



THE COMPOSITE MODEL WITH FIVE GENERATIONS  
OF MASSLESS LEPTONS AND QUARKS

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The Composite Model with Five Generations  
of Massless Leptons and Quarks

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Abstract

A new composite model of leptons and quarks is proposed. The preons in the antisymmetric 15-dim. representation of the hypercolor group  $SU(6)_{HC}$  are simply assigned to the 4 and  $\bar{4}$  representations of the Pati-Salam group  $SU(4)_c$ . The five generations of massless leptons and quarks with no exotics are obtained as the most simple solution to the 't Hooft anomaly-matching, decoupling and n-independence conditions for the unbroken subgroup  $SU(4)_L \times SU(4)_R \times U(1) \times U(1) \times U(1)$  of the original entire global chiral symmetry  $SU(8)_L \times SU(8)_R \times U(1)$ . The symmetry of the effective Lagrangian at the composite level can be identified as the partially grand unification group  $SU(4)_c \times SU(2)_L \times SU(2)_R$ . The color number 4 and the family number 5 are the natural results of the assignment of the preon hypercolor representation.

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1. Introduction

The possible compositeness of leptons and quarks has been investigated recently and many different kinds of models have been proposed (1). The earlier models which are simply based on the quantum number counting are so "naïve" that they could not answer anything about the binding mechanism, and particularly, why the composite fermions can be so light in comparison to their binding scale (2). It is 't Hooft who suggested a very attractive idea that the quarks and leptons can be considered as the hypercolor singlet composites of 3 spin-1/2 preons, and that unbroken chiral symmetries of the fundamental Lagrangian of the preon system in essence can protect the composite fermions from acquiring a mass of the order of  $\Lambda_{HC}$  ( $\Lambda_{HC}$  is the energy scale of the hypercolor binding force) (3). It is also proposed that the currents of unbroken chiral symmetries must satisfy so-called the 't Hooft consistency conditions, namely, the anomaly matching, decoupling and n-independence conditions (3).

The 't Hooft consistency conditions have put very stringent constraints on the constructions of realistic composite models of leptons and quarks. The more severe restrictions also come from phenomenological requirements. These constraints make many of the models suggested so far quite unrealistic. They either could not satisfy all of the 't Hooft consistency conditions or could not reproduce the observed "minimal" 3 generations of massless leptons and quarks. Some models predict a lot of massless exotic leptons and quarks which can be eliminated only quite artificially and so on.

In this paper we would like to suggest a new composite model of leptons and quarks in which the preons are simply assigned into the antisymmetric 15-dim. representation of the hypercolor group  $SU(6)_{HC}$  with multiplicity

8 and the following quantum numbers:

	$SU(6)_{HC}$	$SU(4)_c$	$SU(3)_c$	B-L	T+V	T-V	Q
$T^i$	$\square$	$\square$	$\square$	1/3	1/3	1	1/3
$T^0$	$\square$	$\square$	$\square$	1	-1	1/3	-1/3
$V_i$	$\square$	$\square$	$\square$	-1/3	1/3	-1	0
$V_0$	$\square$	$\square$	$\square$	1	1	1/3	-1
							2/3

where preons  $T^i, V_i$  are something like the original rishons in the Harari-Seiberg model (14), but two more preons  $T^0$  and  $V_0$  are added to the present model. ( $T^i, T^0$ ) and ( $V_i, V_0$ ) now construct the 4 and  $\bar{4}$  representations of the Pati-Salam  $SU(4)_c$  group (4) with B-L as fourth color (5). T+V is the total preon number and T-V is the difference between the T preon number and the V preon number. The five generations of massless leptons and quarks without any exotics can be obtained in this model as the most simple solution to the 't Hooft consistency condition for the unbroken subgroup  $SU(4)_L \times SU(4)_R \times U(1)_{T+V} \times U(1)_{T-V}$  of the original entire chiral symmetries  $SU(8)_L \times SU(8)_R \times U(1)_{T+V}$  with the structures

$$\begin{aligned} \sum_e T^a V_e T^i &= U^i \\ \sum_e T^a V_e V_i &= \bar{d}_i \end{aligned} \quad \begin{aligned} \sum_e T^a V_e T^0 &= \nu \\ \sum_e T^a V_e V_0 &= e^+ \end{aligned} \quad (1)$$

where a  $\mathbf{3}(i,0)$  is the  $SU(4)_c$  color index. At the composite level the model has  $SU(4)_c \times SU(2)_L \times SU(2)_R$  symmetries which can be identified as a partially grand-unification group (5). An interesting feature of this model is that the color number 4 naturally comes from the hypercolor properties of the preons. In fact it is the maximal number allowed by the asymptotic freedom condition for the hypercolor group  $SU(6)_{HC}$ .

The paper is organized as follows. In section 2 the selection of unbroken chiral symmetries is discussed. The emphases are put on the phenomenological requirements of the physics. In section 3 the solutions to the 't Hooft consistency conditions for the selected unbroken chiral symmetries are presented. The details of the model construction are given in section 4. Finally in section 5 we summarize the main features of the model and discuss them.

## 2. Selection of unbroken chiral symmetries

The crucial point in the construction of a composite model of leptons and quarks is to have unbroken chiral symmetries. It is the unbroken chiral symmetries which in essence protect the leptons and quarks from acquiring a mass of the order of  $\Lambda_{HC}$ . And the solutions to the 't Hooft consistency conditions for these unbroken chiral symmetries provide the possible candidates of the observed massless leptons and quarks. Furthermore some parts of the unbroken chiral symmetries later have to be gauged and physically identified as the symmetry group of the strong, electromagnetic and weak interactions. In some sense all important physics is related to the selection of unbroken chiral symmetries.

The possibilities of the original entire chiral symmetries of the fundamental Lagrangian of the preon system as unbroken chiral symmetries have been discussed in the vector-like hypercolor theories by 't Hooft (3) and in the left-right asymmetric (chiral) theories by many others (6). Although several solutions to the 't Hooft consistency conditions have been found in the left-right asymmetric (chiral) theories no massless composite fermion solutions exists in the vector-like hypercolor theories.

therefore some arbitrariness is unavoidable. At the same time Georgi's survival assumption (11) must be applied to the case that the fundamental spectator fermions and the composite fermions "mate" to form a real representation in order to get realistic spectrum of leptons and quarks.

Of course, we can only gauge some parts of the entire global chiral symmetries in such a way that the preon representations themselves are free from the anomalies and over them the electric charge and color SU(3) charges are vector-like (12). In this case, the entire global chiral symmetries must be large enough, for example SU(16), and the preons in fact have exactly the same quantum numbers as those of one generation of leptons and quarks. Therefore the "flavor" problem is not solved.

One way out of this problem is to suppose that the breakdown of the entire global chiral symmetries leaves an unbroken subgroup which is directly selected as the symmetry group of the strong, electromagnetic and weak interactions. Furthermore, the preon representations are assumed to decompose in a vector-like way under the given symmetry breaking (8), that is

$$24 \longrightarrow 4 + \bar{4}$$

It is clear that in this case it is easy to satisfy the anomaly-free condition and to guarantee the electric charge and color SU(3) charges to be vector-like. Since the cancellation of the anomalies in the left-right symmetric electroweak theories are more natural (13), the best candidate for the symmetry group of the strong, electromagnetic and weak interactions in this program therefore is selected as the  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  or its partially grand unification group  $SU(4)_c \times SU(2)_L \times SU(2)_R$  (5). This

However, at present there is no completely reliable method for dealing with very strong gauge couplings and we don't know any sufficient condition for a given chiral symmetry to remain exact. We also don't know how a given chiral symmetry breaks and which subgroup remains unbroken. Thus, the selection of unbroken chiral symmetries only can be considered as a working assumption (of course, the 't Hooft anomaly-matching condition which is necessary condition for an unbroken chiral symmetry has to be satisfied). Especially for the purpose of constructing a realistic composite model it would not be necessary to select the original entire global chiral symmetries as unbroken chiral symmetries (7,8), and the best method is to select the unbroken chiral symmetries from phenomenological requirements.

One important point in this respect is that the resulting theories must be free from the anomalies for the selected unbroken chiral symmetry (if it is gauged) and have vector-like electric charge and color SU(3) charges (9). Unfortunately, the entire global chiral symmetries of the fundamental Lagrangian of the preon system are generally unitary symmetries, and the preons are in their fundamental representation. Thus, at the preon level (also at the composite level if the 't Hooft consistency conditions are satisfied) the resulting theories are not automatically free from anomalies of the currents of the entire global chiral symmetries. The electric charge and the color SU(3) charges are also not automatically vector-like over the preon representations and over the composite representations.

Although by introducing the spectator fermions the entire global chiral symmetries can be gauged without the above problems (3). However, there is no conceivable method to fix the numbers of the spectator fermions (10),

selection of unbroken chiral symmetries is a "minimal" selection in the sense that any unbroken chiral symmetry has to "minimally" contain the symmetry group of the strong, electromagnetic and weak interactions as a subgroup.

As pointed out by Harari and Seiberg, an accidental global  $SU(2)_L \times SU(2)_R$  symmetry can arise in the effective Lagrangian of the massless composites as a result of the proper choice of the preon contents even there is only a global  $U(1)$  symmetry in the fundamental Lagrangian of the preon system (14). Therefore we are led to consider a preon system with the following preon contents (all preons are assumed the left-handed fermions without loss the generality)

preons representation of the hypercolor group  $SU(k)_{HC}$  multiplicity

$P_1$	$R$	$2n$
$P_2$	$\bar{R}$	$2n$

where  $R \times R \times R$  must contain a hypercolor singlet in order to make the hypercolor singlet composites of three spin-1/2 preons, and the color number  $k$ , "Flavor" number  $n$  and representation  $R$  have to be proper choosed to satisfy the asymptotic freedom condition for the hypercolor group  $SU(k)_{HC}$ , namely

$$11k - 4nc(R) \geq 0 \tag{2}$$

where  $c(R)$  is so-called index for the given representation  $R$  (15). The entire global chiral symmetry of the fundamental Lagrangian in this preon system is  $SU(2n) \times SU(2n) \times U(1)$ . As discussed before, this entire global

chiral symmetry is assumed to be broken down to the unbroken part  $SU(n) \times SU(n) \times U(1) \times U(1)$  with the preon representation decomposition as

$$2n \longrightarrow n + \bar{n}$$

3. Solutions to the 't Hooft consistency conditions

All possible hypercolor singlet composites of three spin-1/2 preons are given in Table 1 with the corresponding 't Hooft indices. It is convenient to introduce the  $\Sigma$ -indices and the  $\delta$ -indices which are defined as

$$\begin{aligned} \Sigma k &= k + k' \\ \delta k &= k - k' \end{aligned} \tag{3}$$

etc. It is not difficult to write down the anomaly-matching conditions for the unbroken part  $SU(n) \times SU(n) \times U(1)_{T+V} \times U(1)_{T-V}$ . They are 7 homogeneous equations for the  $\delta$ -indices and 3 equations for the  $\Sigma$ -indices. The coefficients of these equations are the anomaly factors  $A(\mathcal{P}_i)$ , the indices  $c(\mathcal{P}_i)$  and the dimensions  $D(\mathcal{P}_i)$  for the given representations  $\mathcal{P}_i$  of  $SU(n) \times SU(n) \times U(1)_{T+V} \times U(1)_{T-V}$  group.

The decoupling conditions can be divided into four sets according to the conserved  $U(1)_{T+V}$  and  $U(1)_{T-V}$ . They are

$$\begin{aligned} a) \quad \delta k_+ &= -\delta k_- = -\delta s_+ = \delta s_- , & \delta k_0 &= 0 , \\ \Sigma k_+ &= \Sigma k_- , \\ \Sigma s_+ &= \Sigma s_- , \\ \Sigma k_0 &= -\Sigma k_- + \Sigma s_- , \end{aligned}$$

b)  $\delta U_+ = -\delta U_- = -\delta u_+ = \delta u_- , \quad \delta U_0 = 0 ,$

$\Sigma U_+ = \Sigma U_- ,$

$\Sigma u_+ = \Sigma u_- ,$

$\Sigma U_0 = -\Sigma U_+ + \Sigma U_- ;$

c)  $\delta \ell_+ = -\delta \ell_- = -\delta m_+ = \delta m_- , \quad \delta \ell_0 = \delta m_0 = 0 ,$

$\Sigma \ell_+ = \Sigma \ell_- = \Sigma m_+ = \Sigma m_- = \frac{1}{2} \Sigma m_0 ,$  no constraint on  $\Sigma \ell_0 ;$

and

d)  $\delta b_+ = -\delta b_- = -\delta a_+ = \delta a_- , \quad \delta b_0 = \delta c_0 = 0 ,$

$\Sigma b_+ = \Sigma b_- = \Sigma a_+ = \Sigma a_- = \frac{1}{2} \Sigma c_0 ,$  no constraint on  $\Sigma b_0 .$

All derivations are quite straight-forward; the important remarks are the following:

- 1) Substituting the decoupling conditions for the  $\delta_-$ -indices into the 7 anomaly-matching equations for the  $\delta$ -indices, it is found that all 7 homogeneous equations are automatically satisfied. That means, the anomaly-matching conditions do not give any new constraint on the  $\delta$ -indices. Especially the left-right symmetric conditions (all  $\delta$ -indices are zero) could not be derived from the anomaly-matching conditions and decoupling conditions.
- 2) The decoupling conditions automatically guarantee the n-independence of the anomaly-matching conditions. That means, the n-independence may not be an independent condition.

- 3) After using the decoupling conditions the anomaly-matching conditions for the  $\Sigma$ -indices can be simplified as

$27 (\Sigma \ell_- - \Sigma U_-) - 9 (\Sigma s_- - \Sigma u_-) - 2 (\Sigma m_0 - \Sigma c_0) + (\Sigma \ell_0 - \Sigma b_0) = 0 \quad (4)$

$9 (\Sigma \ell_+ + \Sigma U_+) - 3 (\Sigma s_+ + \Sigma u_+) - 2 (\Sigma m_0 + \Sigma c_0) + (\Sigma \ell_0 + \Sigma b_0) = \frac{1}{3} D(R) \quad (5)$

It is found that the only factor dependence of the hypercolor property of the preons in the above equations is the dimension D(R) of the preon hypercolor representation R. This is expected. In fact the spectra of the composites (therefore the hyperflavor properties of the composites) in the vector-like hypercolor theories are independent of the assignment of the hypercolor representation R of the preons if  $R \times R \times R$  contains a hypercolor singlet. Therefore the composite side of the 't Hooft anomaly-matching and the decoupling conditions do not have any change when the assignment of the preon hypercolor representation R is changed, and the only change in the preon side of the 't Hooft anomaly-matching conditions is a factor related to the dimension of the preon hypercolor representation.

- 4) The solutions to the 't Hooft consistency conditions are not necessarily symmetric under the exchange between the T preons and V preons. That means,  $\Sigma t = \Sigma U' , \quad \Sigma s = \Sigma u$  etc. are not necessarily true.

- 5) Except for the indices  $\ell_0, b_0, \ell'_0, b'_0$  associated with the fundamental representations of  $SU(n) \times SU(n) \times U(1)_{T+V} \times U(1)_{T-V}$  the non-zero indices for the antisymmetric representations always imply non-zero indices for the symmetric representations via the decoupling conditions. Since the symmetric representations in most of the models lead to the exotic composite fermions, therefore the natural elimination of all exotic composites can be realized by taking all indices to be zero except for the indices  $\ell_0, b_0, \ell'_0, b'_0$ . Moreover, in this case the 't Hooft consistency conditions have the simple

solution

$$2L_0 = 2b_0 = \frac{2}{3}D(R) \quad , \quad \delta L_0 = \delta b_0 = 0 \quad ,$$

or

$$L_0 = L'_0 = b_0 = b'_0 = \frac{D(R)}{3} \quad \text{and} \quad (6)$$

all other indices are zero.

This solution is the left-right symmetric and symmetric under the exchange between the T preons and V preons.

#### 4. Model construction

The solution to the 't Hooft consistency conditions that

$$L_0 = L'_0 = b_0 = b'_0 = \frac{D(R)}{3}$$

and all other indices are zero

has a very important feature that the only non-zero indices are associated with the composite fermions in the fundamental representations of  $SU(n)_L \times SU(n)_R \times U(1)_{T+V} \times U(1)_{T-V}$ . Furthermore, if the vector-like subgroup  $SU(n)$  of  $SU(n)_L \times SU(n)_R$  is gauged, then an accidental global  $SU(2)$  symmetry can arise at the composite fermion level. The overall symmetry of the effective Lagrangian for the massless composite fermions is, therefore,  $SU(n) \times SU(2)_L \times SU(2)_R \times U(1)_{T+V}$ . Now it is clear that a realistic solution can be obtained if we take  $n = 4$  and identify the resulting

$SU(4) \times SU(2)_L \times SU(2)_R$  as the partially grand unification group  $SU(4)_c \times SU(2)_L \times SU(2)_R$  (5), where  $SU(4)_c$  is the Pati-Salam  $SU(4)_c$  group with B-L as the fourth color (4,5). In this case we get the left-handed

"up-(down-) quark" and "up-(down-) lepton" in the representation

$$SU(4)_c \quad SU(2)_L \quad SU(2)_R \quad U(1)_{T-V}$$

$$\square \quad \square \quad \square \quad 1 \quad 1 \quad 1$$

with the 't Hooft index  $L_0$  ( $b_0$ ), and the number of family is  $D(R)/3$ .

The above discussions are complete general if the preons are assigned into such a representation R of the hypercolor group  $SU(k)_{HC}$  that

- 1) R x R x R contains a hypercolor singlet,
- 2) the dimension D(R) of the representation R is the multiple of 3,
- and 3) the asymptotic freedom condition for the hypercolor group  $SU(k)_{HC}$

$$11k - 4n C(R) \geq 0 \quad (\text{with } n \geq 4)$$

is true.

It is found that these conditions are sufficient to fix the assignment of the preon hypercolor representation almost uniquely. The preons have to be assigned into the antisymmetric 15-dim. representation of the hypercolor group  $SU(6)_{HC}$ . As a result of this assignment we get exactly five generations of massless leptons and quarks. The color number  $n = 4$  in this case is also due to this assignment. In fact it is the maximal number allowed by the asymptotic freedom condition for the hypercolor group  $SU(6)_{HC}$  and the anti-symmetric 15 dim. representation. Finally, the complete quantum number set of the preons is easily determined and as is given in section 1.

#### 5. Summary and Discussion

In this paper a new composite model of leptons and quarks is proposed. The preons in the antisymmetric 15-dim. representation of the hypercolor group



$SU(6)_{HC}$  are simply assigned into the 4 and  $\bar{4}$  representations of the Pati-Salam group  $SU(4)_c$ . The five generations of massless leptons and quarks with no exotics can be obtained as the most simple solution to the 't Hooft anomaly-matching, decoupling and n-independence conditions for the unbroken subgroup  $SU(4)_L \times SU(4)_R \times U(1)_{T+V} \times U(1)_{T-V}$  of the original entire global chiral symmetry  $SU(8)_L \times SU(8)_R \times U(1)_{T+V}$  of the fundamental Lagrangian of the preon system. The symmetry of the effective Lagrangian at the composite level can be identified as the partially grand unification group  $SU(4)_c \times SU(2)_L \times SU(2)_R$ . The color number 4 and the family number 5 are the natural results of the assignment of the preon hypercolor representation.

But some problems remain. For example,

- 1) The original entire global chiral symmetry  $SU(2n) \times SU(2n) \times U(1)$  in this paper is assumed to be broken down to the unbroken subgroup  $SU(n) \times SU(n) \times U(1) \times U(1)$ . Can we find a reliable method to deal with this chiral symmetry breaking problem?
- 2) The masses of the five generations of leptons and quarks can be negligible in comparison to the energy scale  $\Lambda_{HC}$ , how can we produce the observed mass pattern of leptons and quarks at low energy?
- 3) What is the meaning of the conserved  $U(1)_{T-V}$ ?

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References and Footnotes

1) For extensive references to previous work see the review by M. Peskin, Cornell preprint CINS 81/516.

2) S.J. Brodsky and S.D. Drell, SLAC preprint SLAC-PUB-2534 (1980).

3) G. 't Hooft, in Recent Developments in Gauge Theory, edited by G. 't Hooft et al. (Plenum Press, New York, 1980) p. 135.

4) J.C. Pati and A. Salam, Phys. Rev. D8, 1240 (1973); Phys. Rev. D10, 275 (1974).

5) R.E. Marshak and R.N. Mohapatra, Phys. Lett. 91B, 222 (1980).

6) S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. B173, 208 (1980); T. Banks, S. Yankielowicz and A. Schwimmer, Phys. Lett. 96B, 67 (1980); I. Bars and S. Yankielowicz, Phys. Lett. 101B, 159 (1981).

7) S. Weinberg, Texas preprint (1981); C.H. Albright, B. Schrempf and F. Schrempf, Phys. Lett. 108B, 291 (1982); Fermilab-Pub 82/14-THY; DESY 82/012.

8) S.F. King, Phys. Lett. 107B, 201 (1981). The author has discussed the 't Hooft anomaly constraints in the rishon model. By assuming the full flavor symmetry  $SU(6)_L \times SU(6)_R$  to be partially broken to  $SU(3)_L \times SU(3)_R$  a simple solution of one generation of massless quarks has been found. But unfortunately the question where are leptons could not be answered with satisfaction.

9) S. Barr and A. Zee, University of Washington Preprint RLO-1388-845 (1981).

10) A. Davidson and J. Sonnenschein, Weizmann preprint WIS-82/42 (Sept. 1982).

11) H. Georgi, Nucl. Phys. B156, 126 (1979).

12) I. Bars, Yale preprint, YTP 82-12.

13) R.E. Marshak, VPI preprint (1981).

14) H. Harari and N. Seiberg, Phys. Lett. 96B, 269 (1981); Phys. Lett. 102B, 263 (1981).

15) R. Slansky, Los Alamos Preprint LA-UR-80-3495.

Table Caption

Table 1. The hyper-color singlet composites with the corresponding 't Hooft indices.

Table 1

Composite fermion	$\frac{1}{2}$ Hoofst index	$SU(m)$	$SU(n)$	$U(1)_{TW}$	$U(1)_{T-V}$	Composite fermion	$\frac{1}{2}$ Hoofst index	$SU(m)$	$SU(n)$	$U(1)_{TW}$	$U(1)_{T-V}$
TTT	$t_+$	$\square\square\square$	1	1	3	$T^*T^*T^*$	$t'_+$	1	$\square\square\square$	-1	-3
	$t_0$	$\square\square$	1	1	3		$t'_0$	1	$\square\square$	-1	-3
	$t_-$	$\square$	1	1	3		$t'_-$	1	$\square$	-1	-3
	$s_+$	$\square$	$\square\square$	1	3		$s'_+$	$\square\square$	$\square$	-1	-3
	$s_-$	$\square$	$\square$	1	3		$s'_-$	$\square$	$\square$	-1	-3
TTV	$m_+$	$\square$	$\square\square$	1	1	$T^*T^*V^*$	$m'_+$	$\square\square$	$\square$	-1	-1
	$m_-$	$\square$	$\square$	1	1		$m'_-$	$\square$	$\square$	-1	-1
	$n_0$	$\square$	$\square\square$	1	1		$n'_0$	$\square\square$	$\square$	-1	-1
	$l_+$	$\square$	1	1	1		$l'_+$	1	$\square\square\square$	-1	-1
	$l_0$	$\square$	1	1	1		$l'_0$	1	$\square$	-1	-1
	$l_-$	$\square$	1	1	1		$l'_-$	1	$\square$	-1	-1

Table 1 (continued)

Composite fermion	$\frac{1}{2}$ Hoofst index	$SU(m)$	$SU(n)$	$U(1)_{TW}$	$U(1)_{T-V}$	Composite fermion	$\frac{1}{2}$ Hoofst index	$SU(m)$	$SU(n)$	$U(1)_{TW}$	$U(1)_{T-V}$
VVV	$v_+$	$\square\square\square$	1	1	-3	$V^*V^*V^*$	$v'_+$	1	$\square\square\square$	-1	3
	$v_0$	$\square\square$	1	1	-3		$v'_0$	1	$\square\square$	-1	3
	$v_-$	$\square$	1	1	-3		$v'_-$	1	$\square$	-1	3
	$u_+$	$\square$	$\square\square$	1	-3		$u'_+$	$\square\square$	$\square$	-1	3
	$u_-$	$\square$	$\square$	1	-3		$u'_-$	$\square$	$\square$	-1	3
VVT	$a'_+$	$\square$	$\square\square$	1	-1	$V^*V^*T^*$	$a_+$	$\square\square$	$\square$	-1	1
	$a'_-$	$\square$	$\square$	1	-1		$a_-$	$\square$	$\square$	-1	1
	$c'_0$	$\square$	$\square\square$	1	-1		$c_0$	$\square\square$	$\square$	-1	1
	$b'_+$	$\square$	1	1	-1		$b_+$	1	$\square\square\square$	-1	1
	$b'_0$	$\square$	1	1	-1		$b_0$	1	$\square$	-1	1
	$b'_-$	$\square$	1	1	-1		$b_-$	1	$\square$	-1	1

