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Longitudinal-Transverse Mode Coupling in Localized Structures with Orbit Dependent Higher Order Mode Losses

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Introduction

The vertical "PETRA instability" was theoretically described in terms of transverse mode coupling [1,2,3]. In the meantime, transverse mode coupling has been extensively treated by several authors [4,5,6,7,8]. Since current limitations still present in PETRA are connected with satellite resonances [9], the question arises whether localized structures can couple the transverse head-tail modes to the longitudinal shape-modes near satellite resonances and can this lead to an unstable collective motion.

The mathematical properties of transverse mode functions leading to transverse mode coupling hold also in the case of longitudinal-transverse mode coupling. The interaction between transverse and longitudinal collective motion is governed by the transverse impedances and that part of the longitudinal impedance which depends on the orbit position, both related by Maxwell's equations. The expected effect will therefore strongly depend on closed orbit deviations, a characteristic property of the observed current limitations in PETRA. The effect essentially differs from those effects which have recently been studied for localized structures [10,11].

In this article the effect is studied theoretically in the framework of the Vlasow equation [12].

Vlasow equation

longitudinal

If ϑ denotes the longitudinal angular coordinate of a particle with respect to the equilibrium particle, the perturbation of the longitudinal distribution F obeys a Vlasow equation

$$\left\{ \frac{\partial}{\partial t} + \omega_s \frac{\partial}{\partial \Psi} \right\} F = \frac{\partial}{\partial r} W_0(r)\, \omega_s \frac{F_{\shortparallel}[G,\vartheta,t]}{h\,\overline{U}_c} \sin\Psi \qquad (1)$$

$W_0(r)$ $\hat{=}$ stationary distribution
ω_s $\hat{=}$ circular sychrotron frequency
h $\hat{=}$ harmonic number
$\overline{U}_c$ $\hat{=}$ peak voltage multiplied with cosine of phase angle

$F_{\shortparallel}$ describes the longitudinal "force" as a linear functional of the transverse distribution that will be defined later.

The longitudinal coordinate has been parameterized according to

$$\vartheta = r \cos\Psi \qquad (2)$$

transverse

In the the transverse case we introduce the "quasi-time"[2] defined by

$$\tau = \frac{\varphi(s(t))}{\omega_\beta} \qquad (3)$$

$s \hat{=}$ longitudinal coordinate along the ring
$\varphi(s) \hat{=}$ phase advance
$\omega_\beta \hat{=}$ $\omega_o Q_\beta$
$\omega_o \hat{=}$ circular revolution frequency
$Q_\beta \hat{=}$ transverse Q-value

Instead of the transverse coordinate x we introduce the Courant-Snyder coordinate z with help of the amplitude function $\beta(s)$

$$z = x/\sqrt{\beta} \qquad (4)$$

The coordinate z will be parameterized according to

$$z = \rho \cdot \cos\varphi \qquad (5)$$

The Vlasow equation for the transverse perturbation $G(\rho,\varphi;r,\Psi)$ then reads

$$\left\{\frac{\partial}{\partial\tau} + \omega_\beta \frac{\partial}{\partial\varphi} + \omega_s \frac{\partial}{\partial\Psi}\right\} G = \frac{F_\perp[F,\vartheta,\tau]}{E}\, \omega_\beta \beta^{3/2}\, W_o(r) \frac{\partial}{\partial\rho} U_o(\rho) \sin\varphi \qquad (6)$$

$U_o(\rho) \hat{=}$ stationary transverse distribution
$E \hat{=}$ energy.

The transverse "force" $F_\perp$ is a linear functional of the longitudinal perturbation F.

Since the forces $F_{\parallel}$ and $F_\perp$ are generated within localized objects, they have an explicit time dependence as expressed in Eqs. (1) and (6). Therefore the time dependence of F and G is not trivial.

We make the following "ansatz"

time dependence of longitudinal distribution: $e^{-i\omega t}$ (A)
time dependence of transverse distribution: $e^{-i\Omega\tau}$ (B)
with the relation $\Omega = \omega + \omega_o Q_{\beta r}$ (C)

where $Q_{\beta r}$ is the integer part of the transverse Q-value.

The function $e^{i\Omega t - i\omega t}$ is periodic in t and τ with period $2\pi/\omega_o$ because of the periodicity property of $\tau(t)$.

We consider the longitudinal force $F_{\parallel}[G,\vartheta,t]$ which can be formally written as

$$F_{\parallel}[G,\vartheta,t] = \sum_p \tilde{F}_{\parallel}(t,p)\, e^{ip\vartheta} \tilde{g}(p)\, e^{-i\Omega\tau} \qquad (7a)$$

Correspondingly for the transverse case we have

$$F_\perp[F,\vartheta,\tau] = \sum_p \tilde{F}_\perp(\tau,p)\, e^{ip\vartheta} \tilde{f}(p)\, e^{-i\omega t} \qquad (7b)$$

Here $\tilde{g}(p)$, $\tilde{f}(p)$ are Fourier transforms of transverse and longitudinal density functions to be defined later. The "ansatz" (A), (B), (C) "solves" (1) and (6) if we keep only the "dominant" component in the Fourier expansion of $\tilde{F}_{\parallel}(t,p)$, $\tilde{F}(\tau,p)$, namely the component of $e^{\pm i(\Omega\tau - \omega t)}$.

longitudinal and transverse mode functions

For the definition of longitudinal and transverse mode functions we introduce

longitudinal

$$F(\vartheta,t) = f(\vartheta)\, e^{-i\omega t} \qquad (8a)$$

$$f(\vartheta) = \int_{-\pi}^{+\pi} d\Psi \int_0^\infty dr r\, f(r,\varphi)\, \delta(\vartheta - r\cos\Psi) \qquad (8b)$$

$$\tilde{f}(q) = \int_{-\infty}^{+\infty} d\vartheta\, f(\vartheta)\, e^{-i\vartheta q} \qquad (8c)$$

$$f(r,\Psi) = \sum_{n=-\infty}^{+\infty} f_n(r)\, e^{in\Psi} \qquad (8d)$$

and similarly

transverse

$$G(\rho,\varphi;r,\Psi;\tau) = G_\beta(\rho,\varphi;r,\Psi)\, e^{-i\Omega\tau} \qquad (8e)$$

$$G_\beta(\rho,\varphi;r,\Psi) = \sum G_{\beta m}(\rho,\varphi;r)\, e^{im\Psi} \qquad (8f)$$

$$G_{\beta m} = A(\rho)\, g_m(r) \left(\cos\varphi + i\, \frac{\Omega - m\omega_s}{\omega_\beta} \sin\varphi\right) \qquad (8g)$$

$$g(\vartheta) = \int_{-\infty}^{+\infty} d\Psi \int dr r \left(\sum_{m=-\infty}^{+\infty} g_m(r)\, e^{im\Psi}\right) \delta(\vartheta - \vartheta\cos\varphi) \qquad (8h)$$

$$\tilde{g}(q) = \int_{-\infty}^{+\infty} d\vartheta\, g(\vartheta)\, e^{-iq\vartheta} \qquad (8i)$$

In addition we define the transverse dipole moment according to

$$\overline{Z}\, g_m(r) = \int_{-\pi}^{+\pi} d\varphi \int_0^{\infty} d\rho\rho\, G_{\beta m}(\rho,\varphi;\, r)\,\rho\cos\varphi \qquad (9)$$

longitudinal and transverse forces

The longitudinal forces are determined by the longitudinal impedance $Z_{\parallel}$. This impedance can be written as[13]

$$Z_{\parallel} = Z_{0\parallel} + Z_{1\parallel}\frac{x^2}{b^2} \qquad (10)$$

where b is the aperture radius of the object.
The impedance $Z_{1\parallel}$ effects a longitudinal force which depends on the transverse position x.

If there is a closed orbit deviation x_{co} at the location of the object, then betatron oscillations of particles lead to a longitudinal force that depends linearly on the betatron displacement x_β according to[14]

$$F_{\parallel} \sim 2 Z_{1\parallel}\frac{x_{co}\cdot x_\beta}{b^2} \quad . \qquad (11a)$$

Since the longitudinal impedance $Z_{1\parallel}$ leads to a transverse impedance

$$Z_{\perp} = \frac{C Z_{1\parallel}}{b^2\omega} \qquad (11b)$$

$\omega \mathrel{\hat{=}}$ spectral frequency
$C \mathrel{\hat{=}}$ velocity of light

a longitudinal collective oscillation can excite betatron oscillations.

The fundamental integral equations

With help of the relations (1) to (11) we can write down the fundamental integral equations for f and g:

$$(\omega - n\omega_s) f_n(r) = \frac{2I\omega_s\sqrt{\overline{\beta}_0}\,\overline{Z}\,\overline{x}_{co}}{h\overline{U}_c b^2}[i]^{n-1}\frac{n}{r}\frac{\partial W_0}{\partial r}\sum_{p=\infty}^{+\infty}\frac{\overline{Z}^-_{1\parallel}(p)}{p} I_n(pr)\tilde{g}(p) \qquad (12)$$

$$[\Omega - (\omega_\beta + m\omega_s)]\overline{Z}\, g_m(r) = \frac{I\sqrt{\overline{\beta}_0}\,C x_{co}}{4\pi E/e b^2}[i]^{m-1} W_0(r)\sum_{p=-\infty}^{+\infty}\frac{\overline{Z}^+_{1\parallel}(p)}{p} I_m(pr)\tilde{f}(p) \qquad (13)$$

Here $\overline{\beta}_0$, $\overline{x}_{co}$ are the maximum values of the amplitude function and the closed orbit deviation in the rf-region.

The impedances $\overline{Z}^{\pm}_{1\parallel}$ are defined by

$$\overline{Z}^{\pm}_{1\parallel} = \sum_{\ell=1}^{N} Z_{1\parallel\ell}\, e^{\pm\Delta(S_\ell)}\sqrt{\frac{\beta_\ell}{\overline{\beta}_0}}\,\frac{x_{co\ell}}{\overline{x}_{co}} \qquad (14)$$

The index ℓ runs over the positions of localized objects (rf-section) with impedance $Z_{1\parallel\ell}$, amplitude function β_ℓ and closed orbit deviation $x_{co\ell}$

The function $\Delta(s)$ is given by

$$\Delta(s) = \Omega\tau(s) - \omega t(s) \qquad (15)$$

In the sense of the approximations applied this should be replaced by

$$\Delta(s) = \varphi_r(s) \qquad (16)$$

where $\varphi_r(s)$ is the phase advance corresponding to the integer part of the Q-value.

For the derivation of (12), (13), (14) compare for instance ref. 2 or ref. 3.
Writing

$$\lambda_n = \omega - n\omega_s \qquad (17)$$

$$\lambda_m = \Omega - (\omega_\beta + m\omega_s) \qquad (18)$$

and making use of

$$\tilde{f}(p) = 2\pi\sum_{n=-\infty}^{+\infty}[-i]^n \int dr'r'\, f_n(r')\, I_n(pr') \qquad (19)$$

$$\tilde{g}(p) = 2\pi\sum_{n=-\infty}^{+\infty}[-i]^m \int dr'r'\, g_n(r')\, I_n(pr') \qquad (20)$$

we obtain from equs. (12) and (13)

$$\lambda_n f_n = 4\pi n I\,\frac{\overline{Z}\,\overline{x}_{co}\sqrt{\overline{\beta}_0}\,\omega_s}{b^2 h\overline{U}_c}\,\frac{1}{r}\frac{\partial}{\partial r}W_0(r)\sum_p\frac{\overline{Z}^-_{1\parallel}}{p}I_n(pr)\sum_n[i]^{n-m-1}\int_0^{\infty} dr'r' I_m(pr')g_m(r') \qquad (21a)$$

$$\lambda_m f_m = \frac{I\sqrt{\overline{\beta}_0}\,\overline{x}_{co}\,c}{2E/e\,\overline{Z}\,b^2}W_0(r)\sum_p\frac{\overline{Z}^+_{1\parallel}}{p}I_m(pr)\sum_m[i]^{m-n-1}\int_0^{\infty} dr'r' I_n(pr')f_n(r') \qquad (21b)$$

Besides the functions $f_n(r)$, $g_n(r)$ we introduce the adjoint ones

$$f_n(r) = -f_n^+ \frac{1}{r}\frac{\partial}{\partial r} W_0(r) \qquad (22a)$$

$$g_n(r) = g_m^+(r)\, W_0(r) \qquad (22b)$$

with $$2\pi \int_0^\infty dr\, r\, W_0(r) = 1 \qquad (22c)$$

The scalar product of a pair f, f' resp. g, g' is then defined as

$$(f,f') = \int_0^\infty dr\, r\, f^{+*} f \qquad (23a)$$

$$(g,g') = \int_0^\infty dr\, r\, g^{+*} g \qquad (23b)$$

According to (22), (23) we put

$$f_n(r) = A_n \ell_n(r)$$
$$g_m(r) = B_m L_m(r) \qquad (24a)$$

with $$(\ell_m, \ell_m) = (L_m, L_m) = 1 \qquad (24b)$$

for all m.

Finally we introduce

$$k_n(p) = \sqrt{2\pi} \int_0^\infty dr\, r\, \ell_n(r)\, I_n(pr) \qquad (25a)$$

$$k_m(p) = \sqrt{2\pi} \int_0^\infty dr\, r\, L_m(r)\, I_m(pr) \qquad (25b)$$

Multiplying equ. (21a) with f_n^+, equ. (21b) with L_m^+ and using (25) we find

$$\lambda_n A_n = \frac{2n I \bar{z} \sqrt{\bar{\beta}_0}\, \bar{x}_{co}\, \omega_s}{b^2 h \bar{U}_c} \sum_m [i]^{n-m-1} \sum_p \frac{\bar{Z}_{1\shortparallel}^+(p)}{p} k_n^*(p)\, k_m(p)\, B_m \qquad (26a)$$

$$\lambda_m B_m = \frac{I\sqrt{\bar{\beta}_0}\, \bar{x}_{co}\, c}{4\pi E/e\, \bar{z}\, b^2} \sum_n [i]^{m-n-1} \sum_p \frac{\bar{Z}^+(p)}{p} k_m^*(p)\, k_n(p)\, A_n \qquad (26b)$$

Since the functions h(p), k(p), as a function of p, have the same parity properties, the system (26) has similar properties as in the case of transverse mode coupling. Concentrating on the coupling of longitudinal and transverse modes only, one obtains from (26)

$$\begin{pmatrix} \lambda_n & -2n I \dfrac{\bar{z}\, \bar{x}_{co} \sqrt{\bar{\beta}_0}\, \omega_s}{b^2 h \bar{U}_c} D_{nm}^- \\ -\dfrac{I\sqrt{\bar{\beta}_0}\, \bar{x}_{co}\, c}{4\pi E/e\, \bar{z}\, b^2} (D_{nm}^-)^* & \lambda_n - \Delta \end{pmatrix} \begin{pmatrix} A_n \\ B_m \end{pmatrix} = 0 \qquad (27)$$

with $$\Delta = \delta\omega_\beta + (m-n)\, \omega_s \,, \qquad (28)$$

$\delta\omega_\beta$ is the circular betatron frequency corresponding to the fractional part of the Q-value.

We introduce the abbreviation

$$|M|^2 = \frac{n I^2 \bar{x}_{co}^2\, \bar{\beta}_0 \omega_s c}{2\pi E/e\, b^4\, h \bar{U}_c} |D^-|^2 \qquad (29)$$

An instability occurs if

$$n < 0$$
$$|m| - |n| \quad \text{odd} \qquad (30)$$
$$|M| > |\tfrac{\Delta}{2}|$$

From (14) follows

$$D_{nm}^{\pm} = \sum_{\ell=1}^{N} D_{\ell nm}\, e^{\pm i \Delta(s_\ell)} \frac{x_{co\ell}}{\bar{x}_{co}} \sqrt{\frac{\beta_\ell}{\bar{\beta}_0}} \qquad (31a)$$

with $$D_{\ell nm} = \sum_p \frac{Z_{1\shortparallel\ell}}{p} h_n^*(p)\, k_m(p) \qquad (31b)$$

From the parity properties of h, k as functions of p follows

$$D_{\ell nm} = 2 \sum_{p=0}^{\infty} R_\ell\, Z_{1\shortparallel}(p) \frac{h_n^*(p)\, k_m(p)}{p} \qquad (32)$$

At this point we must introduce the normalized functions $\ell(r)$, $L(r)$ solving (12), (13). As an "approximate" solution we put

$$L_m(r) = \left(\frac{m}{|m|}\right)^{|m|} \frac{\sqrt{2}}{\sqrt{\pi}\sqrt{|m|!}} \frac{1}{\sigma^2} \left(\frac{r}{\sigma}\right)^{|m|} e^{-r/\sigma^2} \qquad (33)$$

with

$$\sigma = \frac{\sigma_s}{R}/\sqrt{2} \qquad (34)$$

$\sigma_s \hat{=}$ r.m.s. bunch length
$R \hat{=}$ machine radius

Similarly we put

$$\ell_m(r) = \sqrt{2}\, \frac{L_m(r)}{\sigma} \qquad (35)$$

and obtain

$$k_m(p) = \frac{1}{\sqrt{2}^{|m|}\sqrt{|m|!}} \left(p\, \frac{\sigma_s}{R}\right)^{|m|} e^{-p^2 (\frac{\sigma_s}{R})^2} \qquad (36a)$$

and

$$h_n(p) = \sqrt{2}\, \frac{k_n(p)}{\sigma} \qquad (36b)$$

with

$$\mathrm{Re}\, Z_{1''\ell} = R_{1''\ell} \qquad (37)$$

and

$$R_{eff\,\ell} = 2\, \Sigma\, R_{1''\ell}\, e^{-p^2 (\frac{\sigma_s}{R})^2} \qquad (38)$$

We can express D by differentiating $R_{eff\,\ell}\, (\frac{\sigma_s}{R})$ with respect to $(\frac{\sigma_s}{R})^2$.

Going back to (27) we find lower thresholds if Δ is small. Putting $n = -|n|$ according to (30) and specializing $m = -|m|$ one obtains from (28)

$$\Delta = \delta\omega_\beta - (|m| - |n|)\, \omega_s \qquad (39)$$

which becomes small for $|m| > |n|$ if $\delta\omega_\beta$ is near a satellite frequency

$$\delta\omega_{\beta SAt} = (|m| - |n|)\, \omega_s \qquad (40)$$

Since an instability occurs only if $|m| - |n|$ is odd (see 30) a blow-up occurs only near odd satellite frequencies. Table I shows the relation between modes and satellite frequencies

transverse m	longitudinal n	satellite frequency
- 2	- 1	ω_s
- 4	- 1	$3\,\omega_s$
- 3	- 2	ω_s
- 5	- 2	$3\,\omega_s$

Tab. 1

In order to make use of the differentiating process mentioned above we put

$$v = (|m| + |n| - 1)/2 \qquad (41)$$

We find from (32) and (36)

$$D_{\ell nm} = \frac{1}{\frac{\sigma_s}{R}} \frac{\sqrt{2}}{\sqrt{2}^{|m|}\sqrt{2}^{|n|}\sqrt{|m|!}\sqrt{|n|!}} \left(\frac{\sigma_s}{R}\right)^{|n|+|m|} (-1)^v \frac{d^v}{d\left[\left(\frac{\sigma_s}{R}\right)^2\right]^v} R_{eff\,\ell} \qquad (42)$$

If in a limited range we assume a "power law" for $R_{eff\,\ell}$ as a function of σ_s

$$R_{eff\,\ell} = \mathring{R}_{eff\,\ell} \left(\frac{\sigma_s}{\sigma_{so}}\right)^{-2\mu} \quad , \quad \mu > 0 \qquad (43)$$

we obtain

$$(-1)^v \frac{d^v}{d\left[\left(\frac{s}{R}\right)^2\right]^v} = \mu(\mu+1) \ldots (\mu+v-1)\, \frac{\sigma_s}{R}^{-2v} R_{eff\,\ell} \qquad (44)$$

and therefore

$$D_{\ell nm} = \frac{\sqrt{2}\, \mu(\mu+1)\ldots(\mu+v-1)}{\sqrt{2}^{(|m|+|n|)}\sqrt{|m|!}\sqrt{|n|!}} R_{eff\,\ell} \qquad (45)$$

Relation (29) can then be rewritten as

$$|M| = \varkappa \frac{I\,\overline{R}_{eff}}{\sqrt{E/e\,h\,\overline{U}_c}} \frac{|\overline{x}_{co}|}{b} \sqrt{\frac{\overline{\beta}_o}{b}} \sqrt{\omega_s \cdot \omega_b} \cdot \Sigma \qquad (46a)$$

with

$$\omega_b = \frac{c}{b} \cdot 2\pi \qquad (46b)$$

$$\varkappa = \frac{1}{2\pi} \frac{\sqrt{2|n|}\,\mu(\mu+1)\ldots(\mu+\nu-1)}{\sqrt{2}^{(|m|+|n|)}\sqrt{|m|!}\sqrt{|n|!}} \qquad (46c)$$

$\overline{R}_{eff}$ is the impedance of the whole structure. The quantity Σ is given by

$$\Sigma = \{\sum_{\ell=1}^{N} (\frac{R_{eff\ell}}{R_{eff}} \frac{x_{co}}{\overline{x}_{co}} \sqrt{\frac{\beta_\ell}{\beta_o}} \cos\Delta(s_\ell))^2 + (\frac{R_{eff\ell}}{\overline{R}_{eff}} \frac{x_{co}}{\overline{x}_{co}} \sqrt{\frac{\beta_\ell}{\beta_o}} \sin\Delta(s_\ell))^2\}^{1/2} \qquad (46c)$$

Numerical estimates

We apply the maximum value of Σ, i.e. $\Sigma = 1$ for PETRA and choose the following machine parameters:

$E = 7$ GeV	$h = 3840$
$I = 5$ mA	$U_c = 25$ MV
$\overline{x}_{co} = 2.5$ mm	$n = -1$
$\overline{\beta}_o = 20$ m	$m = -2$
$b = 4$ cm	$2\mu = 1.5$
$\omega_s = 50$ kHz	$\frac{R_{eff}}{b^2} = 1.3 \cdot 10^8\,\Omega/cm^2$ for 56 5-cell and 56 7-cell cavities[15]

This yields

$$|M|/kHz \approx 1 \qquad \text{or} \qquad \Delta \approx 2\text{ kHz}$$

near the first order satellite resonance.

For HERA the effect only appears above the operational bunch intensities if one assumes a distance $\Delta = 2$ kHz.

Experimental results

A "coherent" satellite resonance can be distinguished easily from an incoherent ("normal") satellite resonance if the transverse motion is dominated by a dipole oscillation. An identification of a higher head-tail mode is rather hard.

For the instabilities treated in this article the higher head-tail modes are accompanied by collective longitudinal oscillations. Especially in the case considered numerically the instability leads to longitudinal dipole oscillations which can be easily detected.

In PETRA longitudinal dipole oscillations or longitudinal shape oscillations in coincidence with a transverse blow-up near odd satellite resonances have not been observed.

The reason for this may be that Σ is much smaller than 1 due to the symmetry of the machine and the symmetric locations of the rf cavities.

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