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On multi-bunch instabilities for fractionally filled rings

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Introduction

In the case of a "symmetric" filling of possible buckets in the ring the solution of multi-bunch motion driven by parasitic cavity modes can be described in terms of "normal modes" of the bunch system. The growth rates for the normal modes are then explicitly related to "families" of parasitic modes.

For arbitrary fractional fillings of the ring, however, the system cannot be solved explicitly and the relation between "mode" growth rates and parasitic cavity modes becomes rather complicated.

Thus, the important question arises, whether growth rates for fractional fillings can be related to those for symmetric fillings.

Another important question is connected with the necessary bandwidth for a damper system:

If for a symmetric filling only a few modes are unstable, one can design a damper system operating only in that (restricted) frequency range corresponding to the frequency range of the unstable modes.

Is such a damper system then sufficient to damp the multi-bunch instabilities also in a fractional filling?

It will be shown in this note that the growth rates of a fractional filling can be related to the growth rates of the symmetric filling.

Any damper system keeping a symmetric multi-bunch filling stable also suffices to damp instabilities in a fractional filling.

General equations

Let y_i be the coordinate (transverse or longitudinal) of the i-th bunch in a symmetric filling. The index i runs from 1 to B, B being the number of bunches. The symmetric filling is defined by:

a. All filled buckets contain equal number of particles,

b. the filled buckets are evenly spaced along the ring.

The eigenstates of a symmetric multi-bunch system interacting with parasitic cavity modes are solutions of an equation of the general form [1] :

$$\lambda\vec{y} = L[\vec{y}] \qquad (1)$$

where λ is the eigenvalue (related to eigenfrequency), $\vec{y}$ is the eigenvector:

$$\vec{y} = \begin{pmatrix} y_1 \\ y_i \\ \vdots \\ y_B \end{pmatrix} \qquad (2)$$

and $L[\vec{y}]$ is a linear form of $\vec{y}$, containing the impedances of the parasitic cavity modes.

If the intensity of each bunch is increased by a factor K, L transforms like

$$L[K\cdot\vec{y}] \longrightarrow KL[\vec{y}] \qquad (3)$$

The solutions of (1) are the "normal modes" $\vec{y}_k$ for the corresponding eigenvalues λ_k, $k = 1 \ldots B$.

The vectors $\vec{y}_k$ form an orthogonal base in the B-dimensional coordinate space M.

Let us condider a fractional filling, i.e. a filling having a gap of missing bunches. Without restricting generality we can assume that only the first $N < B$ bunches are present. The coordinate space of these bunches is called R. Any vector $\vec{y}_p \in R$ of the form

$$\vec{y}_p = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} \qquad (4)$$

can be represented by a vector $\vec{y}_M \in M$ with

$$\vec{y}_M = \begin{pmatrix} x_1 \\ x_N \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (5)$$

This is possible because of the property of L, namely

$$L[\vec{y}_p] = L[\vec{y}_M] \qquad (6)$$

If $y_p \in R$ is an eigenstate according to

$$\lambda\vec{y}_p = L[\vec{y}_p] \qquad (7)$$

this equation can be rewritten as

$$\lambda\vec{y}_p = L[\vec{y}_M] \;;\; \lambda\vec{y}_p \in R \qquad (8)$$

The right band side of (8) also "defines" components for $i > N$ (corresponding to the fields at empty buckets). Let us describe the vector defined by the r.h.s. of (8) in M as

$$\vec{y} = \begin{pmatrix} \lambda y_{p1} \\ \lambda y_{pN} \\ y_{LN+1} \\ \vdots \\ y_{LB} \end{pmatrix} \;;\; \vec{y}_L \in M \qquad (9)$$

Since the vectors $\vec{y}_k$ form a base in M, we can write

$$\vec{y}_M = \sum_{k=1}^{B} c_k\vec{y}_k \;;\; \langle\vec{y}_k,\vec{y}_\ell\rangle = B\delta_{k\ell} \qquad (10)$$

and from (8) follows:

$$\vec{y}_L = \sum_{k=1}^{B} c_k\lambda_k\vec{y}_k \qquad (11)$$

with

$$\underline{P}\,\vec{y}_L = \lambda\vec{y}_p \qquad (12)$$

if $\underline{P}$ is the projector on the space R.

Multiplying equ. (12) with $\vec{y}_M^{+}$ yields

$$\vec{y}_M^{+}\cdot\vec{y}_L = \sum_{k=1}^{B} c_k\lambda_k\vec{y}^{+}\cdot\vec{y}_k = \sum_{k=1}^{B} |c_k|^2 B\lambda_k \qquad (13)$$

From (5) and (9) follows

$$\vec{y}_M^{+}\cdot\vec{y}_L = \lambda|\vec{y}_p|^2 \qquad (14)$$

and one obtains

$$\lambda = \sum_{k=1}^{B} \frac{|c_k|^2 B}{|y_p|^2} \lambda_k \qquad (15)$$

From equ. (10) we derive

$$|\vec{y}_M|^2 = |\vec{y}_p|^2 = B \sum_{k=1}^{B} |c_k|^2 \qquad (16)$$

so that equ. (15) finally reads

$$\lambda = \frac{\sum_{k=1}^{B} |c_k|^2 \lambda_k}{\sum_{k=1}^{B} |c_k|^2} \qquad (17)$$

For a common increase of the intensity according to (3) we find

$$\lambda \longrightarrow K\lambda \qquad (18)$$

Relation (17) resp. (18) holds for any eigenstate of the fractional filling.

In order to demonstrate the validity of equ. (17) we consider a single bunch and a single parasitic resonance near a multiple of the revolution frequency. This parasitic mode leads to an eigenvalue λ_m for a certain mode m and to an eigenvalue λ_m^* for the mode $m^* = B-m$, all the other eigenvalues vanish.

For a single bunch we find

$$c_k = 1/B \quad ; \quad k = 1 \ldots\ldots B \qquad (19)$$

therefore we obtain from (17)

$$\lambda = \frac{1}{B} (\lambda_m + \lambda_{B-m}) \qquad (20)$$

and this relation precisely describes the "Robinson instability" for a single bunch at single resonance. In addition, we notice a "reduction of the intensity" due to the factor 1/B corresponding to a reduction of B bunches to a single bunch. Putting K=B (20) together with (18) describes the instability of a single bunch having the same intensity as the multi-bunch filling.

Let us go back to equ. (17). Although the coefficients c_k are unknown, we can estimate the growth rates of the fractional filling:

A. If the maximum growth rate of the symmetric filling is

$$\delta_M = \text{Max } |\text{Im}\lambda_k| \qquad (21)$$

we obtain from (17) and (18)

$$|\text{Im}\lambda| \leq K\delta_M \qquad (22)$$

B. If (for instance due to a damper system) all eigenstates $\vec{y}_k$ are stable ($\text{Im}\lambda_k \geq 0$) then also all states of the fractional filling are stable.

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References

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