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# THE PION MASS: LOOKING FOR ITS ORIGINS

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# The pion mass: looking for its origins

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ABSTRACT

After explaining why pions are special excitations in QCD, I discuss how the pion mass reflects directly the dynamical scale of the strong interactions ($\Lambda_{QCD}$) and the scale of breaking of the weak interactions ($\Lambda_F$). To actually calculate the pion mass, however, requires understanding the origin of the quark masses and so I compare and contrast approaches to this latter problem, based on composite models and on superstrings.

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## I. The pion mass in QCD-dynamical issues

Yukawa's /1/ fundamental idea that pion exchange was responsible for the nuclear potential makes the pions special among the, now long, list of mesons. Although we now know that the description of nuclear forces is much more complicated, it remains true, nevertheless, that because the pion is the lightest of all mesons, the exchange of pions among nucleons gives rise to the longest range part of the nuclear potential: $V(r) \sim \frac{1}{r} e^{-m_\pi r}$
In Quantumchromodynamics (QCD) we now understand that pions, and other mesons, are quark-antiquark bound states. Nevertheless, even here, pions remain very special excitations again, principally, because of their very small mass. Indeed, the pion mass reflects a variety of dynamical issues - not all connected to strong interactions - central to elementary particle physics. It is these issues that I would like to discuss here in this Symposium in honor of Prof. Yukawa.

The reason that pions are special hadrons in QCD is that they are approximate Nambu Goldstone bosons of a spontaneously broken chiral symmetry. The masses of the u and d quarks are known /2/ to be much smaller than the dynamical scale of QCD, $\Lambda_{QCD}$. Hence, their neglect is a reasonable first approximation. In this limit, it is easy to see that the QCD Lagrangian has a global chiral symmetry. Clearly

$$\mathcal{L}_{QCD}^{ch.\,lim.} = -(\bar{u}\,\bar{d})\left\{\gamma^\mu \frac{1}{i}\left(\partial_\mu - ig\frac{\lambda_i}{2}A_{\mu i}\right)\right\}\binom{u}{d} - \frac{1}{4}F_i^{\mu\nu}F_{i\mu\nu} \tag{1}$$

is invariant under the $U(2)_L \times U_R(2)$ transformations:

$$\begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \to e^{i T_a \alpha^a_{L,R}} \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} \qquad T_a = \left(\frac{\tau_a}{2}, 1\right) \tag{2}$$

In fact, actually, effects connected with the chiral anomaly /3/ reduce this symmetry only to $SU(2)_L \times SU_R(2) \times U(1)_{L+R}$. At any rate, it is clear that QCD in this limit has a global chiral symmetry. This symmetry, however, is not respected by the vacuum and is therefore spontaneously broken.

Condensates of quark bilinears

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle \neq 0$$

form in the QCD vacuum, which break the $SU(2)_L + SU_R(2) \times U(1)_{L+R}$ symmetry to $U(2)_{L+R}$. Because of this spontaneous breakdown, three massless Nambu Goldstone bosons must appear in the bound state spectrum of QCD and it is natural to associate the triplet of pions ( $\pi^+, \pi^-, \pi^0$ ) with these excitations. In the limit in which $m_d$ and $m_u$ vanish, the pion mass vanishes. Restoring these small mass parameters in the theory will give the pions their small mass.

Because the masses of the u and d quarks are really much smaller than $\Lambda_{QCD}$, it is possible to compute the pion mass perturbatively. The perturbing Lagrangian

$$\mathcal{L}_{pert.} = -m_u \bar{u}u - m_d \bar{d}d \tag{3}$$

taken in lowest order, yields for the pion mass the formula:

$$m_\pi^2 = -\langle\pi|\mathcal{L}_{pert.}|\pi\rangle = \langle\pi|m_u\bar{u}u + m_d\bar{d}d|\pi\rangle \tag{4}$$



This formula can be related to the vacuum expectation values which gave rise to the spontaneous breakdown of the chiral symmetry, by using the fact that the pions are the (approximate) Nambu Goldstone bosons of this breakdown. The symmetry that breaks down in QCD is $SU(2)_{R-L}$. Hence, the associated global symmetry currents

$$A^\mu_a = (\bar{u}\,\bar{d})\,\gamma^\mu\gamma_5\,\frac{\tau_a}{2}\begin{pmatrix} u \\ d \end{pmatrix} \tag{5}$$

connect the vacuum state to the one pion state, of momentum P:

$$\langle 0|A^\mu_a|\pi_b\rangle = i f_\pi P^\mu \delta_{ab} \tag{6}$$

The proportionality constant $f_\pi$ can be determined from the pion weak decay $\pi \to \mu\nu_\mu$, since the currents $A^\mu_a$ are also precisely the weak currents that appear in the standard electroweak theory /4/. Using straightforward current algebra techniques, one can transform Eq. (4) into an expression involving vacuum expectation values:

$$m_\pi^2 = \frac{1}{f_\pi^2}\langle 0|[Q^5_a,[Q^5_a,\mathcal{L}_{pert.}]]|0\rangle \tag{7}$$

This is the, well known, Dashen's formula /5/, where

$$Q^5_a = \int d^3x \; A^0_a(x) \tag{8}$$

are the axial charges, which are connected with the symmetry breakdown. Calculating the triple commutator yields, finally, for the pion mass the expression

$$m_\pi^2 = (m_u + m_d)\left[-\frac{\langle 0|\bar{u}u|0\rangle}{f_\pi^2}\right] \tag{9}$$

where one has used that the vacuum expectation of $\bar{u}u$ and $\bar{d}d$ are the same.

In general, one uses Eq. (9), taking $m_\pi$ and $f_\pi$ from experiment, to predict $m_u+m_d$ or $\langle 0|\bar{u}u|0\rangle$, given an estimate of the other. For instance, using QCD sum rules /6/ one can extract a value for the quark masses /2/ - at a running scale of 1 GeV:

$$m_u + m_d = 14 \pm 4 \text{ MeV} \tag{10}$$

Then from Eq. (9) one obtains for the quark condensate

$$\langle 0|\bar{u}u|0\rangle = -\left[(225 \pm 25)\,\text{MeV}\right]^3 \tag{11}$$

Alternatively, one can estimate $\langle 0|\bar{u}u|0\rangle$ by means of a QCD lattice calculation. The matter is a little bit complicated, since this condensate does not follow an asymptotic scaling law. Nevertheless, a very recent computation of Barbour et al. /7/ gives a value

$$\left[-\langle 0|\bar{u}u|0\rangle\right]^{1/3} \simeq 220\text{-}250 \text{ MeV} \tag{12}$$

which is perfectly consistent with (11) and, using Eq. (9), implies

$$m_u + m_d \simeq 10\text{-}15 \text{ MeV}, \tag{13}$$

in agreement with the QCD sum rule value. This small value for the quark masses, incidentally, justifies the perturbative approach to compute $m_\pi^2$, which lead to Eq. (9).



I would like, however, to look at Eq. (9) in a different way. Basically there are two pieces in this equation. The term $\langle 0|\bar{u}u|0\rangle / f_\pi^2$ is a purely strong interaction quantity, while the quark masses are external parameters. In principle also $f_\pi$, just like $\langle 0|\bar{u}u|0\rangle$ can be calculated in QCD. Since Eq. (9) is already first order in the quark masses, this calculation can be done in the chiral limit of Eq. (1) and so it can only depend on the dynamical scale, $\Lambda_{QCD}$, of the strong interactions. Thus one has

$$-\frac{\langle 0|\bar{u}u|0\rangle}{f_\pi^2} \sim \Lambda_{QCD} \tag{14}$$

To determine the constant of proportionality requires a difficult dynamical calculation, but one expects that it be of order unity. At any rate, one can write schematically

$$m_\pi^2 \sim (m_u + m_d)\,\Lambda_{QCD} \tag{15}$$

To make further progress in computing, ab initio, the pion mass requires an understanding of the origin of the quark masses, which are more closely rooted in the weak interactions.

Before addressing this question, there is a slight complication that needs to be attended to. The charged and neutral pions do not have the same mass. Radiative effects give an extra electromagnetic contribution to the mesons and this mass shift is calculable, again, perturbatively. One has

$$\delta m_\pi^2 = m_{\pi^+}^2 - m_{\pi^0}^2 = -\langle \pi|\mathcal{L}_{\text{eff.}}^{em}|\pi\rangle \tag{16a}$$

where

$$\mathcal{L}^{em}_{eff}(x) = \frac{e^2}{2}\int d^4y \; \Delta^{\mu\nu}(x-y)\, T\left(J^{em}_\mu(x)\, J^{em}_\nu(y)\right) \tag{16b}$$

with $\Delta^{\mu\nu}$ being the virtual photon propagator. At first sight, the calculation of this purely electromagnetic mass shift via Eqs. (16) may appear dubious. The time ordered product in Eq. (16) is singular as $x \to y$ and so it is not clear whether a finite result will emerge from the calculation. In fact, as I shall show, in the chiral limit one has no need for counterterms and $\delta m^2_\pi$ is directly calculable as it stands.

The potentially divergent pieces in the time ordered product can be easily identified by means of an operator product expansion /8/

$$T\left(J^\mu_{em}(x)\, J^{em}_\mu(y)\right) = C_o(x-y)\,1 + \sum_q C_q(x-y)\, m_q \bar{q} q(y) + C_g(x-y)\, F^{\mu\nu}_i F_{i\mu\nu}(y) + \ldots \tag{17}$$

In the above, I have displayed explicitly only the potentially singular terms, with $C_o \sim ((x-y)^2)^{-3}$ and $C_q \sim C_g \sim ((x-y)^2)^{-2}$. Because of (17) one sees that the divergent pieces in $\mathcal{L}^{em}_{eff}$ can always be absorbed by appropriate counterterms in $\mathcal{L}_{QCD}$, since the operators appearing in (17) are precisely those appearing already in the QCD Lagrangian. That is, it is always possible to write

$$\mathcal{L}^{em}_{eff} = \mathcal{L}^{em}_{finite} + \Delta E\, 1 + \sum_q \Delta m_q \bar{q} q + \delta Z_g\, F^{\mu\nu}_i F_{i\mu\nu} \tag{18}$$



However, in the chiral limit, one can show that these counterterms vanish and $\delta m^2_\pi$ is directly calculable and finite. First of all $\Delta E$ - the vacuum energy - does not contribute to $\delta m^2_\pi$, since it gives a common mass shift to both $m^2_{\pi^+}$ and $m^2_{\pi^0}$. Further, in the chiral limit, since $m_q \to 0$, one needs no fermion mass counterterm. Finally, the last term in Eq. (18) is also absent. The trace anomaly /9/ relates the matrix element of the gluon field strength to the trace of the energy momentum tensor and this trace, between pion states, vanishes, since the pion mass vanishes in the chiral limit:

$$\langle \pi | F^{\mu\nu}_i F_{i\mu\nu} | \pi \rangle \sim \langle \pi | \theta^\mu_\mu | \pi \rangle \sim m^2_\pi \to 0 \tag{19}$$

The actual calculation of the electromagnetic mass shift $\delta m^2_\pi$ was done long ago by Das, Guralnik, Mathur, Low and Young /10/ in a classic paper. Using Eqs. (16) and current algebra techniques, to replace the pion states by the vacuum state, these authors obtained for $\delta m^2_\pi$, in the chiral limit, the expression

$$\delta m^2_\pi = m^2_{\pi^+} - m^2_{\pi^0} = \frac{3e^2}{f^2_\pi}\, \frac{1}{i}\int \frac{d^4q}{(2\pi)^4} \int d\mu^2\, \frac{\mu^2\left(\rho_V(\mu^2) - \rho_A(\mu^2)\right)}{q^2(q^2+\mu^2)} \tag{20}$$

where $\rho_V(\mu^2)$ and $\rho_A(\mu^2)$ are the spectral functions for the Vector and Axial vector current two point functions, respectively. These functions can, in principle, be computed in QCD. In the pre-QCD days of Ref. /10/, Eq. (20) was evaluated approximately by saturating $\rho_V$ and $\rho_A$ by the contributions of the $\rho$ and $A_1$ mesons respectively, using the Kawarabayashi, Suzuki, Fayazuddin, Riazuddin (KSFR) relation to interrelate the relevant coupling constants to $f_\pi$. This yielded the result

$$\delta m_\pi^2 = \frac{3\alpha}{2\pi}\, m_\rho^2 \ln 2 \tag{21}$$

which implies $m_{\pi^+} - m_{\pi^0} \simeq$ 5 MeV, very near to the observed experimental value $m_{\pi^+} - m_{\pi^0} \simeq$ 4.6 MeV.

Two remarks are in order:

(1) Before the advent of QCD, two calculations tried to "improve" on the results of Das et al. /10/ and found that $\delta m_\pi^2$ was divergent! This caused a certain amount of confusion and certainly called into question whether the good result of Eq. (21) might just be an accident. In one of the calculations /11/, contributions to $\delta m_\pi^2$ for $m_u, m_d \neq 0$ were considered. In the other /12/, corrections to $m_{\pi^+}^2 - m_{\pi^0}^2$ of order $e^4$ were studied. In hindsight, it is now easy to understand why the results were divergent, since in both cases one is away from the chiral limit ($m_{\pi^+}^2 \neq 0$ at $O(e^2)$!) and one should expect counterterms. However, since these counterterms correspond to redefinitions of the parameter in the QCD Lagrangian, expressing $\delta m_\pi^2$ entirely in terms of physical bound state masses should properly serve to remove these unwanted infinities. Hence, Eq. (21) should be taken seriously. The chiral limit is a good limit in QCD.



(2) Although Eq. (21) gives a small mass shift of 5 MeV, numerically the radiative effect is not small:

$$\delta m_\pi^2 \simeq (37\ \mathrm{MeV})^2 \tag{22}$$

It is only because $m_u+m_d$ is sufficiently big ($m_u+m_d \simeq 15$ MeV) that the charged and neutral pions are approximately degenerate. If $(m_u+m_d)$ were much smaller than an MeV, the chiral breaking mass shift common to both the $\pi^+$ and $\pi^0$ mesons would be much smaller than the radiative shift ($m_{\pi^\pm}^2 \gg m_{\pi^0}^2$ !) and the tails of nuclear forces due to charged or neutral pion exchange would break isospin badly. Clearly, the value of the u and d quark masses play a crucial role in the strong interactions, although they are parameters outside of these interactions!

Including also the effects of electromagnetism, which along with the quark masses breaks explicitly the global symmetry of $\mathcal{L}_{QCD}$ of Eq. (1), we can write for the pion mass, schematically

$$m_{\pi^0}^2 \sim (m_u + m_d)\,\Lambda_{QCD} \tag{23a}$$

$$m_{\pi^+}^2 \sim (m_u + m_d)\,\Lambda_{QCD} + \alpha\,\Lambda_{QCD}^2 \tag{23b}$$

These formulas show, again, that to understand the pion masses we must understand where the quark masses come from, and ultimately what is their relationship with the dynamical scale of the strong interactions $\Lambda_{QCD}$, as well as to the strength of purely electromagnetic effects.

## II. What is the source for the masses of quarks

To determine the pion mass it is necessary to know the value of $(m_u+m_d)$. Why this sum of masses is near 15 MeV is part of the general puzzle of what physics fixes the values of all quark and lepton masses. The spectrum of fermion masses, as shown in Table I, adopted from Ref. /2/, appears to be quite random, although there is a definite family pattern. Both the interfamily splittings as well as the intrafamily splittings are large. Furthermore, although the u quark is lighter than the d quark, this pattern is reversed for the other two generations.

Table I: Spectrum of charged quarks and leptons, from Ref. 2 *

| | | |
|---|---|---|
| $m_e \sim 0.5$ MeV | $m_\mu$ 105 MeV | $m_\tau \sim$ 1.77 GeV |
| $m_u \sim$ 5 MeV | $m_c \sim 1350$ MeV | $m_t \overset{(!)}{\sim}$ 40 GeV |
| $m_d \sim$ 9 MeV | $m_s \sim 175$ MeV | $m_b \sim$ 5.3 GeV |

* All quark masses are at the scale of 1 GeV. Neutrino masses, if they exist, are certainly much smaller than those of their charged counterparts of the same family.

In the standard electroweak theory /4/ all the numbers in Table I, along with the values of the related weak mixing angles, are free parameters. Nothing fixes these mass values and, a priori, the quark and lepton masses could have any value. The only relevant statement one can make is that the appearance of fermion masses, at all, is connected with the breakdown



of SU(2) x U(1). Because, in the standard model, the left handed fermions are doublets under SU(2) while the right handed fermions are singlets, clearly no SU(2) invariant mass term is allowed. However, such a mass term may arise after spontaneous breakdown. A coupling of left handed fermions and right handed fermions to an SU(2) doublet Higgs field, $\Phi$, is perfectly SU(2) invariant. These couplings can generate a fermion mass, whenever there is a symmetry breakdown. Clearly when $\Phi \to \langle \Phi \rangle$, mass terms ensue out of the couplings of $\Phi$ to the massless fermions in the theory.

Schematically, let $\psi_L$ and $\psi_R$ denote the left and right helicity projections of some quark or lepton field. Then from the couplings of $\Phi$ to these fields (the, so called, Yukawa couplings!):

$$\mathcal{L}_{\text{Yukawa}} = h\,(\bar{\psi}_L \Phi \psi_R + \bar{\psi}_R \Phi^+ \psi_L) \tag{24}$$

it follows, after symmetry breakdown $[\Phi \to \langle \Phi \rangle]$, that the fermion $\psi$ has a mass:

$$m_\psi = h \langle \Phi \rangle \equiv \frac{h\,v}{\sqrt{2}} \tag{25}$$

Although the value of $\langle \Phi \rangle$, or of v, is fixed by the scale of the electroweak breakdown (the Fermi scale $\Lambda_F$)

$$v = (\sqrt{2}\,G_F)^{-1/2} \equiv \Lambda_F \simeq 250 \text{ GeV} \tag{26}$$

the value of h is arbitrary. Hence it follows that also the value of the fermion masses are arbitrary.

More specifically, the couplings of the doublet Higgs $\Phi$ are not family diagonal. Thus, in general, the coupling constant h above should really be replaced by a coupling constant matrix $h^f_{ij}$, where f distinguishes the fermion type (f: l,d,u) and i and j go over the generation indices. This leads then to mass matrices $M^f_{ij}$ which again are arbitrary, because the couplings $h^f_{ij}$ are arbitrary. The quark and lepton masses are the eigenvalues of these mass matrices.

Eventhough in the standard model $m_u$ and $m_d$ are not predicted, since the effective Yukawa couplings $(h_u,h_d)$ are not fixed, nevertheless the formula for the pion mass takes a pleasing form. Neglecting the, numerically small, electromagnetic contribution one has

$$m_\pi^2 \sim (h_u + h_d)\, \Lambda_F \Lambda_{QCD} \qquad (27)$$

That is, the pion mass squared is proportional to both the dynamical scale of the strong interactions, $\Lambda_{QCD}$, as well as to the scale of the breakdown of the electroweak interactions, the Fermi scale $\Lambda_F$. Apart from a dynamically computable coefficient of O(1), the pion mass turns out to be small since the effective Yukawa couplings $h_u$ and $h_d$ are very small numbers. These numbers are of order 2-5 x $10^{-5}$, to give the observed values for $m_u$ and $m_d$. However, no understanding of why these numbers are what they are - in particular why they are so small - is possible within the standard model. The couplings $h_u$ and $h_d$ are fixed by the value of the quark masses and not vice versa!

To predict the quark and lepton masses, and therefore a fortiori $m_\pi^2$,



one must go beyond the standard model. Two routes are clear. Either

(i) The masses of the fermions are not related to Yukawa couplings of fermions to a doublet Higgs boson. This would follow if no real Higgs sector for the standard model existed, but SU(2) x U(1) would dynamically break down. Still, one would expect that the fermion masses be proportional to the Fermi scale of SU(2) x U(1) breakdown. However, the proportionality coefficient $c_f$, where

$$m_f = c_f \Lambda_F . \qquad (28)$$

would then be dynamically determined. In particular, if quarks and leptons are not elementary but bound states of a more fundamental theory, one should be able to calculate the coefficients $c_f$ from the bound state dynamics.

(ii) The mass of the fermions are related to Yukawa couplings. However, the values of the Higgs couplings $h^f_{ij}$ are not arbitrary but are fixed by a new theory which contains the standard model. Since the Higgs sector of the standard model is now assumed to exist, at least for a set of energies well below that of the new theory which subsumes the standard model, one must make sure that this sector is stable. This naturality requirement obtains if there is a low energy supersymmetry /13/, broken at most by masses of the order of the Fermi scale. If $\tilde{m}_f$ denotes the mass of a fermion superpartner, then $m_f - \tilde{m}_f \sim O(\Lambda_F)$. Since the new theory must predict all $h^f_{ij}$, it must, in a sense, be a "theory of everything" (TOE). Superstring theories have the potentiality of being a TOE and thus being able to predict all relevant Yukawa couplings.

I will discuss in the next section in some more detail these contrasting approaches to the origin of fermion masses based, respectively, on compositeness and superstrings. However, here I would like to make some general remarks. Composite models are a bottom up approach to the problem, in which a solution to the mass question is sought via a new layer of dynamics. Superstrings are really a top down approach. Dynamics at a very high scale fix kinematically the Yukawa couplings $h^f_{ij}$, but then all the rest is really determined perturbatively, since these Yukawa couplings are really very much smaller than unity. In either approach it is clear that before one can really expect to obtain a realistic answer to the fermion mass question, and therefore to the question of the origin of the pion mass, enormous practical and conceptual problems must be overcome. In the case of composite models these problems are much clearer (although the solutions are still very distant!) essentially because the models have been investigated already for a number of years. Superstrings are the current vogue and thus are, at the moment, pregnant with promise. It will remain to be seen whether the problems that are now beginning to be delineated in this approach will find appropriate solution.

## III. Composite Models and Superstrings - open problems and criticisms

Imagining that quark and leptons are not elementary but composite objects is not a trivial step, since at present there is no evidence at all for further substructure. Thus, if one tries to understand quark and lepton masses through compositeness, one is faced with the immediate problem of



explaining the discrepancy between the scale of compositeness $\Lambda_c$ and the size of the quark and lepton masses. From a variety of bounds, notably those obtained from (g-2) measurements and from accurate tests of high energy QED processes like $e^+e^- \to e^+e^-$, one knows /14/ that the compositeness scale for quarks and leptons is at least of O (TeV). Thus

$$\Lambda_c \gg m_{q,\ell} \tag{29}$$

Composite models of quark and leptons must therefore be built with certain protective symmetries which force certain states - the quarks and leptons - to have essentially vanishing masses, compared to the typical dynamical scale ($\Lambda_c$) of the theory /15/. Furthermore, the dynamics of these models must also give rise to certain repetitions of these (approximately) massless states, so as to reproduce the family pattern observed in the quarks and leptons.

The combination of the above requirements, of a dynamics which gives (in some limit) $m = 0$ fermions with repetitive family structure, with asking that these excitations have precisely the quantum numbers of quarks and leptons is quite difficult to realize in practice. Thus there is really no composite model paradigm, although there exists a variety of semi-realistic models /15/. However, the more difficult step still to achieve in composite models is the generation of <u>small</u> masses, once a protective symmetry is built into the theory.

Two avenues, broadly speaking, have been followed to generate the small quark and lepton masses. Either one introduces some seed mass m or one breaks spontaneously, a little, the protective symmetries that guaranteed $m_{q,l} = 0$. An example of the first option above is discussed in a recent paper of Masiero et al. /16/, in which massless boundstate fermions, generated by supersymmetry and chirality protection, acquire mass by breaking these symmetries explicitly. L. Mizrachi and I /17/ are currently exploring some of the physics connected with the second option. Generically, introducing a seed mass m always raises the question of what is its origin. More serious, however, is the fact that from a single seed scale m it is essentially very difficult (impossible?) to generate appropriate inter and intra family hierarches.

More promising, perhaps, is imagining that the protective symmetries that guarantee, in first approximation, that $m_{q,l} = 0$ can be broken slightly spontaneously. Here two questions arise:

i) How can one really break spontaneously a symmetry slightly, if there is only one dynamical scale $\Lambda_c$ in the theory? One would need to imagine that in the theory there are condensates which form which are of a scale much below $\Lambda_c$. This is what is proposed in /17/.

ii) Spontaneous breakdown of global symmetries always gives rise to Nambu Goldstone bosons. What happens to these massless excitations? Again here the idea being pursued is that these Goldstone excitations are very weakly coupled - since the symmetry is only slightly broken - and most probably end up by having some small mass, since the global symmetries are eventually explicitly broken by gauge interactions (This is analogous to how the $\pi^+$ got a radiative mass in QCD). These are just speculations at the moment,



but they suggest that if this is the way in which the small quark and lepton masses arise, the experimental tell tale sign of a new level of substructure may well be provided by the observation of some of these pseudo Goldstone bosons. In particular, bosons with both quark and lepton quantum numbers appear particularly promising signals to pursue, since they are an almost generic feature of these classes of models.

In contrast to composite models, which must face dynamical issues immediately (e.g. why Eq. (29) holds?), superstrings appear to hold many advantages. Whether this is really the case, however, remains to be proven and only time and further research will tell. It is not my purpose here to enter into details of these theories, since they have been discussed by Freund /18/ in this Symposium. Nevertheless, I shall indicate the features of superstring theories which are relevant for the issue of quark and lepton masses, following the compactification route suggested by Candelas, Horowitz, Strominger and Witten /19/.

Superstring theories are tendimensional string theories which possess a supersymmetry and which are thought to provide a possible finite theory of gravity. They have no anomalies if the gauge group G associated with the theory is either SO(32) or $E_8 \times E_8$ /20/. In the zero slope limit, which corresponds to physics below the Planck scale $M_{Planck} \simeq 10^{19}$ GeV, these theories reduce to a d = 10 supergravity theory and an associated G supersymmetric Yang Mills theory. The assumption made by Candelas et al. /19/, which connects these theories with reality, is that these theories compactify to four dimensions at a scale below $M_{planck}$ leaving an N =1 supersymmetry unbroken. The unbroken supersymmetry obtains only if the relevant manifold for the theory is $M_4 x K$, where K is a, so called, Calabi-Yau manifold, which has an SU(3) holonomy. The fermions which emerge in 4 dimensions have properties which are fixed by K.

Particularly promising for an understanding of quark and lepton properties is the case when $G = E_8 \times E_8$. In this case, in 4 dimensions the remaining gauge group is $G_4 = E_6^* \times E_8$. where $E_6^*$ is either $E_6$ or a subgroup of $E_6$, depending on the topology of K /21/. Three very nice qualitative features emerge:

(i) The d=4 fermions are in a 27 representation of $E_6$. It is well known that the usual quarks and leptons fit well in this representation /22/, although this also implies the existence of some exotic partners.

(ii) The number of d=4 27's that emerge is related to a topological invariant in K, the Euler characteristic $\chi(K)$. One has /19/

$$n_{27} - n_{\overline{27}} = \frac{1}{2} |\chi(K)| \tag{30}$$

(iii) Yukawa couplings between the d=4 fermions and Higgs fields are computable at the compactification scale $M_{comp}$ from the d=10 gauge interactions. Basically the scalar fields are components of the d=10 gauge fields in which the vector degrees of freedom are along the compact dimensions. By performing harmonic expansions on K and integrating over the compact dimensions one should in principle be able to determine the Yukawa couplings $h^f_{ij}(M_{comp.})$



Although the above three properties rightly have excited great interest in superstrings, one should emphasize that the remaining problems ahead for superstrings are daunting. Of course, just as in composite models, one also has no evidence at all for the idea. That is, there is no evidence for $d > 4$, supersymmetry or strings! Theoretically, however, the main problem is how to pass from the qualitative statements made above to more quantitative statements. Specifically two problems appear very difficult to resolve:

i) What is the correct space K? In principle the superstring theory itself should determine this, but in practice this is too hard to do. The problem is that even admitting that K is a Calabi Yau manifold the choice is enormous, with most spaces giving rise to too many families. Furthermore since these spaces are complicated, the hope of ever doing harmonic analysis on K seems very dim.

ii) How does one really go from $M_{comp}$ to the scale of around a few GeV, where one measures the quark masses? This is a non trivial problem since the supersymmetry at $M_{comp}$ must be broken and one must, furthermore, generate a way, from $M_{comp}$, to break the usual electroweak theory down at a scale $\Lambda_F \ll M_{comp}$.

Both these problems suggest that superstring theories will only have qualitative, but no quantitative, predictions for the quark and lepton masses. Although perhaps more elegant than composite models, superstrings also lack direct predictability regarding the crucial question of the origin of the quark and lepton masses.

## IV. Concluding Remarks

I hope that the above discussion has indicated how the pion mass, whose value is of quintessential importance for understanding long range nuclear forces, is really more crucially dependend on effects beyond the strong interactions. What drives $m_\pi^2$ is the value of the up and down quark masses and these are disconnected from the strong interaction scale $\Lambda_{QCD}$. Although one expects that the quark masses are proportional to the scale $\Lambda_F$ of electroweak breaking, the present options for trying to determine theoretically the constant of proportionality are diametrically different. Either $m_q = c_q \Lambda_F$, where $c_q$ is the dynamical result of a boundstate computation in an underlying theory, one level below that of quarks and leptons. Or $m_q = h_q \Lambda_F$, where $h_q$ is the kinematical result of integrating over compact zero modes of an effective d=10 theory. Both of these options are attractive, but clearly it appears impossible on purely theoretical grounds at the moment to decide among them. Only some further experimental input, like the discovery of supersymmetric particles or of leptoquarks, will swing the balance one way or the other. Let us hope for some experimental hints in the near future, so that one will not need to wait for the centennial of the meson theory to answer the question of what is the origin of the pion mass.



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