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# The Renormalization of the Electroweak Standard Model

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Abstract:

A renormalization scheme for the electroweak standard model is presented in which the electric charge and the masses of the gauge bosons, Higgs particle and fermions are used as physical parameters. The photon is treated such that quantum electrodynamics is contained in the usual form. Field renormalization respecting the gauge symmetry gives finite Green functions. The Ward identities between the Green functions of the unphysical sector allow a renormalization that maintains the simple pole structure of the propagators. Explicit results for the renormalized self energies and vertex functions are given. They can be directly used as building blocks for the evaluation of 1-loop radiative corrections.

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## 1. Introduction

The recent discovery of the W and Z bosons at the $P\bar{P}$ collider at CERN [1] with values for the masses of these particles very close to those predicted by the Glashow-Salam-Weinberg model [2] was an important step in establishing this model as a good candidate for the gauge theory of the electroweak interaction. But also the experiments with low momentum transfers ($|q^2| \ll M_W^2$) [3] and at $e^+e^-$ storage rings [4] contribute to a steady improvement of the determination of the structure and parameters of the electromagnetic and weak interaction. The accuracy of these experiments has reached a level which requires the inclusion of radiative corrections for an adequate theoretical discussion. This will be even more the case when the $e^+e^-$ machines with energies up to 100 GeV, which are dedicated for the investigation of the detailed properties of the electroweak bosons, go into operation [5].

The standard model is a non-Abelian gauge theory of the electroweak interaction [6] where the masses of the particles are generated with help of the Higgs mechanism. The renormalizability of quantum field theories of this class was proved already in 1971 by 't Hooft [7]. This means that those parts occuring in the evaluation of Feynman diagrams of higher order which without regularization would become ultraviolet divergent can be absorbed by renormalization of the fields and couplings. The importance of the renormalization constants is not only to absorb divergences but also to complete the definition of the quantized field theory. The finite parts of the renormalization constants - fixed by the renormalization conditions - influence the results of the calculation of radiative corrections and therefore of physically observable effects. A well understood example of a field theory is quantum electrodynamics. For this theory exists a generally accepted renormalization scheme using the electric charge e and the mass of the electron m as physical parameters. Electroweak theories contain much more fields and parameters than QED, moreover their structure is more complicated because of their non-Abelian, non-simple, spontaneously broken gauge symmetry. The choice of the renormalized parameters and their definition via measurable quantities as well as the definition of the Weinberg angle is not unique beyond the tree level.

Consequently several different schemes have been proposed in the literature [8 - 17]. The greater part [10, 12 - 15] deals with processes where $|q^2| \ll M_W^2$, like μ decay and ν scattering; ref.s [9, 11, 16, 17] consider high $q^2$ $e^+e^-$ annihilation. An attempt for their characterization can be made using the following criteria:

- Schemes with and without field renormalization; S matrix elements but not the Green functions are finite in the latter case.
- Field renormalization with and without respecting the original gauge symmetry; Green functions are finite but have complicated properties under gauge transformations in the latter case.
- Determination of the parameters from low energy experiments like μ decay and $\overset{(-)}{\nu}_\mu e$ scattering or from high energy experiments i.e. measurements of the W, Z masses.

Of course, not all papers on electroweak radiative corrections fit simply into these categories.

In this paper we present a renormalization scheme for the standard electroweak model which is defined by the following conditions:

i) The physical parameters are the electric charge e, the masses of the W and Z bosons, the Higgs mass and the fermion masses. e is defined as the strength of the electromagnetic coupling in the Thomson limit, the masses as the position of the poles of the renormalized propagators. Our parameters are directly accessible to experiment, since only the measurement of the fine structure constant α and of masses is required. The determination of masses in direct resonance production experiments is only very little influenced by radiative corrections. Bare masses and couplings do not occur in our scheme. This avoids possible confusions in calculating cross sections in higher orders. Since our scheme uses $M_W$, $M_Z$ it seems especially well suited for the analysis of experiments at high energies. The Weinberg angle $\Theta_W$ and the Fermi constant $G_F$ are no fundamental parameters. In our scheme $c = \cos\Theta_W = M_W/M_Z$, $s = \sqrt{1 - c^2}$ are only shorthand notations to simplify the formulas.



ii) On-shell photons couple as the usual real photon to the electron without any admixture of $Z^0$ contributions. Therefore the QED subpart of the model is realized in a simple way. Consequently photonic radiative corrections can be treated separately and in the same way as in QED calculations [18].

iii) Complete field renormalization respecting gauge invariance and the use of the 't Hooft-Feynman gauge lead to UV finite renormalized Green functions reflecting the gauge symmetry structure. We investigate the restrictions of the Slavnov-Taylor identities for the renormalization of the unphysical Green functions. Especially we perform a renormalization of the gauge fixing parameters in such a way that the poles of all these renormalized Green functions are situated at $M_Z^2$, $M_W^2$, 0.

Sect. 2 of this paper contains the definition of the complete Lagrangian and its parameters; sect. 3 the discussion of the Slavnov-Taylor resp. Ward identities; sect. 4 the renormalization conditions. The complete list of the Feynman rules including the counter terms can be found in the appendix A. In sect. 5 and 6 we list the 1-loop formulas for the renormalization constants, the renormalized self energies and mixing propagators as well as the fermion gauge boson vertex functions and present the numerical results for these quantities. They are needed e.g. for the calculation of the radiative corrections to $e^+e^-$ annihilation, deep inelastic lepton scattering with $|q^2| \cong M_W^2$. A first application of these results has been the calculation of the forward-backward asymmetry in μ pair production at PETRA/PEP energies [19].

## 2. The renormalized Lagrangian and the Feynman rules of the standard electroweak model

### 2.1 The classical Lagrangian, parameters and fields

Gauge theories of the electroweak interaction are constructed in such a way that at low energies and in lowest order the experimentally successful Fermi model is recovered. In the case of the standard model [2] the universality of the weak interaction is realized in the form of the gauge group SU(2) × U(1). The gauge symmetry is spontaneously broken such that the electromagnetic gauge invariance $U(1)^{em}$ is maintained. For this a minimal Higgs mechanism with a SU(2) doublet of scalar fields is used. It allows to predict from low energy experiments the masses $M_W$, $M_Z$ of the heavy gauge bosons $W^{\pm}$, Z. The existence and main properties of these particles have recently been confirmed by experiments at the $P\bar{P}$ collider [1].

The classical Lagrangian of the standard model $\mathcal{L}_C$ is composed of the gauge, Higgs and fermion part:

$$\mathcal{L}_C = \mathcal{L}_{YM} + \mathcal{L}_H + \mathcal{L}_F \ . \qquad (2.1)$$

According to the gauge group SU(2) × U(1) we have an isotriplet $W^a_\mu(x)$ and an isosinglet $B_\mu(x)$ of gauge fields with gauge coupling constants $g_2$ and $g_1$ leading to the Yang-Mills Lagrangian:

$$\mathcal{L}_{YM} = -\frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \varepsilon^{abc} W^b_\mu W^c_\nu)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \ . \qquad (2.2)$$

The complex Higgs doublet $\phi(x)$

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^o(x) \end{pmatrix} = \begin{pmatrix} \phi^+(x) \\ (v + \eta(x) + i\chi(x))/\sqrt{2} \end{pmatrix}$$

with hypercharge Y = 1 is coupled to the gauge bosons and has a self coupling:

$$\mathcal{L}_H = (D_\mu\phi)^+(D^\mu\phi) - \frac{\lambda}{4}(|\phi|^2)^2 + \mu^2|\phi|^2 \qquad (2.3)$$



with the covariant derivative:

$$D_\mu = \partial_\mu - ig_2\, I^a\, W^a_\mu + ig_1 \frac{Y}{2} B_\mu \ . \qquad (2.4)$$

The left-handed fermion fields $\psi^L_{i\sigma}(x)$ are grouped into doublets (i = doublet index, σ = component of the doublet) of the weak isospin, the right-handed fields $\psi^R_{i\sigma}(x)$ into singlets, the hypercharges respecting the Gell-Mann Nishijima relation $Q = I^3 + Y/2$. The Lagrangian $\mathcal{L}_F$ which describes the interaction between the fermions, the gauge fields and the scalars then has the form*):

$$\begin{aligned} \mathcal{L}_F = \sum_i \{ & \bar{\psi}^L_{i\sigma} i\gamma^\mu D^{\sigma\sigma'}_\mu \psi^L_{i\sigma'} + \bar{\psi}^R_{i\sigma} i\gamma^\mu D^{\sigma\sigma'}_\mu \psi^R_{i\sigma'} \\ & + (- g_{i+}\bar{\psi}^R_{i+}\phi^o\psi^L_{i+} - g_{i-}\bar{\psi}^R_{i-}\phi^{o*}\psi^L_{i-} \\ & + g_{i+}\bar{\psi}^R_{i+}\phi^+\psi^L_{i-} - g_{i-}\bar{\psi}^R_{i-}\phi^-\psi^L_{i+} + h.c.)\} \ . \end{aligned} \qquad (2.5)$$

This completes the construction of $\mathcal{L}_C$ in terms of the fields $W^a_\mu$, $B_\mu$, $\phi$, $\psi^L$, $\psi^R$ and the parameters

$$g_2,\ g_1,\ \lambda,\ \mu^2,\ g_{i\sigma} \ . \qquad (2.6)$$

$\mathcal{L}_C$ is invariant under local transformations of the group SU(2) × U(1):

$$g = \exp[ig_2 I^a\theta_a(x) - i\frac{g_1}{2} Y\theta_Y(x)] \ ,$$

namely

$$\frac{\delta\mathcal{L}_C}{\delta\theta^\alpha(x)} = 0 \ , \quad \alpha = \{a, Y\} \ . \qquad (2.7)$$

*) We do not write explicitly colour indices and the Cabbibo transformation of the quark fields i.e. we assume the coupling matrix $g_{ij} = g_i\delta_{ij}$ to be diagonal.

A formulation where the physical content of the theory is more - but the symmetry less - transparent can be obtained by performing the Weinberg transformation of the gauge fields:

$$W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2} ,$$

$$Z_\mu = \frac{M_W}{M_Z} W_\mu^3 + \left(1 - \frac{M_W^2}{M_Z^2}\right)^{1/2} B_\mu , \qquad (2.8)$$

$$A_\mu = - \left(1 - \frac{M_W^2}{M_Z^2}\right)^{1/2} W_\mu^3 + \frac{M_W}{M_Z} B_\mu$$

and using the following parameters:

$$e,\ M_W,\ M_Z,\ M_H,\ m_{i\sigma} \qquad (2.9)$$

with

$$e = \frac{g_1 g_2}{(g_1^2 + g_2^2)^{1/2}} , \quad M_W = g_2\mu/\sqrt{\lambda} , \quad M_Z = (g_1^2 + g_2^2)^{1/2}\mu/\sqrt{\lambda} ,$$

$$M_H = \sqrt{2}\,\mu , \quad m_{i\sigma} = g_{i\sigma}\mu\sqrt{2/\lambda} . \qquad (2.10)$$

Each of the parameters (2.9) is directly accessible to experiments since for their determination measurements of the Thomson scattering cross section (for the electric charge e) and of the masses of the W boson, Z boson, Higgs boson and the fermions are required. This is the reason why we prefer the set of more physical fields (2.8) and parameters (2.9) as the basis for the formulation of the electroweak Lagrangian $\mathcal{L}_C$. This set is especially well suited for the comparison of the results of the standard model with high energy experiments. It may be that for low energy processes the use of other parameters like the Fermi constant $G_F$ and the Weinberg angle $\Theta_W$ is more convenient [10, 12, 13]. The relation between $M_W$, $M_Z$ and $G_F$, $\Theta_W$ to lowest order is:

$$G_F/\sqrt{2} = \pi\alpha/2M_W^2\,(1 - M_W^2/M_Z^2) ,$$

$$\cos\Theta_W = M_W/M_Z . \qquad (2.11)$$

Depending on the specific renormalization scheme some of the relations (2.8) - (2.11) may get corrections from higher order contributions.



### 2.2 Gauge fixing and ghost fields

For the systematic treatment of the quantization of $\mathcal{L}_C$ and higher order calculations it is convenient to choose a renormalizable gauge. We introduce linear gauge fixings $F^\alpha(W,B,\phi)$ of the 't Hooft type. Written in the physical fields (2.8) they read:

$$F^\pm = (\xi_1^W)^{-1/2}\partial^\mu W_\mu^\pm \mp iM_W(\xi_2^W)^{1/2}\phi^\pm ,$$

$$F^Z = (\xi_1^Z)^{-1/2}\partial^\mu Z_\mu - M_Z(\xi_2^Z)^{1/2}\chi , \qquad (2.12)$$

$$F^\gamma = (\xi_1^\gamma)^{-1/2}\partial^\mu A_\mu .$$

Then we add to $\mathcal{L}_C$ the term

$$\mathcal{L}_{fix} = -\frac{1}{2}\sum_\alpha (F^\alpha)^2 \qquad (2.13)$$

and introduce the Faddeev-Popov ghost fields $u^\alpha(x)$ resp. $u^\pm(x)$, $u^Z(x)$, $u^\gamma(x)$ with the Lagrangian [20]:

$$\mathcal{L}_{FP} = \bar{u}^\alpha(x)\frac{\delta F^\alpha}{\delta\theta^\beta(x)}u^\beta(x) = \bar{u}^\alpha K^{\alpha\beta}u^\beta . \qquad (2.14)$$

A particular choice of the gauge parameters $\xi$ is:

$$\xi_1^W = \xi_2^W = \xi_1^Z = \xi_2^Z = \xi_1^\gamma = 1 . \qquad (2.15)$$

This 't Hooft-Feynman gauge has the advantage that at least to lowest order the poles of the longitudinal parts of the gauge boson propagators, the unphysical Higgs fields $\phi^\pm$, $\chi$ and the ghost fields are situated at $M_W^2$ or $M_Z^2$ and that no gauge field - Higgs field mixing occurs.

With $\mathcal{L}_{fix}$ and $\mathcal{L}_{FP}$ we have completed the construction of a renormalizable Lagrangian

$$\mathcal{L} = \mathcal{L}_C + \mathcal{L}_{fix} + \mathcal{L}_{FP} \qquad (2.16)$$

for the standard electroweak model. The Feynman rules which follow from this Lagrangian are presented in appendix A.

## 2.3 Multiplicative Renormalization

The Lagrangian (2.16) is the starting point for the calculation of Green functions and S matrix elements including radiative corrections. Since symmetry arguments were important in the construction of $\mathcal{L}_C$ we perform the multiplicative renormalization of $\mathcal{L}$ in such a way that the gauge symmetry is respected:

$$\begin{aligned}
W^a_\mu &\to (Z_2^W)^{1/2} W^a_\mu , \quad B_\mu \to (Z_2^B)^{1/2} B_\mu , \\
\phi &\to (Z^\phi)^{1/2} \phi , \\
\psi^L_{i\sigma} &\to (Z_L^i)^{1/2} \psi^L_{i\sigma} , \quad \psi^R_{i\sigma} \to (Z_R^{i\sigma})^{1/2} \psi^R_{i\sigma} , \\
g_2 &\to Z_1^W (Z_2^W)^{-3/2} g_2 , \quad g_1 \to Z_1^B (Z_2^B)^{-3/2} g_1 , \\
\lambda &\to Z^\lambda (Z^\phi)^{-2} \lambda , \quad \mu^2 \to (\mu^2 - \delta\mu^2)(Z^\phi)^{-1} , \\
v &\to (Z^\phi)^{1/2} (v - \delta v) , \\
g_{i\sigma} &\to (Z^\phi)^{-1/2} Z_1^{i\sigma} g_{i\sigma} , \\
\xi^\alpha_{1,2} &\to 1 + \delta\xi^\alpha_{1,2} , \quad u^a \to (\tilde{Z}_2^W)^{1/2} u^a , \quad u^B \to (\tilde{Z}_2^B)^{1/2} u^B .
\end{aligned} \tag{2.17}$$

These definitions of renormalized fields and parameters induce corresponding expressions for the fields (2.8) i.e. $W^\pm_\mu$, $Z_\mu$, $A_\mu$ and the parameters (2.9). Writing

$$Z_i = 1 + \delta Z_i \tag{2.18}$$

we obtain $\mathcal{L} \to \mathcal{L} + \delta\mathcal{L}$ where the expression for $\mathcal{L}$ in the renormalized quantities is identical with the original one, but now contains the renormalized physical parameters and fields. The quantities $\delta Z_i$, $\delta v$, $\delta\mu^2$, $\delta\xi^\alpha_i$ occur in the counter term Lagrangian $\delta\mathcal{L}$. Their finite parts have to be fixed by the explicit renormalization conditions. Before doing this we study the restrictions which are imposed on the renormalization procedure by the Slavnov-Taylor identities of the theory.

The Feynman rules belonging to $\mathcal{L}$ and the counter terms from $\delta\mathcal{L}$ are listed in app. A.



# 3. Slavnov-Taylor identities

## 3.1 The Becchi-Rouet-Stora transformation

The original gauge invariance of $\mathcal{L}_C$ is lost after the introduction of $\mathcal{L}_{fix}$ and $\mathcal{L}_{FP}$. The complete Lagrangian $\mathcal{L}$ is invariant under gauge transformations involving also ghost fields $u^\alpha(x)$. For the sake of compactness we use the following condensed notation for the fields and their transformations:

$$\Phi_s = \{W^a_\mu(x), B_\mu(x), \phi(x), \psi^L_{i\sigma}(x), \psi^R_{i\sigma}(x)\} , \tag{3.1}$$

$$\delta\Phi_s = (\Delta^\alpha_s + g^\alpha T^\alpha_{st} \Phi_t)\, \delta\theta^\alpha . \tag{3.2}$$

The inhomogeneous term $\Delta^\alpha_s$ acts only on the gauge field part of $\Phi$, $T^\alpha$ denotes the representation matrices of the SU(2) × U(1) generators. The Becchi-Rouet-Stora transformation [21] under which $\mathcal{L}$ is invariant is constructed in such a way that the parameters of the infinitesimal gauge transformation $\delta\theta^\alpha$ contain the ghost fields:

$$\delta\theta^\alpha(x) = u^\alpha(x)\cdot\bar{\lambda} . \tag{3.3}$$

($\bar{\lambda}$ is independent of $\alpha$ and has ghost number -1). Since (3.2) together with (3.3) defines a gauge transformation $\mathcal{L}_C$ is still invariant. The transformation of the ghost fields $u^\alpha$, $\bar{u}^\alpha$ is defined in such a way that $\mathcal{L}_{fix} + \mathcal{L}_{FP}$ is also invariant:

$$\delta\bar{u}^\alpha = F^\alpha \cdot \bar{\lambda} , \tag{3.4}$$

$$\delta u^\alpha = (K^{-1})^{\alpha\beta} \delta K^{\beta\gamma} u^\gamma = -\frac{1}{2} C^{\alpha\beta\gamma} u^\beta u^\gamma \bar{\lambda} , \tag{3.5}$$

where $F^\alpha$ is the linear gauge fixing (2.12), K the Faddeev-Popov kernel (2.14) and $C^{\alpha\beta\gamma}$ the structure constant of the gauge group.

## 3.2 The Slavnov-Taylor identities

The BRS symmetry of $\mathcal{L}$ induces symmetry relations between the Green functions of the theory. They can be derived in a compact form with the help of the path integral formalism. The generating functional W of the Green functions $\tau_{s_1 \ldots s_n} = \langle 0 | T\Phi_{s_1} \ldots \Phi_{s_n} | 0 \rangle$:

$$\tau_{s_1 \ldots s_n} = i^{-n} \left. \frac{\delta^n W}{\delta j_{s_1} \ldots \delta j_{s_n}} \right|_{j=0} \tag{3.6}$$

is defined by:

$$W[j, \omega, \bar{\omega}] = \int D\Phi_s \, D\bar{u} \, Du \, \exp\{i \int d^4x [\mathcal{L} + \Phi_s j_s + \bar{u}^\alpha \omega^\alpha + \bar{\omega}^\alpha u^\alpha]\} \, . \tag{3.7}$$

Here we have introduced sources $j_s$ for the fields $\Phi_s$ and sources $\omega^\alpha$, $\bar{\omega}^\alpha$ for the (anti)-ghost fields. From the invariance of $\mathcal{L}$, $D\Phi_s$ and $D\bar{u}Du$ under BRS transformations one obtains for W the identity:

$$\left. \{ iF^\alpha[\frac{\delta}{i\delta j}] + j_t(\Delta_t^\beta + gT_{tt'}^\beta \frac{\delta}{i\delta j_{t'}}) \frac{\delta^2}{i\delta\omega^\alpha i\delta\bar{\omega}^\beta} \} W[j,\omega,\bar{\omega}] \right|_{\omega=\bar{\omega}=0} = 0 \, . \tag{3.8}$$

From this Slavnov-Taylor identity [22] follow the desired relations between the Green functions $\tau$ by taking suitable derivatives with respect to the sources $j_s$ and putting afterwards all $j_s = 0$.

A special class of relations, those which do not directly contain Green functions of ghost fields, results if the gauge fixing operator $F^\alpha$ is applied to eq. (3.8):

$$\left. F^\alpha[\frac{1}{i}\frac{\delta}{\delta j}] \, F^\beta[\frac{1}{i}\frac{\delta}{\delta j}] \, W[j] \right|_{j=0} = i \, \delta^{\alpha\beta} W[0] \, . \tag{3.9}$$

This equation relates the 2-point functions, namely the gauge boson propagators $\Delta_{\mu\nu}^{\alpha\beta}(k)$, the gauge boson Higgs mixing propagators $\Delta_\mu^{\alpha i}(k)$ and the unphysical Higgs propagators $\Delta^{ij}(k)$:



$$\begin{aligned} k^\mu k^\nu \Delta_{\mu\nu}^W(k) - 2 M_W k^\mu \Delta_\mu^{W\phi}(k) + M_W^2 \Delta^\phi(k) &= -i \, , \\ k^\mu k^\nu \Delta_{\mu\nu}^Z(k) + 2iM_Z k^\mu \Delta_\mu^{Z\chi}(k) + M_Z^2 \Delta^\chi(k) &= -i \, , \\ k^\mu k^\nu \Delta_{\mu\nu}^\gamma(k) &= -i \, , \\ k^\mu k^\nu \Delta_{\mu\nu}^{\gamma Z}(k) + iM_Z k^\mu \Delta_\mu^{\gamma\chi}(k) &= 0 \, . \end{aligned} \tag{3.10}$$

We decompose the gauge field propagators into their transverse and longitudinal parts

$$\Delta_{\mu\nu}^{\alpha\beta}(k) = (-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}) \Delta_T^{\alpha\beta}(k^2) - \frac{k_\mu k_\nu}{k^2} \Delta_L^{\alpha\beta}(k^2) \, , \tag{3.11}$$

make use of the Lorentz covariance of the gauge Higgs mixing propagators

$$\Delta_\mu^{\alpha i}(k) = ik_\mu \Delta^{\alpha i}(k^2) = ik_\mu \frac{1}{k^2 - M_\alpha^2} \Sigma^{\alpha i}(k^2) \frac{1}{k^2 - M_i^2} \, , \tag{3.12}$$

and split off the free parts of the propagators

$$\Delta(k) = i/(k^2 - M^2 + \Sigma(k)) \, , \tag{3.13}$$

$$\Delta_{T,L}^{\alpha\beta}(k) = \frac{-i}{k^2 - M_\alpha^2} \Sigma_{T,L}^{\alpha\beta}(k^2) \frac{1}{k^2 - M_\beta^2} \, . \tag{3.12}$$

We get the following relations between the self energies:

$$\begin{aligned} k^2(\Sigma_L^W - 2 M_W \Sigma^{W\phi}) - M_W^2 \Sigma^\phi &= 0 \, , \\ k^2(\Sigma_L^Z + 2iM_Z \Sigma^{Z\chi}) - M_Z^2 \Sigma^\chi &= 0 \, , \\ k^2 \Sigma_L^\gamma &= 0 \, , \\ k^2(\Sigma_L^{\gamma Z} + iM_Z \Sigma^{\gamma\chi}) &= 0 \, . \end{aligned} \tag{3.14}$$

As a consequence of $U(1)^{em}$ gauge invariance the longitudinal photon self energy vanishes identically as is the case in pure QED.

By adding the appropriate counter terms (app. A) we arrive at identities for the renormalized self energies $\hat{\Sigma}$:

$$k^2(\hat{\Sigma}_L^W - 2\,M_W\hat{\Sigma}^{W\phi}) - M_W^2\hat{\Sigma}^{\phi} = (k^2 - M_W^2)[k^2(\delta Z_2^W - \delta\xi_1^W) - M_W^2(\delta\xi_2^W + \delta Z_2^{\phi}) - \delta M_W^2] \ ,$$

$$k^2(\hat{\Sigma}_L^Z + 2iM_Z\hat{\Sigma}^{Z\chi}) - M_Z^2\hat{\Sigma}^{\chi} = (k^2 - M_Z^2)[k^2(\delta Z_2^Z - \delta\xi_1^Z) - M_Z^2(\delta\xi_2^Z + \delta Z_2^{\phi}) - \delta M_Z^2] \ ,$$

$$\hat{\Sigma}_L^{\gamma} = k^2(\delta Z_2^{\gamma} - \delta\xi_1^{\gamma}) \ , \qquad (3.15)$$

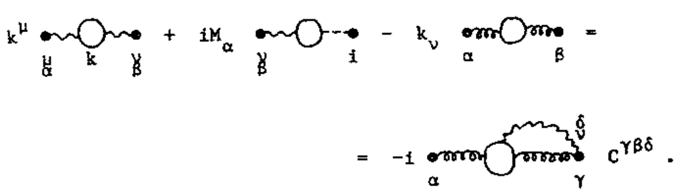
$$(\hat{\Sigma}_L^{\gamma Z} + iM_Z\hat{\Sigma}^{\gamma\chi}) = -\,k^2(\delta Z_2^{\gamma Z} - \delta\xi_1^{\gamma Z}) + M_Z^2(\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z} + \tfrac{1}{2}\delta\xi_2^{\gamma Z} - \tfrac{1}{2}\delta\xi_1^{\gamma Z}) \ .$$

One consequence of these results is that the number of independent renormalization conditions for the unphysical propagators $\Delta_{\mu\nu,L}^{\alpha\beta}$, $\Delta_{\mu}^{\alpha i}$, $\Delta^{ij}$ is reduced. But eq. (3.15) is compatible with a renormalization where the poles of these propagators are located at $M_W^2$, $M_Z^2$, 0. This means that the structure which is realized in lowest order in the Feynman gauge can be maintained in all orders by a suitable renormalization of the gauge fixing parameters $\xi_{1,2}^{\alpha}$.

We do not work out the relations like (3.9) between the "unphysical" parts of higher Green functions since we do not need them for the investigation of the restrictions on the renormalization constants in the unphysical sector. Instead we have a look at the ghost propagators. Differentiation of eq. (3.8) with respect to the sources of the gauge fields yields:

$$[k^{\mu}\Delta_{\mu\nu}^{\alpha\beta}(k) + iM_{\alpha}\Delta_{\nu}^{\beta i}(k) - k_{\nu}G^{\alpha\beta}(k)]\ \delta^{(4)}(k-k') =$$

$$-\,iC^{\gamma\beta\delta}\int d^4q\ G^{\alpha\gamma,\delta}_{\nu}(k,-k'+q,-q) \ , \qquad (3.16)$$




Here $G^{\alpha\beta}(k)$ denotes the ghost propagator and $G^{\alpha\beta,\delta}_{\nu} = \langle 0|(T\,u^{\alpha}\bar{u}^{\beta}W_{\nu}^{\delta})|0\rangle$ the gauge field - ghost three point function. Eq. (3.16) is in lowest order the usual relation between the longitudinal part of the gauge field propagator and the ghost propagators. In 1-loop order eq. (3.16) allows to



determine the ghost self energies from $\Sigma_L^{\alpha\beta}$, $\Sigma^{\alpha i}$ and the diagram on the r.h.s.. An important consequence of eq. (3.16) is that renormalization in the ghost sector can be performed in such a way that the poles of the ghost propagators remain at $M_W^2$, $M_Z^2$, 0. The explicit form of the identities (3.16) reads in 1-loop order for the self energies:

$$\Sigma_L^{\gamma}(k^2) - \tilde{\Sigma}^{\gamma}(k^2) = -\,\frac{\alpha}{4\pi}\,k^2\,B_o(k^2;M_W,M_W) \ ,$$

$$\Sigma_L^{\gamma Z}(k^2) - \tilde{\Sigma}^{\gamma Z}(k^2) = \frac{\alpha}{4\pi}\frac{c}{s}\,(k^2 - M_Z^2)\,B_o\,(k^2;M_W,M_W) \ ,$$

$$\Sigma_L^{Z\gamma}(k^2) + iM_Z\Sigma^{\gamma\chi}(k^2) - \tilde{\Sigma}^{Z\gamma}(k^2) = \frac{\alpha}{4\pi}\frac{c}{s}\,k^2\,B_o(k^2;M_W,M_W) \ , \qquad (3.17)$$

$$\Sigma_L^{Z}(k^2) + iM_Z\Sigma^{Z\chi}(k^2) - \tilde{\Sigma}^{Z}(k^2) = -\,\frac{\alpha}{4\pi}\frac{c^2}{s^2}\,(k^2 - M_Z^2)\,B_o(k^2;M_W,M_W) \ ,$$

$$\Sigma_L^{W}(k^2) - M_W\,\Sigma^{W\phi}(k^2) - \tilde{\Sigma}^{W}(k^2) =$$

$$= -\,\frac{\alpha}{4\pi}\,(k^2 - M_W^2)[\frac{c^2}{s^2}\,B_o(k^2;M_W,M_Z) + B_o(k^2;M_W,0)] \ ,$$

where the singular 1-loop integral $B_o$ is defined in eq. (5.4). Adding the appropriate counter terms of app. A we obtain the identities for the renormalized self energies:

$$\hat{\Sigma}_L^{\gamma} - \hat{\tilde{\Sigma}}^{\gamma} = k^2(\delta Z_2^{\gamma} - \delta\tilde{Z}^{\gamma} - \tfrac{1}{2}\delta\xi_1^{\gamma} - \frac{\alpha}{4\pi}\,B_o(k^2;M_W,M_W)) \ ,$$

$$\hat{\Sigma}_L^{\gamma Z} - \hat{\tilde{\Sigma}}^{\gamma Z} = -\,k^2(\delta Z_2^{\gamma Z} - \delta\tilde{Z}^{\gamma Z} - \tfrac{1}{2}\delta\xi_1^{\gamma Z}) + \tfrac{1}{2}M_Z^2(\delta Z_2^{\gamma Z} - \delta\tilde{Z}^{\gamma Z} - \delta\xi_2^{\gamma Z}) +$$

$$+\,\frac{\alpha}{4\pi}\,\frac{c}{s}(k^2 - M_Z^2)\,B_o(k^2;M_W,M_W) \ , \qquad (3.18)$$

$$\hat{\Sigma}_L^{\gamma Z} + iM_Z\hat{\Sigma}^{\gamma\chi} - \hat{\tilde{\Sigma}}^{Z\gamma} = -\,k^2(\delta Z_2^{\gamma Z} - \delta\tilde{Z}^{\gamma Z} - \tfrac{1}{2}\delta\xi_1^{\gamma Z}) + \tfrac{1}{2}M_Z^2(\delta Z_2^{\gamma Z} - \delta\tilde{Z}^{\gamma Z} - \delta\xi_1^{\gamma Z} + \delta\xi_2^{\gamma Z})$$

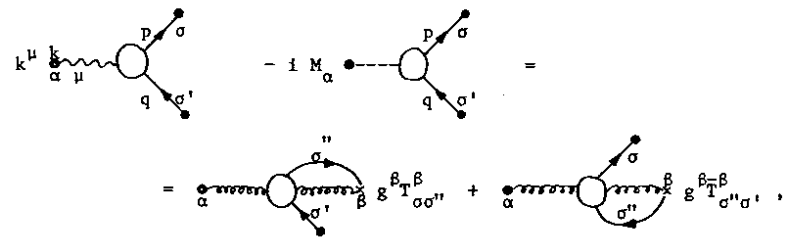
$$+\,\frac{\alpha}{4\pi}\,\frac{c}{s}\,k^2\,B_o(k^2;M_W,M_W) \ ,$$

$$\hat{\Sigma}_L^{Z} + iM_Z\hat{\Sigma}^{Z\chi} - \hat{\tilde{\Sigma}}^{Z} = (k^2 - M_Z^2)(\delta Z_2^{Z} - \delta\tilde{Z}^{Z} - \tfrac{1}{2}\delta\xi_1^{Z} - \frac{\alpha}{4\pi}\,\frac{c^2}{s^2}\,B_o(k^2;M_W,M_W)) \ ,$$

$$\hat{\Sigma}_L^W - M_W \hat{\Sigma}^{W\phi} - \hat{\tilde{\Sigma}}^W = (k^2-M_W^2)[\delta Z_2^W - \delta\tilde{Z}^W - \frac{1}{2}\delta\xi_1^W - \frac{\alpha}{4\pi}(\frac{c^2}{s^2} B_o(k^2;M_W,M_Z) + B_o(k^2;M_W,0))] . \tag{3.18'}$$

### 3.3 Generalized Ward identities

The identities (3.10) and (3.16) relate unphysical parts of Green functions. In analogy to QED where we know a Ward identity [23] between the $ee\gamma$-vertex and the electron propagator we can derive from eq. (3.8) by differentiating twice with respect to the sources of the fermion fields identities relating fermion vertices to fermion propagators:




$$k^\mu \tau_\mu^{\alpha,\sigma\sigma'}(k,p,q) - iM_\alpha \tau^{\phi\sigma\sigma'}(k,p,q) =$$
$$= iG^{\alpha\beta}(k)[g^\beta T^\beta_{\sigma\sigma''} S_F^{\sigma''\sigma'}(q) + S_F^{\sigma\sigma''}(p) g^\beta \bar{T}^\beta_{\sigma''\sigma'}]$$
$$+ ig^\beta T^\beta_{\sigma\sigma''} \int d^4r\, G_{con}^{\alpha\beta\ \sigma''\sigma'}(k,r-p,r,q) \tag{3.19}$$
$$+ i\int d^4r\, G_{con}^{\alpha\beta,\sigma\sigma''}(k,q-r,p,r)\, g^\beta \bar{T}^\beta_{\sigma''\sigma'}$$

with

$$\bar{T} = -\gamma_0 T^T \gamma_0 .$$

The physical content of these identities can be seen by evaluating them in 1-loop approximation and inserting the results (3.17). Neglecting terms of order $\alpha \cdot m_{i\sigma}/(\sqrt{k^2}, M_W, M_Z)$ i.e. for high energies, the following generalisations of the ordinary QED Ward identity are valid:



$$k^\mu(\Lambda_\mu^{\gamma ee}(k,p,q) + \Lambda_\mu^{\gamma\nu\nu}(k,p,q)) = e[\Sigma^e(p) - \Sigma^e(q)] , \tag{3.20}$$
$$k^\mu(\Lambda_\mu^{\gamma uu}(k,p,q) + \Lambda_\mu^{\gamma dd}(k,p,q)) = e[\frac{2}{3}(\Sigma^u(p) - \Sigma^u(q)) + \frac{1}{3}(\Sigma^d(p) - \Sigma^d(q))]$$

and similar relations for the other fermion generations. The vertex functions $\Lambda_\mu^{\alpha\sigma\sigma'}$ are the 1-loop contributions to the amputated 3-point Green functions $\tau_\mu$, and the fermion self energies $\Sigma^{i\sigma}$ follow from the propagators:

$$iS_F^{-1} = \not{p} - m_{i\sigma} + \Sigma^{i\sigma}(p) = \not{p} - m_{i\sigma} + \not{p}\Sigma_V^{i\sigma} + \not{p}\gamma_5\Sigma_A^{i\sigma} + m_{i\sigma}\Sigma_S^{i\sigma} . \tag{3.21}$$

As a result the QED identities for each separate charged fermion are replaced by similar identities for the fermion doublets. In addition we have found analogous relations for the vertices of the fermions and the heavy gauge boson Z (written for the first fermion doublet):

$$k^\mu(\Lambda_\mu^{Z\nu\nu}(k,p,q) + \Lambda_\mu^{Zee}(k,p,q) = \frac{e}{cs}[\Sigma^\nu(p)\,\frac{1}{2}\cdot\frac{1-\gamma_5}{2} - \frac{1}{2}\cdot\frac{1+\gamma_5}{2}\,\Sigma^\nu(q) +$$
$$+ \Sigma^e(p)(-\frac{1}{2}\cdot\frac{1-\gamma_5}{2} + s^2) + (\frac{1}{2}\cdot\frac{1+\gamma_5}{2} - s^2)\Sigma^e(q)] , \tag{3.22}$$

$$k^\mu(\Lambda_\mu^{Zuu}(k,p,q) + \Lambda_\mu^{Zdd}(k,p,q)) = \frac{e}{cs}[\Sigma^u(p)(\frac{1}{2}\cdot\frac{1-\gamma_5}{2} - \frac{2}{3}s^2) - (\frac{1}{2}\cdot\frac{1+\gamma_5}{2} - \frac{2}{3}s^2)\Sigma^u(q) +$$
$$+ \Sigma^d(p)(-\frac{1}{2}\cdot\frac{1-\gamma_5}{2} + \frac{1}{3}s^2) + (\frac{1}{2}\cdot\frac{1+\gamma_5}{2} - \frac{1}{3}s^2)\Sigma^d(q)] .$$

## 4. Renormalization conditions in the on-shell scheme

The study of the counter terms in the Lagrangian and of the detailed Slavnov-Taylor identities allows us to formulate explicitly the renormalization conditions. Thereby not only the ultraviolet divergencies occuring in the loop expansion are absorbed in the infinite parts of the renormalization constants but also the finite parts are fixed. These lead to physically observable consequences. As already mentioned in the introduction various more or less elaborate renormalizations for the standard model are used in the literature. They differ in the choice of the physical parameters and the prescriptions for the finite parts of the field renormalization constants. Although the results obtained with different consistent renormalizations formally deviate from each other only in higher order terms it may be that radiative corrections calculated with a low energy renormalization scheme and applied to high energy experiments differ sensibly from high energy renormalization calculations.

We propose a high energy renormalization scheme which is defined by the following conditions *):

- The poles of the renormalized propagators lie at $M_W^2$, $M_Z^2$, 0, $M_H^2$, $m_{i\sigma}^2$. This implies for the renormalized self energies:

$$\hat{\Sigma}_T^W(M_W^2) = \hat{\Sigma}_T^Z(M_Z^2) = \hat{\Sigma}^\eta(M_H^2) = \hat{\Sigma}^{i\sigma}(m_{i\sigma}^2) = 0. \qquad (4.1)$$

- According to the residual $U(1)^{em}$ symmetry it is possible to renormalize so that the properties of the photon and the electric charge are defined like in QED:

$$\frac{1}{k^2}\hat{\Sigma}_T^\gamma(k^2)\Big|_{k^2=0} = 0\ , \quad \hat{\Sigma}_T^{\gamma Z}(0) = 0\ , \quad \hat{\Gamma}_\mu^{\gamma ee}(k^2=0, \not p=\not q=m_e) = ie\gamma_\mu\ ^{**)}. \qquad (4.2)$$

---

*) In the following equations only real parts of self energies enter. The imaginary parts are finite by themselves and we define the mass as the real part of the pole position in the propagator.

**) This is a condition for the vector part of the photon vertex $\hat{\Gamma}_\mu^{\gamma ee}$, only. For the axial vector part no seperate condition has to be imposed since $\hat{\Gamma}_{\mu,A}^{\gamma ee}(k^2=0) = 0$ is automatically fulfilled.



- The residues of the propagators of fermions with $I^3 = -1/2$ and of the physical Higgs particle are 1:

$$\left(\frac{1}{\not p - m_{i-}}\hat{\Sigma}^{i-}(p)\right)\Big|_{\not p=m_{i-}} = 0\ , \quad \left(\frac{\partial}{\partial p^2}\hat{\Sigma}^\eta(p^2)\right)\Big|_{p^2=M_H^2} = 0\ . \qquad (4.3)$$

- Vanishing tadpole:

$$\hat{T} = 0. \qquad (4.4)$$

- The poles in the unphysical sector are at $M_W^2$, $M_Z^2$, 0:

$$\hat{\Sigma}_L^W(M_W^2) = \hat{\Sigma}_L^Z(M_Z^2) = \hat{\Sigma}^\phi(M_W^2) = \hat{\Sigma}^\chi(M_Z^2) = 0,$$

$$\frac{1}{k^2}\hat{\Sigma}_L^\gamma\Big|_{k^2=0} = 0. \qquad (4.5)$$

- The residue of the photon ghost propagator is one and the photon ghost-Z ghost mixing propagator vanishes at $k^2 = 0$:

$$\frac{1}{k^2}\hat{\tilde{\Sigma}}^\gamma(k^2)\Big|_{k^2=0}\ , \quad \hat{\tilde{\Sigma}}^{\gamma Z}(0) = 0\ . \qquad (4.6)$$

The conditions (4.1) - (4.6) fix all the renormalization constants of eq. (2.17). We have chosen our scheme in such a way that the following properties hold:

- We use as physical renormalized parameters e, $M_W$, $M_Z$, $M_H$, $m_f$. The experiments at the $P\bar{P}$ collider have determined $M_W$ and $M_Z$ to an accuracy which defines the parameters of the standard model at least as precisely as the low-energy $\mu$ decay and leptonic $\nu$ scattering do it for $G_F$ and $\Theta_W$ (comp. eq. (2.11)). We expect a further increase of the accuracy of $M_W$, $M_Z$ from the SLC and LEP experiments.

- Eq.s (4.2) characterize our procedure as a natural extension of the successful QED renormalization. This means in practice that existing results on photonic corrections [18] can be taken over directly. Especially 1-loop calculations can be divided into real and virtual photonic corrections (the sum of their contributions in physical cross sections is infrared finite) and weak corrections (IR finite by themselves). We may consider $e^2/16\pi^2 = \alpha/4\pi = 1/(4\pi\cdot 137.036)$ as the effective expansion parameter.

- We work only with one field renormalization constant for a symmetry multiplet. Therefore renormalization conserves the gauge transformation properties of the fields and the Green functions. As a consequence of this minimal number of field renormalization constants not all the residues of the renormalized propagators are one. This is the case for the W, Z and the $I^3 = +1/2$ fermions. Therefore wave function renormalization for in- and outgoing particles is needed. These do not occur in the complete amplitudes for physical S matrix elements.

- In $\mathcal{L}_{fix} + \mathcal{L}_{FP}$ we have built in the renormalization constants $\delta\xi^{\alpha}_{1,2}$ and fixed them in such a way that the simple pole structure of the 't Hooft-Feynman gauge survives renormalization. The Slavnov-Taylor identities (3.15) and (3.18) guarantee that with the conditions (4.5) also the poles in the other unphysical propagators $\Delta^{W\phi};\ldots;\ G^{W};\ldots$ are at the same positions. This simplifies considerably the evaluation of Feynman diagrams.

- We have checked that the Ward identities for the fermion gauge boson vertices (3.19) are compatible with our renormalization prescription.

Finally we translate the conditions (4.1) - (4.6) into prescriptions for the singular and finite parts of the renormalization constants (2.17) resp. their combinations (app.A):

$$\Sigma^W_T(M_W^2) = \delta M_W^2 = M_W^2(-2\frac{\delta v}{v} + 2\delta Z_1^W - 3\delta Z_2^W + \delta Z^\phi) ,$$

$$\Sigma^Z_T(M_Z^2) = \delta M_Z^2 = M_Z^2(-2\frac{\delta v}{v} + 2\delta Z_1^Z - 3\delta Z_2^Z + \delta Z^\phi) ,$$

$$\Sigma^\eta(M_H^2) = \delta M_H^2 = M_H^2(-3\frac{\delta v}{v} + \frac{3}{2}\delta Z^\lambda - \delta Z^\phi) + \delta\mu^2 ,$$

$$\Sigma_V^{i\sigma}(m_{i\sigma}) + \Sigma_S^{i\sigma}(m_{i\sigma}) = \frac{\delta m_{i\sigma}}{m_{i\sigma}} = \delta Z_1^{i\sigma} - \frac{\delta v}{v} ; \qquad (4.1')$$



$$\frac{1}{k^2}\Sigma_T^\gamma(k^2)\Big|_{k^2=0} = -\delta Z_2^\gamma ,$$

$$\Sigma_T^{\gamma Z}(0) = M_Z^2(\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z}) ,$$

$$\frac{\delta e}{e} = \delta Z_1^\gamma - \frac{3}{2}\delta Z_2^\gamma ; \qquad (4.2')$$

$$\Sigma_L^e(m_e) + m_e^2(\Sigma_L^{e\prime}(m_e) + \Sigma_R^{e\prime}(m_e) + 2\Sigma_S^{e\prime}(m_e)) = -\delta Z_L^{(e,\nu_e)} ,$$

$$\Sigma_R^e(m_e) + m_e^2(\Sigma_L^{e\prime}(m_e) + \Sigma_R^{e\prime}(m_e) + 2\Sigma_S^{e\prime}(m_e)) = -\delta Z_R^e ,$$

$$\Sigma^{\eta\prime}(M_H^2) = -\delta Z^\phi ; \qquad (4.3')$$

$$\frac{\delta t}{t} = \frac{\delta v}{v} - \frac{\delta\mu^2}{M_H^2} - \frac{1}{2}\delta Z^\lambda , \quad t = M_W M_H^2 \cdot \frac{2s}{e} ; \qquad (4.4')$$

$$\Sigma_L^W(M_W^2) = \delta M_W^2 + M_W^2\delta\xi_1^W ,$$

$$\Sigma_L^Z(M_Z^2) = \delta M_Z^2 + M_Z^2\delta\xi_1^Z ,$$

$$\Sigma^\phi(M_W^2) = \delta M_W^2 + M_W^2\delta\xi_2^W - M_H^2\frac{\delta t}{t} ,$$

$$\Sigma^\chi(M_Z^2) = \delta M_Z^2 + M_Z^2\delta\xi_2^Z - M_H^2\frac{\delta t}{t} ,$$

$$\frac{1}{k^2}\Sigma_L^\gamma(k^2)\Big|_{k^2=0} = \delta\xi_1^\gamma - \delta Z_2^\gamma , \qquad (4.5')$$

$$\frac{1}{k^2}\tilde\Sigma^\gamma(k^2)\Big|_{k^2=0} = -\delta\tilde Z_2^\gamma ,$$

$$\tilde\Sigma^{\gamma Z}(0) = M_Z^2(\delta Z_1^{\gamma Z} - \frac{3}{2}\delta\tilde Z_2^{\gamma Z} + \frac{1}{2}\delta\xi_2^{\gamma Z}) . \qquad (4.6')$$

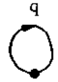
## 5. Explicit results in 1-loop approximation

The intention of this paper is to provide the building blocks needed to compute radiative electroweak corrections to $e^+e^-$ annihilation, deep inelastic scattering and other processes at high energies. We do this by evaluating the explicit 1-loop expressions for the renormalization constants, renormalized self energies and fermion gauge boson vertices. The calculations are performed analytically, thereby neglecting terms which are of the order of magnitude $\alpha m_f^2/(M_W^2, s)$ in the final results. The ultraviolet divergences are treated with the method of dimensional regularization [24]. This is possible since the standard model is free of $\gamma_5$-anomalies. The 4-dimensional integration and the Dirac and tensor structures are replaced by D-dimensional ones:

$$\int \frac{d^4q}{(2\pi)^4} \rightarrow \mu^{4-D} \int \frac{d^Dq}{(2\pi)^D} \qquad (5.1)$$

($\mu$ is introduced for dimensional reasons).

In order to explain our notation we give the results for the scalar tadpole integral:

q 

$$\rightarrow \mu^{4-D} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{q^2-M^2+i\varepsilon} = \frac{-i}{16\pi^2} A(M)$$

$$A(M) = -M^2(\Delta_M+1). \qquad (5.2)$$

The UV divergent part $\Delta_M$ contains the Euler-Mascheroni constant $\gamma$ and has the form:

$$\Delta_M = \frac{2}{4-D} - \gamma - \ln\frac{M^2}{4\pi\mu^2}. \qquad (5.3)$$

The scalar 1-loop self energy integral defines the function $B_o(k^2;M_1,M_2)$:

q+k  q

$$\rightarrow \mu^{4-D} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{(q^2-M_1^2+i\varepsilon)((q+k)^2-M_2^2+i\varepsilon)}$$

$$= \frac{i}{16\pi^2} B_o(k^2;M_1,M_2)$$



with:

$$B_o(k^2;M_1,M_2) = \frac{1}{2}\Delta_{M_1}+\frac{1}{2}\Delta_{M_2}+1-\frac{M_1^2+M_2^2}{M_1^2-M_2^2}\ln\frac{M_1}{M_2} + F(k^2;M_1,M_2)$$

$$= \frac{1}{2}\Delta_{M_1}+\frac{1}{2}\Delta_{M_2} - \int_o^1 dx \ln\frac{x^2k^2-x(k^2+M_1^2-M_2^2)+M_1^2-i\varepsilon}{M_1M_2}. \qquad (5.4)$$

The function $F(k^2;M_1,M_2) = F(k^2;M_2,M_1)$ is finite and vanishes for $k^2 = 0$. Its explict form is written in app. B.1.

### 5.1 The tadpole

The vacuum expectation value v of the Higgs field, which in lowest order is given by $v^2 = \mu^2/\lambda$ gets 1-loop contributions from the diagrams of fig. 1. They lead in the 't Hooft-Feynman gauge to the expressions:

$$T = \frac{1}{16\pi^2}\frac{e}{s}\cdot\frac{1}{2M_W}\{-M_H^2(A(M_W)+\frac{1}{2}A(M_Z)+\frac{3}{2}A(M_H))$$

$$-6M_W^2A(M_W)-4M_W^4-3M_Z^2A(M_Z)-2M_Z^4+4\sum_{i\sigma} m_{i\sigma}^2A(m_{i\sigma})\} \qquad (5.5)$$

$$= \delta t = \frac{2s}{e} M_W M_H^2 \, \delta t/t .$$

The tadpole diagrams of fig. 1 give for example contributions to the self energies. These are absorbed by mass renormalization rending the $\delta M_1^2$ gauge independent.

### 5.2 Unrenormalized self energies and vertex functions

#### a) Gauge boson self energies

The contributions of the diagrams of fig. 2 to the longitudinal and transverse unrenormalized self energies have been computed by [9]. We present them decomposed into the singular parts (defined to be proportional to $\Delta$) and finite parts:

$$\Sigma(k^2) = \Sigma_{sing}(k^2,\Delta) + \Sigma_{fin}(k^2) \ . \qquad (5.6)$$

The explicit expressions are (the index f denotes any fermion $i\sigma$; $v_f = v_{i\sigma}$, $a_f = a_{i\sigma}$ are defined in app. A):

$$\Sigma^{\gamma}_{T,sing} = \frac{\alpha}{4\pi} k^2 \left\{\frac{4}{3} \sum_f Q_f^2 \Delta_f - 3\Delta_W\right\} ,$$

$$(5.7)$$

$$\Sigma^{\gamma}_{T,fin} = \frac{\alpha}{4\pi} \left\{\frac{4}{3} \sum_f Q_f^2 [(k^2+2m_f^2)F(k^2;m_f,m_f) - \frac{k^2}{3}] - (3k^2+4M_W^2)F(k^2;M_W,M_W)\right\} ,$$

$$\Sigma^{\gamma Z}_{T,sing} = \frac{\alpha}{4\pi} \left\{-\frac{4}{3} \sum_f Q_f v_f k^2 \Delta_f + \frac{1}{cs}[k^2(3c^2+\frac{1}{6}) + 2M_W^2]\Delta_W\right\} ,$$

$$(5.8)$$

$$\Sigma^{\gamma Z}_{T,fin} = \frac{\alpha}{4\pi} \left\{-\frac{4}{3} \sum_f Q_f v_f [(k^2+2m_f^2)F(k^2;m_f,m_f) - \frac{k^2}{3}]\right.$$

$$\left. + \frac{1}{cs}[k^2(3c^2+\frac{1}{6}) + M_W^2(4c^2+\frac{4}{3})] F(k^2;M_W,M_W) + \frac{k^2}{9cs}\right\} ;$$

$$\Sigma^{Z}_{T,sing} = \frac{\alpha}{4\pi} \left\{\frac{4}{3} \sum_f (v_f^2+a_f^2)k^2\Delta_{\tilde f} + [k^2(3-\frac{19}{6s^2}+\frac{1}{6c^2}) + M_Z^2(4+\frac{1}{c^2}-\frac{1}{s^2})]\Delta_W\right\} + \quad {*)}$$

$$+ 2 M_Z^2 \,\delta t/t ,$$

$$(5.9)$$

$$\Sigma^{Z}_{T,fin} = \frac{\alpha}{4\pi} \left\{\frac{4}{3} \sum_f [(v_f^2+a_f^2)((k^2+2m_f^2) F(k^2;m_{\tilde f},m_{\tilde f}) - \frac{k^2}{3}) - \frac{3}{8c^2s^2}m_f^2 F(k^2;m_f,m_f)]\right.$$

$$- \frac{c^2}{3s^2}[\frac{2}{3} k^2 + (10k^2+20M_W^2) F(k^2;M_W,M_W)] +$$

$$+ \frac{1}{3s^2c^2}[3M_W^2 F(k^2;M_W,M_W) + \frac{1}{4}(10M_Z^2-2M_H^2+k^2)(1-\frac{M_H^2+M_Z^2}{M_H^2-M_Z^2} \ln\frac{M_H}{M_Z}$$

$$- \ln\frac{M_H M_Z}{M_W^2} + F(k^2;M_H,M_Z)) - \frac{1}{2}M_H^2 \ln\frac{M_H^2}{M_W^2} - \frac{1}{2} M_Z^2 \ln\frac{M_Z^2}{M_H^2} +$$

$$+ \frac{k^2}{6} + (M_H^2-M_Z^2)^2 F(k^2;M_Z,M_H)/4k^2] +$$

$$\left. + \frac{(c^2-s^2)^2}{3s^2c^2} [\frac{k^2}{6} + (2M_W^2+\frac{k^2}{4}) F(k^2;M_W,M_W)]\right\} ;$$

---

*) $\tilde f = f$ for $f \neq \nu_\ell$ and $\tilde f = \ell$ for $f = \nu_\ell$



$$\Sigma^{W}_{T,sing} = \frac{\alpha}{4\pi}\cdot\frac{1}{s^2} \left\{\frac{1}{6} \sum_i{}^{*)} [\Delta_{i+}(k^2-\frac{5}{2}m_{i+}^2-\frac{1}{2}m_{i-}^2) + \Delta_{i-}(k^2-\frac{5}{2}m_{i-}^2-\frac{1}{2}m_{i+}^2)] -\right.$$

$$\left. - [M_W^2(1-\frac{s^2}{c^2}) + \frac{19}{6} k^2] \Delta_W\right\} + 2 M_W^2 \,\delta t/t ,$$

$$(5.10)$$

$$\Sigma^{W}_{T,fin} = \frac{\alpha}{4\pi} \left\{\frac{1}{3s^2} \sum_i [(k^2-\frac{m_{i+}^2+m_{i-}^2}{2})(1-\frac{m_{i+}^2+m_{i-}^2}{m_{i+}^2-m_{i-}^2} \ln\frac{m_{i+}}{m_{i-}}{}^{*)} + F(k^2;m_{i+},m_{i-}))-\right.$$

$$- \frac{k^2}{3} - \frac{(m_{i+}^2-m_{i-}^2)^2}{2k^2} F(k^2;m_{i+},m_{i-})]$$

$$- \frac{c^2}{3s^2} [(7M_Z^2+7M_W^2+10k^2)(1-\frac{M_Z^2}{M_Z^2-M_W^2} \ln\frac{M_Z^2}{M_W^2} + F(k^2;M_Z,M_W))$$

$$+4M_Z^2 \ln\frac{M_Z^2}{M_W^2} + \frac{2}{3}k^2 - \frac{2}{k^2}(M_Z^2-M_W^2)^2 F(k^2;M_Z,M_W)]$$

$$- \frac{4}{3}M_W^2 - \frac{32}{9}k^2 - \frac{1}{3}(4M_W^2+10k^2) F(k^2;0,M_W) + \frac{2}{3}\frac{M_W^4}{k^2} F(k^2;0,M_W)$$

$$+ \frac{s^2}{c^2} [1 - \frac{M_Z^2}{M_Z^2-M_W^2} \ln\frac{M_Z^2}{M_W^2} + F(k^2;M_Z,M_W)]M_W^2$$

$$+ \frac{1}{s^2} [1 - \frac{M_H^2}{M_H^2-M_W^2} \ln\frac{M_H^2}{M_W^2} + F(k^2;M_H,M_W)]M_W^2$$

$$+ \frac{1}{s^2} \left\{\frac{5}{18} k^2 - \frac{1}{3} M_W^2 - \frac{1}{6} M_Z^2 - \frac{1}{6} M_H^2 + \frac{1}{6}(2M_W^2-\frac{1}{2} k^2) \frac{M_Z^2}{M_Z^2-M_H^2} \ln\frac{M_Z^2}{M_W^2}\right.$$

$$- \frac{1}{6}(M_W^2+M_Z^2-\frac{k^2}{2}) F(k^2;M_Z,M_W) + (M_Z^2-M_W^2)^2 F(k^2;M_Z,M_W)/12k^2 +$$

$$+ \frac{1}{6}(2M_W^2-\frac{1}{2}k^2) \frac{M_H^2}{M_H^2-M_W^2} \ln\frac{M_H^2}{M_W^2} - \frac{1}{6} (M_W^2+M_H^2-\frac{k^2}{2}) F(k^2;M_H,M_W) +$$

$$\left.\left. + (M_H^2-M_W^2)^2 F(k^2;M_H,M_W)/12k^2\right]\right\} ;$$

---

*) For lepton doublets with $m_{i+}=0$ replace $\Delta_{i+} \to \Delta_{i-}$ and drop the log term in $\Sigma^{W}_{T,fin}$

$$\Sigma^{\gamma}_{L,sing} = \Sigma^{\gamma}_{L,fin} = 0 \; ; \tag{5.11}$$

$$\Sigma^{Z}_{L,sing} = \frac{\alpha}{4\pi} \{M_Z^2(2\frac{c^2}{s^2} - \frac{1}{c^2s^2} - 2)\,\Delta_W\} + 2M_Z^2\,\delta t/t \; ,$$

$$\Sigma^{Z}_{L,fin} = \frac{\alpha}{4\pi} \{-\frac{1}{c^2s^2}\,[M_Z^2 - \frac{M_Z^2}{M_H^2-M_Z^2}(M_H^2 \ln\frac{M_H^2}{M_W^2} - M_Z^2 \ln\frac{M_Z^2}{M_W^2}) + \tag{5.12}$$

$$+ (M_Z^2 - \frac{(M_H^2-M_Z^2)^2}{4k^2})\,F(k^2;M_H,M_Z)] + 2\,\frac{c^2-s^2}{s^2}\,M_Z^2\,F(k^2;M_W,M_W)\} \; ;$$

$$\Sigma^{\gamma Z}_{L,sing} = \frac{\alpha}{4\pi} \{-2\frac{c}{s}\,M_Z^2\,\Delta_W\} \; ,$$

$$\Sigma^{\gamma Z}_{L,fin} = \frac{\alpha}{4\pi} \{-2\frac{c}{s}\,M_Z^2\,F(k^2;M_W,M_W)\} \; ; \tag{5.13}$$

$$\Sigma^{W}_{L,sing} = \frac{\alpha}{4\pi} \{\frac{c^2-s^2}{c^2s^2}\,M_W^2\,\Delta_W\} + 2M_W^2\,\delta t/t \; ,$$

$$\Sigma^{W}_{L,fin} = \frac{\alpha}{4\pi} \{\frac{c^2-s^2}{c^2s^2}\,M_W^2 + \frac{3s^2-2}{s^4}\,M_Z^2 \ln\frac{M_Z^2}{M_W^2} + \tag{5.14}$$

$$+ \frac{1}{s^2}\,\frac{M_W^2}{M_H^2 - M_W^2}\,M_H^2 \ln\frac{M_H^2}{M_W^2} + 2\,\frac{M_W^4}{k^2}\,F(k^2;0,M_W) +$$

$$+ [\frac{3c^2-1}{c^2s^2}\,M_W^2 + (2\frac{c^2}{s^2} + \frac{1}{4s^2})\frac{(M_W^2 - M_Z^2)^2}{k^2}]\,F(k^2;M_W,M_Z) +$$

$$- \frac{1}{s^2}(M_W^2 - \frac{(M_H^2-M_W^2)^2}{4k^2})\,F(k^2;M_W,M_H)\} \; .$$



b) Gauge boson Higgs boson mixing

The diagrams of fig. 3 contribute to the gauge boson Higgs boson mixing self energies defined in eq. (3.12). The singular and finite parts of them are

$$\Sigma^{\gamma\chi}_{sing} = \frac{\alpha}{4\pi} \{2\,\frac{c}{s}\,M_Z\,\Delta_W\} \; ,$$

$$\Sigma^{\gamma\chi}_{fin} = \frac{\alpha}{4\pi} \{2\,\frac{c}{s}\,M_Z\,F(k^2;M_W,M_W)\} \; ; \tag{5.15}$$

$$\Sigma^{Z\chi}_{sing} = \frac{\alpha}{4\pi} \{2 + \frac{1}{2c^2} + \frac{1}{4c^2s^2}\}\,M_Z\,\Delta_W - M_Z\,\delta t/t \; ,$$

$$\Sigma^{Z\chi}_{fin} = \frac{\alpha}{4\pi}\,M_Z\,\{\frac{3}{4c^2s^2}\,[1 - \frac{1}{M_H^2-M_Z^2}(M_H^2 \ln\frac{M_H^2}{M_W^2} - M_Z^2 \ln\frac{M_Z^2}{M_W^2}) + \tag{5.16}$$

$$+ (1 - \frac{(M_Z^2-M_H^2)^2}{3k^2M_Z^2})\,F(k^2;M_H,M_Z)] + \frac{4s^2-1}{2s^2}\,F(k^2;M_W,M_W)\} \; ;$$

$$\Sigma^{W\phi}_{sing} = \frac{\alpha}{4\pi}\,M_W\,\{\frac{1}{2s^2} - \frac{3}{4c^2s^2}\}\,\Delta_W + M_W\,\delta t/t \; ,$$

$$\Sigma^{W\phi}_{fin} = \frac{\alpha}{4\pi}\,M_W\,\{-\frac{1}{4s^2} - \frac{3}{4c^2s^2} + \tag{5.17}$$

$$+ \frac{3}{4s^2}\,[1 - \frac{M_H^2}{M_W^2-M_H^2} \ln\frac{M_H^2}{M_W^2} - (1 - \frac{(M_H^2-M_W^2)^2}{3k^2M_W^2})\,F(k^2;M_W,M_H)]$$

$$+ \frac{4c^2s^2+5c^2-3}{4c^2s^2}\,[\frac{M_Z^2}{M_W^2-M_Z^2} \ln\frac{M_Z^2}{M_W^2} + F(k^2;M_W,M_Z)]$$

$$- \frac{8c^2+1}{4c^2}\,\frac{M_W^2-M_Z^2}{k^2}\,F(k^2;M_W,M_Z) + (2\,\frac{M_W^2}{k^2} - 1)\,F(k^2;0,M_W)\} \; .$$

c) Higgs boson self energies

The self energy contributions of the Higgs bosons to the physical S matrix elements are only of the order of magnitude $\alpha m_f^2/(M_W^2,s)$. Nevertheless we need them in order to calculate the full set of renormalization constants. The diagrams of fig. 4 give

$$\Sigma^{\eta}_{sing} = \frac{\alpha}{4\pi}\{\frac{2c^2+1}{2c^2s^2}k^2 - \frac{17}{4c^2s^2}M_Z^2 - \frac{17}{2s^2}M_W^2 - \frac{2c^2+1}{8c^2s^2}M_H^2 - \frac{27}{8s^2}\frac{M_H^4}{M_W^2}\}\Delta_W + 3M_H^2\,\delta t/t,$$

$$\Sigma^{\eta}_{fin} = \frac{\alpha}{4\pi}\{\frac{7}{4c^2s^2}(M_Z^2+2c^2M_W^2) - \frac{2c^2+1}{8c^2s^2}M_H^2 - \frac{3}{8s^2}\frac{M_H^4}{M_W^2} \tag{5.18}$$

$$+ \frac{1}{4c^2s^2}[-2k^2+17M_Z^2+\frac{1}{2}M_H^2+\frac{M_H^4}{M_Z^2}]\ln\frac{M_Z^2}{M_W^2} +$$

$$+ \frac{21}{8s^2}\frac{M_H^4}{M_W^2}\ln\frac{M_H^2}{M_W^2} - \frac{9}{4s^2}\frac{M_H^4}{M_W^2}F(k^2;M_H,M_H) +$$

$$+ \frac{1}{s^2}[k^2 - 7M_W^2 - \frac{M_H^4}{2M_W^2}]F(k^2;M_W,M_W) +$$

$$+ \frac{1}{2c^2s^2}[k^2 - 7M_Z^2 - \frac{M_H^4}{2M_H^2}]F(k^2;M_Z,M_Z)\} ;$$

$$\Sigma^{\chi}_{sing} = \frac{\alpha}{4\pi}\frac{2c^2+1}{2c^2s^2}k^2\Delta_W + M_H^2\,\delta t/t ,$$

(5.19)

$$\Sigma^{\chi}_{fin} = \frac{\alpha}{4\pi}\{\frac{1}{2c^2s^2}k^2[1-\frac{1}{M_H^2-M_Z^2}(M_H^2\ln\frac{M_H^2}{M_W^2} - M_Z^2\ln\frac{M_Z^2}{M_W^2}) +$$

$$+ (1 - \frac{(M_H^2-M_Z^2)^2}{2k^2M_Z^2})F(k^2;M_H,M_Z)] + \frac{1}{s^2}k^2F(k^2;M_W,M_W)\} ;$$



$$\Sigma^{\phi}_{sing} = \frac{\alpha}{4\pi}\frac{2c^2+1}{2c^2s^2}k^2\Delta_W + M_H^2\,\delta t/t ,$$

(5.20)

$$\Sigma^{\phi}_{fin} = \frac{\alpha}{4\pi}\{\frac{2c^2+1}{2c^2s^2}k^2 + \frac{1+c^2-4c^2s^2}{2c^2s^2}k^2[\frac{M_Z^2}{M_W^2-M_Z^2}\ln\frac{M_Z^2}{M_W^2} + F(k^2;M_W,M_Z)] +$$

$$+ \frac{k^2M_H^2}{2s^2(M_W^2-M_H^2)}\ln\frac{M_H^2}{M_W^2} - \frac{(8c^2+1)s^2}{4c^2}M_Z^2F(k^2;M_W,M_Z) +$$

$$+ \frac{1}{4s^2}(2k^2 - \frac{(M_W^2-M_H^2)^2}{M_W^2})F(k^2;M_W,M_H) + 2(k^2-M_W^2)F(k^2;0,M_W)\} .$$

d) Ghost self energies

The ghost self energies (diagrams of fig. 5) are:

$$\tilde{\Sigma}^{\gamma}_{sing} = \frac{\alpha}{4\pi}k^2\Delta_W , \quad \tilde{\Sigma}^{\gamma}_{fin} = \frac{\alpha}{4\pi}k^2F(k^2;M_W,M_W) ; \tag{5.21}$$

$$\tilde{\Sigma}^{\gamma Z}_{sing} = \frac{\alpha}{4\pi}\{-\frac{c}{s}(k^2+M_Z^2)\Delta_W\} ,$$

(5.22)

$$\tilde{\Sigma}^{\gamma Z}_{fin} = \frac{\alpha}{4\pi}\{-\frac{c}{s}(k^2+M_Z^2)F(k^2;M_W,M_W)\} ;$$

$$\tilde{\Sigma}^{Z\gamma}_{sing} = \frac{\alpha}{4\pi}\{-\frac{c}{s}k^2\Delta_W\} , \quad \tilde{\Sigma}^{Z\gamma}_{fin} = \frac{\alpha}{4\pi}\{-\frac{c}{s}k^2F(k^2;M_W,M_W)\} ; \tag{5.23}$$

$$\tilde{\Sigma}^{Z}_{sing} = \frac{\alpha}{4\pi}\{\frac{c^2}{s^2}k^2 - (1+\frac{1}{4c^2s^2}-\frac{1}{2s^2})M_Z^2\}\Delta_W + M_Z^2\,\delta t/t ,$$

(5.24)

$$\tilde{\Sigma}^{Z}_{fin} = \frac{\alpha}{4\pi}\{-\frac{M_Z^2}{4c^2s^2}[1-\frac{1}{M_H^2-M_Z^2}(M_H^2\ln\frac{M_H^2}{M_W^2} - M_Z^2\ln\frac{M_Z^2}{M_W^2}) + F(k^2;M_H,M_Z)] +$$

$$+ \frac{c^2}{s^2}(k^2 - \frac{2s^2-1}{2c^2}M_Z^2)F(k^2;M_W,M_W)\} ;$$

$$\tilde{\Sigma}^W_{sing} = \frac{\alpha}{4\pi} \{\frac{1}{s^2} k^2 + \frac{2c^2-1}{4c^2s^2} M_W^2\} \Delta_W + M_W^2 \, \delta t/t \, , \tag{5.25}$$

$$\tilde{\Sigma}^W_{fin} = \frac{\alpha}{4\pi} \{\frac{1}{s^2} k^2 + \frac{2c^2-1}{4c^2s^2} M_W^2 - \frac{1}{s^2} (\frac{c^2}{s^2} k^2 + \frac{3c^2-1}{4c^2s^2} M_W^2) \ln \frac{M_Z^2}{M_W^2} +$$

$$+ \frac{1}{4s^2} \frac{M_H^2 M_W^2}{M_H^2-M_W^2} \ln \frac{M_H^2}{M_W^2} + k^2 F(k^2;M_W,0) +$$

$$+ (\frac{c^2}{s^2} k^2 + \frac{3c^2-1}{4c^2s^2} M_W^2) F(k^2;M_W,M_Z) - \frac{1}{4s^2} M_W^2 F(k^2;M_W,M_H)\} .$$

e) Fermion self energies

Because of Lorentz covariance we can decompose the self energies $\Sigma^{i\sigma}(k)$ of the fermions:

$$\Sigma^{i\sigma}(k) = \not k \, \Sigma_V^{i\sigma}(k^2) + \not k \, \gamma_5 \, \Sigma_A^{i\sigma}(k^2) + m_{i\sigma} \Sigma_S^{i\sigma}(k^2). \tag{5.26}$$

The diagrams of fig. 6 give the following contributions to the invariant functions $\Sigma^{i\sigma}_{V,A,S}$:

$$\Sigma_V^{i\sigma} = - \frac{\alpha}{4\pi} [Q^2_{i\sigma}(2B_1(k^2;m_{i\sigma},\lambda)+1) + (v^2_{i\sigma}+a^2_{i\sigma})(2B_1(k^2;m_{i\sigma},M_Z) + 1) + \tag{5.27}$$

$$+ \frac{1}{4s^2} (2B_1(k^2;m_{i\sigma'},M_W) + 1)] ,$$

$$\Sigma_A^{i\sigma} = - \frac{\alpha}{4\pi} [ \quad 0 \quad + 2v_{i\sigma}a_{i\sigma}(2B_1(k^2;m_{i\sigma},M_Z) + 1) -$$

$$- \frac{1}{4s^2}(2B_1(k^2;m_{i\sigma'},M_W) + 1)] ,$$

$$\Sigma_S^{i\sigma} = - \frac{\alpha}{4\pi} [Q^2_{i\sigma}(4B_o(k^2;m_{i\sigma},\lambda) - 2) + (v^2_{i\sigma} - a^2_{i\sigma})(4B_o(k^2;m_{i\sigma},M_Z) - 2) + 0] .$$

The photon contribution was calculated with a small photon mass $\lambda$ in order to regularize possible infrared divergencies. The functions $B_o$ and $B_1$ are defined in eq.s (5.4) and (B.2). Instead of the vector and axial vector



parts of the self energies $\Sigma_{V,A}$ it may be more convenient to use the right- and left-handed parts:

$$\Sigma_R = (\Sigma_V + \Sigma_A), \quad \Sigma_L = (\Sigma_V - \Sigma_A). \tag{5.27'}$$

f) Fermion gauge boson vertex functions

The vertex functions*) $\Gamma_\mu^{\alpha\sigma\sigma'}(k^2,p,q)$ contain for $|k^2| >> m^2_{\sigma,\sigma'}$ and $p^2 = m_\sigma^2$, $q^2 = m_{\sigma'}^2$, only vector and axial vector parts. The Feynman diagrams of fig. 7 yield (diagrams containing Higgs exchanges are neglected):

$$\Gamma_\mu^{\gamma\sigma\sigma}(k^2) = - i \, e \, Q_\sigma \, \gamma_\mu \tag{5.28}$$

$$- i \, e \, Q_\sigma \, \gamma_\mu \frac{\alpha}{4\pi} Q_\sigma^2 [\Delta_\sigma - 2\ln \frac{m_\sigma^2}{\lambda^2} + 4 + \Lambda_1(k^2,m_\sigma)]$$

$$- i \, e \, Q_\sigma[(v_\sigma^2 + a_\sigma^2)\gamma_\mu - 2v_\sigma a_\sigma \gamma_\mu \gamma_5] \frac{\alpha}{4\pi}[\Delta_Z - \frac{1}{2} + \Lambda_2(k^2,M_Z)]$$

$$- i \, e \, Q_{\sigma'} \gamma_\mu (1-\gamma_5) \frac{\alpha}{4\pi} \frac{1}{4s^2}[\Delta_W - \frac{1}{2} + \Lambda_2(k^2,M_W)]$$

$$- i \, e \, I_\sigma^3 \gamma_\mu (1-\gamma_5) \frac{\alpha}{4\pi} \frac{3}{2s^2}[\Delta_W - \frac{1}{6} + \Lambda_3(k^2,M_W)] \quad {**)} ,$$

$$\Gamma_\mu^{\gamma\nu\nu}(k^2) = i \, e \, \gamma_\mu(1-\gamma_5) \frac{\alpha}{4\pi} \frac{1}{4s^2}[\Delta_W - \frac{1}{2} + \Lambda_2(k^2,M_W)] \tag{5.29}$$

$$- i \, e \, \gamma_\mu(1-\gamma_5) \frac{\alpha}{4\pi} \frac{3}{4s^2}[\Delta_W - \frac{1}{6} + \Lambda_3(k^2,M_W)] ,$$

---

*) In the following we drop the fermion family index i since Cabbibo rotation is not involved.

**) In eq.s (5.28, 30) σ' denotes the isospin partner of the fermion σ.

$$\Gamma_\mu^{Z\sigma\sigma}(k^2) = i\ e\ \gamma_\mu(v_\sigma - a_\sigma\gamma_5) \tag{5.30}$$
$$+ i\ e\ \gamma_\mu(v_\sigma - a_\sigma\gamma_5)\ \frac{\alpha}{4\pi}\ Q_\sigma^2[\Delta_\sigma - 2\ln\frac{m_\sigma^2}{\lambda^2} + 4 + \Lambda_1(k^2,m_\sigma)]$$
$$+ i\ e[v_\sigma(v_\sigma^2 + 3a_\sigma^2)\gamma_\mu - a_\sigma(3v_\sigma^2 + a_\sigma^2)\gamma_\mu\gamma_5]\ \frac{\alpha}{4\pi}\ [\Delta_Z - \frac{1}{2} + \Lambda_2(k^2,M_Z)]$$
$$+ i\ e\ \gamma_\mu(1-\gamma_5)\ \frac{\alpha}{4\pi}\ \frac{v_{\sigma'}+a_{\sigma'}}{4s^2}[\Delta_W - \frac{1}{2} + \Lambda_2(k^2,M_W)]$$
$$+ i\ e\ I_\sigma^3\ \gamma_\mu(1-\gamma_5)\ \frac{\alpha}{4\pi}\ \frac{3c}{2s^3}[\Delta_W - \frac{1}{6} + \Lambda_3(k^2,M_W)]\ ,$$

$$\Gamma_\mu^{Z\nu\nu}(k^2) = i\ \frac{e}{2sc}\ \gamma_\mu\ \frac{1-\gamma_5}{2}\ \{1 + \frac{\alpha}{4\pi}\ \frac{1}{4s^2c^2}\ [\Delta_Z - \frac{1}{2} + \Lambda_2(k^2,M_Z)]$$
$$+ \frac{\alpha}{4\pi}\ \frac{2s^2-1}{2s^2}\ [\Delta_W - \frac{1}{2} + \Lambda_2(k^2,M_W)] \tag{5.31}$$
$$+ \frac{\alpha}{4\pi}\ \frac{3c^2}{s^2}\ [\Delta_W - \frac{1}{6} + \Lambda_3(k^2,M_W)]\}\ ,$$

$$\Gamma_\mu^{We\nu}(k^2) = i\ \frac{e}{\sqrt{2}s}\ \gamma_\mu\ \frac{1-\gamma_5}{2}\ \{1 + \frac{\alpha}{4\pi}\cdot\frac{2s^2-1}{4s^2c^2}\ [\Delta_Z - \frac{1}{2} + \Lambda_2(k^2,M_Z)]$$
$$+ \frac{\alpha}{4\pi}\cdot 3[\Delta_W + \frac{5}{6} + \Lambda_4^{m_e}(k^2;M_W,0)] \tag{5.32}$$
$$+ \frac{\alpha}{4\pi}\cdot\frac{3c^2}{s^2}[\Delta_W + \frac{5}{6} + \frac{M_Z^2}{M_Z^2 - M_W^2}\ \ln\frac{M_W^2}{M_Z^2} + \Lambda_4(k^2;M_Z,M_W)]\}\ ,$$

$$\Gamma_\mu^{Wdu}(k^2) = \frac{ie}{2\sqrt{2}s}\ \gamma_\mu(1 - \gamma_5)\cdot\{1 + \frac{\alpha}{4\pi}Q_uQ_d[\Delta_d - 2\ln\frac{m_d^2}{\lambda^2} + 4 + 3\ln\frac{m_d}{m_u} +$$
$$+ \frac{1}{2}\Lambda_1(k^2,m_d) + \frac{1}{2}\Lambda_1(k^2,m_u)] +$$
$$+ \frac{\alpha}{4\pi}(v_u+a_u)(v_d+a_d)[\Delta_Z - \frac{1}{2} + \Lambda_2(k^2,M_Z)] +$$
$$+ \frac{\alpha}{4\pi}3Q_u[\Delta_W + \frac{5}{6} + \Lambda_4^{m_u}(k^2;M_W,0)] - \frac{\alpha}{4\pi}3Q_d[\Delta_W + \frac{5}{6} + \Lambda_4^{m_d}(k^2;M_W,0) +$$
$$+ \frac{\alpha}{4\pi}\ \frac{3c^2}{s^2}[\Delta_W + \frac{5}{6} + \frac{M_Z^2}{M_Z^2 - M_W^2}\ \ln\frac{M_W^2}{M_Z^2} + \Lambda_4(k^2;M_Z,M_W)]\}\ .$$



The invariant functions $\Lambda_{1,\ldots,4}$ together with some of their properties are presented in app. B.3.

For the renormalization of the electric charge we need the $\gamma$ee-vertex at $k^2 = 0$, $p^2 = q^2 = m_e^2$. Its explicit form is:

$$\Gamma_\mu^{\gamma ee}(0) = i\ e\ \gamma_\mu + i\ e\ \gamma_\mu\ \frac{\alpha}{4\pi}[\Delta_e - 2\ln\frac{m_e^2}{\lambda^2} + 4] \tag{5.33}$$
$$+ i\ e\ \frac{\alpha}{4\pi}[(v_e^2 + a_e^2)\gamma_\mu - 2v_ea_e\gamma_\mu\gamma_5][\Delta_Z - \frac{1}{2}]$$
$$+ i\ e\ \gamma_\mu\frac{1-\gamma_5}{2}\ \frac{\alpha}{4\pi}\ \frac{3}{2s^2}[\Delta_W - \frac{1}{6}].$$

For later use we give also the neutrino-photon vertex at $k^2 = 0$:

$$\Gamma_\mu^{\gamma\nu\nu} = -\ i\ e\ \gamma_\mu\ \frac{1-\gamma_5}{2}\ \frac{\alpha}{4\pi}\ \frac{1}{s^2}\ \Delta_W\ . \tag{5.34}$$

### 5.3 Renormalization constants

The prescription for the calculation of the renormalization constants from the unrenormalized self energies and vertex functions have been defined in eq.s (4.1') to (4.6'). We find for the mass renormalization $\delta M_W^2$, $\delta M_Z^2$ of the heavy gauge bosons

$$\delta M_W^2 = \delta M_{W,sing}^2 + \delta M_{W,fin}^2 = \Sigma_{T,sing}^W(M_W^2) + \text{Re}\,\Sigma_{T,fin}^W(M_W^2) , \tag{5.35}$$

$$\delta M_Z^2 = \delta M_{Z,sing}^2 + \delta M_{Z,fin}^2 = \Sigma_{T,sing}^Z(M_Z^2) + \text{Re}\,\Sigma_{T,fin}^Z(M_Z^2) \tag{5.36}$$

explicit expressions by using eq.s (5.9) and (5.10). Eq.s (4.1') tell us that with $\delta M_W^2$, $\delta M_Z^2$ also the following combination of $\delta e$, $\delta Z_1^Z$, $\delta Z_2^Z$ is determined:

$$\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} = \frac{s^2}{c^2-s^2}\left(2\frac{\delta e}{e} - 2\,\delta Z_1^Z + 3\,\delta Z_2^Z\right) . \tag{5.37}$$

Eq.s (4.2') together with (5.7), (5.8), (5.33) give for the photon field renormalization constant:

$$\delta Z_2^\gamma = \frac{\alpha}{4\pi}\left[-\frac{4}{3}\sum_{i\sigma}(Q_{i\sigma}^2\,\Delta_{i\sigma}) + 3\,\Delta_W + \frac{2}{3}\right] , \tag{5.38}$$

a combination of the $(\gamma,Z)$ renormalization constants:

$$\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z} \equiv \frac{sc}{c^2-s^2}\left(\delta Z_1^Z - \delta Z_2^Z - \delta Z_1^\gamma + \delta Z_2^\gamma\right) = -\frac{\alpha}{4\pi}\frac{2c}{s}\,\Delta_W \tag{5.39}$$

and the charge renormalization:

$$\frac{\delta e}{e} \equiv \delta Z_1^\gamma - \frac{3}{2}\,\delta Z_2^\gamma = \frac{\alpha}{4\pi}\left[\frac{2}{3}\sum_{i\sigma}(Q_{i\sigma}^2\,\Delta_{i\sigma}) - \frac{7}{2}\,\Delta_W - \frac{1}{3}\right] . \tag{5.40}$$



A comparison of eq. (5.40) with (5.38) shows that

$$\frac{\delta e}{e} = -\frac{1}{2}\,\delta Z_2^\gamma - \frac{\alpha}{4\pi}\cdot 2\Delta_W . \tag{5.41}$$

This means that the familiar QED relation is modified by the non-Abelian couplings of the gauge bosons.

The four eq.s (5.37) to (5.40) allow the separate determination of $\delta Z_1^\gamma$, $\delta Z_2^\gamma$, $\delta Z_1^Z$, $\delta Z_2^Z$:

$$\delta Z_1^\gamma = \frac{\alpha}{4\pi}\left[-\frac{4}{3}\sum_{i\sigma}(Q_{i\sigma}^2\,\Delta_{i\sigma}) + \Delta_W + \frac{2}{3}\right] , \tag{5.42}$$

$$\delta Z_2^\gamma = \frac{\alpha}{4\pi}\left[-\frac{4}{3}\sum_{i\sigma}(Q_{i\sigma}^2\,\Delta_{i\sigma}) + 3\Delta_W + \frac{2}{3}\right] ,$$

$$\delta Z_1^Z = \frac{\alpha}{4\pi}\left[-\frac{4}{3}\sum_{i\sigma}(Q_{i\sigma}^2\,\Delta_{i\sigma}) + \left(7 - 6\frac{c^2}{s^2}\right)\Delta_W + \frac{2}{3}\right] + \frac{c^2-s^2}{s^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) ,$$

$$\delta Z_2^Z = \frac{\alpha}{4\pi}\left[-\frac{4}{3}\sum_{i\sigma}(Q_{i\sigma}^2\,\Delta_{i\sigma}) + \left(7 - 4\frac{c^2}{s^2}\right)\Delta_W + \frac{2}{3}\right] + \frac{c^2-s^2}{s^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) .$$

Together with these constants also $\delta Z_1^W$, $\delta Z_2^W$, $\delta Z_1^{\gamma Z}$, $\delta Z_2^{\gamma Z}$ are determined. Explicit expressions may be obtained with help of eq. (A.1).

The mass and field renormalization of the leptons according to eq.s (4.1') and (4.3') treats the charged, massive leptons and the neutral, massless neutrinos in an unsymmetric way. This is a consequence of spontaneous breaking of SU(2) × U(1) and of chiral symmetry. As a result the neutrinos remain massless and left-handed after renormalization, whereas the charged leptons suffer mass renormalization:

$$\frac{\delta m_{i\sigma}}{m_{i\sigma}} = \frac{\alpha}{4\pi}\left\{\frac{1}{4s^2}\left(\Delta_W - \frac{1}{2}\right) + 2(v_{i\sigma}^2 + a_{i\sigma}^2)\left(\Delta_Z - \frac{1}{2}\right) - 4(v_{i\sigma}^2 - a_{i\sigma}^2)\left(\Delta_Z + \frac{1}{2}\right) - 3\left(\Delta_{i\sigma} + \frac{4}{3}\right)\right\} = -\frac{\delta v}{v} + \delta Z_1^{i\sigma} . \tag{5.43}$$

The residue of the electron propagator was put equal to one for both the L and R parts. This gives:

$$\delta Z_L^{(e,\nu)} = -\frac{\alpha}{4\pi}\{\Delta_e - 2\ln\frac{m_e^2}{\lambda^2} + 4 + (v_e + a_e)^2(\Delta_Z - \frac{1}{2}) + \frac{1}{2s^2}(\Delta_W - \frac{1}{2})\},$$

$$\delta Z_R^e = -\frac{\alpha}{4\pi}\{\Delta_e - 2\ln\frac{m_e^2}{\lambda^2} + 4 + (v_e - a_e)^2(\Delta_Z - \frac{1}{2})\}. \quad (5.44)$$

In the case of quarks we have two right-handed singlets associated with one left-handed doublet. The two mass renormalization constants are determined by eq. (5.43). The doublet renormalization constant $\delta Z_L^{(d,u)}$ and the singlet renormalization constant $\delta Z_R^d$ for the $I_3 = -1/2$ members are fixed by the analogous conditions as in the lepton case: L and R residues in the d-propagator are put equal to one.

$\delta Z_R^u$ is determined such that the residues of the L and R parts in the u-propagator are still equal (but $\neq 1$). This yields:

$$\delta Z_L^{(d,u)} = -\frac{\alpha}{4\pi}\{Q_d^2(\Delta_d - 2\ln\frac{m_d^2}{\lambda^2} + 4) + (v_d + a_d)^2(\Delta_Z - \frac{1}{2}) + \frac{1}{2s^2}(\Delta_W - \frac{1}{2})\},$$

$$\delta Z_R^d = -\frac{\alpha}{4\pi}\{Q_d^2(\Delta_d - 2\ln\frac{m_d^2}{\lambda^2} + 4) + (v_d - a_d)^2(\Delta_Z - \frac{1}{2})\}, \quad (5.45)$$

$$\delta Z_R^u = -\frac{\alpha}{4\pi}\{Q_u^2(\Delta_u - 2\ln\frac{m_u^2}{\lambda^2} + 4) + (v_u - a_u)^2(\Delta_Z - \frac{1}{2}) - \delta(u,d)\},$$

with

$$\delta(u,d) = Q_u^2(\ln\frac{M_Z^2}{m_u^2} - 2\ln\frac{m_u^2}{\lambda^2}) - Q_d^2(\ln\frac{M_Z^2}{m_d^2} - 2\ln\frac{m_d^2}{\lambda^2}) + \frac{3}{2}. \quad (5.46)$$

One might wonder whether this unsymmetric treatment leads to physical consequences. This, however, is not the case since in the calculation of S matrix elements the field renormalization constants drop out.

The Higgs mass $M_H$ and the Higgs field are renormalized using the prescriptions (4.1') and (4.3') for $\delta M_H^2$ and $\delta Z^\phi$ together with the expression (5.18) for the unrenormalized Higgs self energy. This gives:



$$\left.\frac{\delta M_H^2}{M_H^2}\right|_{sing} = \frac{\alpha}{4\pi}\Delta_W(\frac{3 + 6c^2}{8c^2s^2} - \frac{17}{4c^2s^2}\frac{M_Z^2 + 2c^2M_W^2}{M_H^2} - \frac{27}{8s^2}\frac{M_H^2}{M_W^2}) + 3\frac{\delta t}{t},$$

$$\left.\frac{\delta M_H^2}{M_H^2}\right|_{fin} = \frac{\alpha}{4\pi}\{-\frac{2c^2 + 1}{8c^2s^2} + \frac{7}{4c^2s^2}\frac{M_Z^2 + 2c^2M_W^2}{M_H^2} - \frac{3}{8s^2}\frac{M_H^2}{M_W^2} +$$

$$+ \frac{1}{4c^2s^2}(-\frac{3}{2} + 17\frac{M_Z^2}{M_H^2} + \frac{M_H^2}{M_Z^2})\ln\frac{M_Z^2}{M_W^2} + \frac{21}{8s^2}\frac{M_H^2}{M_W^2}\ln\frac{M_H^2}{M_W^2} +$$

$$+ \frac{1}{s^2}(1 - 7\frac{M_W^2}{M_H^2} - \frac{M_H^2}{2M_W^2})\mathrm{Re}\ F(M_H^2;M_W,M_W) +$$

$$+ \frac{1}{2c^2s^2}(1 - 7\frac{M_Z^2}{M_H^2} - \frac{M_H^2}{2M_W^2})\mathrm{Re}\ F(M_H^2;M_Z,M_Z) - \frac{9}{4s^2}\frac{M_H^2}{M_W^2}\}; \quad (5.47)$$

$$\delta Z^\phi_{sing} = -\frac{\alpha}{4\pi}\Delta_W\frac{2c^2 + 1}{2c^2s^2},$$

$$\delta Z^\phi_{fin} = \frac{\alpha}{4\pi}\mathrm{Re}\{\frac{1}{2c^2s^2}\ln\frac{M_Z^2}{M_W^2} - \frac{1}{s^2}F(M_H^2;M_W,M_W) - \frac{1}{2c^2s^2}F(M_H^2;M_Z,M_Z) +$$

$$+ \frac{1}{s^2}(7M_W^2 - M_H^2 + \frac{M_H^4}{2M_W^2})F'(M_H^2;M_W,M_W) + \quad (5.48)$$

$$+ \frac{1}{2c^2s^2}(7M_Z^2 - M_H^2 + \frac{M_H^4}{2M_Z^2})F'(M_H^2;M_Z,M_Z) + \frac{9}{4s^2}\frac{M_H^4}{M_W^2}F'(M_H^2;M_H,M_H)\}.$$

The renormalization of the gauge fixing parameters $\xi$ follows from eq. (4.5') together with the expressions (5.11 - 14), (5.19, 20) for the longitudinal parts of the gauge boson self energies and the self energies of the unphysical Higgs fields:

$$\delta\xi_1^\gamma = \delta Z_2^\gamma , \tag{5.49}$$

$$\delta\xi_1^Z = \frac{\alpha}{4\pi}\frac{1}{c^2 s^2}\,\mathrm{Re}\,\{(2c^2 - 4c^2 s^2 - 1)\Delta_W - 1 + \frac{1}{M_H^2 - M_Z^2}(M_H^2 \ln\frac{M_H^2}{M_W^2} - M_Z^2 \ln\frac{M_Z^2}{M_W^2})$$
$$+ 2c^2(c^2 - s^2)\,F(M_Z^2;M_W,M_W) - (1 - \frac{(M_H^2 - M_Z^2)^2}{4M_Z^4})\,F(M_Z^2;M_H,M_Z)\}$$
$$- \frac{\delta M_Z^2}{M_Z^2} + 2\,\frac{\delta t}{t} , \tag{5.50}$$

$$\delta\xi_1^W = \frac{\alpha}{4\pi}\frac{1}{c^2 s^2}\,\mathrm{Re}\,\{(c^2 - s^2)(\Delta_W + 1) + (3 - \frac{2}{s^2})\ln\frac{M_Z^2}{M_W^2} + c^2\frac{M_H^2}{(M_H^2 - M_W^2)}\ln\frac{M_H^2}{M_W^2}$$
$$+ 2c^2 s^2\,F(M_W^2;0,M_W) + (2s^4 c^2 + 3c^2 - 1 + \frac{s^4}{4})\,F(M_W^2;M_W,M_Z) +$$
$$+ c^2(\frac{(M_H^2 - M_W^2)^2}{4M_W^4} - 1)\,F(M_W^2;M_W,M_H)\} - \frac{\delta M_W^2}{M_W^2} + 2\frac{\delta t}{t} , \tag{5.51}$$

$$\delta\xi_2^Z = \frac{\alpha}{4\pi}\frac{1}{2c^2 s^2}\,\mathrm{Re}\,\{(2c^2 + 1)\Delta_W + 1 - \frac{M_H^2}{M_H^2 - M_Z^2}\ln\frac{M_H^2}{M_W^2} + \frac{M_Z^2}{M_H^2 - M_Z^2}\ln\frac{M_Z^2}{M_W^2} +$$
$$+ 2c^2\,F(M_Z^2;M_W,M_W) + (1 - \frac{(M_H^2 - M_Z^2)^2}{2M_Z^4})F(M_Z^2;M_H,M_Z)\} - \frac{\delta M_Z^2}{M_Z^2} , \tag{5.52}$$

$$\delta\xi_2^W = \frac{\alpha}{4\pi}\frac{1}{2c^2 s^2}\,\mathrm{Re}\,\{(2c^2 + 1)\Delta_W + 2c^2 + 1 + (4c^2 - \frac{1 + c^2}{s^2})\ln\frac{M_Z^2}{M_W^2} +$$
$$+ c^2\,\frac{M_H^2}{M_W^2 - M_H^2}\ln\frac{M_H^2}{M_W^2} + \frac{1}{2}(5 - 9s^2 - \frac{1}{s^2})\,F(M_W^2;M_W,M_Z) +$$
$$+ c^2(1 - \frac{(M_W^2 - M_H^2)^2}{2M_W^4})\,F(M_W^2;M_W,M_H)\} - \frac{\delta M_W^2}{M_W^2} . \tag{5.53}$$



Finally we have to renormalize the ghost fields. Eq.s (4.6') and (5.21, 22) give:

$$\delta\tilde{Z}_1^\gamma = s^2\,\delta\tilde{Z}_1^W + c^2\,\delta\tilde{Z}_1^B = \frac{1}{2}\,\delta Z_2^\gamma - \frac{\alpha}{4\pi}\,\Delta_W , \tag{5.54}$$

$$\delta\tilde{Z}_1^{\gamma Z} = cs\,(\delta Z_1^W - \delta Z_1^B) = -\,2\delta Z_1^{\gamma Z} + 3\delta Z_2^{\gamma Z} - \delta\xi_2^{\gamma Z} + \frac{\alpha}{4\pi}\frac{c}{s}\,\Delta_W . \tag{5.55}$$

With these expressions we have determined all the renormalization constants of our renormalization scheme of the standard model. They can be used together with the counterterms and the explicit 1-loop calculations to derive the finite renormalized Green functions of the model. In the next section we present these results for the self energies of the gauge bosons and the fermions as well as the fermion gauge boson vertices.

## 6. Renormalized self energies and vertex functions

### 6.1 Gauge boson self energies

In order to give an impression of the influence of the 1-loop contributions on the magnitude of radiative corrections we present in this section the formulas for the renormalized self energies and vertex functions and numerical results for these quantities. They have been calculated with the following set of parameters (if possible taken from [25]):

$$\alpha = (137.036)^{-1},$$
$$M_W = 82 \text{ GeV}, \quad M_Z = 93 \text{ GeV}, \quad M_H = 100 \text{ GeV},$$
$$m_e = 0.511 \text{ MeV}, \quad m_\mu = 105.66 \text{ MeV}, \quad m_\tau = 1784 \text{ MeV},$$
$$m_u = 5 \text{ MeV}, \quad m_d = 7 \text{ MeV}, \quad m_s = 150 \text{ MeV} \; [26],$$
$$m_c = 1.5 \text{ GeV}, \quad m_b = 4.5 \text{ GeV}, \quad m_t = 30 \text{ GeV}.$$

$M_W$ and $M_Z$ are chosen in agreement with the results of the $P\bar{P}$ collider experiments [1]. The Higgs mass is still unknown, we consider 100 GeV as a representative value for this parameter. The results do not critically depend on it unless $M_H$ is smaller than 10 GeV or bigger than 1 TeV. The hadronic contributions to the self energies are calculated from quark loops using the effective quark masses given above. The results are for energies between 10 and 100 GeV in agreement with direct evaluation of these hadronic contributions to the photon self energy [27].
From the expressions (5.7 - 14) for the unrenormalized self energies and the renormalization constants (5.35 - 42) together with the prescriptions for the renormalization we obtain for the renormalized transverse parts of the gauge boson self energies the following formulas:

$$\hat{\Sigma}_T^\gamma(k^2) = \Sigma_T^\gamma(k^2) - k^2\left(\frac{\partial}{\partial k^2}\Sigma_T^\gamma(k^2)\right)\Big|_{k^2=0}, \qquad (6.1)$$

$$\hat{\Sigma}_T^{\gamma Z}(k^2) = \Sigma_T^{\gamma Z}(k^2)_{fin} - k^2\frac{c}{s}\mathrm{Re}\left(\frac{\Sigma_T^Z(M_Z^2)}{M_Z^2} - \frac{\Sigma_T^W(M_W^2)}{M_W^2}\right)\Big|_{fin}, \qquad (6.2)$$



$$\hat{\Sigma}_T^Z(k^2) = (\Sigma_T^Z(k^2) - \Sigma_T^Z(M_Z^2))_{fin} + (k^2 - M_Z^2)\left(\frac{\alpha}{6\pi} + \frac{c^2 - s^2}{s^2}\mathrm{Re}\left(\frac{\Sigma_T^Z(M_Z^2)}{M_Z^2} - \frac{\Sigma_T^W(M_W^2)}{M_W^2}\right)\Big|_{fin}\right), \qquad (6.3)$$

$$\hat{\Sigma}_T^W(k^2) = (\Sigma_T^W(k^2) - \Sigma_T^W(M_W^2))_{fin} + (k^2 - M_W^2)\left(\frac{\alpha}{6\pi} + \frac{c^2}{s^2}\mathrm{Re}\left(\frac{\Sigma_T^Z(M_Z^2)}{M_Z^2} - \frac{\Sigma_T^W(M_W^2)}{M_W^2}\right)\Big|_{fin}\right). \qquad (6.4)$$

The numerical results are presented in fig. 8 - 11 for $|k^2| < (200 \text{ GeV})^2$. The real parts of the diagonal self energies $\hat{\Sigma}_T^\gamma$, $\hat{\Sigma}_T^Z$, $\hat{\Sigma}_T^W$, compared to the free inverse propagators, are not small but yield 10% as typical order of magnitude. The imaginary parts get positive contributions from the fermion loops but negative from the gauge loops. The imaginary parts of the W and Z self energy depend strongly on the energy. Therefore the approximation using a constant imaginary part in the vicinity of the resonance leading to the usual Breit-Wigner type form of the modulus square of the propagator is not really justified. In fig. 12, 13 we show a comparison between a Breit-Wigner distribution using $M_W$ and $\mathrm{Im}\hat{\Sigma}_T^W(M_W^2)$ resp. $M_Z$ and $\mathrm{Im}\hat{\Sigma}_T^Z(M_Z^2)$ and the corresponding quantities resulting from (6.3) and (6.4). We find for the distributions of the W and Z FWHM values which are 10% bigger than $\mathrm{Im}\hat{\Sigma}_T(M^2)/M = \Gamma$. This means that for the determination of the width of the W and Z a careful analysis of the experimental distributions is necessary.

In the case of the W self energy we have contributions of loops containing photons. The physical channel $W \to W + \gamma$ has its threshold at $k^2 = M_W^2$. Consequently we observe in fig. 11 the peak in the real part and the structure in the imaginary part. The magnitudes of these effects depend on the details of the $WW\gamma$ coupling. In a model where the W is coupled to the photon in the form of a minimal substitution it would be different from that of the standard model.

The diagonal gauge boson self energies are very large compared to $\alpha/\pi$ and therefore will give the main contributions besides bremsstrahlung to the radiative corrections in $e^+e^-$ annihilation. Compared to these the $\gamma Z$ mixing is much smaller and in our renormalization scheme typically of the order of magnitude of 1%. In our scheme we do not use the Weinberg angle $\Theta_W$ as a fundamental parameter but as a short hand for $\sin^2\Theta_W = (1 - M_W^2/M_Z^2)$. The results shown in fig. 9 might be interpreted as contributions to an effective running i. e. energy depending Weinberg angle.

The residue of the renormalized Z propagator is different from 1. We obtain:

$$\frac{\partial}{\partial k^2}\hat{\Sigma}_T^Z(k^2)\Big|_{k^2 = M_Z^2} = - 0.086 + i0.029 .$$

For comparison with other renormalization schemes we present also $\hat{\Sigma}_T^W(0)$ and $\hat{\Sigma}_T^Z(0)$:

$$\hat{\Sigma}_T^Z(0) = 0.077\, M_Z^2, \quad \hat{\Sigma}_T^W(0) = 0.076\, M_W^2 . \tag{6.5}$$

These values enter into the calculation of radiative corrections to low energy processes.

## 6.2 Fermion self energies

We described the renormalization prescription for the lepton and quark self energies in sect. 5.3. Together with the unrenormalized expressions (5.27) and the renormalization constants (5.43 - 46) we obtain the renormalized self energies from the equations:

$$\hat{\Sigma}^{i\sigma}(k) = \not{k}\,\frac{1 - \gamma_5}{2}\left(\Sigma_L^{i\sigma}(k^2) + \delta Z_L^i\right) + \not{k}\,\frac{1 + \gamma_5}{2}\left(\Sigma_R^{i\sigma}(k^2) + \delta Z_R^{i\sigma}\right) + \tag{6.6}$$
$$+ m_{i\sigma}\left(\Sigma_S^{i\sigma}(k^2) - m_{i\sigma}(\delta Z_R^{i\sigma} + \delta Z_L^i)/2 - \delta m_{i\sigma}\right) .$$

We illustrate the results with help of the $\nu_e$ self energy and the electron self energy. The corresponding invariant functions are shown in figs. 14 and 15. For the neutrinos only left-handed contributions exist. They are



in our renormalization scheme infrared divergent. Therefore in fig. 14 the IR finite expression $\hat{\Sigma}_L^{\nu}(p^2) - \hat{\Sigma}_L^{\nu}(0)$ is drawn for timelike momenta $p^2$. We find for the self energy only weak $p^2$ dependence.

The real and imaginary parts of the invariant functions $\hat{\Sigma}_V^e$, $\hat{\Sigma}_A^e$, $\hat{\Sigma}_S^e$ of the electron self energy are presented in fig.s 15 In the case of $\mathrm{Re}\hat{\Sigma}_V^e$ and $\mathrm{Re}\hat{\Sigma}_S^e$ we have subtracted the IR divergent part $\frac{\alpha}{4\pi}(2\ln(m_e^2/\lambda^2) - 4)$. We find that $\hat{\Sigma}_A$ and $\hat{\Sigma}_V$ are small, only $\mathrm{Re}\hat{\Sigma}_S^e(p^2)$ reaches a level of several percent.

## 6.3 Renormalized gauge boson fermion vertices

The following list of the renormalized vertex functions contains vector and axial vector couplings only and is valid for on shell fermions and $|k^2| \gg m_f^2$. As in sect. 5.2 we write down the formulas only for the first lepton and quark multiplet.

a) Electromagnetic current:

$$\hat{\Gamma}_\mu^{\gamma ee}(k^2) = ie\gamma_\mu(F_V^{\gamma e} - \gamma_5 F_A^{\gamma e}) ,$$

$$\hat{\Gamma}_\mu^{\gamma\nu\nu}(k^2) = ie\gamma_\mu(1 - \gamma_5)F^{\gamma\nu} ,$$

$$\hat{\Gamma}_\mu^{\gamma dd}(k^2) = - ieQ_d\gamma_\mu(F_V^{\gamma d} - \gamma_5 F_A^{\gamma d}) ,$$

$$\hat{\Gamma}_\mu^{\gamma uu}(k^2) = - ieQ_u\gamma_\mu(F_V^{\gamma u} - \gamma_5 F_A^{\gamma u}) . \tag{6.7}$$

The form factors contain the functions $\Lambda_{1,2,3,4}$ given in Appendix B.3:

$$F_V^{\gamma e} = 1 + \frac{\alpha}{4\pi}[\Lambda_1(k^2,m_e) + (v_e^2 + a_e^2)\Lambda_2(k^2,M_Z) + \frac{3}{4s^2}\Lambda_3(k^2,M_W)] ,$$

$$F_A^{\gamma e} = \frac{\alpha}{4\pi}[2v_e a_e\Lambda_2(k^2,M_Z) + \frac{3}{4s^2}\Lambda_3(k^2,M_W)] ,$$

$$F^{\gamma\nu} = \frac{\alpha}{4\pi}\frac{1}{4s^2}\left[\Lambda_2(k^2,M_W) - 3\Lambda_3(k^2,M_W)\right] ; \qquad (6.8)$$

$$F_V^{\gamma d} = 1 + \frac{\alpha}{4\pi}[Q_d^2\Lambda_1(k^2,m_d) + (v_d^2 + a_d^2)\Lambda_2(k^2,M_Z) - \frac{1}{2s^2}\Lambda_2(k^2,M_W) + \frac{9}{4s^2}\Lambda_3(k^2,M_W)] ,$$

$$F_A^{\gamma d} = \frac{\alpha}{4\pi}[2v_d a_d\Lambda_2(k^2,M_Z) - \frac{1}{2s^2}\Lambda_2(k^2,M_W) + \frac{9}{4s^2}\Lambda_3(k^2,M_W)] ,$$

$$F_V^{\gamma u} = 1 + \frac{\alpha}{4\pi}[Q_u^2\Lambda_1(k^2,m_u) + (v_u^2 + a_u^2)\Lambda_2(k^2,M_Z) - \frac{1}{8s^2}\Lambda_2(k^2,M_W) + \frac{9}{8s^2}\Lambda_3(k^2,M_W) + \delta(u,d)] \;^{*)} ,$$

$$F_A^{\gamma u} = \frac{\alpha}{4\pi}[2v_u a_u\Lambda_2(k^2,M_Z) - \frac{1}{8s^2}\Lambda_2(k^2,M_W) + \frac{9}{8s^2}\Lambda_3(k^2,M_W)] . \qquad (6.9)$$

b) Weak neutral current:

$$\hat{\Gamma}_\mu^{Zee}(k^2) = ie\gamma_\mu(F_V^{Ze} - \gamma_5 F_A^{Ze}) ,$$

$$\hat{\Gamma}_\mu^{Z\nu\nu}(k^2) = ie\gamma_\mu(1 - \gamma_5)F^{Z\nu} ,$$

$$\hat{\Gamma}_\mu^{Zdd}(k^2) = ie\gamma_\mu(F_V^{Zd} - \gamma_5 F_A^{Zd}) ,$$

$$\hat{\Gamma}_\mu^{Zuu}(k^2) = ie\gamma_\mu(F_V^{Zu} - \gamma_5 F_A^{Zu}) \qquad (6.10)$$

with the form factors:

$$F_V^{Ze} = v_e + \frac{\alpha}{4\pi}[v_e\Lambda_1(k^2,m_e) + v_e(v_e^2 + 3a_e^2)\Lambda_2(k^2,M_Z) + \frac{1}{8s^3c}\Lambda_2(k^2,M_W) - \frac{3c}{4s^3}\Lambda_3(k^2,M_W)] ,$$

*) $\delta(u,d)$ is defined in eq. (5.46).



$$F_A^{Ze} = a_e + \frac{\alpha}{4\pi}[a_e\Lambda_1(k^2,m_e) + a_e(3v_e^2 + a_e^2)\Lambda_2(k^2,M_Z) + \frac{1}{8s^3c}\Lambda_2(k^2,M_W) - \frac{3c}{4s^3}\Lambda_3(k^2,M_W)] ,$$

$$F^{Z\nu} = \frac{1}{4sc}\{1 + \frac{\alpha}{4\pi}[-\ln\frac{M_Z^2}{m_e^2} - \frac{9}{2} + 2\ln\frac{m_e^2}{\lambda^2} + \frac{1}{4s^2c^2}\Lambda_2(k^2,M_Z) + \frac{2s^2-1}{2s^2}\Lambda_2(k^2,M_W) + \frac{3c^2}{s^2}\Lambda_3(k^2,M_W)]\} ; \qquad (6.11)$$

$$F_V^{Zd} = v_d + \frac{\alpha}{4\pi}[v_d Q_d^2\Lambda_1(k^2,m_d) + v_d(v_d^2 + 3a_d^2)\Lambda_2(k^2,M_Z) + \frac{1 - 2Q_u s^2}{8s^3c}\Lambda_2(k^2,M_W) - \frac{3c}{4s^3}\Lambda_3(k^2,M_W)] ,$$

$$F_A^{Zd} = a_d + \frac{\alpha}{4\pi}[a_d Q_d^2\Lambda_1(k^2,m_d) + a_d(3v_d^2 + a_d^2)\Lambda_2(k^2,M_Z) + \frac{1 - 2Q_u s^2}{8s^3c}\Lambda_2(k^2,M_W) - \frac{3c}{4s^3}\Lambda_3(k^2,M_W)] ,$$

$$F_V^{Zu} = v_u + \frac{\alpha}{4\pi}[v_u Q_u^2\Lambda_1(k^2,m_u) + v_u(v_u^2 + 3a_u^2)\Lambda_2(k^2,M_Z) + v_u\delta(u,d) - \frac{1 + 2Q_d s^2}{8s^3c}\Lambda_2(k^2,M_W) + \frac{3c}{4s^3}\Lambda_3(k^2,M_W)] ,$$

$$F_A^{Zu} = a_u + \frac{\alpha}{4\pi}[a_u Q_u^2\Lambda_1(k^2,m_u) + a_u(3v_u^2 + a_u^2)\Lambda_2(k^2,M_Z) + a_u\delta(u,d) - \frac{1 + 2Q_d s^2}{8s^3c}\Lambda_2(k^2,M_W) + \frac{3c}{4s^3}\Lambda_3(k^2,M_W)] . \qquad (6.12)$$

c) Weak charged current:

$$\hat{\Gamma}_\mu^{We\nu}(k^2) = i\frac{e}{2\sqrt{2}s}\gamma_\mu(1-\gamma_5)\,F^{We\nu} ,$$

$$\hat{\Gamma}_\mu^{Wdu}(k^2) = i\frac{e}{2\sqrt{2}s}\gamma_\mu(1-\gamma_5)\,F^{Wdu} , \qquad (6.13)$$

with the formfactors

$$F^{We\nu} = 1 + \frac{\alpha}{4\pi}\{\frac{3(3c^2-1)}{2s^2} + \ln\frac{m_e^2}{M_W^2} + 2\ln\frac{m_e^2}{\lambda^2} + (\frac{2s^2-1}{2s^2} + \frac{3c^2}{s^4})\ln\frac{M_W^2}{M_Z^2} +$$

$$+ \frac{2s^2-1}{4s^2c^2}\Lambda_2(k^2,M_Z) + 3\Lambda_4^{m_e}(k^2;M_W,0) + \frac{3c^2}{s^2}\Lambda_4(k^2;M_Z,M_W)\} ,$$

$$F^{Wdu} = 1 + \frac{\alpha}{4\pi}\{Q_uQ_d[3\ln\frac{m_d}{m_u} + \frac{1}{2}\Lambda_1(k^2,m_d) + \frac{1}{2}\Lambda_1(k^2,m_u) + \frac{s^2}{c^2}\Lambda_2(k^2,M_Z)] + \qquad (6.14)$$

$$+ \frac{2s^2-1}{4s^2c^2}\Lambda_2(k^2,M_Z) + 3Q_u\Lambda_4^{m_u}(k^2,M_W,0) - 3Q_d\Lambda_4^{m_d}(k^2,M_W,0) +$$

$$+ \frac{3c^2}{s^2}\Lambda_4(k^2,M_Z,M_W) - \frac{1}{3}(\ln\frac{M_Z^2}{m_d^2} - 2\ln\frac{m_d^2}{\lambda^2} + \frac{9}{2}) + \frac{3}{s^2} + (\frac{1}{2s^2} - \frac{3c^2}{s^4})\ln\frac{M_Z^2}{M_W^2}\} .$$

d) Examples: The electron and ν photon formfactors, the electron Z boson formfactor, the Weν-formfactor

For illustration we present the weak contributions (the parts with $\Lambda_{2,3}$ in eq.s (6.7 - 13) ) to the vector and axialvector photonic formfactor of the electron in fig. 16 and 17 for $|k^2| \leq (150\ \mathrm{GeV})^2$. The vector part $F^{\gamma e}_{V,weak}$ vanishes at $k^2 = 0$ as a consequence of charge renormalization, the axial vector part $F^{\gamma e}_{A,weak}(0) = 0$ because of the Ward identity. For the $k^2$ values given above the real and imaginary parts of these formfactors are typically of the order of magnitude of $10^{-3}$ e.



The γν vertex vanishes in lowest order but gets contributions from 1-loop diagrams (b, c of fig. 17) containing the W exchange and the non-Abelian gauge boson coupling. The resulting formfactor $F^{\gamma\nu}(k^2)$ shown in fig. 18 has for $|k^2| \leq (150\ \mathrm{GeV})^2$ also values up to some $10^{-3}$ e.

The non-photonic contributions to the e Z boson formfactors are shown in fig. 19 a,b. They have the same characteristics as the other formfactors. Compared to the self energy effect of the weak bosons the weak contributions to the vertex corrections give effects in $e^+e^-$ annihilation processes which are one order of magnitude smaller.

The Weν- and the corresponding quark formfactors obtain a contribution from the WWγ-coupling (see fig. 7c). This shows a pronounced structure around $k^2 = M_W^2$ and reaches an order of magnitude of $-40\cdot\frac{\alpha}{4\pi}$ for the real and imaginary part (see fig. 20). Together with the corresponding contribution (the Wγ-loop) to the W self energy it may lead to interesting effects in W exchange dominated reactions for timelike large momentum transfers.

We conclude this section with some remarks on the box diagrams with two weak bosons. In contrast to the self energy and vertex diagrams they are both UV and IR finite and consequently in the 1-loop approximation not directly influenced by the renormalization scheme. Their contribution to 1-loop radiative corrections to S matrix elements is in the energy range considered of the same order of magnitude as those of the vertex corrections.

## 7. Conclusion and Outlook

In this paper we have worked out a renormalization scheme for the standard electroweak model characterized by the following properties: use of the electric charge e and particle masses as physical parameters; minimal number of field renormalization constants respecting the SU(2)×U(1) symmetry; the simple pole structure of the 't Hooft-Feynman gauge is maintained after renormalization in a way consistent with the Slavnov-Taylor identities. We have calculated the explicit results and presented numerical values for the 1-loop corrected propagators and the fermion gauge boson vertices. These are the building blocks needed for the calculation of electroweak radiative corrections to $e^+e^-$ annihilation, deep inelastic lepton scattering and $P\bar{P}$ annihilation at high energies. The dominant weak contributions of these corrections result from the diagonal gauge boson self energies.

Applications of these results to $e^+e^-$ annihilation into μ pairs at PETRA/PEP energies have already been published [19], those for higher energies and to Bhabha scattering are in preparation [28]. The use of this high energy renormalization scheme for the calculation of low energy processes like μ decay and elastic ν scattering will allow to study the renormalization scheme dependence of the 1-loop predictions of the standard electroweak theory.



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Appendix A: Feynman rules and counter terms

We present the Feynman rules of the standard model using $e$, $M_W$, $M_Z$, $M_H$, $m_{i\sigma}$ as parameters and

$$c = M_W/M_Z,\ s = (1-M_W^2/M_Z^2)^{1/2},\ v_{i\sigma} = (I^3_{i\sigma}-2s^2Q_{i\sigma})/2sc,\ a_{i\sigma} = I^3_{i\sigma}/2sc$$

as abbreviations for writing out the couplings. We combine the renormalization constants $\delta Z_i^W$, $\delta Z_i^B$, $\delta\xi_i^W$, $\delta\xi_i^B$, $\delta\tilde{Z}^W$, $\delta\tilde{Z}^B$ to those for the photon, Z boson and mixing terms:

$$\delta Z_i^\gamma = s^2\,\delta Z_i^W + c^2\,\delta Z_i^B ,$$

$$\delta Z_i^Z = c^2\,\delta Z_i^W + s^2\,\delta Z_i^B , \qquad \text{similar for } \delta\xi_i,\ \delta\tilde{Z} , \qquad \text{(A.1)}$$

$$\delta Z_i^{\gamma Z} = cs\,(\delta Z_i^W - \delta Z_i^B) = \frac{cs}{c^2-s^2}(\delta Z_i^Z - \delta Z_i^\gamma) .$$

The renormalization constants $\delta M_W^2$, $\delta M_Z^2$, $\delta M_H^2$, $\delta m_{i\sigma}$, $\delta t$ are defined in eq.s (4.1') and (4.4'). This gives the following list of Feynman rules and counter terms:

$$\eta\ \text{---}\times \quad = -\,i\,e\,\frac{2s}{e}\,M_W\,M_H^2\,\frac{\delta t}{t} ,$$

$$= \frac{i}{k^2-M^2}\{-g_{\mu\nu};\ 1;\ 1;\ \not{k}+m_{i\sigma}\} ,$$

$$W^a_\mu \sim\times\sim W^a_\nu = i\,g_{\mu\nu}\,[(k^2-M_a^2)\,\delta Z_2^a - \delta M_a^2] - ik_\mu k_\nu\delta\xi_1^a;\ a = \pm,Z,\gamma ,$$

$$Z_\mu \sim\times\sim A_\nu = i\,g_{\mu\nu}\,[-k^2\delta Z_2^{\gamma Z} + M_Z^2(\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z})] + i\,k_\mu k_\nu\delta\xi_1^{\gamma Z} ,$$



$$\{W^+_\mu;Z_\mu;A_\mu\}\ \{\phi^-;\chi;\chi\} =$$

$$\sim\times\text{---}\ (k) \quad = k_\mu\cdot\frac{1}{2}\,M_{W,Z}\,\{+\,i(\delta\xi_1^W-\delta\xi_2^W);\ \delta\xi_1^Z-\delta\xi_2^Z;\ -\delta\xi_1^{\gamma Z}+\delta\xi_2^{\gamma Z}\} ,$$

$$\eta - \times - \eta = -i[(k^2-M_H^2)\,\delta Z^\phi - \delta M_H^2] ,$$

$$\{\phi^+;\chi\}\ \{\phi^-;\chi\}$$

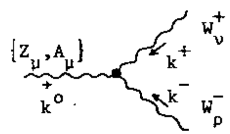
$$\text{---}\times\text{---} = -i[(k^2-M_{W,Z}^2)\delta Z^\phi - \delta M_{W,Z}^2 - M_{W,Z}^2\,\delta\xi_2^{W,Z} + M_H^2\frac{\delta t}{t}] ,$$

$$u^a \cdots\times\cdots \bar{u}^a = -i[k^2(\delta\tilde{Z}^a - \tfrac{1}{2}\delta\xi_1^a) - M_a^2(\delta\tilde{Z}^a + \frac{\delta M_a^2}{M_a^2} + \tfrac{1}{2}\,\delta\xi_2^a)];\ a = \pm,Z,\gamma ,$$

$$u^\gamma \cdots\times\cdots \bar{u}^Z = i[k^2(\delta\tilde{Z}^{\gamma Z} - \tfrac{1}{2}\delta\xi_1^{\gamma Z}) + M_Z^2(\delta Z_1^{\gamma Z} - \tfrac{3}{2}\delta Z_2^{\gamma Z} + \tfrac{1}{2}\delta\tilde{Z}^{\gamma Z} + \tfrac{1}{2}\delta\xi_2^{\gamma Z})] ,$$

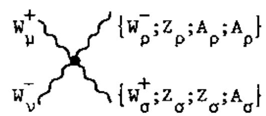
$$u^Z \cdots\times\cdots \bar{u}^\gamma = i[k^2(\delta\tilde{Z}^{\gamma Z} - \tfrac{1}{2}\delta\xi_1^{\gamma Z}) + M_Z^2(\delta Z_1^{\gamma Z} - \tfrac{3}{2}\delta Z_2^{\gamma Z} + \tfrac{1}{2}\delta\tilde{Z}^{\gamma Z})] ,$$

$$i\sigma \longrightarrow\times\longrightarrow i\sigma = -i[\not{k}(\delta Z_L^i\,\frac{1-\gamma_5}{2} + \delta Z_R^{i\sigma}\,\frac{1+\gamma_5}{2}) - m_{i\sigma}(\tfrac{1}{2}\delta Z_L^i + \tfrac{1}{2}\delta Z_R^{i\sigma}) - \delta m_{i\sigma}] ,$$



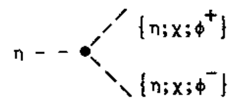

$$= -\,ie\{-\tfrac{c}{s};\ 1\}[g_{\mu\nu}(k^+-k^o)_\rho + g_{\nu\rho}(k^--k^+)_\mu +$$

$$+\ g_{\rho\mu}(k^o-k^-)_\nu](1 + \delta Z_1^W) ,$$

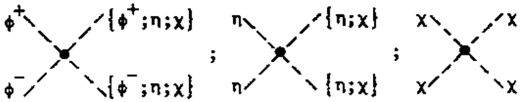



$$= -i2e^2\{\frac{1}{s^2};\ \frac{c^2}{s^2};\ -\frac{c}{s};\ 1\}[g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}]\cdot$$

$$\cdot(1 + 2\delta Z_1^W - \delta Z_2^W) ,$$

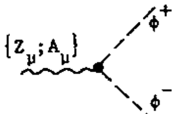



$$= -i\,\frac{e}{2s}\,\frac{M_H^2}{M_W}\{3;1;1\}\left(1-\frac{\delta v}{v}+\delta Z^{\lambda}\right),$$

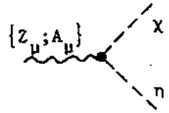



$$= -i\,\frac{e^2}{4s^2}\,\frac{M_H^2}{M_W^2}\{2;1;1;3;1;3\}\cdot(1+\delta Z^{\lambda}),$$

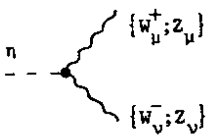

$$= \frac{e}{2s}\,(p-q)_{\mu}\{\mp i;1\}\,(1+\delta Z^{\phi}+\delta Z_1^W-\delta Z_2^W),$$



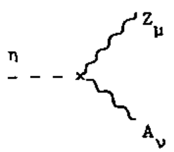

$$= -ie(p-q)_{\mu}\left\{\frac{s^2-c^2}{2cs};\,1\right\}\Big\{1+\delta Z^{\phi}+\Big\{\delta Z_1^Z-\delta Z_2^Z+\frac{2cs}{c^2-s^2}(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z});\ \delta Z_1^{\gamma}-\delta Z_2^{\gamma}+\frac{c^2-s^2}{2cs}(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z})\Big\}\Big\},$$




$$= \frac{e}{2cs}(p-q)_{\mu}\,\{1+\delta Z^{\phi}+\delta Z_1^Z-\delta Z_2^Z;\ \delta Z_2^{\gamma Z}-\delta Z_1^{\gamma Z}\},$$

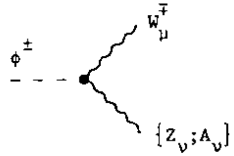



$$= i\,\frac{e}{s}\,M_W g_{\mu\nu}\left\{1;\,\frac{1}{c^2}\right\}\left(1+\frac{1}{2}\delta Z^{\phi}+\frac{\delta M_{W;Z}}{M_{W;Z}}+\delta Z_1^{W;Z}-\frac{1}{2}\,\delta Z_2^{W;Z}\right),$$

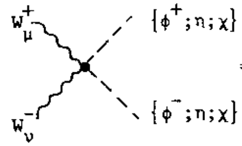



$$= -i\,\frac{e}{c^2 s}\,M_W g_{\mu\nu}(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z}),$$

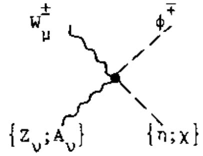




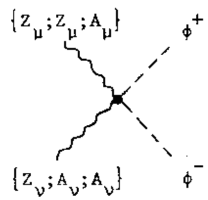

$$= -i\,e\,M_W g_{\mu\nu}\left\{\frac{s}{c};\,1\right\}\left(1+\frac{1}{2}\delta Z^{\phi}+\frac{\delta M_W}{M_W}+\frac{1}{2}\delta Z_2^W+\delta Z_2^B\right),$$



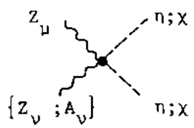

$$= i\,\frac{e^2}{2s^2}\,g_{\mu\nu}(1+\delta Z^{\phi}+2\delta Z_1^W-2\delta Z_2^W),$$



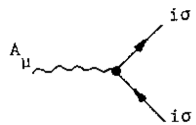

$$= -i\,\frac{e^2}{2}\,g_{\mu\nu}\left\{\frac{1}{c};\,\frac{1}{s}\right\}\{1;\ \pm i\}(1+\delta Z^{\phi}+\delta Z_1^W-\delta Z_2^W+\delta Z_1^B-\delta Z_2^B)$$




$$= ie^2 g_{\mu\nu}\begin{cases}\dfrac{(c^2-s^2)^2}{2c^2s^2}\Big(1+\delta Z^{\phi}+2\delta Z_1^Z-2\delta Z_2^Z-\dfrac{4cs}{c^2-s^2}(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z})\Big), \\ \dfrac{s^2-c^2}{cs}\Big(1+\delta Z^{\phi}+\delta Z_1^{\gamma}-\delta Z_2^{\gamma}+\delta Z_1^Z-\delta Z_2^Z+\dfrac{1}{2cs(c^2-s^2)}(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z})\Big), \\ 2\,\Big(1+\delta Z^{\phi}+2\delta Z_1^{\gamma}-2\delta Z_2^{\gamma}+\dfrac{c^2-s^2}{cs}(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z})\Big),\end{cases}$$



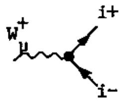

$$= i\,\frac{e^2}{2c^2s^2}\,g_{\mu\nu}\{1+2\delta Z_1^Z-\delta Z_2^Z+\delta Z^{\phi};\ \delta Z_2^{\gamma Z}-\delta Z_1^{\gamma Z}\},$$



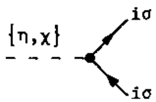

$$= -ieQ^{i\sigma}\gamma_{\mu}\left(1+\delta Z_1^{\gamma}-\delta Z_2^{\gamma}+\delta Z_L^{i}\,\frac{1-\gamma_5}{2}+\delta Z_R^{i\sigma}\,\frac{1+\gamma_5}{2}\right)+$$

$$-\,ie\gamma_{\mu}(v_{i\sigma}-\gamma_5 a_{i\sigma})(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z}),$$

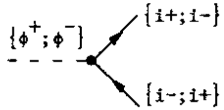

$$= ie\gamma_\mu(v_{i\sigma} - \gamma_5 a_{i\sigma})(1 + \delta Z_1^Z - \delta Z_2^Z) +$$
$$+ i\,\frac{e}{cs}\,\gamma_\mu\left[(I_3^\sigma - s^2 Q^{i\sigma})\,\frac{1-\gamma_5}{2}\,\delta Z_L^i - s^2 Q^{i\sigma}\,\frac{1+\gamma_5}{2}\,\delta Z_R^{i\sigma} + csQ^{i\sigma}(\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z})\right],$$



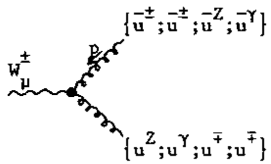

$$= i\,\frac{e}{\sqrt{2}s}\,\gamma_\mu\,\frac{1-\gamma_5}{2}(1 + \delta Z_1^W - \delta Z_2^W + \delta Z_L^i)\ ,$$



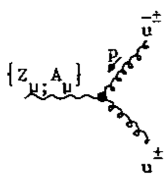

$$= -\,\frac{e}{2s}\,\frac{m_{i\sigma}}{M_W}\,\{i;2I_3^\sigma\gamma_5\}(1 + \frac{\delta m_{i\sigma}}{m_{i\sigma}} + \frac{\delta v}{v} + \frac{1}{2}\,\delta Z_L^i + \frac{1}{2}\,\delta Z_R^{i\sigma})\ ,$$




$$= i\,\frac{e}{\sqrt{2}s}\,\frac{1}{M_W}\,(\frac{1 \mp \gamma_5}{2}m_{i+}(1 + \frac{\delta m_{i+}}{m_{i+}} + \frac{\delta v}{v} + \delta Z_1^{i+} + \frac{1}{2}\delta Z_R^{i+} + \delta Z_L^i) +$$
$$+ \frac{1 \pm \gamma_5}{2}m_{i-}(1 + \frac{\delta m_{i-}}{m_{i-}} + \frac{\delta v}{v} + \delta Z_1^{i-} + \frac{1}{2}\delta Z_R^{i-} + \frac{1}{2}\delta Z_L^i)),$$

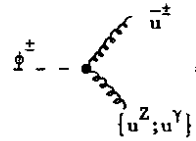



$$= \pm\ i\ e\ p_\mu\{\frac{c}{s};\ -1;\ -\frac{c}{s};\ 1\}(1 + \frac{1}{2}\,\delta\tilde{Z}^W + \frac{1}{2}\,\delta\tilde{Z}^B +$$
$$+ \delta Z_1^W - \delta Z_2^W - \frac{1}{2}\{\delta\xi_1^W;\ \delta\xi_1^W;\ \delta\xi_1^B;\ \delta\xi_1^B\})\ ,$$

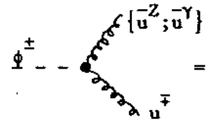



$$= \mp\ iep_\mu\{\frac{c}{s};\ -1\}(1 + \delta\tilde{Z}^W - \frac{1}{2}\delta\xi_1^W + \delta Z_1^W - \delta Z_2^W)\ ,$$

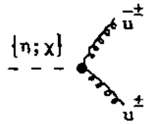




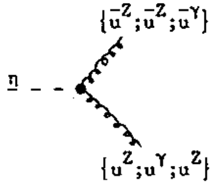

$$= ieM_W\{\frac{s^2-c^2}{2cs};\ 1\}(1 - \frac{\delta v}{v} + \delta Z^\phi + \frac{1}{2}\,\delta\tilde{Z}^W +$$
$$+ \{\frac{c^2}{c^2-s^2};\ \frac{1}{2}\}(\delta Z_1^W - \frac{3}{2}\,\delta Z_2^W + \frac{1}{2}\,\delta\tilde{Z}^W + \delta\xi_2^W) +$$
$$+ \{\frac{-s^2}{c^2-s^2};\ \frac{1}{2}\}(\delta Z_1^B - \frac{3}{2}\,\delta Z_2^B + \frac{1}{2}\,\delta\tilde{Z}^B + \delta\xi_2^W))\ ,$$




$$= i\,\frac{e}{2cs}\,M_W\{1 - \frac{\delta v}{v} + \delta Z^\phi + \delta Z_1^Z - \frac{3}{2}\delta Z_2^Z + \frac{1}{2}\delta\tilde{Z}^W + \frac{1}{2}\delta\tilde{Z}^Z + \frac{1}{2}\delta\xi_2^W;$$
$$-\delta Z_1^{\gamma Z} + \frac{3}{2}\delta Z_2^{\gamma Z} - \frac{1}{2}\delta\tilde{Z}^{\gamma Z}\}\ ,$$




$$= -\ i\,\frac{e}{2s}M_W\{1;\ \pm i\}(1 - \frac{\delta v}{v} + \delta Z^\phi + 2\delta Z_1^W - 3\delta Z_2^W + \delta\tilde{Z}^W + \frac{1}{2}\delta\xi_2^W)\ ,$$




$$= -\ i\,\frac{e}{2c^2 s}\,M_W\begin{cases}(1 - \frac{\delta v}{v} + \delta Z^\phi + 2\delta Z_1^Z - 3\delta Z_2^Z + \delta\tilde{Z}^Z + \frac{1}{2}\,\delta\xi_2^Z)\ , \\ (-\ \delta Z_1^{\gamma Z} + \frac{3}{2}\,\delta Z_2^{\gamma Z} - \frac{1}{2}\,\delta\tilde{Z}^{\gamma Z} - \frac{1}{2}\,\delta\xi_2^{\gamma Z})\ , \\ (-\ \delta Z_1^{\gamma Z} + \frac{3}{2}\,\delta Z_2^{\gamma Z} - \frac{1}{2}\,\delta\tilde{Z}^{\gamma Z})\ .\end{cases}$$

Appendix B:

1) The finite part of the scalar self energy $F(k^2;M_1,M_2)$:

$$F(k^2;M_1,M_2) = 1 + \left(\frac{M_1^2-M_2^2}{k^2} - \frac{M_1^2+M_2^2}{M_1^2-M_2^2}\right) \ln \frac{M_2}{M_1} + \tag{B.1}$$

$$+ \begin{cases} + \frac{1}{k^2}[((M_1+M_2)^2-k^2)((M_1-M_2)^2-k^2)]^{1/2} \cdot \\ \quad \cdot \ln \frac{\sqrt{(M_1+M_2)^2-k^2} + \sqrt{(M_1-M_2)^2-k^2}}{\sqrt{(M_1+M_2)^2-k^2} - \sqrt{(M_1-M_2)^2-k^2}} \quad , \quad k^2 < (M_1-M_2)^2 \\ - \frac{2}{k^2}[(M_1+M_2)^2-k^2]^{1/2}[k^2-(M_1-M_2)^2]^{1/2} \cdot \\ \quad \cdot \arctan \frac{\sqrt{k^2-(M_1-M_2)^2}}{\sqrt{(M_1+M_2)^2-k^2}} \quad , \quad (M_1-M_2)^2 < k^2 < (M_1+M_2)^2 \\ - \frac{1}{k^2}[k^2-(M_1+M_2)^2]^{1/2}[k^2-(M_1-M_2)^2]^{1/2} \cdot \\ \quad \cdot \left\{ \ln \frac{\sqrt{k^2-(M_1+M_2)^2} + \sqrt{k^2-(M_1-M_2)^2}}{\sqrt{k^2-(M_1-M_2)^2} - \sqrt{k^2-(M_1+M_2)^2}} - i\pi \right\} \quad , \quad k^2 > (M_1+M_2)^2 . \end{cases}$$

2) The function $B_1$ $(k^2;M_1,M_2)$ is defined by:

$$\frac{i}{16\pi^2} k_\mu \; B_1(k^2;M_1,M_2) = \mu^{4-D} \int \frac{d^Dq}{(2\pi)^D} \; \frac{q_\mu}{(q^2-M_1^2+i\varepsilon)((q+k)^2-M_2^2+i\varepsilon)}$$

and related to $B_o$:

$$2k^2 \; B_1(k^2;M_1,M_2) = A(M_2) - A(M_1) + (M_2^2-M_1^2-k^2)B_o(k^2;M_1,M_2) \; . \tag{B.2}$$

This gives for equal masses:

$$B_1(k^2;M,M) = -\frac{1}{2} \; B_o(k^2;M,M) \; . \tag{B.2'}$$



3) The vertex functions $\Lambda_{1,\ldots,4}$:

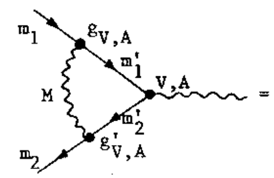
The contributions of the photonic diagrams of fig. 7a to the vertex function $\Gamma_\mu$ (of sect. 5.2f) for $|k^2| \gg m^2$, $\not p = \not q = m$ have already been calculated [18]. After splitting off the UV divergent parts and the coupling constants remains the function $\Lambda_1(k^2,m)$:

$$\Lambda_1(0,m) = 0 \qquad \text{for } k^2 = 0 \; ,$$

$$\Lambda_1(k^2,m) = -2\ln\frac{|k^2|}{\lambda^2}\left(\ln\frac{|k^2|}{m^2}-1\right) + \ln\frac{|k^2|}{m^2} + \ln^2\frac{|k^2|}{m^2} + \tag{B.3}$$

$$+ \begin{cases} 4\left(\frac{\pi^2}{3}-1\right) + 2\pi i\left(\ln\frac{k^2}{\lambda^2} - \frac{3}{2}\right) & \text{for } k^2 \gg m^2, \\ 4\left(\frac{\pi^2}{12}-1\right) & \text{for } -k^2 \gg m^2. \end{cases}$$

The diagrams of fig.7b describing the exchange of the heavy bosons Z and W lead to the following integral:




$$= \mu^{4-D} \int \frac{d^Dl}{(2\pi)^D} \; \frac{\gamma_\nu(g_V'-g_A'\gamma_5)(-\not q-\not l+m_2')\gamma_\mu(V-A\gamma_5)(\not p-\not l+m_1')\gamma^\nu(g_V-g_A\gamma_5)}{(l^2-M^2)((p-l)^2-m_1'^2)((q+l)^2-m_2'^2)}$$

$$= \frac{i}{16\pi^2} \gamma_\mu(\lambda_V-\lambda_A\gamma_5) \cdot \begin{cases} [\Delta_M - \frac{1}{2}] & \text{for } k^2 = 0 \; , \\ [\Delta_M - \frac{1}{2} + \Lambda_2(k^2,M)] & \text{for } k^2 \gg m_{1,2}^2, m_{1,2}'^2 \end{cases}$$

with:

$$\lambda_V = V(g_Vg_V' + g_Ag_A') + A(g_Vg_A' + g_V'g_A) \; ,$$
$$\lambda_A = A(g_Vg_V' + g_Ag_A') + V(g_Vg_A' + g_V'g_A) \; ,$$

and

$$\Lambda_2(k^2,M) = -\frac{5}{2} + \ln w + 2(1+w)^2 \frac{d}{dw}\left(\frac{J(w)}{1+w}\right), \quad w = \frac{M^2}{k^2}.$$

The parameter integral

$$J(w) = \int_0^1 dx \int_0^1 dy\, y \ln\left[w(1-y) - y^2 x(1-x) - i\varepsilon\right]$$

can be evaluated with help of the dilogarithm

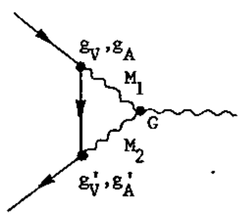
$$Sp(z) = -\int_0^1 dt\, \frac{\ln(1-zt)}{t}$$

and yields for $\Lambda_2$ the expression:

$$\Lambda_2(k^2,M) = -\frac{7}{2} - 2w - (2w+3)\ln(-w) + 2(1+w)^2\left[Sp(1+\frac{1}{w}) - \frac{\pi^2}{6}\right] \quad \text{for } k^2 < 0,$$

$$\Lambda_2(k^2,M) = -\frac{7}{2} - 2w - (2w+3)\ln(w) + 2(1+w)^2\left[\ln(w)\ln(\frac{w+1}{w}) - Sp(-\frac{1}{w})\right]$$
$$- i\pi\left[3 + 2w - 2(1+w)^2\ln(\frac{1+w}{w})\right] \quad \text{for } k^2 > 0. \qquad \text{(B.4)}$$

In fig. 19 we show $\Lambda_2(k^2,M_Z)$, $\Lambda_2(k^2,M_W)$, $\Lambda_3(k^2,M_W)$, and $\Lambda_4(k^2;M_Z,M_W)$. In a similar way we have obtained the invariant functions belonging to the diagram 7c containing the triple boson vertex:




$$= \frac{i}{16\pi^2}\gamma_\mu(\lambda_1 - \lambda_2\gamma_5)\cdot\Lambda(k^2;M_1,M_2)$$

with

$$\lambda_1 = (g_V g'_V + g_A g'_A)G, \quad \lambda_2 = (g_V g'_A + g_A g'_V)G$$



and

$$\Lambda(k^2;M_1,M_2) = \begin{cases} 3\left[\Delta_M - \frac{1}{6} + \Lambda_3(k^2,M)\right] & \text{for } M_1 = M_2 = M, \\ 3\left[\frac{\Delta_{M_1}+\Delta_{M_2}}{2} + \frac{5}{6} - \frac{M_1^2+M_2^2}{M_1^2-M_2^2}\ln\frac{M_1}{M_2} + \Lambda_4(k^2;M_1,M_2)\right] & \text{for } M_1 \neq M_2. \end{cases} \qquad \text{(B.5)}$$

The remaining functions have the properties

$$\Lambda_3(0,M) = \Lambda_4(0,M_1,M_2) = 0,$$

$$\Lambda_4(k^2,M_1,M_2) = \Lambda_4(k^2,M_2,M_1).$$

They read for $|k^2| \gg m_f^2 \quad (w = M^2/k^2)$:

$$\Lambda_3(k^2,M) = \frac{5}{6} - \frac{2w}{3} + \frac{2w+1}{3}\sqrt{1-4w}\,\ln\frac{\sqrt{1-4w}+1}{\sqrt{1-4w}-1} + \frac{2}{3}w(w+2)\left(\ln\frac{\sqrt{1-4w}+1}{\sqrt{1-4w}-1}\right)^2 \quad \text{for } k^2 < 0,$$

$$\Lambda_3(k^2,M) = \frac{5}{6} - \frac{2w}{3} + \frac{2}{3}(2w+1)\sqrt{4w-1}\,\arctan\frac{1}{\sqrt{4w-1}} - \frac{8}{3}w(w+2)\left(\arctan\frac{1}{\sqrt{4w-1}}\right)^2 \quad \text{for } 0 < k^2 < 4M^2, \qquad \text{(B.6)}$$

$$\Lambda_3(k^2,M) = \frac{5}{6} - \frac{2w}{3} + \frac{2w+1}{3}\sqrt{1-4w}\,\ln\frac{1+\sqrt{1-4w}}{1-\sqrt{1-4w}} + \frac{2}{3}w(w+2)\left[\ln^2\left(\frac{1+\sqrt{1-4w}}{1-\sqrt{1-4w}}\right) - \pi^2\right]$$
$$- i\pi\left[\frac{2w+1}{3}\sqrt{1-4w} + 2\ln\frac{1+\sqrt{1-4w}}{1-\sqrt{1-4w}}\right] \quad \text{for } k^2 > 4M^2$$

and with $w_1 = M_1^2/k^2$, $w_2 = M_2^2/k^2$:

$$\Lambda_4(k^2;M_1,M_2) = \frac{1}{6} + \frac{w_1+w_2}{w_1-w_2}\ln\frac{M_1}{M_2} - \frac{w_1-w_2}{3}\ln\frac{M_1}{M_2} + \frac{w_1+w_2+1}{3}\left(\ln\frac{M_1}{M_2} - 1\right)$$
$$+ \frac{w_1+w_2+1}{3}\left[x_1\ln\frac{x_1}{x_1-1} + x_2\ln\frac{-x_2}{1-x_2}\right]$$
$$- \frac{2}{3}(w_1+w_2+w_1w_2)\ln\frac{x_1}{x_1-1}\ln\frac{-x_2}{1-x_2}, \qquad \text{(B.7)}$$

$$x_{1,2} = \begin{cases} \frac{1-w_1+w_2}{2} \pm \frac{1}{2}\sqrt{(1-w_1+w_2)^2 - 4w_2} & \text{for } k^2 < (M_1-M_2)^2 \text{ and } k^2 > (M_1+M_2)^2, \\ \frac{1-w_1+w_2}{2} \pm \frac{i}{2}\sqrt{4w_2 - (1-w_1+w_2)^2} & \text{for } (M_1-M_2)^2 < k^2 < (M_1+M_2)^2. \end{cases}$$

The imaginary part of $\Lambda_4$ is obtained from (B.7).

$$\mathrm{Im}\,\Lambda_4(k^2;M_1,M_2) = -\pi\cdot\theta\left(k^2 - (M_1+M_2)^2\right)\cdot$$

$$\cdot\left\{\frac{w_1+w_2+1}{3}\sqrt{(1-w_1+w_2)^2 - 4w_2}\right.$$

$$\left.+\frac{2}{3}(w_1+w_2+w_1w_2)\left(\ln\frac{x_1}{1-x_1} + \ln\frac{x_2}{1-x_2}\right)\right\}.$$

In the case $M_2 = 0$, i. e. if one of the bosons is a photon, the mass m of the fermion coupled to the photon has to be respected. In that special case we have to use the following expression, valid for $|k^2| >> m^2$ $(w = M^2/k^2)$:

$$\Lambda_4^m(k^2;M,0) = \frac{1}{2} - \frac{w}{3} + \frac{1-w^2}{3}\ln\frac{w-1}{w} + \frac{2}{3}\ln\frac{M^2}{m^2} +$$

$$+\frac{w}{3}\left[\ln^2\left(\frac{M^2-k^2}{m^2}\right) - \left(\ln\frac{M^2}{m^2}\right)^2 + 2\,\mathrm{Sp}\left(\frac{1}{1-w}\right)\right].$$

For $k^2 \cong M^2$ replace $M^2 \to M^2 - iM\Gamma$.

Fig. 20 shows $\Lambda_4^m(k^2;M,0)$ for $m = m_e$, $M = M_W$.



Figure Captions

Fig. 1: 1-loop tadpole diagrams.

Fig. 2: 1-loop gauge boson self energy diagrams*).

Fig. 3: Diagrams for gauge boson Higgs boson mixing.

Fig. 4: 1-loop Higgs boson self energy diagrams.

Fig. 5: 1-loop ghost self energy diagrams.

Fig. 6: 1-loop fermion self energy diagrams.

Fig. 7: 1-loop gauge boson fermion vertex diagrams.

Fig. 8: Real and imaginary parts of the renormalized transverse photon self energy $\hat{\Sigma}_T^\gamma(k^2)$. The curve shows $\hat{\Sigma}_T^\gamma(k^2)/k^2$.

Fig. 9: Photon Z boson mixing. Presented is $\hat{\Sigma}_T^{\gamma Z}(k^2)/k^2$.

Fig. 10, 11: Renormalized W, Z self energy. The curves are $\mathrm{Re}\hat{\Sigma}_T^{W,Z}(k^2)/(k^2 - M_{W,Z}^2)$ and $\mathrm{Im}\,\hat{\Sigma}_T^{W,Z}(k^2)/\mathrm{Im}\,\hat{\Sigma}_T^{W,Z}(M_{W,Z}^2)$.

Fig. 12, 13: Comparison between a Breit-Wigner distribution with $M_{W,Z}$ and $\mathrm{Im}\,\hat{\Sigma}_T^{W,Z}(M^2_{W,Z})$ as paramters (- - - - -) and the square of the modulus of the renormalized W, Z propagators (———).

Fig. 14 a,b: Real (a) and imaginary (b) parts of the electron self energy. Presented are the invariant functions $\hat{\Sigma}_S^e$, $\hat{\Sigma}_A^e$, $\hat{\Sigma}_V^e$.

Fig. 15: Real (———) and imaginary (- - - - -) parts of the neutrino self energy subtracted at $p^2 = 0$.

*) In fig.s 2 - 7 tadpole diagrams are omitted.

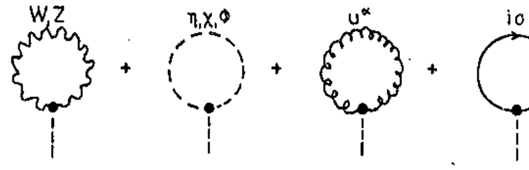



Fig. 1

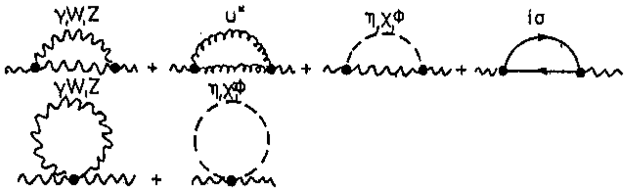



Fig. 2


Fig. 3

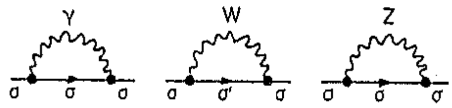



Fig. 4

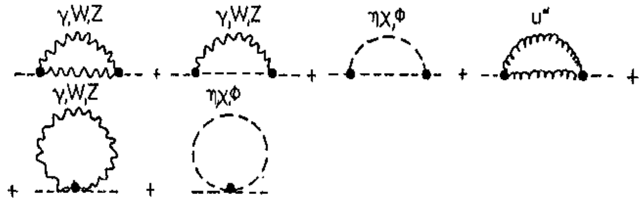



Fig. 5

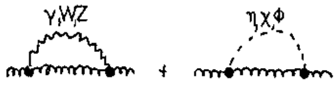



Fig. 6

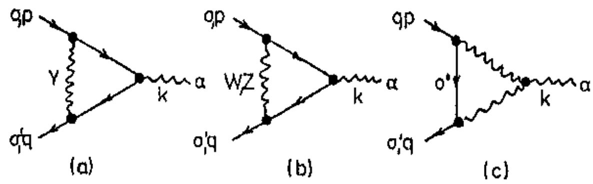



Fig. 7



Fig. 16 a,b: Real (———) and imaginary (- - - - -) parts of the weak corrections to the vector (a) and axial vector (b) photon formfactor of the electron.

Fig. 17: Real (———) and imaginary (- - - - -) parts of the electromagnetic formfactor of the neutrino.

Fig. 18 a,b: Real (———) and imaginary (- - - - -) parts of the weak contributions to the vector (a) and axial vector (b) Z boson electron formfactor.

Fig. 19: Real and imaginary parts of the vertex integral $\Lambda_2(k^2,M)$ for $M = M_W$, $M_Z$ and $\Lambda_3(k^2,M_W)$, $\Lambda_4(k^2;M_Z,M_W)$.

Fig. 20: Real (———) and imaginary (- - - - -) parts of the photonic invariant function $\Lambda_4^m(k^2,M_W,0)$ for $m = m_e$.

Fig. 8

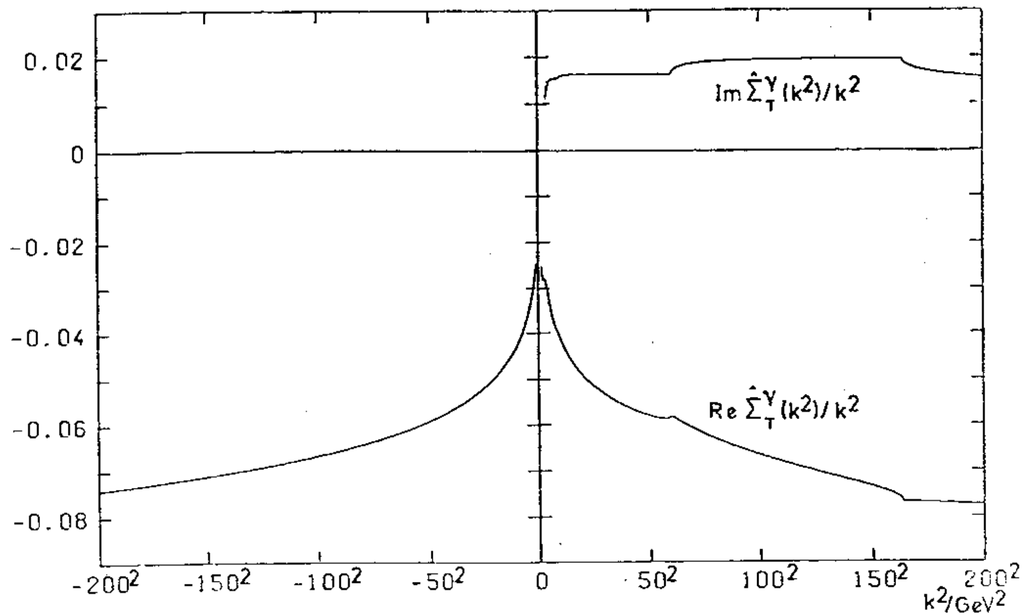



Fig. 10

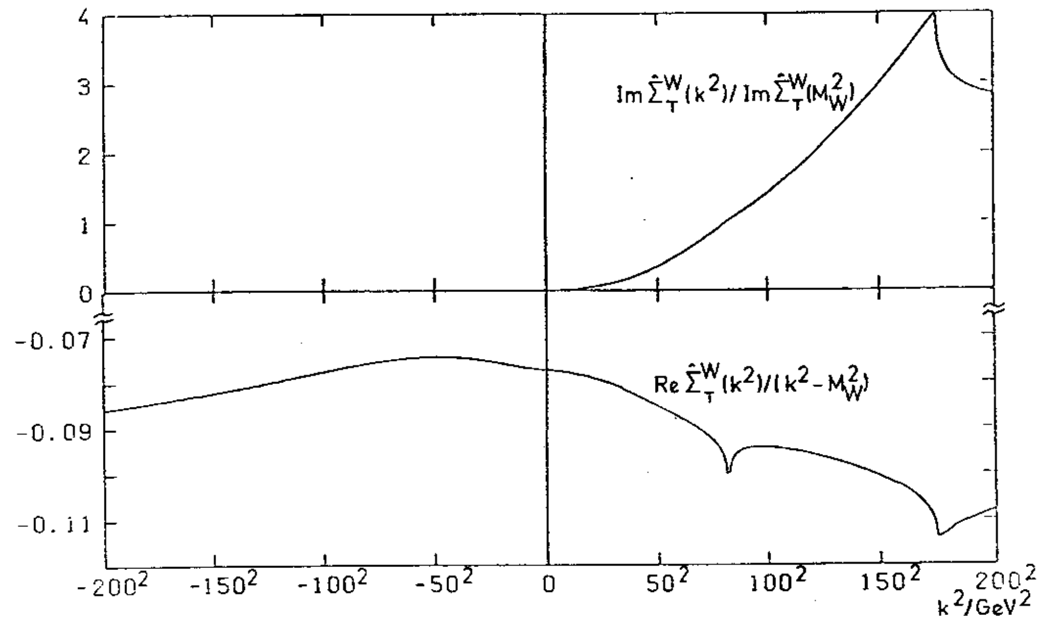



Fig. 9

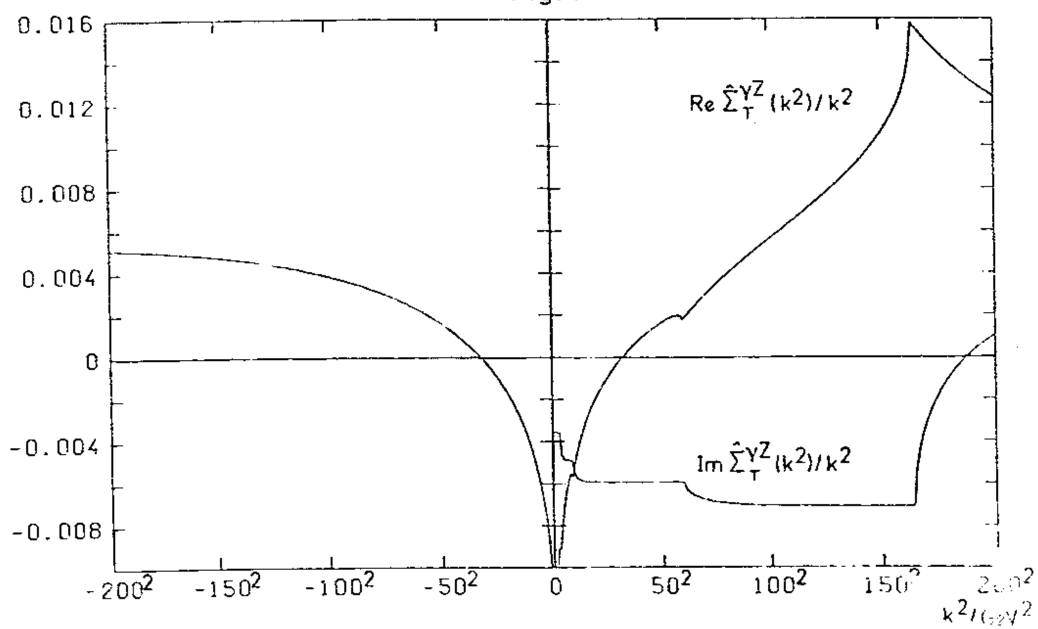



Fig. 11

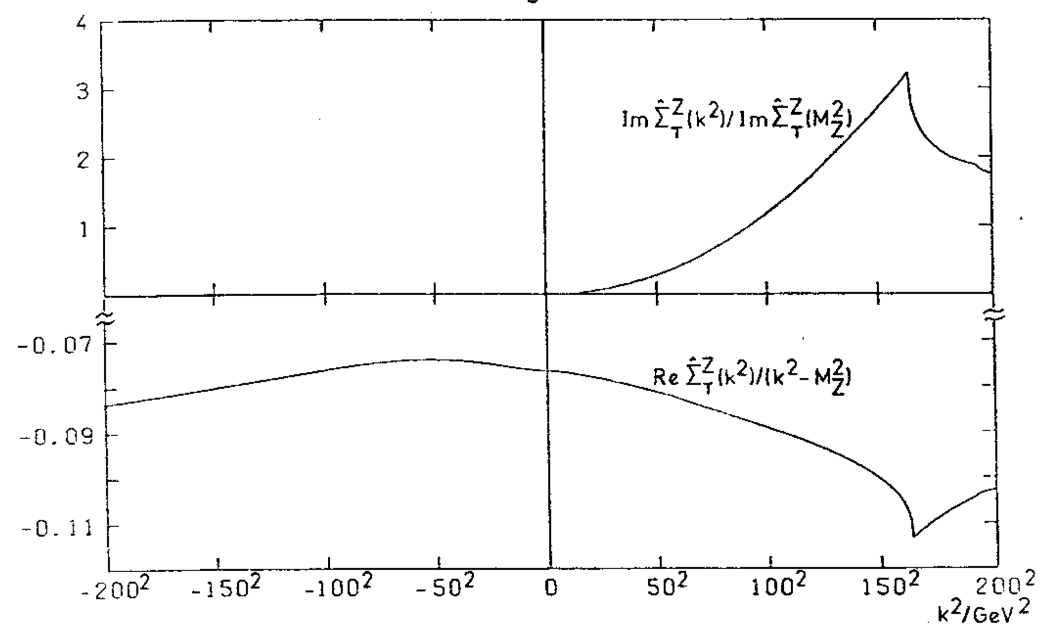
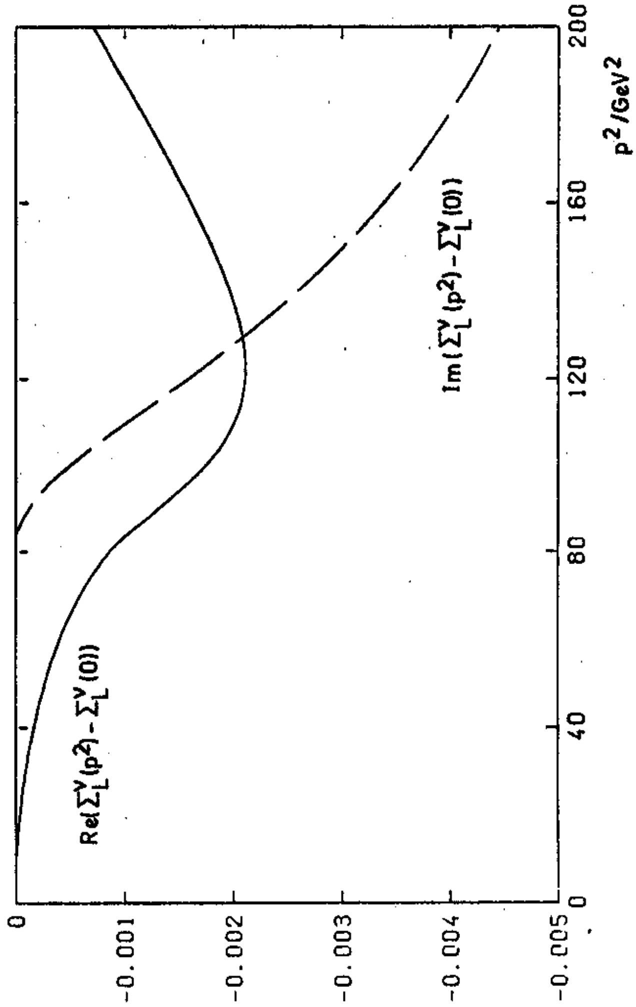
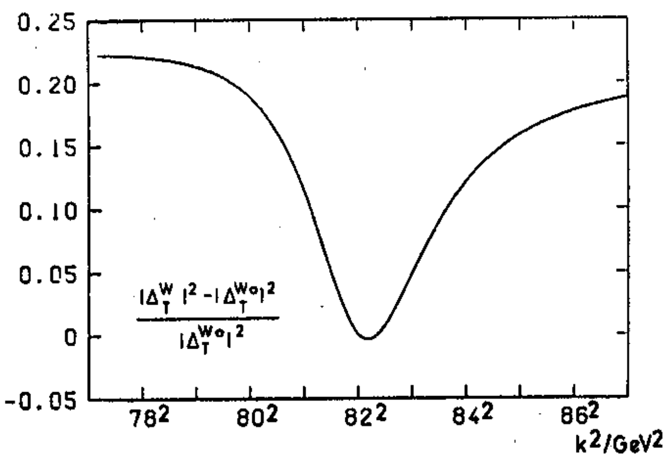


Fig. 14



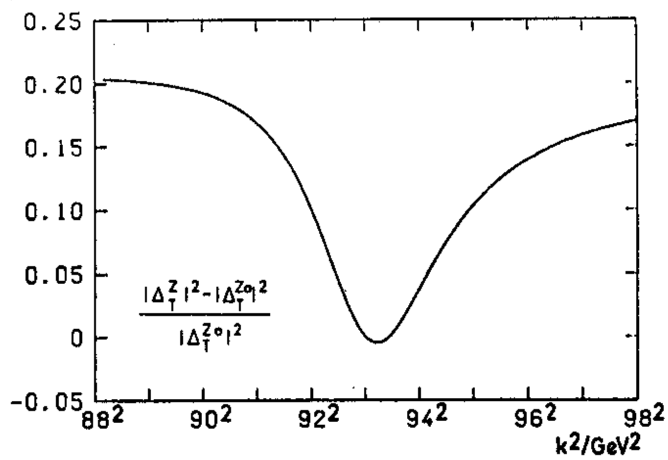
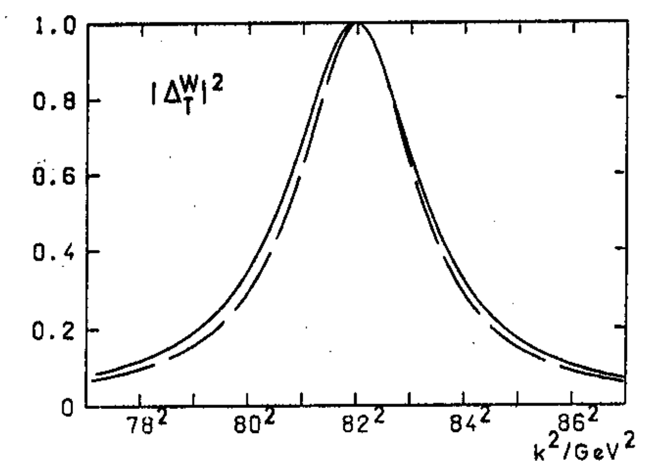




Fig. 12

Fig. 13



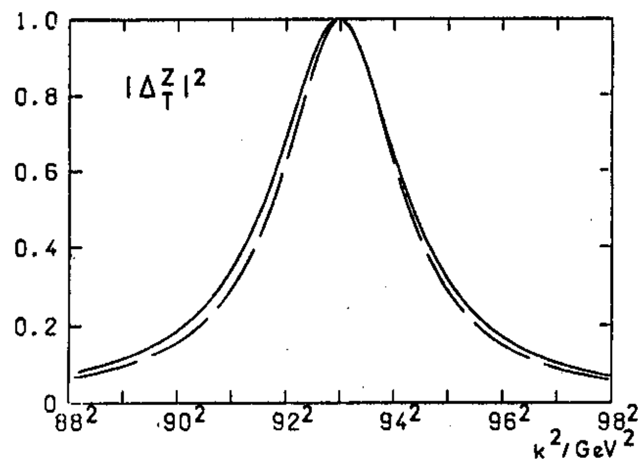



Fig. 16 a

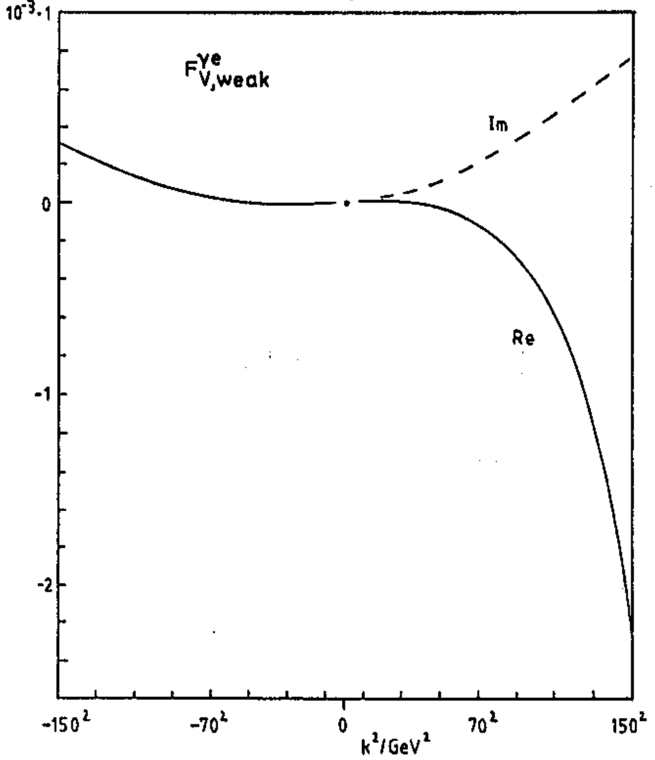



Fig. 16 b

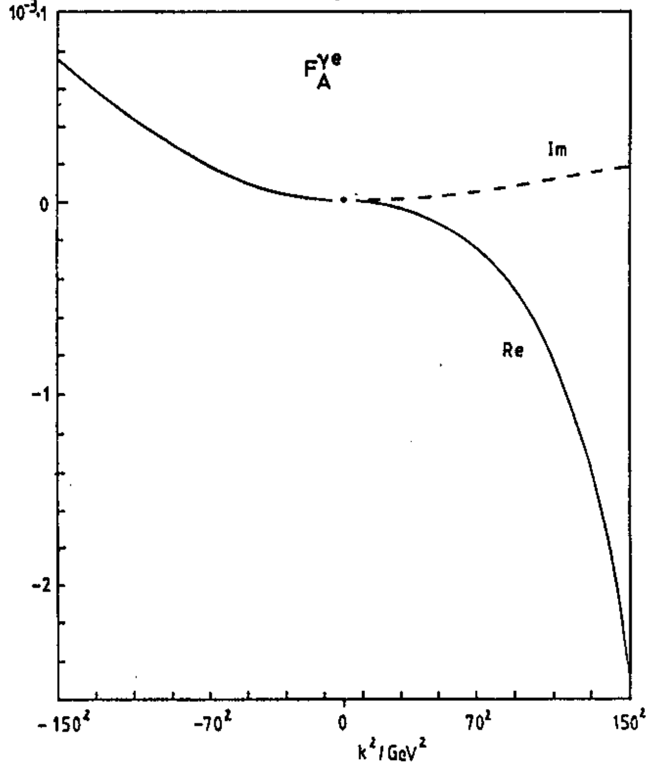



Fig. 15a

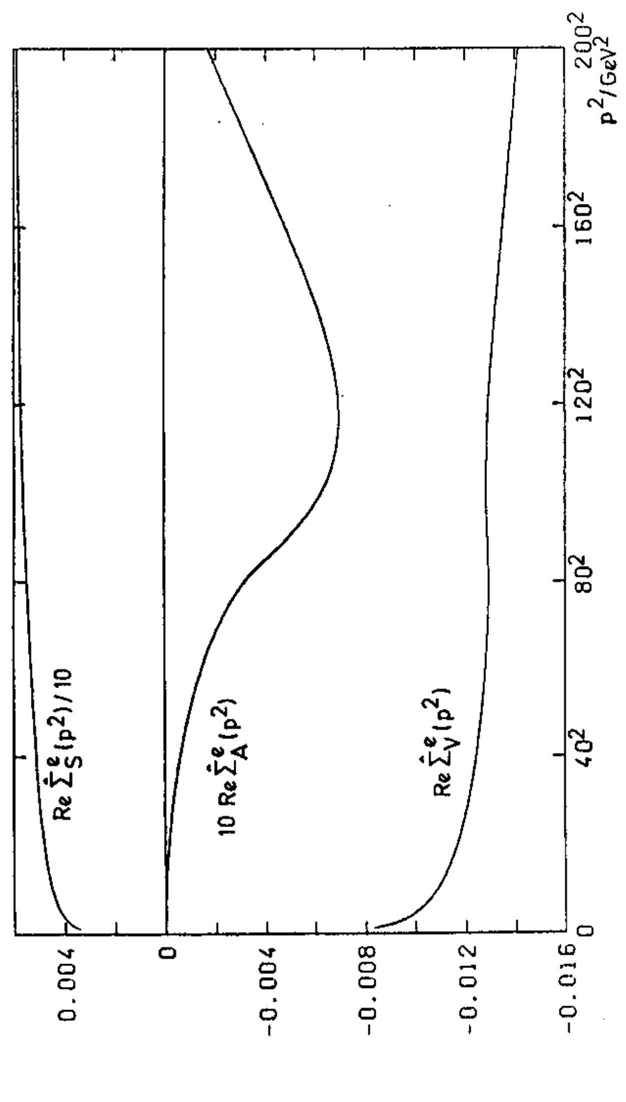



Fig. 15b

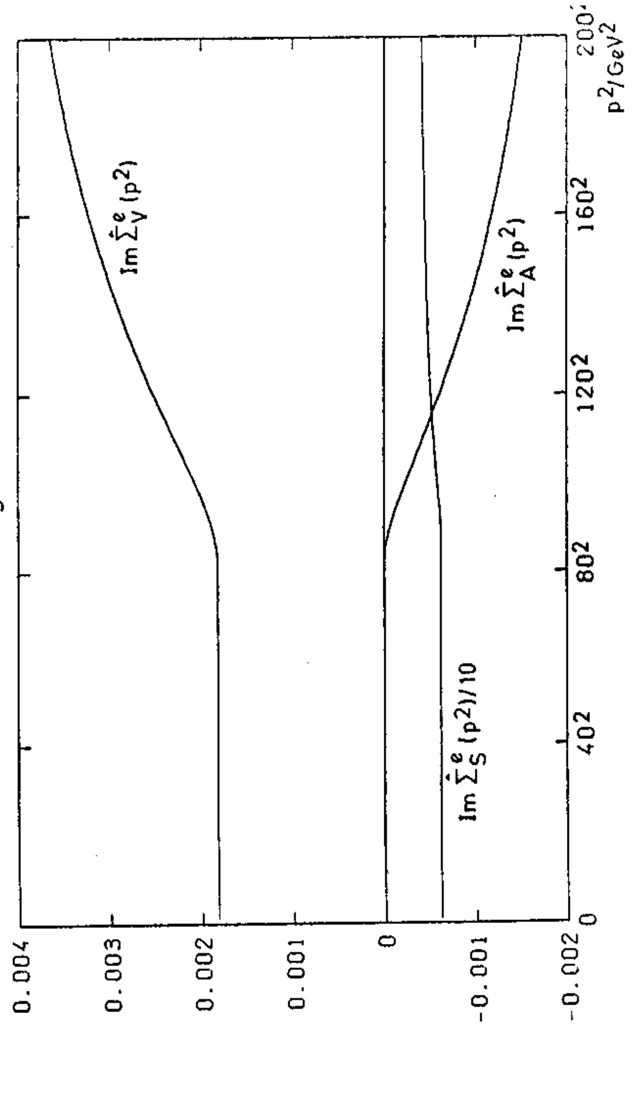


Fig. 17

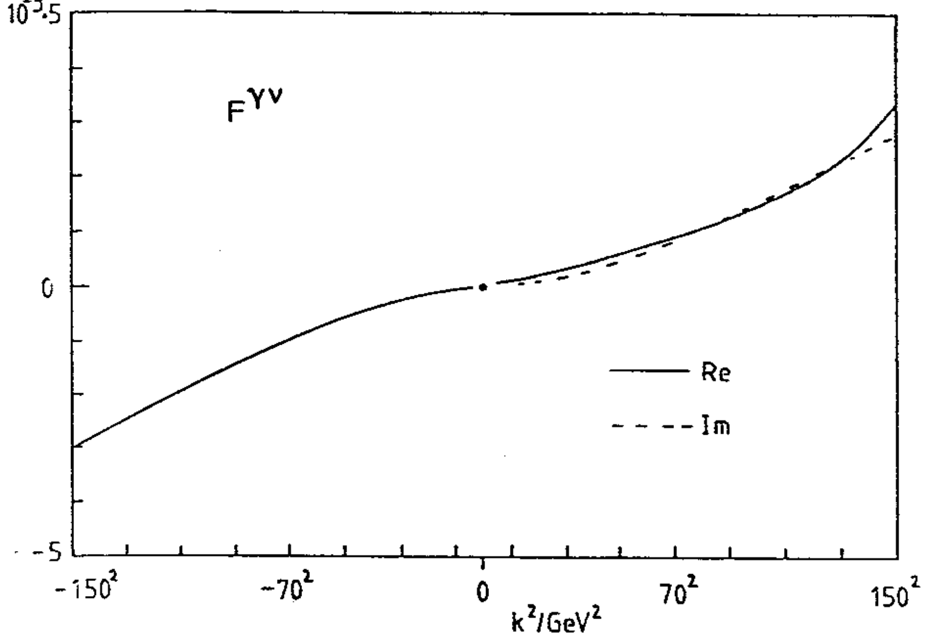



Fig. 18 a

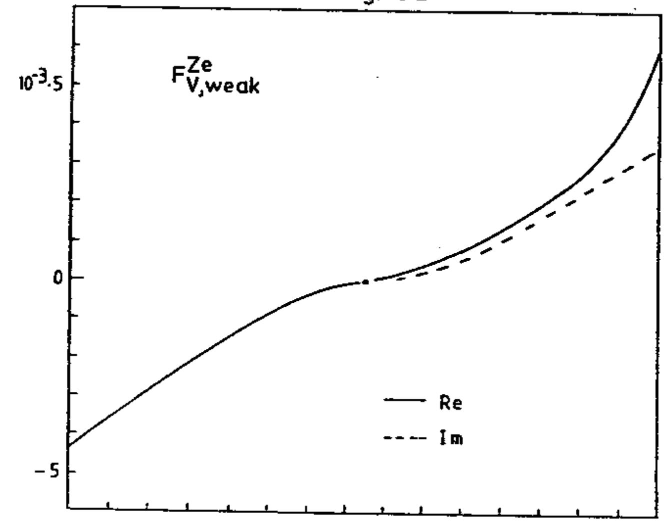



Fig. 18 b

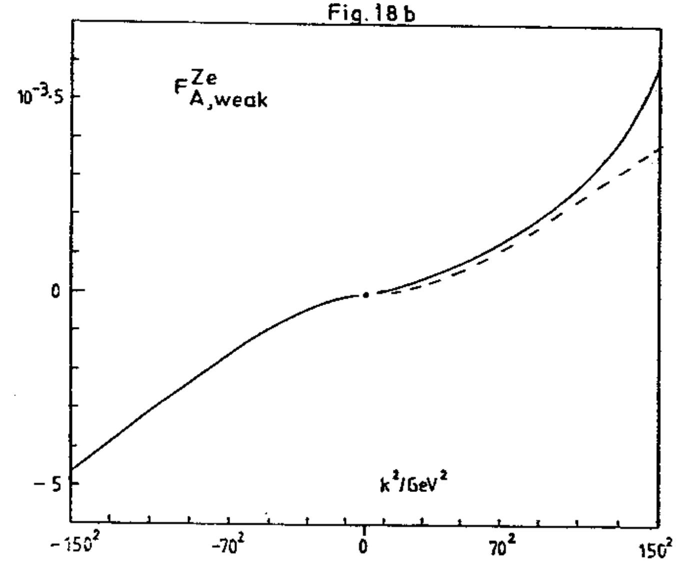
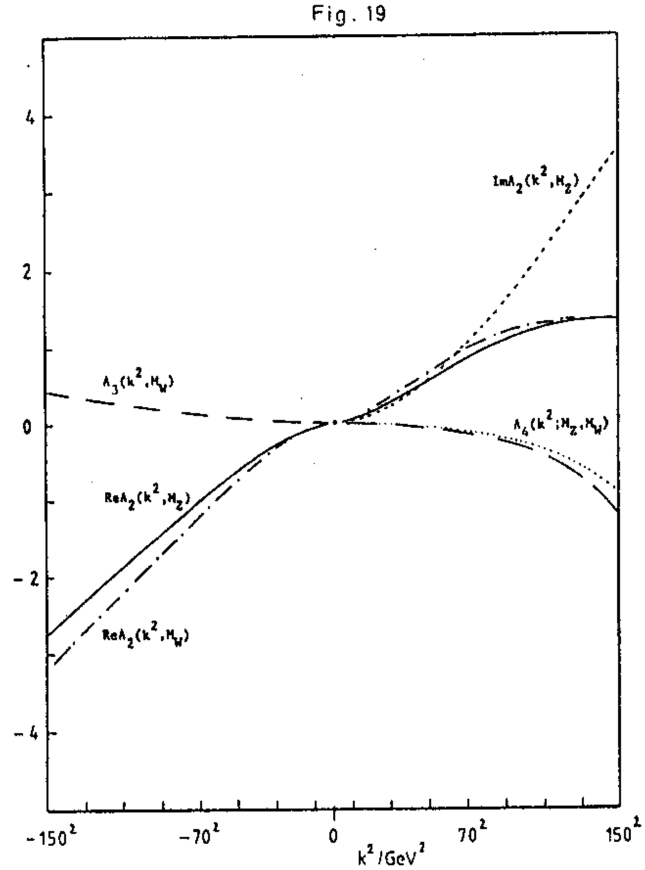
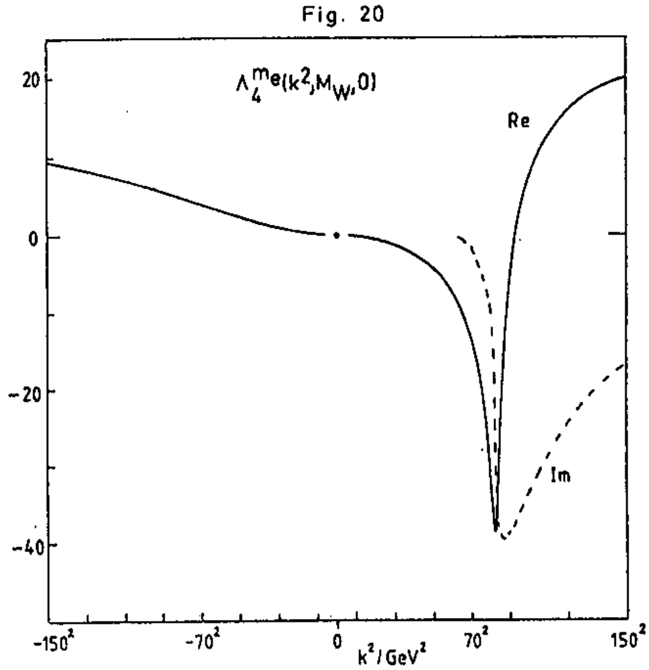


Fig. 19



4
2
0
- 2
- 4
ImΛ2(k2,MZ)
Λ3(k2,MW)
Λ4(k2;MZ,MW)
ReΛ2(k2,MZ)
ReΛ2(k2,MW)
-150²
-70²
0
70²
150²
k²/GeV²


Fig. 20



Λ4me(k2,MW,0)
Re
Im
20
0
-20
-40
-150²
-70²
0
70²
150²
k²/GeV²