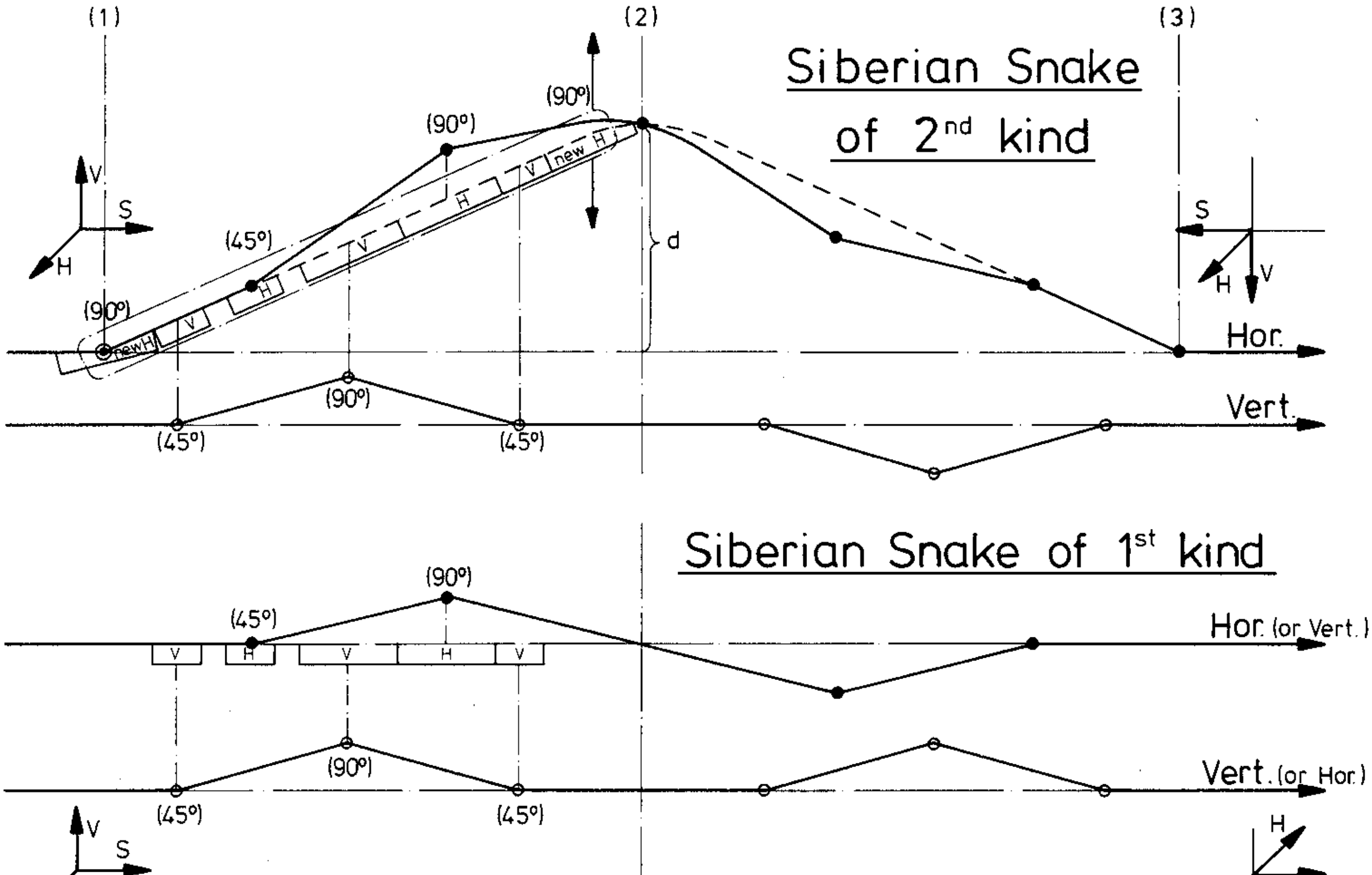


Fig. 2 "Old" Siberian Snakes of 1st and 2nd kind (spin rotation in braces)

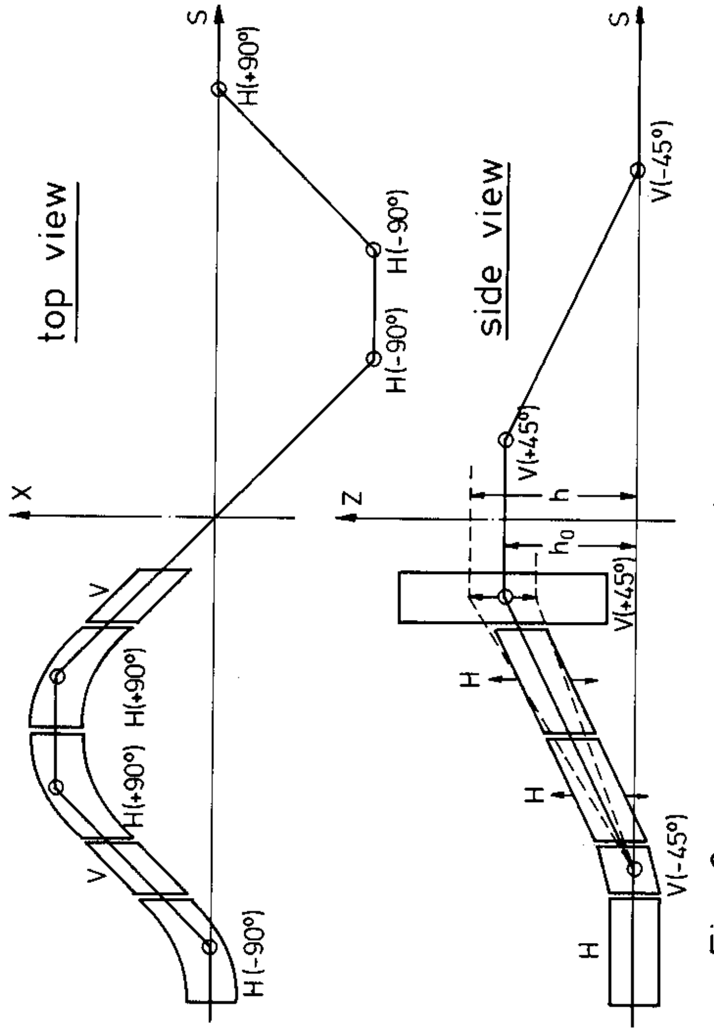


Fig. 3 Siberian Snake of 1st kind (spin rotation in braces)

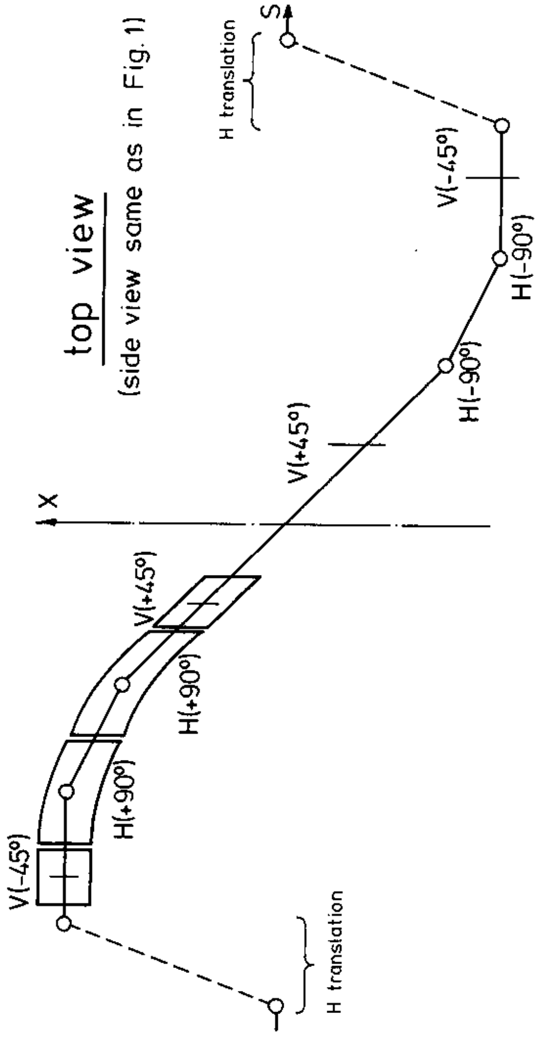


Fig. 4 Siberian Snake of 2nd kind (spin rotation in braces)

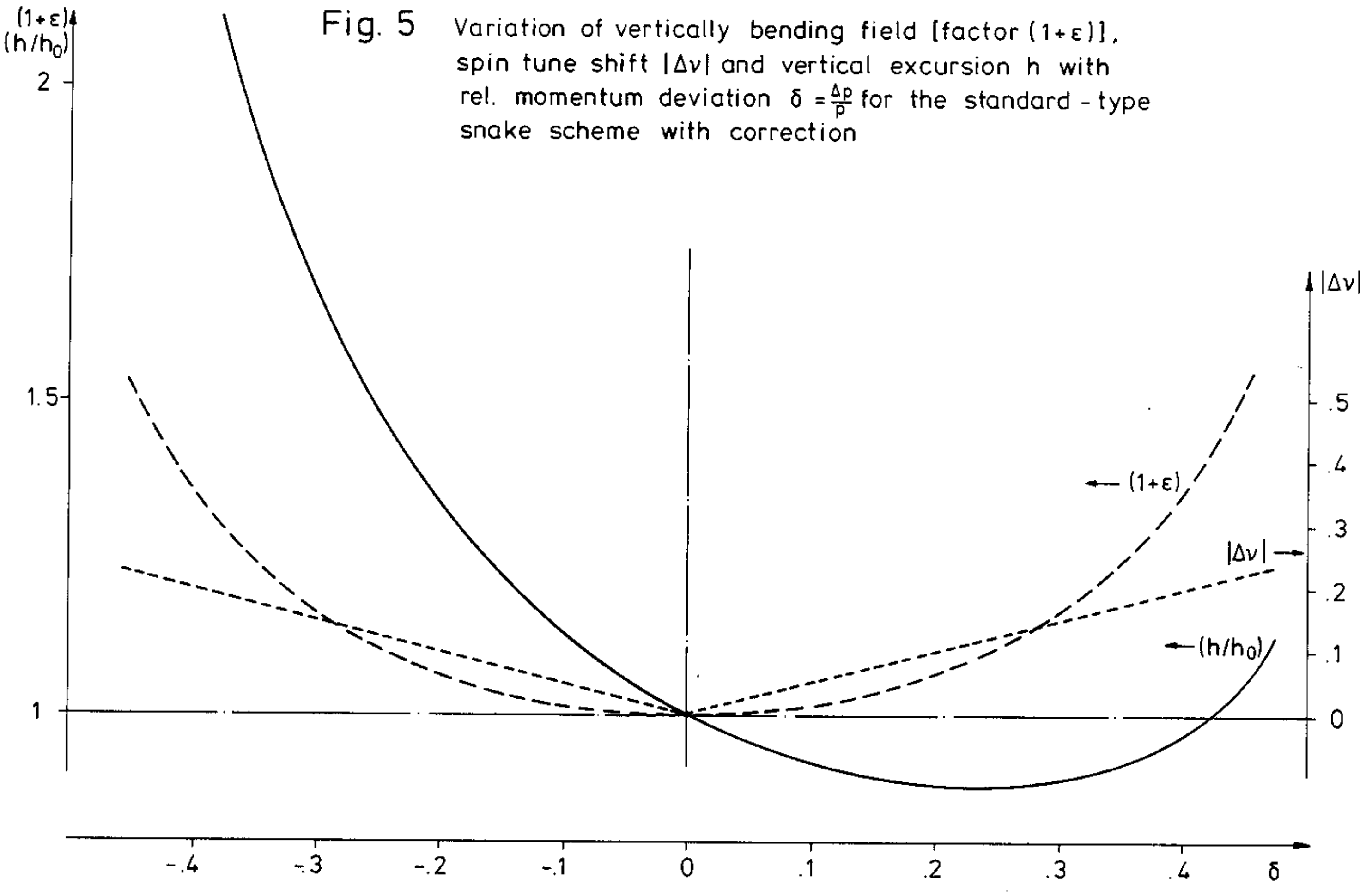


Fig. 5 Variation of vertically bending field [factor $(1+\varepsilon)$], spin tune shift $|\Delta\nu|$ and vertical excursion h with rel. momentum deviation $\delta = \frac{\Delta p}{p}$ for the standard-type snake scheme with correction

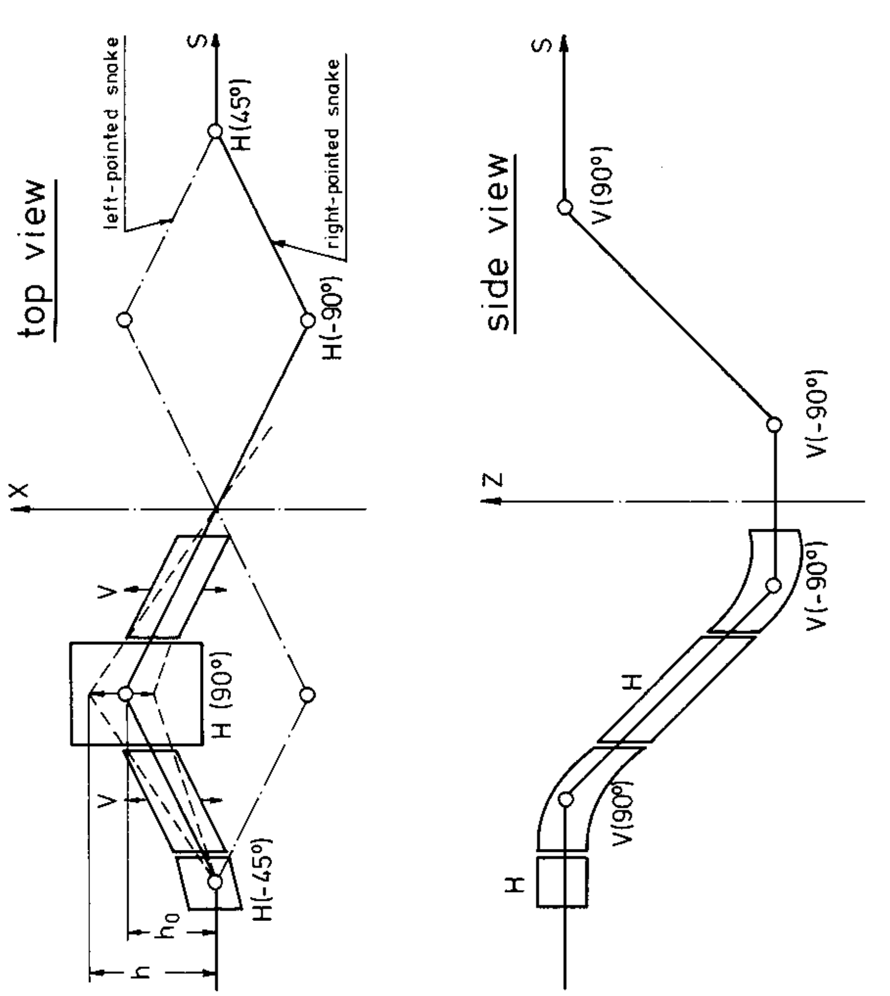


Fig. 6 Left- and right- pointed snake (spin rotation in braces)

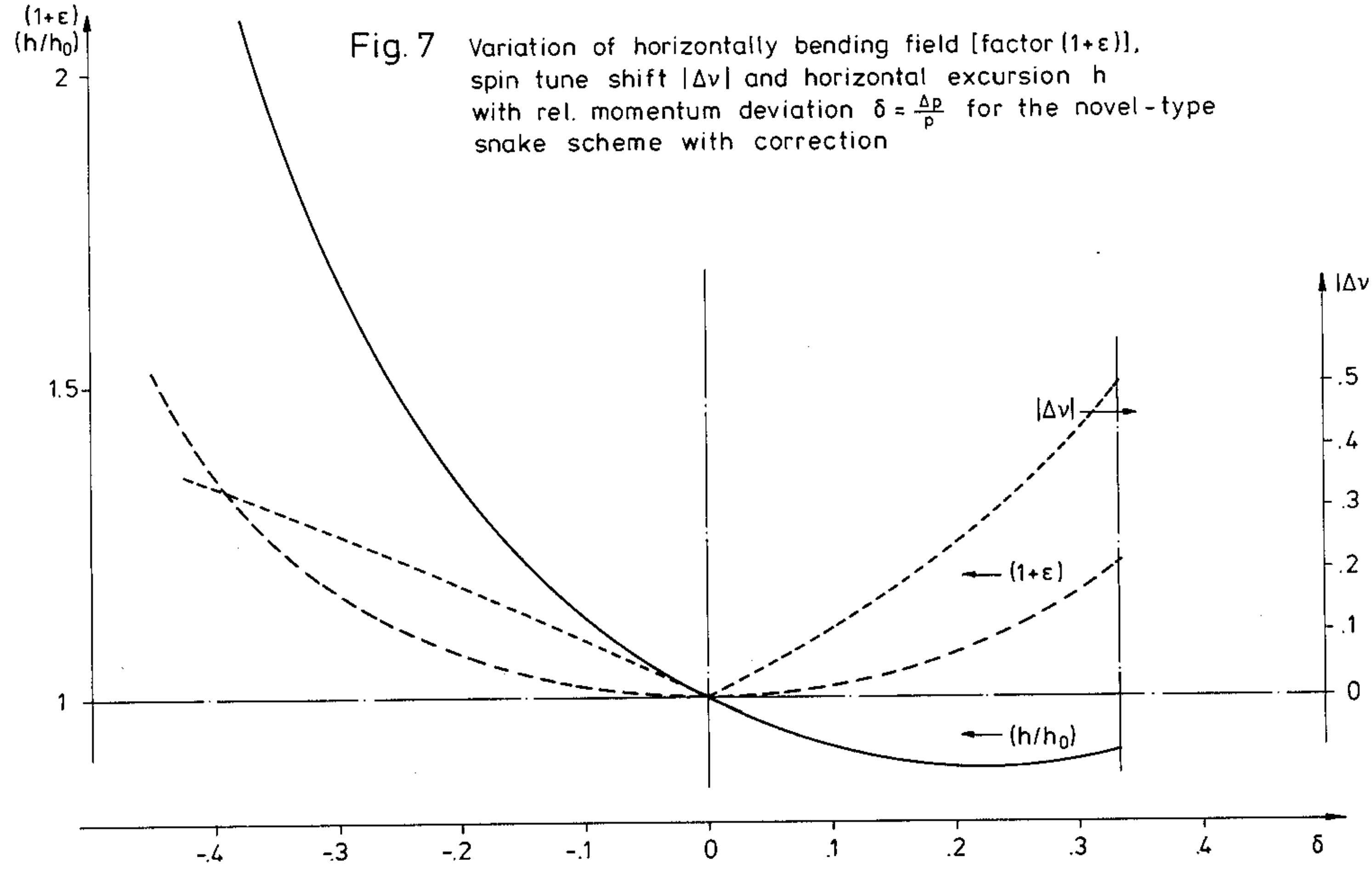


Fig. 7 Variation of horizontally bending field [factor $(1+\varepsilon)$], spin tune shift $|\Delta\nu|$ and horizontal excursion $h$ with rel. momentum deviation $\delta = \frac{\Delta p}{p}$ for the novel-type snake scheme with correction