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Abstract

I summarize Monte Carlo results, which were recently obtained using Symanzik improved actions.

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The current lattice calculations are necessary but also exploratory with respect to behaviour of QCD at $\beta \sim 6$ and with respect to possible connections to very large β. Error analysis and precise connections to very large β are still to come.

The numerical solution of the KONDO Problem (see paper by Wilson RMP 47 (1975) 773) sets the standards for

a) Error analysis
b) matching to perturbation theory at small g
c) making predictions that are subsequently confirmed within theoretical error.

(K. Wilson, Cargèse 83)

I. Introduction

Accepting the point of view that present Monte Carlo (MC) studies of euclidean quantum field theories are mainly exploratory, a major challenge becomes to study possible improvements of the present methods. Sources of systematic errors and limitations to numerical simulations are:

a) statistical noise;
b) finite size effects;
c) finite lattice spacing effects.

Wilson /1/ suggested block spin transformations and MC renormalization group (MCRG) techniques. The MCRG has been applied to the 2d O(3) non-linear σ-model by Shenker and Tobochnik /2/, and recently spin-wave techniques were employed /3/. For more details the reader is refered to the original literature.

I will report on MC results, which were obtained using Symanzik /4,5/ improved actions. Symanzik suggested a systematic procedure for constructing lattice actions, which minimize the cutoff dependence (c), and approach more rapidly the continuum limit. There are many possible actions on the lattice which formally converge to the same continuum limit when the lattice spacing $a \to o$. In case of renormalizable interactions, for small lattice spacing, a given action on the lattice is equivalent to a local theory on the continuum with a local effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_0 + a^2 \mathcal{L}_1 + a^4 \mathcal{L}_2 + \ldots$$

$\mathcal{L}_o$ is the ordinary renormalizable continuum Lagrangian in d dimensions, $\mathcal{L}_1$ contains operators of dimension d + 2, etc. ... . Because of "irrelevant" operators the renormalization group equations contain non-universal scaling violating terms. n-point Green-functions calculated on the lattice obey

$$\left\{-a\frac{\partial}{\partial a} + \bar{\beta}(g)\frac{\partial}{\partial g} + n\,\bar{\gamma}(g)\right\} G_n(p_1,\ldots,p_n;\, g, a) = \sum_{m=1}^{\infty} c_m(g) \left(\frac{a^2}{\xi_r^2}\right)^m \ln\left(\frac{a}{\xi_r}\right). \tag{1}$$

Here $\xi_r = \alpha\xi$ is the "relevant range of interaction" /6/, and ξ is the correlation length (inverse mass gap). This means the parameter α should be adjusted, such that the coefficient c_1 is numerically close to 1. Typically for 4d lattice gauge theories is $\alpha \approx 4$.

The suggestion of Symanzik is to choose the lattice action in such a way that the r.h.s. of equation (1) is at most of order

$$O\left(\frac{a^4}{\xi_r^4} \ln\frac{a}{\xi_r}\right). \tag{2}$$

For the simplest case of mass ratios equations (1), (2) imply

$$\frac{m_1}{m_2} = O\left(\frac{a^2}{\xi_r^2}\ln\frac{a}{\xi_r}\right) \to O\left(\frac{a^4}{\xi_r^4}\ln\frac{a}{\xi_r}\right). \tag{3}$$

In other words: <u>Scaling in</u> the <u>general</u> sense of equation (1) is improved. In the limit $g \to o$ the $\bar{\beta}$- and $\bar{\gamma}$-functions approach their universal 2-loop values. For instance

$$\bar{\beta}_{univ}(g) = -\beta_0 g^3 - \beta_1 g^5 + O(g^7)$$

implies for any physical mass

$$m = const \cdot \Lambda_L, \text{ with} \tag{4.a}$$

$$\Lambda_L = a^{-1}(\beta_0 g^2)^{-\beta_1/2\beta_0^2} \exp\left(-\frac{1}{2\beta_0 g^2}\right)\left(1 + O(g^2)\right). \tag{4.b}$$

Equations (4) characterize <u>asymptotic scaling</u>, which is violated by the $O(g^2)$ corrections. There are no theoretical reasons, to expect asymptotic scaling to be improved.

Symanzik improved actions can be calculated in perturbation theory, in $1/N$ expansion or, in principle, also by Monte Carlo checks of scaling ("trial and error"). For the 2d non-linear σ-model and 4d non-abelian gauge theories perturbative calculations were carried out in Ref. /7,5, 8-11/. For the σ-model tree improved (TI) and 1-loop improved (1LI) actions are known, for gauge theories the TI action /8,9/ is known and some of the calculations for the 1LI action were done recently /11/.

In this lecture I report on MC simulations with (perturbatively) Symanzik improved actions. Section II reviews results /12-14/ for the 2d O(3) σ-model, and section III summarizes results /15-19/ as obtained for 4d non-abelian gauge theories.

II. 2d O(3) Non-Linear σ-Model

For this model the TI action (TIA) was calculated by Martinelli et al. /7/ and the 1LI action (1LIA) was calculated by Symanzik /5/. For the 1LIA the relevant numerical integrals are given in Ref. /12/.

MC calculations were done with the TIA /13,14/ and with the 1LIA /12,14/. The results are summarized in the following.

1.) Mass gap: Results for all three actions as obtained in Ref. /14/ are given in Figure 1. The MC data for the TIA are well consistent with those of Ref. /13/

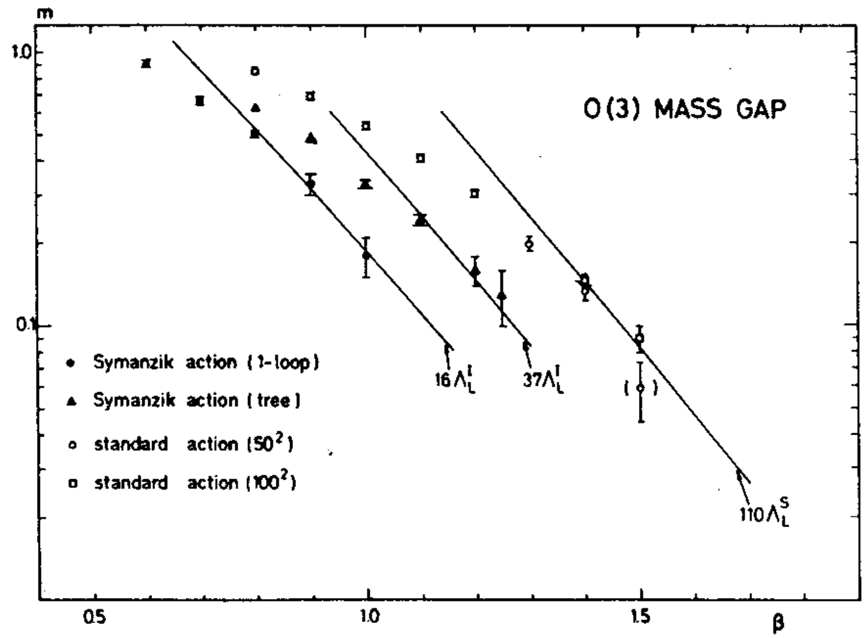

Fig. 1

and the MC data for the 1LIA are an extension of those of Ref. /12/. For the 1LIA and the TIA we find - on a 50^2 lattice - asymptotic scaling windows, whereas (on this lattice) there is no scaling signal with the SA. This becomes even more obvious from the plot of the corresponding defects in Fig. 2. On larger lattices the situation for the SA is unclear, as has

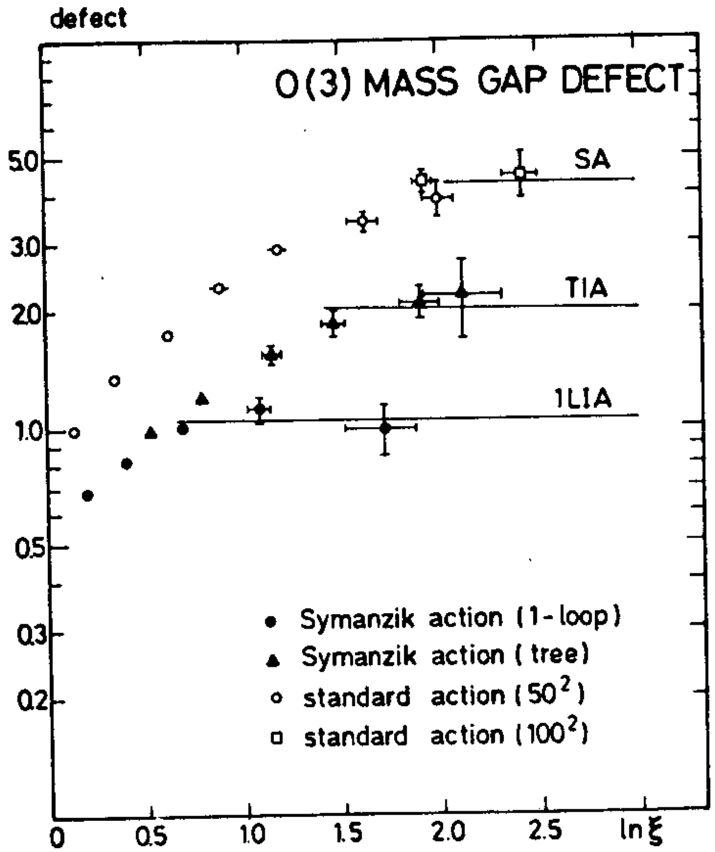

Fig. 2

been reviewed in Ref. /20/.

For a quantitative comparison of the asymptotic estimates one needs the Λ-scales. Several authors /21, 5, 13, 20/ did independent calculations and agree on

$$\Lambda_L^{improved} / \Lambda_L^{standard} = 2.219 \; . \qquad (5)$$

The Λ-scales for TIA and 1LIA are (asymptotically) identical. Therefore the TIA gives an estimate by a factor 2.3 higher than the estimate with the 1LIA action, and the SA estimate is again by a factor 1.3 higher than the TIA estimate. The large discrepancies are argued to be due to higher perturbative corrections to the Λ-scales (cf. equation (4.b)). There are good reasons to believe, that the exact result is close to the estimate of Lüscher /22/, which is $\gtrsim$ 1.3 times the 1LIA estimate.

2.) Energy-momentum dispersion: The energy-momentum dispersion for momentum

$$K = \frac{2\pi n}{L} \;, \quad (n = \pm 1, \pm 2, \ldots, \pm 10;\; L = 50)$$

was investigated in Ref. /14/. Results for SA, TIA and 1LIA are given in Figures 3, 4, 5 respectively.

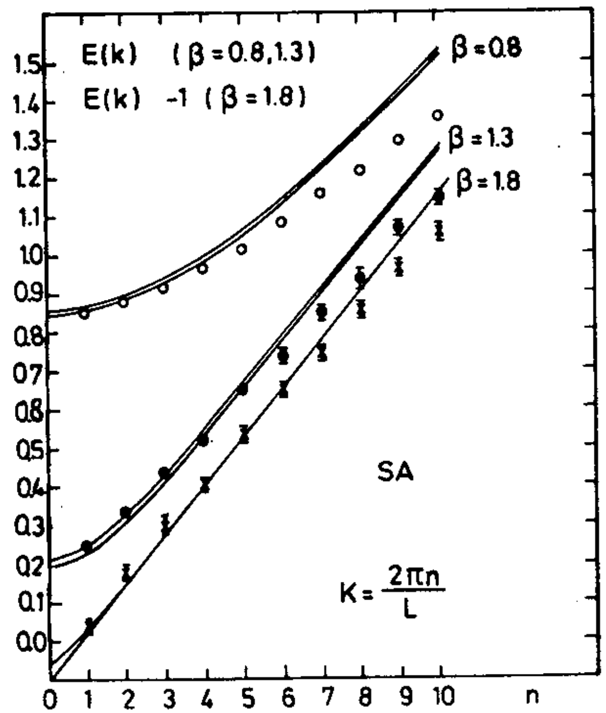

Fig. 3

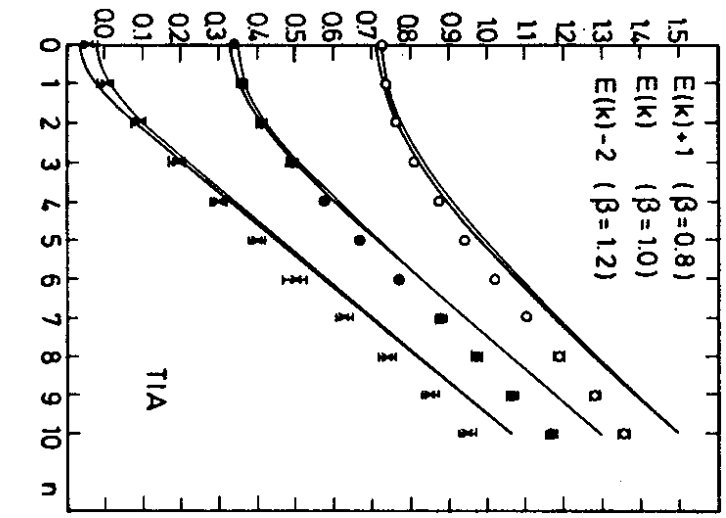

Fig. 4

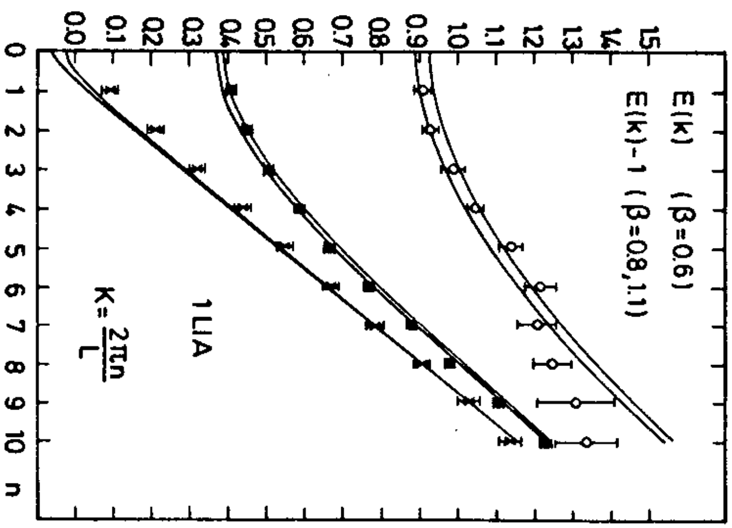

Fig. 5

The full lines are the expected behaviour

$$E(\vec{K}) = \sqrt{\vec{K}^2 + m^2}$$

as calculated from the mass gap. SA and TIA behave similarly, but there is considerable improvement in case of the 1LIA. This is conjectured to be due to a number of (numerically small) off-diagonal terms in this action, which are also expected to improve rotation invariance. Rotation invariance still remains to be checked.

3.) Magnetic susceptibility: MC results for the TIA where obtained in Ref. /13,14/ and for the 1LIA in Ref. /12,14/. Figure 6 summarizes for all

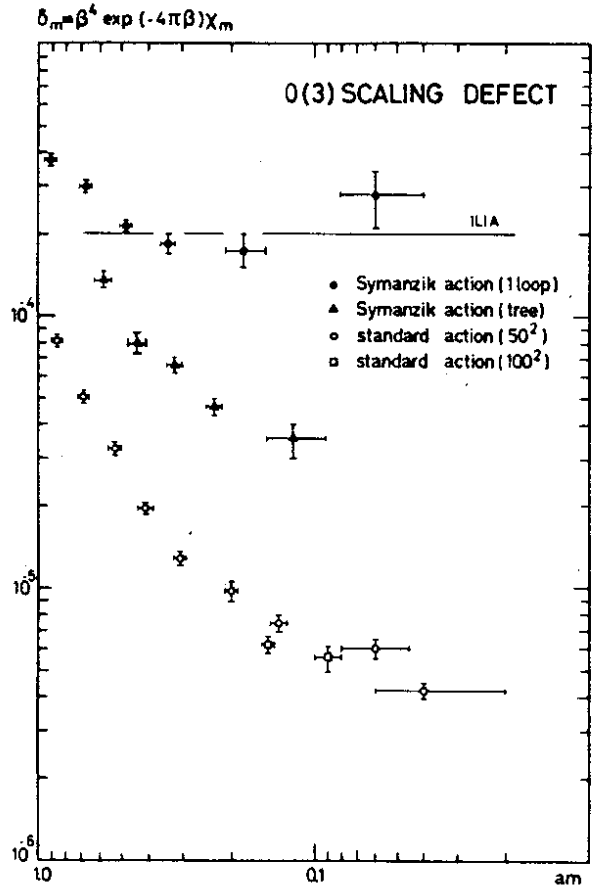

Fig. 6

three actions results for the defects, as obtained in Ref. /14/. Only the 1LIA exhibits an asymptotic scaling window.

4.) 4-point dimensionless coupling constant: For SA and TIA the 4-point dimensionless coupling constant at zero momentum

$$\lambda(\beta) = \frac{\int d^2x\, d^2y\, d^2z \, \langle \vec{s}(x)\cdot\vec{s}(y) \; \vec{s}(z)\cdot\vec{s}(0)\rangle_c}{\chi^2 \; \xi^2} = \frac{\chi_4}{\chi^2 \, \xi^2} \underset{(\beta\to\infty)}{\longrightarrow} \lambda^*$$

(χ magnetic susceptibility) was investigated by Falcioni et al. /13/. The value of λ^* is relevant to establish the convergence of the lattice theory to the continuum limit. In fact, while χ and ξ are rapidly varying functions of β, $\lambda_{LATT}(\beta)$ is expected to behave as

$$\lambda_{LATT}(\beta) = \lambda^* \left(1 + \mathcal{O}\left(\frac{a^2}{\xi_{\gamma}^2} \ln \frac{a}{\xi_{\gamma}}\right)\right)$$

exactly in the same way as the ratios of masses (3).

The most reliable estimates of λ^* comes from the so called pseudo ε-expansion /23/ of the linear σ-model. One obtains

$$\lambda^* = -6.66 \pm 0.06. \qquad (6)$$

Figure 7 (from Ref. /13/) compares this value with MC data for the TIA (black circles, β_{NEW}) and the SA (open circles, β_{OLD}). The full line is the high temperature expansion in case of the SA, and the dashed line is the corresponding Padé approximant (3,2). All MC data are obtained on rather small sized lattices, otherwise statistical noise would prevent any clear signal. The largest lattice is 24^2, corresponding to the highest β-values, all other used lattices are smaller. In fact in Ref. /14/ χ_4 was measured on a 50^2 lattice, but the signal remained within the statistical noise.

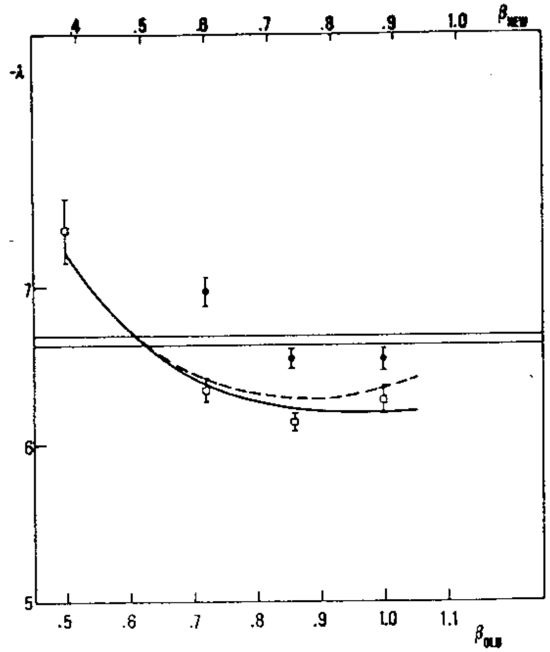

Fig. 7

In accordance with our theoretical expectations Figure 7 indicates that λ^* is approached more rapidly in case of the TIA than for the SA.

5.) Scaling in general: In Ref. /14/ the general scaling behaviour (1) of mass gap and two point functions was investigated. MC data were taken for the normalized two point function as a function of the lattice momentum:

$$R_2 = \frac{S_2(p)}{S_2(0)} = \frac{S_2(p)}{\chi} \qquad (7)$$

with $p = \frac{2\pi n}{L}$, $n = \pm1, \pm2, \ldots, \pm10$. The solution of the renormalization group equation

$$\left[-m\frac{\partial}{\partial m} + \bar{\beta}(g)\frac{\partial}{\partial g}\right] R_2(g,m) = 0$$

is

$$R_2 = r_2\left(\ln(m) + \int^{g}\frac{dg'}{\bar{\beta}(g')}\right). \qquad (8.a)$$

This means only a function of <u>one</u> variable

$$x = \ln(m) + \int^{g}\frac{dg'}{\bar{\beta}(g')} . \qquad (8.b)$$

This property can be checked graphically. A convenient choice is to plot for fixed β $\ln R_2$ as a function of ln(p), and to try to bring the curves belonging to different β-values on top of each other by a shift $\Delta(\beta)$ in ln(p). If this can be achieved there is scaling. Figures 8 and 9 give the results of these attempts for SA and 1LIA. In both cases we find good evidence for scaling in general (with lattice corrections). Within the statistical noise no improvement is visible.

Finally in Ref. /14/ the lattice $\bar{\beta}$-function was calculated from various observables. The 1LIA comes closest to $\bar{\beta}_{univ}$, than the TIA follows and the SA is rather far off. This is consistent with assuming the 1LIA asymptotic mass gap estimate to be closest to the exact value, which is presumably between the TIA estimate and the 1LIA estimate.

6.) <u>Summary</u>: We found improvement of the energy-momentum dispersion /14/ (1LIA) and of the 4-point dimensionless coupling constant /13/ (TIA). Both results are in agreement with theoretical expectations for Symanzik improved actions. Within the statistical errors scaling in general /14/ was not seen to be improved. Asymptotic scaling is considerably improved /12,13/, particularly for the 1LIA /12,14/. This is an "experimental" result and not expected to be related to Symanzik's improvement program.

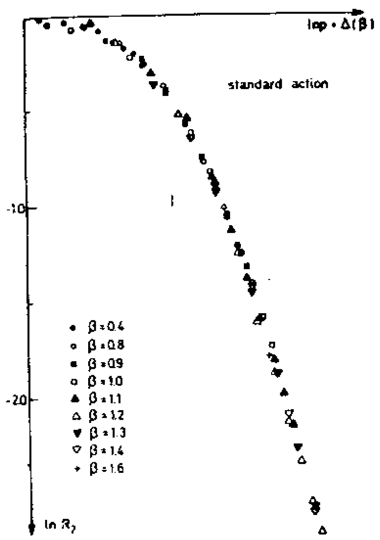

Fig. 8

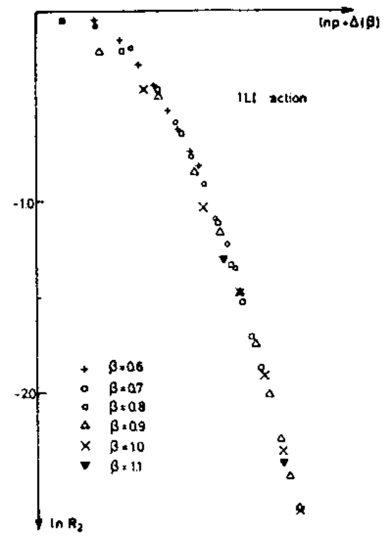

Fig. 9

III. 4d Non-Abelian Gauge Theories

For SU(N) lattice gauge theories the TIA is known due to tree-level calculations by P. Weisz /8/ and (necessary) 1-loop calculations by Curci et al. /9/. The 1LIA is not yet completely known, but relevant work has already been done /11/.

MC calculations for the TIA were done with gauge groups SU(2) /15-18/ and SU(3) /19/.

III.1 Gauge Group SU(2):

1.) The string tension:

A first result was reported by Belforte et al. /15/. These authors claimed an improvement of asymptotic scaling and gave from MC measurements of Creutz /24/ ratios the continuum estimate

$$\sqrt{K} = (17.9 \pm 1.0)\ \Lambda_L^{TI} \qquad (9)$$

A slightly lower result $\sqrt{K} \approx 16.4\ \Lambda_L^{TI}$ was independently obtained by Fukugita et al. /15/.

As the MC statistics of Belforte et al. /15/ is rather poor (50 sweeps with measurements after a thermalization of 200 sweeps at each β-value), we /17/ decided to carry out a similar calculation on an 8^4 lattice with a more reasonable statistics (2000 sweeps for measurements after a thermalization of 200 sweeps at each β-value). On a physical scale in units of Λ_L^{TI} our main results are given in Figure 10.

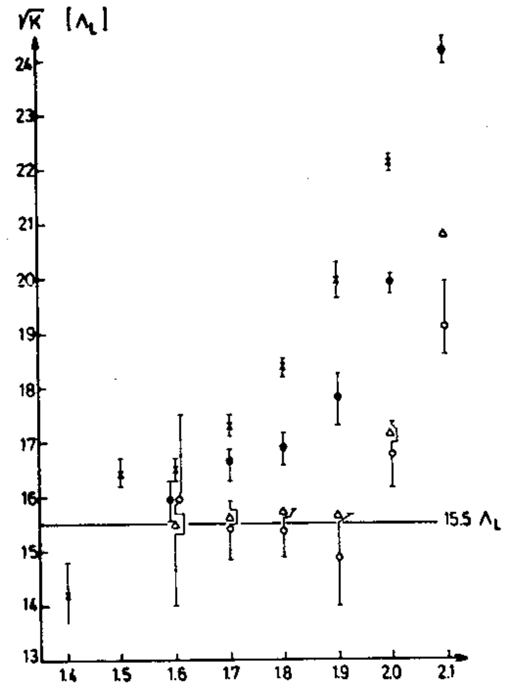

Fig. 10

Depicted are results for $\chi(3,3)$, $\chi(4,3)$ and $\chi(4,4)$. The final estimate

$$\sqrt{K} \approx 15.5\ \Lambda_L^{TI} \qquad (10)$$

is based on $\chi(4,4)$. The triangles Δ correspond to values as obtained from improved definitions /11/ $\hat{\chi}(3,3)$ of the Creutz ratio $\chi(3,3)$. The improved Creutz ratios work well: $\hat{\chi}(3,3)$ results are well compatible with non-improved $\chi(4,4)$ results.

As we observe $\quad \chi(4,4) < \chi(4,3) < \chi(3,3)$

one should regard equation (10) as an upper bound for the string tension. Consistency of the $\chi(4,4)$ data with asymptotic scaling indicates that this bound may be rather good. Gutbrod and Montvay /18/ have, however, analyzed the full quark-antiquark potential with high statistics on an 15^4 lattice. This analysis indicates violations of asymptotic scaling and a further lowering of the string tension to

$$\sqrt{K} \approx 12.5 \; \Lambda_L^{TI} \tag{11}$$

In Figure 11 (from Ref. /18/) this is the full line $\Lambda^{SI}_{Latt} = 0.08 \sqrt{K}$.

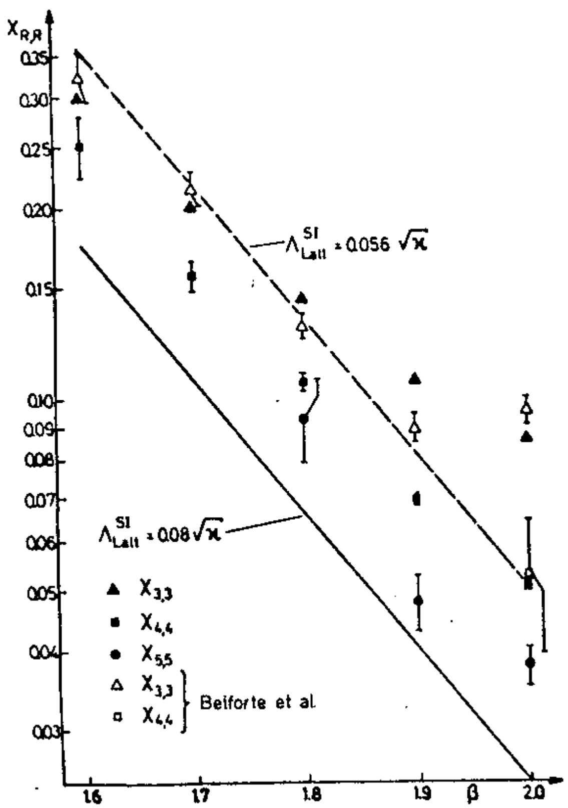

Fig. 11

Finally we would like to compare with string tension results as obtained using the SA. The perturbative calculations /10,11/ of the Λ-ratio give

$$\Lambda_L^{TI} / \Lambda_L^{SA} = 4.13 \tag{12}$$

MC results for the SA string tension have at present (similar to the above string tension results for the TIA) a decreasing tendency. Early results (for instance /24,25/) were around $\sqrt{K} \approx 76 \; \Lambda_L^{SA}$, whereas a high statistics calculation /17/ of the Creutz ratio $\chi(4,4)$ gives $\sqrt{K} \approx 69 \; \Lambda_L^{SA}$, and the potential analysis of Gutbrod and Montvay /18/ yields $\sqrt{K} \approx 56 \; \Lambda_L^{SA}$. If one takes "corresponding" results for calculating the Λ-ratio from the string tension, the number ≈ 4.5 is obtained in about 10% agreement with equation (12). Further clarification is desirable. In particular violations of asymptotic scaling would make numbers like (10,11) meaningless. Calculation of mass ratios may, however, still be feasible.

2. The glueball spectrum

SU(2) mass spectrum calculations with Symanzik improved actions were carried out in Ref. /16,17/. Recently I also became aware of Ref. /26,27/.

In Ref. /16,17/ MC calculations based on 21 Wilson loops of length ≤ 8 were done on an 5^3 8 lattice. On-diagonal correlations were considered and a very high statistics (up to 160 000 sweeps at some β-values) is available. Figure 12 (from Ref. /17/) gives our final $m(0^+)$ estimate

$$m(0^+) = (50 \pm 5) \tag{13}$$

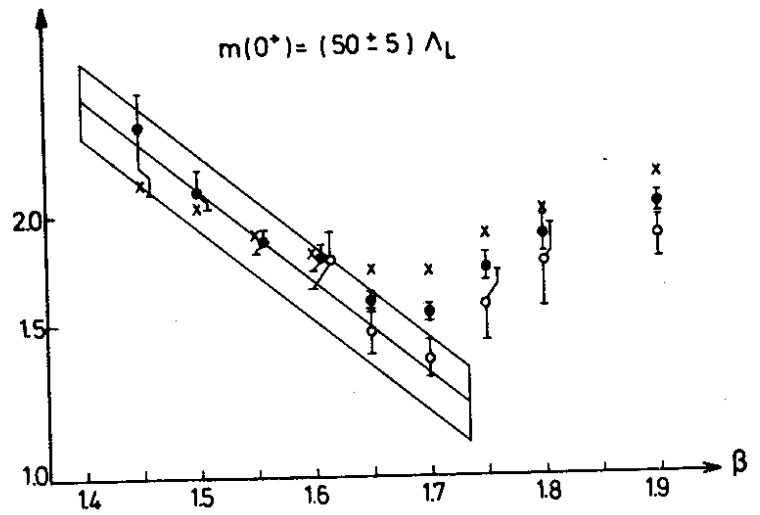

Fig. 12

Depicted are mass gap results as obtained from correlations of the "best" operators up to distance t = 2. The best operator is by definition the operator, which gives the lowest $m(0^+)$ result from correlations at distance t = 1. Our $m(0^+)$ result is consistent with a recent investigation by Mütter and Schilling /26/. A similar result was also obtained by Fukugita et al. /27/, who used, however, a different action.

In Ref. /16,17/ we have also calculated masses for the excited glueball states 0^-, 1^+ and 2^+. In contrast to our results for the 0^+ state, we find no (asymptotic) scaling for any of the excited states. The lowest excited state is 2^+. Only this state allows us to get some distance t = 2 results out of the statistical noise. In Figure 13 (from Ref. /17/) the $m(2^+)$ results as obtained from the "best" operator are depicted.

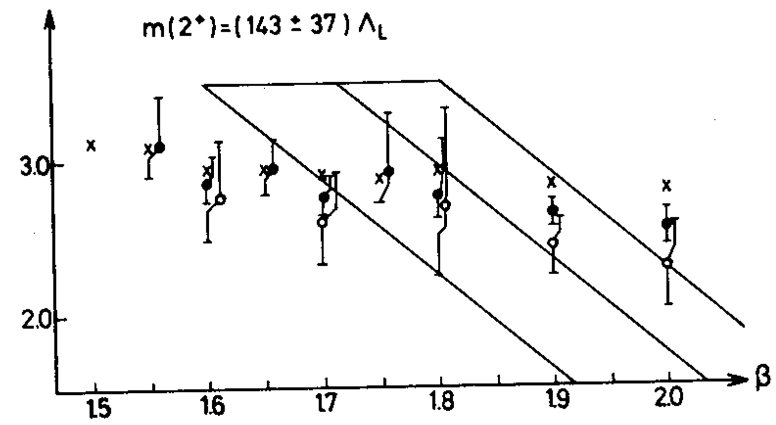

Fig. 13

With our improved statistics the earlier (weak) signal /16/ for asymptotic scaling disappeared nearly completely. There are severe problems in estimating reliable (statistical!) error bars of the noisy correlations at distance t = 2. See Ref. /17/ for a detailed discussion.

Mass ratios at distance t = 1 are, however, slightly improved /16,17/. In the relevant β-region we find for these ratios the following orders of magnitude:

$$m(2^+) \approx 1.6\, m(0^+) \quad (14.a)$$

$$m(0^-) \approx 2.2\, m(0^+) \quad (14.b)$$

$$m(1^+) \approx 2.8\, m(0^+) \quad (14.c)$$

In Ref. /17/ we have also calculated correlations between momentum $\vec{K}$ eigenstates. The TIA does not improve the relativistic energy-momentum dispersion as compared with the SA. Let us introduce

$$m(\vec{K}) = \sqrt{E(\vec{K})^2 - \vec{K}^2}$$

Figure 14 gives results for the lowest 2^+ momentum eigenstates, as obtained

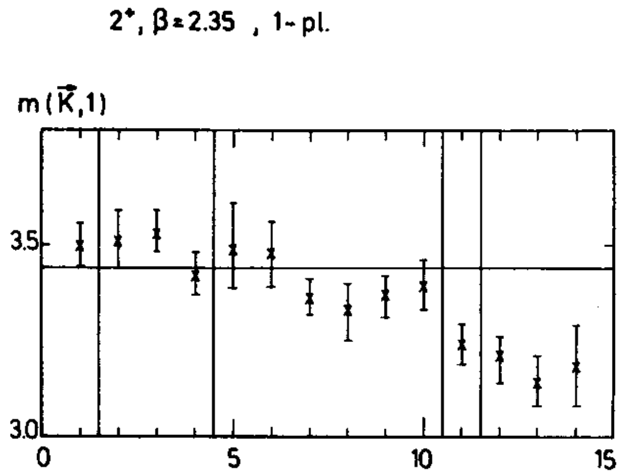

Fig. 14

with the SA on an 8^4 lattice. For the lowest 10 momenta we find $m(\vec{K})$= const. within statistical errors. As equivalent results look statistically rather independent, one can use momentum eigenstates to improve the MC statistics. A similar observation has been made in Ref. /28/.

III.2 Gauge Group SU(3)

First results were reported by Forcrand and Roiesnel /19/. They estimate string tension, $m(0^{++})$ mass gap and deconfinement temperature.

Their string tension

$$\sqrt{K} = (26 \pm 2) \cdot \Lambda_L^{TI} \qquad (15)$$

relies on Creutz ratios up to $\chi(3,3)$. Correlations between several planar Wilson loops at distance 0, 1, 2 lead to the mass gap

$$m(0^{++}) = (58 \pm 9)\,\Lambda_L. \qquad (16)$$

Figure 15 (from Ref. /11/) also exhibits results at distance 1 for the 2^{++}

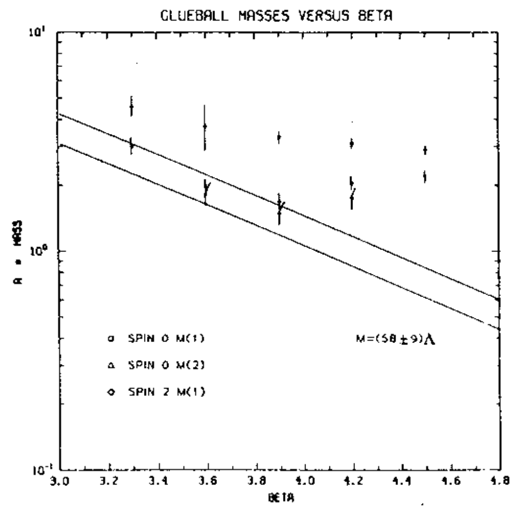

Fig. 15

state, which are lower than in case of the SA.

Finally they find some overshooting of scaling behaviour for the deconfinement temperature. An estimate still compatible with asymptotic scaling is

$$T_c = (12 \pm 1.2)\,\Lambda_L \qquad (17)$$

Comparing equations (15-17) to previous results with the SA consistency with universality is found within rather large uncertainties.

III.3 Summary and Conclusions

SA and TIA give similar results in MC calculations. There is a moderate improvement of glueball mass ratios at distance t = 1. Further the TIA exhibits asymptotic scaling at a slightly smaller correlation length than the SA. In summary a^2/ξ_r^2 corrections seem to be rather small.

Only with the SA an MCV spectrum calculation is on save grounds /17/, because the TIA has severe problems with the transfer matrix /29/. In connection with this it was found, that the TIA has no restoration of Lorentz invariance at distance t = 1 /17/. In view of the modest other practical improvements, this is an important point favouring the SA for MC calculations.

Acknowledgements

I would like to thank I. Montvay and P. Weisz for useful discussions.

Note added

After writing up this report, I got aware of a number of new preprints /30/ concerning the discussed topics.

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