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ABSTRACT

Some recent Monte Carlo calculations in the SU(2) Higgs-model with a scalar doublet field are reviewed. Questions about the dependence on the scalar self-coupling are discussed in the framework of a strong self-coupling expansion. The numerical results are consistent with an asymptotically free continuum limit at vanishing bare gauge coupling.

1. INTRODUCTION

Most of the recent efforts in Monte Carlo simulations of lattice gauge field theories [1,2] are concentrated on the study of pure SU(N) gauge theories and QCD-like theories with SU(N) gauge fields and a number of spin-$\frac{1}{2}$ fermion fields. The numerical study of quantum field theories containing scalar matter fields received up to now only a relatively limited amount of interest. The detailed investigation of pure gauge field theories is certainly the basis for any future understanding of the physical theories with matter fields, therefore it is unavoidable. The extraordinary numerical efforts invested in the latest simulations of QCD with quarks is motivated by the great challenge represented by the complex and experimentally well measured hadron spectrum. The fermionic matter

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fields are, however, notoriously difficult for numerical studies. Scalar matter fields are, from the numerical point of view, much simpler.

In the standard $SU(3) \otimes SU(2) \otimes U(1)$ theory of strong-electroweak interactions scalar fields play a very important rôle, because they are responsible, via the "Higgs-mechanism", for the masses of all the particles. Experimentally the "Higgs-sector" of the standard model is unknown, the Higgs-particle and its couplings are not yet directly observed. Therefore, the numerical study of field theories with scalar matter fields, in particular the Higgs-sector of the standard model, is both interesting and important. Since from the technical point of view scalar fields are simpler, it is conceivable that the study of quantum field theories containing scalar matter fields can contribute rather substantially to our understanding of the gauge-matter interactions.

One of the possible reasons for the limited interest in performing Monte Carlo simulations with scalar fields is, perhaps, the almost rigorously proven triviality of the simplest renormalizable scalar field theory. Namely, as a result of almost 15 years of hard work [3], we almost definitely know, that the single-component ϕ^4-theory in the four-dimensional continuum is trivial, i.e. equivalent to a free field theory. This fact, and the large number of apparently free parameters, discredited the Higgs-sector, too, because neglecting fermions and electromagnetism, the Higgs-sector of the standard model is the SU(2)-gauged version of a four-component (doublet) scalar field theory with ϕ^4 self-interaction. It is, however, a priori not clear what is the consequence of the triviality of ϕ^4 interaction for the "standard" SU(2) Higgs-model with doublet scalar field. There are, in principle, several possibilities:

i) the standard Higgs-model, too, is trivial in the four-dimensional continuum;

ii) its continuum limit is non-trivial but λ-independent (the ϕ^4 self-coupling λ is "irrelevant");

iii) it is non-trivial and λ-dependent.

In the first case, the ϕ^4 self-coupling "spoils" the otherwise nice gauge-interaction too, and the only possible continuum limit is a free theory of massive, spin-1 vector bosons ("W-bosons") and a single massive scalar ("Higgs-boson"). In the other two cases the ϕ^4 coupling has to be, presumably, asymptotically free, because the gauge interaction is asymptotically free. At high energies the gauge coupling is negligible and we are left with the "asymptotically trivial" (i.e. asymptotically free) ϕ^4-theory. In the interesting case ii) the continuum theory has one independent parameter less than the bare theory. Therefore, the Higgs-boson mass, for instance, is a function of the W-boson mass and of the (renormalized) gauge coupling constant.

2. STRONG SELF-COUPLING EXPANSION

2.1 Motivation

The standard Higgs-model has three bare coupling parameters: $\beta \equiv 4/g^2$ is the SU(2) gauge coupling, $\varkappa$ the hopping parameter of the scalar doublet field and λ is the ϕ^4 self-coupling (the precise definitions see below in the lattice action). In renormalized perturbation theory the renormalized ϕ^4 coupling λ_{ren} is a free parameter which can be traded, for instance, against the value of the mass of physical Higgs-particle m_H. At tree level the relation is

$$m_H = m_W \frac{\sqrt{8\lambda_{ren}}}{g_{ren}} . \qquad (1)$$

Here m_W is the W-boson mass and g_{ren} denotes the renormalized SU(2) coupling constant. The low energy phenomenology (below $\sim$100 GeV)

is rather insensitive to λ_{ren} (or to the Higgs-mass m_H). If, however, m_H is very large ($m_H \sim 1$ TeV), then the ϕ^4 self-coupling of the Higgs-field implies a strongly interacting Higgs-sector, which can produce rich, non-perturbative phenomena in the few-hundred GeV range [4,5].

According to the tree-level formula (1), for $\lambda \to \infty$ the Higgs-mass goes to infinity. Of course, due to the strong interaction, the tree level relation is not valid and therefore m_H can stay finite. In fact, there are large-N expansion arguments [6] implying that a "Higgs-remnant" with the quantum numbers of the physical Higgs-particle remains in the spectrum even for $\lambda \to \infty$. A first rough Monte Carlo investigation of the correlations showed [7], that $m_H/m_W \simeq O(1)$ is possible even at infinitely strong bare self-coupling $\lambda = \infty$. A more detailed numerical study of the λ-dependence was also carried out recently [8], showing a remarkable universal behaviour of the mass gaps for different λ-values. Plotting, for instance, the W-mass in lattice units (am_W) for fixed $\beta = 2.3$ and $\lambda = 0.1$, 0.5, 1.0 and ∞, as a function of an appropriately chosen third variable, one obtains Fig. 1. This shows, that the λ-dependence in the given λ-range is surprisingly weak, in fact, it is too weak to be seen by the limited numerical accuracy in Ref. 8). The interesting question is, of course, whether the λ-dependence goes away completely in the continuum limit or not. We shall see below, that a powerful tool for the study of λ-dependence is the strong self-coupling expansion (SSCE).

2.2 Lattice Action

The gauge field is described on the lattice, as usual, by the link variables $U(x,\mu) \in SU(2)$ (x = lattice point, $\mu = \pm1, \pm2, \pm3, \pm4$ lattice directions). The doublet field on the lattice points can be represented by its length $\rho_x \geq 0$ and by an angular variable $\alpha_x \in SU(2)$. Since α_x is equivalent to the

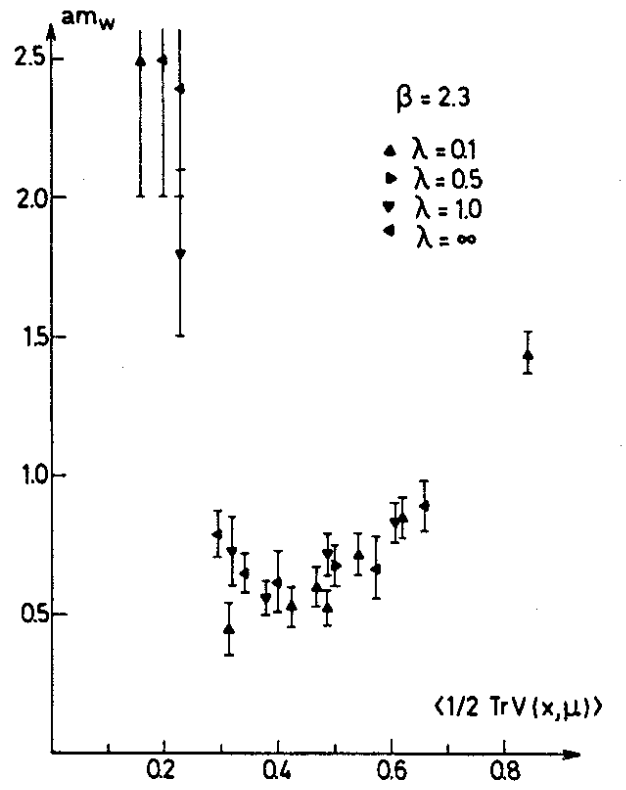

Fig. 1. The W-mass as a function of $\langle \mathrm{Tr}\, V(x,\mu)\rangle$ according to Ref. 8).

local gauge freedom, it is possible to introduce the gauge invariant link variables $V(x,\mu) \equiv \alpha^{+}_{x+\hat{\mu}} U(x,\mu) \alpha_x$ ($\hat{\mu}$ = unit vector in direction μ). In terms of the physical degrees of freedom the lattice action is

$$S^{(s)}_{\lambda\beta\kappa} = \beta \sum_P \left(1 - \tfrac{1}{2}\mathrm{Tr}\, V_P\right) + \\ + \sum_x \left\{ -3 \ln \rho_x + \lambda s^4 \left[\rho_x^2 - s^{-2}\left(1 - \frac{1}{2\lambda}\right)\right]^2 - \\ - \kappa s^2 \sum_\mu \rho_{x+\hat{\mu}} \rho_x \mathrm{Tr}\, V(x,\mu) \right\} . \tag{2}$$

Here $\sum_P$ means a summation over plaquettes. The arbitrary scale factor $s>0$ in $S^{(s)}_{\lambda\beta\kappa}$ corresponds to the rescaling $\rho_x \to s\rho_x$ of the integration variable ρ_x . The integration measure for the action (2) is $d\rho_x$ times the SU(2) Haar-measure $d^3V(x,\mu)$. In the limit $\lambda \to \infty$ the length is fixed to the value $\rho_x = s^{-1}$. For s = 1, the $\lambda = \infty$ limit of the action is

$$S^{\lambda=\infty}_{\beta,\kappa} = \beta \sum_P \left(1 - \tfrac{1}{2}\mathrm{Tr}\, V_P\right) - \kappa \sum_{x,\mu} \mathrm{Tr}\, V(x,\mu) . \tag{3}$$

2.3 General Scheme of SSCE

The action in Eq. (2) can be splitted up into its $s = 1$, $\lambda = \infty$ limit (3) at the hopping parameter value $s^2\kappa$ plus the rest. The strong self-coupling expansion is obtained by expanding into powers of the coupling term $s^2\kappa\,(\rho_{x+\hat{\mu}}\rho_x - 1)\,\mathrm{Tr}\, V(x,\mu)$ and then performing the integrations $d\rho_x$ over the length of the Higgs-field. The occurring integrals can be expressed by the parabolic cylinder function $D_p(z)$ like

$$J_k(\lambda) = \int_{-\sqrt{\lambda} + (2\sqrt{\lambda})^{-1}}^{\infty} dx \; e^{-x^2} \left(1 + \frac{x}{\sqrt{\lambda}} - \frac{1}{2\lambda}\right)^{1+\frac{k}{2}} = \tag{4}$$

$$= \exp\left[-\frac{1}{4}\left(\frac{1}{\sqrt{2\lambda}} - \sqrt{2\lambda}\right)^2\right] \frac{\Gamma\left(2+\frac{k}{2}\right)}{\sqrt{2}\,(2\lambda)^{1/2+k/4}} D_{-2-\frac{k}{2}}\left(\frac{1}{\sqrt{2\lambda}} - \sqrt{2\lambda}\right).$$

In the expectation values only the ratios (k = 1,2,...)

$$i_k = \frac{J_k(\lambda)}{J_0(\lambda)} = \frac{\Gamma(2+\frac{k}{2})\, D_{-2-\frac{k}{2}}\left(\frac{1}{\sqrt{2\lambda}} - \sqrt{2\lambda}\right)}{(2\lambda)^{k/4}\, D_{-2}\left(\frac{1}{\sqrt{2\lambda}} - \sqrt{2\lambda}\right)} \qquad (5)$$

appear. These behave asymptotically like

$$i_k \rightarrow \begin{cases} (\lambda\rightarrow\infty) & 1 + \frac{k(k-2)}{16\lambda} + o(\lambda^{-2}) \; ; \\ (\lambda\rightarrow 0) & \Gamma(2+\frac{k}{2})\left(1 - \lambda\frac{k(k+6)}{4} + o(\lambda^2)\right) . \end{cases} \qquad (6)$$

From the first line it can be seen, that for $\lambda\rightarrow\infty$ the expansion is similar to a series expansion into the inverse powers of λ. For small λ, however, every term in the series remains finite.

In general, the terms of the SSCE series are given by some correlation functions at $\lambda=\infty$. In particular, as we shall see below, in many important quantities the expectation value of $\mathrm{Tr}\,V(x,\mu)$ appears. Therefore, it is natural to choose the freedom in the scale factor s by requiring

$$\langle \mathrm{Tr}\,V(x,\mu)\rangle_{\lambda\beta\kappa} = \langle \mathrm{Tr}\,V(x,\mu)\rangle_{\lambda=\infty,\beta,s^2\kappa} . \qquad (7)$$

This means that SSCE is done (for fixed β) along the curves with constant link expectation value $\langle \mathrm{Tr}\,V(x,\mu)\rangle$. Since the phase transition between the confinement-like and Higgs-like phases (for fixed β and different λ) occurs at nearly the same value of $\langle \mathrm{Tr}\,V(x,\mu)\rangle$ [8], the choice in Eq. (7) implies that the expansion is done along curves which do not cross the phase transition surface. This is, of course, important for an optimal convergence radius.

2.4 Examples: Average ρ and Two-Link Correlations

The second order expansion for the average Higgs-field length is, to a good approximation,

$$\langle\rho_y\rangle_{\lambda\beta\kappa} = i_1 + 8\kappa i_1(i_2 - i_1^2)\langle \mathrm{Tr}\,V(x,\mu)\rangle_{\lambda=\infty} + \qquad (8)$$
$$+ 4\kappa^2\left(\langle \mathrm{Tr}\,V(x,\mu)\rangle_{\lambda=\infty}\right)^2 \left(i_3 i_2 + 7 i_3 i_1^2 + 13 i_1 i_2^2 + 30 i_1^5 - 51 i_2 i_1^3\right) + \cdots$$

The complete second order contains, in addition, an s-dependent piece which is proportional to some specific combinations of the connected 2-link correlations $\langle \mathrm{Tr}\,V(x_1,\mu_1)\,\mathrm{Tr}\,V(x_2,\mu_2)\rangle^c_{\lambda=\infty}$. For the choice of s in Eq. (7), however, this is only a small correction to the second order contribution given above. As another example, the lowest non-trivial order of the connected two-link correlation function is

$$\langle \mathrm{Tr}\,V(y_1\nu_1)\,\mathrm{Tr}\,V(y_2\nu_2)\rangle^c_{\lambda\beta\kappa} = \langle \mathrm{Tr}\,V(y_1\nu_1)\,\mathrm{Tr}\,V(y_2\nu_2)\rangle^c_{\lambda=\infty(\beta)}$$
$$+ \kappa(i_1^2 - s^2)\sum_{x,\mu>0}\langle \mathrm{Tr}\,V(y_1\nu_1)\,\mathrm{Tr}\,V(y_2\nu_2)\,\mathrm{Tr}\,V(x,\mu)\rangle^c_{\lambda=\infty} + \cdots$$

Such expressions can be used, for instance, for the SSCE of the mass gaps.

The comparison of Eq. (8) with the Monte Carlo data [8] at λ = 1.0 and 0.1 is shown by Table I. As it can be seen, the agreement of this low order SSCE and the Monte Carlo results is impressive. As a preliminary numerical study has shown, Eq. (8) qualitatively describes the average Higgs-field length $\langle\rho_y\rangle$ even at λ = 0.01. Higher order expansions for the I_W = 0 and I_W = 1 mass gaps, and a detailed comparison to numerical data will be published elsewhere [9]. The present conclusion concerning SSCE is:

i) $\lambda \gtrsim 0.1$ seems to be the region of strong self-coupling, where low orders of SSCE are sufficient;

Table I.

The second order SSCE for the average Higgs-field length $\langle \rho \rangle_{SC}$ compared to the Monte Carlo data $\langle \rho \rangle_{MC}$[8] at $\lambda = 1.0; 0.1$ and for $\beta = 2.3; \infty$.

$\lambda = 1.0$					
$\varkappa(\beta=2.3)$	$\langle \rho \rangle_{SC}$	$\langle \rho \rangle_{MC}$	$\varkappa(\beta=\infty)$	$\langle \rho \rangle_{SC}$	$\langle \rho \rangle_{MC}$
0.2	1.059	1.060	0.22	1.069	1.070
0.3	1.114	1.115	0.24	1.080	1.081
0.31	1.151	1.152	0.25	1.096	1.092
0.32	1.178	1.178	0.26	1.122	1.124
0.35	1.246	1.241	0.27	1.148	1.149
0.4	1.347	1.331	0.28	1.173	1.173
			0.3	1.219	1.216
			0.32	1.289	1.256
$\lambda = 0.1$					
$\varkappa(\beta=2.3)$	$\langle \rho \rangle_{SC}$	$\langle \rho \rangle_{MC}$	$\varkappa(\beta=\infty)$	$\langle \rho \rangle_{SC}$	$\langle \rho \rangle_{MC}$
0.19	1.336	1.353	0.155	1.280	1.290
0.195	1.423	1.450	0.16	1.291	1.302
0.2	1.523	1.558	0.163	1.306	1.319
0.205	1.594	1.636	0.165	1.330	1.346
0.21	1.663	1.712	0.167	1.360	1.379
0.22	1.780	1.845	0.17	1.408	1.430
0.3	2,535	2.628	0.175	1.484	1.514
			0.18	1.554	1.589

ii) SSCE may well be a convergent expansion for all $\lambda > 0$ values;

iii) a natural variable, instead of the hopping parameter $\varkappa$ (for fixed β and fixed lattice size) is $L \equiv \langle \frac{1}{2} \mathrm{Tr} V(x,\mu) \rangle$.

The practical advantage of SSCE is, that it allows to concentrate the numerical study to $\lambda = \infty$. The time consuming coverage

of the whole 3-parameter space $(\lambda, \beta, \varkappa)$ with the measured points is not necessary. Combined with the usual lattice perturbation theory in the gauge coupling g (at $g \to 0$), the SSCE may also help to pin down the question of λ-dependence in the continuum limit. A completely analytic study is also possible if, in addition, a hopping parameter expansion is done in the remaining $(\lambda = \infty, \beta = \infty)$ non-linear σ-model. This latter procedure is equivalent to the combination of the usual "high temperature expansion"[10] in the $\beta = \infty$ ϕ^4-model with the g = 0 perturbation theory.

3. SOME RECENT NUMERICAL RESULTS

In two recent papers[7,8] the correlations in W-boson and Higgs-boson channels and the static energy of an external colour charge pair were investigated in the standard Higgs-model by numerical Monte Carlo simulation. (For references to earlier numerical stuJies in the standard Higgs-model see these papers. For some new results see also the lecture of J. Jersák in these Proceedings). The numerical study of the correlations is useful for the understanding of the phase structure and of the continuum limit. In general, it is relatively easy to determine the correlations (much easier than e.g. the plaquette-plaquette correlations in pure gauge theory). In the vicinity of the phase transition between the confinement-like and Higgs-like regions, however, the long range correlations require large lattices and the critical slowing down (or metastability) is very dangerous. Sometimes surprisingly long runs are needed in order to be reasonably sure that the results refer to the equilibrium situation.

As examples of some new, high statistics results on 12^4 lattices with the full SU(2) group[11] let us consider the W-boson and Higgs-boson masses in lattice units in two points. At ($\lambda = \infty$, $\beta = 2.3$, $\varkappa = 0.41$), which is above the phase transition surface, one obtains $am_W = 0.507(14)$ and $am_H = 0.79(3)$. Lorentz-invariance is well satisfied in this point. In particular, one obtains from the zero momentum correlations $am_W = 0.505(15)$ and from the p=1 (in lattice units) correlations $am_W = 0.510(48)$. In another point ($\lambda = \infty$, $\beta = 2.3$, $\varkappa = 0.39$), below the phase transition surface, the result is: $am_W = 1.27(8)$ and $am_H = 0.39(2)$.

In Ref. 8) the static energy of an external colour charge pair (in short, "potential") was investigated in detail on 12^4 lattice and using the icosahedral approximation for SU(2). The aim was to obtain information about the renormalization group trajectories (RGT's), since along a RGT the potential can be rescaled to a common, physical curve. (This method was used for the study of scaling in pure SU(2) gauge theory in Ref. 12).) It turns out 8), that the potential shape sensitively depends on the value of m_H/m_W, therefore the RGT's can be determined quite well. An important point is, that the rescaling of the potential is possible, to a good accuracy, along curves in the λ = const. planes. This and the weak dependence of the masses on λ (if plotted like in Fig. 1) suggests, that the continuum limit can be λ-independent. Of course, this question has to be considered in more detail in future studied. In particular, the crucial point is, whether comparing the appropriate λ = const. RGT's, the λ -dependence weakens for growing β . For fixed λ the scaling properties of the standard lattice Higgs-model are qualitatively similar to the situation in QCD with a single quark mass parameter, i.e. there is an asymptotically free fixed point at $\beta = \infty$ and a (λ -dependent) critical hopping parameter $\varkappa = \varkappa_{crit}(\lambda)$. The different RGT's are parametrized by the mass parameter in the Higgs-potential: above the phase transition line there are the trajectories with spontaneous symmetry breaking, below the phase transition line the trajectories describing a confining theory with scalar matter fields (see Fig. 2).

Once the questions concerning the continuum limit are cleared, it becomes possible to calculate such phenomenologically interest-

ing quantities like the ratio of the Higgs-boson mass to the W-boson mass m_H/m_W. Since the renormalized SU(2) coupling is weak phenomenologically ($g^2_{ren} \gtrsim 0.5$ at the energy scale of m_W), one has to perform the Monte Carlo calculation at high β. This makes the calculation difficult. In a first attempt one can simplify the shape of the RGT's: as a zeroth approximation one can assume that the RGT goes nearly parallel to the phase transition surface between $\beta = \infty$ and the considered large β , and then it departs almost perpendicularly towards $\varkappa = \infty$.

In the Monte Carlo calculation I took $\lambda = 1.0$; $\beta = 8.0$ and $\varkappa = 0.30$ on 10^4 lattice with the full SU(2) group. The measured value of the renormalized gauge coupling, as determined by the Coulomb-potential at lattice distances 1-5, is $\alpha = 0.034(3)$. This roughly corresponds to the expected value $3g^2_{ren}/(16\pi) \simeq 0.03$. From 10000 measured sweeps, I obtained for the masses in lattice units $am_H = 1.4(2)$ and $am_W = 0.23(3)$, therefore

$$\frac{m_H}{m_W} \sim 6 \qquad (10)$$

I also checked in similar runs, that this ratio does not change appreciably between $\varkappa = 0.28$ and $\varkappa = 0.32$. Eq. (10) gives for the physical Higgs-boson mass $m_H \sim$ 500-600 GeV, but this has to be considered only as a first estimate. Completely neglected are here the virtual fermion loops and the electromagnetic U(1)-coupling. Furthermore, there is an unknown (presumably large) error due to finite lattice size effects. One has to study, in the future, also the scaling in this β-range (the precise shape of the physical RGT) and the question of λ-dependence.

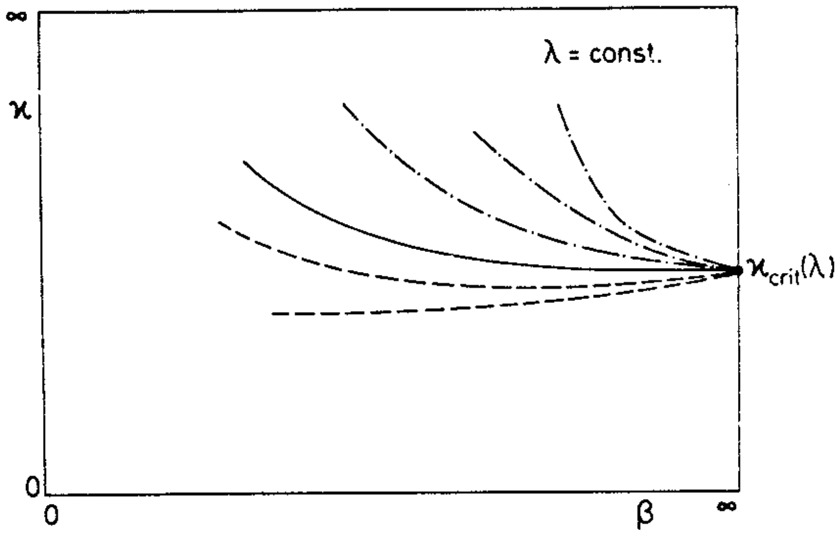

Fig. 2

Fig. 2. The schematic shape of the RGT's in a λ = const. plane. The full line gives the position of the phase transition. The dashed-dotted lines are the RGT's in the Higgs-phase, the dashed ones the RGT's in the confinement-like phase.

4. DISCUSSION

An important parameter in the standard Higgs-model is the ratio of the Higgs-boson mass to the W-boson mass $\xi \equiv m_H/m_W$. According to the Monte Carlo simulations [7,8,11] ξ is greater than 1 in the Higgs phase and smaller than 1 in the confinement phase. For the phenomenological value of the weak SU(2) coupling a first numerical estimate gives $\xi \sim 6$. If in the continuum limit (λ = const., $\beta \to \infty$, $\kappa \to \kappa_{crit}(\lambda)$) the second alternative mentioned in the introduction is realized (i.e. if λ is irrelevant) then, besides the Λ-parameter for the SU(2) gauge coupling, ξ is the only free physical parameter of the theory. Every other quantity,

like for instance $m_W/\Lambda_{SU(2)}$ or the value of the ϕ^4 coupling at some specific point (λ_{ren}), is a function of $\Lambda_{SU(2)}$ and ξ. The renormalization group equations for the renormalized Green's functions are valid with only two coupling parameters. In the usual weak coupling perturbation theory of the standard Higgs-model there are three free parameters. In order to take into account the non-perturbative constraint implied by the requirement of the existence of a mathematically well defined continuum theory, one has to impose on the three parameters of renormalized perturbation theory some external constraint. This is similar to the situation in pure ϕ^4 theory, where $\lambda_{ren} = 0$ is such an external constraint.

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