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QUARK AND LEPTON MASSES IN A SIX DIMENSIONAL SO(12) MODEL*

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ABSTRACT

I calculate Yukawa couplings in a six dimensional Einstein-Yang-Mills theory based on SO(12). I argue that the fermion masses may follow the pattern $m_t >> m_b, m_c, m_\tau >> m_s, m_\mu >> m_d, m_u, m_e$.

There have been many attempts to predict the spectrum of fermion masses and their mixing angles in a four dimensional framework, as for example by discrete symmetries, grand unification, family unification, technicolor or constituent models. So far, no conclusive model has emerged which predicts more than (at best!) a few relations between fermion masses like $m_b(M_X) = m_\tau(M_X)$. Higher dimensional models can naturally account for several fermion generations, the number of generations being given by the chirality index[1] related to topology and symmetry of internal space. Since the hierarchical structure of the fermion mass matrices seems to suggest a generation pattern, one may conjecture that this is again related to properties of internal space in the context of a higher dimensional theory.

On the other hand, the fermion mass matrices may be among the few testable predictions of higher dimensional models. In contrast to usual four dimensional models, higher dimensional theories often have only one fundamental spinorial quantity. This eliminates the possibility of many free Yukawa couplings and typically leads to models with only very few or no free parameters describing the coupling of fermions. In a theory of gravity and gauge interactions, for example, the fermion interactions are contained in the minimal covariant kinetic term

$$L_\psi = i\bar{\psi}\gamma^{\hat{\mu}}D_{\hat{\mu}}\psi \qquad (1)$$

In addition, four dimensional gauge fields and scalar fields are often different components of the same higher dimensional field. Relations between the fermion masses and the mass of the W-boson should therefore not surprise us.

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In this talk I report on a calculation of Yukawa couplings for quarks and leptons in a six dimensional SO(12) gauge theory coupled to gravity[2]. After spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ this would result in predictions for ratios of fermion masses compared to the W-boson mass.

Let me start with the action

$$S = -\int d^6\delta\, \hat{g}_6^{\frac{1}{2}} \{\delta\hat{R} + \frac{1}{8}G^{AB}_{\hat{\mu}\hat{\nu}}G^{AB\hat{\mu}\hat{\nu}} + \varepsilon - L_\psi\} \qquad (2)$$

$$L_\psi = i\bar{\psi}_1\gamma^{\hat{\mu}}D_{\hat{\mu}}\psi_1 + i\bar{\psi}_2\gamma^{\hat{\mu}}D_{\hat{\mu}}\psi_2 \qquad (3)$$

with $G^{AB}_{\hat{\mu}\hat{\nu}}$ the six dimensional SO(12) gauge field strength and ψ_1 and ψ_2 Majorana-Weyl spinors with opposite six dimensional helicity and belonging to the two inequivalent 32 dimensional spinor representations of SO(12). (For details and conventions see ref.2.) This model has no anomalies[3]. I investigate solutions of the field equations with four dimensional Poincaré symmetry and unbroken gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. For simplicity I assume that spacetime is a direct product of Minkowski space M^4 and a two dimensional internal space. Then the most general solution has internal space S^2 with gauge fields in an SO(3) symmetric monopole configuration on S^2:[4]

$$A_\varphi = -\frac{i}{2g}\,(\pm 1 - \cos\vartheta)\begin{pmatrix} \begin{matrix}0 & m\\ -m & 0\end{matrix} & & & & & \\ & \begin{matrix}0 & m\\ -m & 0\end{matrix} & & & & \\ & & \begin{matrix}0 & p\\ -p & 0\end{matrix} & & & \\ & & & \begin{matrix}0 & p\\ -p & 0\end{matrix} & & \\ & & & & \begin{matrix}0 & p\\ -p & 0\end{matrix} & \\ & & & & & \begin{matrix}0 & n\\ -n & 0\end{matrix}\end{pmatrix}$$

$$A_\vartheta = A_\mu = 0 \qquad (4)$$

Here m,n and p are integers with n+p even. They label the different monopole configurations which are all topologically trivial according to $F_1(Spin(12))=0$. This means that all solutions can be obtained from each other by continuous deformations. For generic m,n and p the four dimensional gauge symmetry is

$$K = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \times U(1)_q \times SU(2)_G \qquad (5)$$

Here $U(1)_q$ is the abelian group commuting with the SO(10) subgroup of SO(12) and $SU(2)_G$ corresponds to isometry transformations on S^2. For specific values of the monopole numbers the symmetry may be larger, like $SU(5) \times U(1) \times U(1)_q \times SU(2)_G$ for the case m = p or $SO(10) \times U(1)_q \times SU(2)_G$ for m = p = 0. Since the solutions with lower symmetry can be obtained continuously from the SO(10) symmetric solutions, one may interprete them as spontaneous symmetry breaking of SO(10).

I will concentrate on an SU(5) symmetric example with n=3, m=p=1. Harmonic expansion of the six dimensional spinors leads to three massless chiral fermion generations. Under the generation group $SU(2)_G \times U(1)_q$ the fermions of the type u, d, u^c and e^c in the 10 of SU(5) transform as a doublet with charge $q = \frac{1}{2}$ plus a singlet with $q = -\frac{1}{2}$. In contrast, the fields e, ν and d^c in the $\overline{5}$ of SU(5) transform as a triplet with $q = -\frac{1}{2}$. (In addition the model leads to massless right handed neutrinos at this stage.) One observes that quarks and leptons within the same generation transform differently with respect to the generation group $SU(2)_G \times U(1)_q$. This feature is generic for all m, p ≠ 0. It has important consequences for predictions on mass ratios, since Yukawa couplings will depend on $SU(2)_G \times U(1)_q$ transformation properties.

For a calculation of Yukawa couplings one needs to identify candidates for the weak Higgs doublet responsible for breaking of $SU(2)_L \times U(1)_Y$. They must be contained in the SO(12) gauge fields, noting that the adjoint of SO(12) contains a complex SO(10) - 10 plet with $q = \pm 1$. One therefore has two independent $SU(2)_L$ doublets H_1 and H_2 within the six dimensional gauge fields. The internal components of six dimensional gauge fields transform as four dimensional scalars and their harmonic expansion leads to an infinite tower of scalar doublets with various $SU(2)_G$ quantum numbers. However, only the lowest modes can have Yukawa couplings to the chiral fermions whereas all higher modes decouple due to $SU(2)_G$ symmetry. At this stage, the low energy Higgs field must be a combination

$$\varphi = \gamma_i H_i \tag{6}$$

with $\sum_i |\gamma_i|^2 = 1$.

In my example (n=3, p=m=1) the lowest modes transform as a $SU(2)_G$ singlet (H_1) and a triplet (H_2^-, H_2^o, H_2^+). Comparison with the quantum numbers of quarks and leptons immediately implies that H_1 can only couple to the bilinear of the $SU(2)_G$ singlet, charge two third quark and antiquark, which I will identify with the top quark:

$$L_{Yuk,1} \sim t_L t_L^c H_1 + \text{ h.c.} \tag{7}$$

A similar analysis for H_2 gives the general form

$$L_{Yuk,2} \sim (b_L b_L^c + \tau_L \tau_L^c) H_2^- + (c_L, u_L) \begin{pmatrix} H_2^- & H_2^o \\ H_2^o & H_2^+ \end{pmatrix}^* \begin{pmatrix} c_L^c \\ u_L^c \end{pmatrix} + \text{ h.c.} \tag{8}$$

where the different coupling constants still have to be calculated. No Yukawa couplings are allowed for s,μ,d and e as a consequence of the symmetries of the model.

Since the different doublets have all different $SU(2)_G$ quantum numbers, their mixing will be small if $SU(2)_G$ is broken at a scale M_G somewhat below the scale M_c characteristic for spontaneous compactification[5]. Let me assume that the leading contribution to the low energy doublet d comes from H_1 and that the admixtures from H_2^-, H_2^o and H_2^+ are suppressed by coefficients r_i involving different powers of the ratio M_G/M_c so that

$$\langle H_1 \rangle \gg \langle H_2^- \rangle \gg \langle H_2^o \rangle, \langle H_2^+ \rangle \tag{9}$$

Once d gets a vacuum expectation value, this would result in a mass hierarchy

$$m_t \gg m_b, m_\tau, m_c \gg m_s, m_\mu, m_u, m_d, m_e \tag{10}$$

resembling already a little bit the observed pattern.

One may go one step further and explicitly calculate the Yukawa couplings of H_1 and H_2. One finds that they are all proportional to the four dimensional gauge coupling with ratios being given by ratios of $SU(2)_G$ - Clebsch-Gordon-coefficients. The approximation $\langle H_2^o \rangle = \langle H_2^+ \rangle = 0$ results in the following predictions for fermion masses (M_W in the W-boson mass)

$$m_t = 2M_W \tag{11}$$

$$m_b = m_\tau \tag{12}$$

$$m_c = \sqrt{\frac{4}{3}}\, m_b \tag{13}$$

$$m_\mu = m_s = m_d = m_u = m_e = 0 \tag{14}$$

Especially eq.(13) can certainly not be considered a good approximation to a realistic spectrum.

In any case, the monopole solution (4) should not be taken as a realistic ground state (even in the limit of unbroken $SU(2)_L \times U(1)_Y$). Its symmetry is too large - SU(5) and the generation group have to be spontaneously broken - and the spectrum of bosonic fields contains tachyons indicating an instability. To obtain a more realistic model with a possible approximate ground state where all gauge symmetries except $SU(3)_C \times SU(2)_L \times U(1)_Y$ are spontaneously broken, one may introduce[2] a six dimensional scalar field in the fifth rank totally antisymmetric tensor representation 792 of SO(12). With respect to the $SO(10) \times U(1)_q$ subgroup this field decomposes

$$792 \to 126_\circ + \overline{126}_\circ + 210_{\pm 1} + 120_\circ \tag{15}$$

The $SU(3)_C \times SU(2)_L \times U(1)_Y$ singlets in 126 and 210 can be responsible for spontaneous symmetry breaking, including the generation group. In the approximation of an unbroken abelian symmetry $U(1)_q$ (which is a combination of $U(1)_{\tilde{q}}$, $U(1)_G$, $U(1)_R$ and $U(1)_{B-L}$ with charge $\tilde{q} = I_{3G} + \frac{1}{2}q + aI_{3R}$) one can again perform a quantum number analysis for the various Higgs doublets. One finds that the only two sorts of fields which can mix with H_1 are H_2^- and h, a combination of scalar doublets from the 792-scalar. The mass matrices for the quarks and charged leptons are

$$M_U = \begin{matrix} a_{33}H_1 & a_{32}h & 0 \\ a_{23}h & a_{22}H_2^{-*} & 0 \\ 0 & 0 & 0 \end{matrix} \tag{16}$$

$$M_D = \begin{matrix} b_{33}H_2^- & 0 & b_{32}h^* \\ 0 & b_{22}h^* & 0 \\ 0 & 0 & 0 \end{matrix} \tag{17}$$

$$M_E = \begin{matrix} c_{33}H_2^- & 0 & 0 \\ 0 & c_{22}h^* & 0 \\ c_{13}h^* & 0 & 0 \end{matrix} \tag{18}$$

The non zero Yukawa couplings a_{ij}, b_{ij}, c_{ij} will be calculable for any given ground state with $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{\tilde{q}}$ symmetry. This symmetry implies that the first generation remains massless

$$m_d = m_u = m_e = 0 \tag{19}$$

For $H_1 >> H_2^- >> h$ one finds the pattern

$$m_t >> m_b, m_c, m_\tau >> m_\mu, m_s >> m_u, m_d, m_e \tag{20}$$

In addition, the admixture of h leads to non vanishing mixing angles and CP violating phase. This scenario predicts mixing angles of order $h/H_2^- \sim m_s/m_b \sim$ a few percent for the third generation. Finally, the fermions of the first generation acquire masses once $U(1)_{\tilde{q}}$ is spontaneously broken. These masses put a lower limit on the corresponding symmetry breaking scale. In a realistic model, corrections from $U(1)_{\tilde{q}}$ breaking would replace the zeros in (16)-(18) by entries of order 1-50 MeV. The qualitative structure of these mass matrices is therefore fixed and it would be interesting to investigate if this is consistent with the observed phenomenology for mixing angles and CP violation.

To conclude, let me resume the general scenario how I imagine that higher dimensional theories could explain the hierarchical structure of fermion masses. One observes four groups for the masses of fermions:

$$\begin{matrix} m_t \\ m_b, m_c, m_\tau \\ m_s, m_\mu \\ m_d, m_u, m_e \end{matrix} \tag{21}$$

The overall scale changes from one group to the next roughly by a factor 20. I emphasize that these groups do not correspond exactly to the generations. For example, the second group contains quarks of the second and the third generation. In four dimensional models where generations appear essentially as repetitions, this feature is difficult to understand. I propose an explanation where these groups are induced by quantum numbers of a generation group. Quarks and leptons within the same generation may transform differently with respect to this generation group, as I have shown above.

In usual four dimensional gauge theories Yukawa couplings are free parameters and one may take them as small as one needs. This gives only little predictivity, but these models can accomodate the observed spectrum. In higher dimensional models, four dimensional Yukawa couplings are not free. Typically, they turn out to be of the same order of magnitude as the gauge coupling g or they vanish due to symmetries.At first sight, this seems to run into conflict with a hierarchical mass pattern. However, higher dimensional models typically contain several or many fields with the quantum numbers of the Higgs doublet. As we have seen above, different doublets may couple to different fermions. The idea is that there are essentially four sorts of fields H_1, H_2, H_3, H_4 where H_1 couples only to top and H_2 to the second group, whereas H_3 does not couple to the first generation. If the low energy doublet is mainly H_1 with small admixtures of H_2, H_3 and H_4 obeying

$$\langle H_1 \rangle : \langle H_2 \rangle : \langle H_3 \rangle : \langle H_4 \rangle \sim 20 \tag{22}$$

the qualitative structure of the mass hierarchies would be obtained. Already at this stage the scenario is very restrictive for higher dimensional model building. It is not easy to find models with appropriate quantum numbers for the doublets H_i so that the above pattern of couplings is realized. However, if electron and top quark are allowed to couple to the same scalar field, one typically expects a prediction $m_e \approx m_t$ since all Yukawa couplings are of the same order. Thus a serious problem arises for models not consistent with the above scenario.

It may seem that I only have replaced the problem of explaining small Yukawa couplings by a similar one for the small mixings between scalar fields. The second one, however, will be easier to solve, at least if the model exhibits somewhat different length scales. If the generation group is spontaneously broken at a scale M_G smaller than the compactification scale M_c ($M_G/M_c \sim 1/20$, for example), the mixing of fields with different generation quantum numbers would be suppressed by different powers of M_G/M_c. The hierarchical structure of fermion masses would then be explained by a fine structure of scales near the compactification scale.

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