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by

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Current Algebra and Weak Decays Involving Axions

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Abstract

We reexamine the predictions obtained by the current algebra method for weak decays involving axions, notably for the processes $K^+ \to \pi^+ a$ and $\pi^+ \to a e^+ \nu$. We find, contrary to claims made in the literature and to naive expectations that these processes are in fact heavily suppressed. Our results depend on a subtle cancellation which originates after one properly identifies the physical axion and pion states.

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Together with T.T. Wu, we have recently constructed [1] |F0| a variant of the standard axion model [2], which avoids previous bounds on the existence of axions. This new axion model was motivated by the existence of a narrow positron line in the produced positron spectra in heavy ion collisions at GSI [3], which appears to be correlated with an equally narrow electron line [4]. Such signals would naturally arise if a particle of mass near 1.7 MeV were produced nearly at rest in the heavy ion collision and then subsequently decayed rapidly into e^+e^- pairs. The presumption that such a particle might be the standard axion is, however, not tenable, since then either $\psi \to \gamma a$ or $\Upsilon \to \gamma a$ would occur at rates about two orders of magnitude above the present experimental limits. In the new axion model of Ref [1], the axion is only strongly coupled to up quarks and electrons so that one neatly avoids all quarkonic bounds. Furthermore most other axion bounds are also avoided, since the axion decays very rapidly ($\tau \sim 6\times10^{-13}$ sec) into e^+e^- pairs. A possible exception is provided by weak decays involving axions, where at first sight there appears to be trouble. The purpose of this note is to discuss these decays in detail and to show that indeed, contrary to naive expectations, they are heavily suppressed. Hence, this reinforces the claim made in Ref [1] that the new axion model is perfectly viable. Besides this more partisan point of view, we hope that our discussion will serve to clarify this issue, whose treatment in the existing literature is full of contradictory and confusing claims.

To set the stage, we reproduce the results of a paper of Suzuki [5], who has analyzed the predictions from current algebra for weak decays involving a pseudoscalar particle ϕ – to be identified with the GSI excitation – which couples to the light quarks. Suzuki's analysis, which is perfectly correct, for the decays $K^+ \to \pi^+\phi$ and $\pi^+ \to \phi e^+ \nu$ gives

$$\frac{\Gamma(K^+\to\pi^+\phi)}{\Gamma(K^+\to\pi^+\pi^0)} \simeq f_\pi^2 \left\{ \left(\frac{g_u}{2m_u} + \frac{g_d}{2m_d}\right) r + \left(\frac{g_u}{2m_u} - \frac{g_d}{2m_d}\right) \right\}^2 \quad (1)$$

$$\Gamma(\pi^+ \to \phi e^+ \nu) \simeq \frac{G_F^2 m_\pi^5}{768\pi^3} \left[2 f_\pi^2 \left(\frac{g_u}{2m_u} - \frac{g_d}{2m_d}\right)^2 \right] \quad (2)$$

Here g_u and g_d are the coupling of ϕ to the up and down quarks and $r \simeq 20$ is the $\Delta I = 1/2$ enhancement. Taking these results at face value for the new axion model, puts this model in difficulty. As we shall discuss in more detail below, the combinations $g_u / 2m_u \pm g_d / 2m_d$ represent the isoscalar and isovector couplings of the axion respectively, which for the new axion model are:

$$\left(\frac{g_u}{2m_u} - \frac{g_d}{2m_d}\right) \simeq \frac{m_u x}{(m_u+m_d)f} \simeq 10^{-4}\ MeV^{-1} \tag{3}$$

$$\left(\frac{g_u}{2m_u} + \frac{g_d}{2m_d}\right) \simeq 0 \ .$$

Hence one would have

$$B(K^+ \to a\pi^+) \simeq 2\times 10^{-5} \tag{4}$$

$$B(\pi^+ \to a e^+\nu) \simeq 2\times 10^{-6} \tag{5}$$

Strictly speaking, the first branching ratio above is not in contradiction with experiment since, to our knowledge, there is no bound for $K^+ \to a\pi^+$, $a \to e^+e^-$ with $M_{e^+e^-} \leqslant 5MeV$ [6]. On the other hand the $\pi^+ \to ae^+\nu$, $a \to e^+e^-$ decay appears to be in contradiction with the present bound for $\pi^+ \to e^+e^-e^+\nu$[7] of 5×10^{-9} [F1]. Thus it is important to ascertain how reliable these results are theoretically. We will show that, even though the results (1) and (2) are reasonable estimates for a pseudoscalar excitation ϕ, <u>for an axion a correct treatment yields completely different answers</u>. Indeed for an axion these decay rates are heavily suppressed and vanish in the chiral limit!

The new axion model [1], in effect, has a Peccei-Quinn symmetry (PQ) [2], which acts <u>only</u> on the u and d quarks. By coupling u_R and d_R to two different Higgs fields ϕ_1 and ϕ_2 one insures that the total Lagrangian has an allowed U(1) chiral symmetry. When ϕ_1 and ϕ_2 get vacuum expectation values this global symmetry is spontaneously broken, giving rise to a Nambu-Goldstone boson - the axion [9]. Because the $U(1)_{PQ}$ symmetry is anomalous, when strong interactions are included, the axion picks up a small mass. For our considerations it will be important to discuss in some detail how this mass generation occurs. This we do, following the general treatment of Bardeen and Tye [10].

It suffices to isolate the coupling of the axion to the u and d quarks. One has [10]

$$\mathcal{L}_{a\bar{q}q} = -m_u \bar{u}\, e^{i\gamma_5 \frac{xa}{f}} u - m_d \bar{d}\, e^{i\gamma_5 \frac{a}{xf}} d \tag{6}$$

The above shows that a chiral rotation of the u and d quarks can be undone by a translation of the Goldstone axion field a. The parameter f in (6) is the weak interaction scale: $f = (\sqrt{2}\, G_F)^{-1/2} \approx 250$ GeV, while x is a free parameter which is the ratio of the vacuum expectation values of ϕ_2 to ϕ_1. Expanding the exponential in (6) one sees that the coupling of axion to u and d quarks is proportional, respectively, to x and x^{-1}. Hence for x large the axion, at the Lagrangian level, couples mostly to up quarks. The chiral current associated with the $U(1)_{PQ}$ symmetry of (6) is just [F2]

$$J_\mu^S = f\partial_\mu a + \frac{x}{2}\bar{u}\gamma_\mu\gamma_5 u + \frac{1}{2x}\bar{d}\gamma_\mu\gamma_5 d \tag{7}$$

This current is not conserved in the presence of the strong interactions [11]

$$\partial^\mu J_\mu^S = \frac{1}{2}\left(x+\frac{1}{x}\right)\frac{g^2}{16\pi^2} F_a^{\mu\nu}\tilde{F}_{a\,\mu\nu} \tag{8}$$

and so cannot be used to do current algebra computations. However, as Bardeen and Tye [10] showed, one can construct another current $\tilde{J}_\mu$ which is anomaly free and soft (i.e. it has a vanishing divergence for m_u, $m_d \to 0$). With this current one can proceed to do current algebra calculations. For the case at hand, $\tilde{J}_\mu$ is simply

$$\begin{aligned}\tilde{J}_\mu &= J_\mu^S - \frac{1}{2}\left(x+\frac{1}{x}\right)\left[\frac{m_d}{m_u+m_d}\bar{u}\gamma_\mu\gamma_5 u + \frac{m_u}{m_u+m_d}\bar{d}\gamma_\mu\gamma_5 d\right] \\ &= f\partial_\mu a + \frac{xm_u - \frac{1}{x}m_d}{m_u+m_d}\left[\frac{\bar{u}\gamma_\mu\gamma_5 u}{2} - \frac{\bar{d}\gamma_\mu\gamma_5 d}{2}\right]\end{aligned} \tag{9}$$

so that the quark piece of this current is just proportional to the isospin axial current A_3^μ. Letting

$$\lambda = \frac{xm_u - \frac{1}{x}m_d}{m_u + m_d} \tag{10}$$

one has

$$\tilde{J}_\mu = f\partial_\mu a + \lambda A_{3\mu} \quad (11)$$

The divergence of $\mathfrak{J}_\mu$ is readily computed since $\mathfrak{J}^S_\mu$, apart from the anomaly, is divergenceless. Using Eq (6) one finds:

$$\partial^\mu \tilde{J}_\mu = (x+\tfrac{1}{x})\frac{m_u m_d}{m_u+m_d}\left[\bar{u}\, e^{i\gamma_5 \frac{xa}{f}} i\gamma_5 u + \bar{d}\, e^{i\gamma_5 \frac{a}{xf}} i\gamma_5 d\right] . \quad (12)$$

A similar calculation for the isospin current yields

$$\partial^\mu A_{3\mu} = m_u \left[\bar{u}\, e^{i\gamma_5 \frac{xa}{f}} i\gamma_5 u\right] - m_d \left[\bar{d}\, e^{i\gamma_5 \frac{a}{xf}} i\gamma_5 d\right] . \quad (13)$$

Note that the particular choice made in (9) for $\mathfrak{J}_\mu$ guarantees that $\mathfrak{J}_\mu$ is divergenceless when either m_u or $m_d \to 0$. This satisfies the physical requirement that if any quark is massless, then the axion remains a real Nambu-Goldstone boson. More important for our purposes is that $\mathfrak{J}_\mu$ is such that the off diagonal sigma term vanishes. That is [F3]

$$\langle 0|-i[\tilde{Q}, \partial_\mu A_3^\mu]|0\rangle = \langle 0|-i[Q_3^5, \partial \tilde{J}^\mu_\mu]|0\rangle = 0 \quad (14)$$

while

$$\langle 0|-i[Q_3^5, \partial_\mu A_3^\mu]|0\rangle = -(m_u+m_d)\langle 0|\bar{u}\, e^{i\gamma_5 \frac{xa}{f}} u|0\rangle \quad (15)$$

$$\langle 0|-i[\tilde{Q}, \partial_\mu \tilde{J}^\mu]|0\rangle = -(x+\tfrac{1}{x})^2 \frac{m_u m_d}{m_u+m_d}\langle 0|\bar{u}\, e^{i\gamma_5 \frac{xa}{f}} u|0\rangle . \quad (16)$$

These results imply that the currents $\mathfrak{J}^\mu$ and A_3^μ only connect the physical axion and π^0 fields to vacuum, so that

$$\langle 0|\tilde{J}_\mu|\pi^0\rangle = 0$$
$$\langle 0|A_{3\mu}|a\rangle = 0 \quad (17)$$

where $|\pi^0\rangle$ and $|a\rangle$ are physical states of definite masses. Writing

$$\langle 0|\tilde{J}_\mu|a\rangle = i f_{phys} k_\mu \quad (18)$$
$$\langle 0|A_{3\mu}|\pi^0\rangle = i f_\pi k_\mu$$

then Dashen's theorem [12] implies

$$f_\pi^2 m_\pi^2 = \langle 0|-i[Q_3^5, \partial_\mu A_3^\mu]|0\rangle = -(m_u+m_d)\langle 0|\bar{u}\, e^{i\gamma_5 \frac{xa}{f}} u|0\rangle \quad (19)$$

$$f_{phys}^2 m_a^2 = \langle 0|-i[\tilde{Q}, \partial_\mu \tilde{J}^\mu]|0\rangle = -(x+\tfrac{1}{x})^2 \frac{m_u m_d}{m_u+m_d}\langle 0|\bar{u}\, e^{i\gamma_5 \frac{xa}{f}} u|0\rangle$$

and yields the well known formula for the axion mass (applicable to our effective one generation model)

$$m_a^2 = m_\pi^2 \frac{f_\pi^2}{f_{phys}^2}(x+\tfrac{1}{x})^2 \frac{m_u m_d}{m_u+m_d} \quad (20)$$

Note that if the off diagonal sigma terms did not vanish the currents $\mathfrak{J}^\mu$ and A_3^μ would have connected the vacuum state to both the π^0 and a states, and one would have had to find a linear combination of them that obeyed Eqs (17). For the choice made by Bardeen and Tye [10] this is not necessary. However, the axion field appearing in Eq (11) is not the physical axion field. Since (17) must hold, it follows that

$$a = \cos\theta\, a_{phys} + \sin\theta\, \pi^0_{phys} \quad (21)$$

where

$$\sin\theta = -\lambda \frac{f_\pi}{f} \quad (22)$$

and

$$f_{phys} = f\cos\theta \simeq f . \quad (23)$$

Therefore the current $\mathfrak{J}_\mu$ which connects the axion to vacuum is

$$\tilde{J}_\mu = f_{phys}\partial_\mu a_{phys} + \lambda(A_{3\mu} - f_\pi \partial_\mu \pi^0_{phys}) . \quad (24)$$

The presence of a small piece of the pion field in (24) has important consequences for weak decays involving axions. It is this piece which removes most of the contributions of these decays in the chiral limit.

Let us consider first the decay $\pi^+ \to a e^+ \nu$. To calculate this process one needs to know the matrix element of the weak vector current $V^-_\mu = \bar{u}\gamma_\mu d$ between the axion and the π^+. On general grounds, this matrix element can be written as

$$\langle a | V_-^{\mu}(0) | \pi^+ \rangle = f_+(q^2)\,(P_a + P_\pi)^\mu + f_-(q^2)\,(P_a - P_\pi)^\mu \tag{25}$$

although only the f_+ form factor is relevant for the total rate. For an ordinary pseudoscalar particle ϕ, coupled as indicated before to the u and d quarks, one can estimate the relevant form factor $f_+^\phi(0)$ very simply by remarking that

$$\mathcal{L}_{int} = g_u\, \bar{u}\, i\gamma_5 u\, \phi + g_d\, \bar{d}\, i\gamma_5 d\, \phi \tag{26}$$

$$= \left(\frac{g_u}{2m_u} - \frac{g_d}{2m_d}\right) \phi\, \partial^\mu A_{3\mu} + \cdots$$

Whence, recalling that $\partial^\mu A_{\mu 3}$ is dominated by the π^0 pole, it follows that

$$\langle \phi | V_-^{\mu}(0) | \pi^+ \rangle \simeq \left(\frac{g_u}{2m_u} - \frac{g_d}{2m_d}\right) f_\pi \langle \pi^0 | V_-^{\mu}(0) | \pi^+ \rangle \tag{27}$$

This identifies $f_+^\phi(0)$ as

$$f_+(0) = \sqrt{2}\left(\frac{g_u}{2m_u} - \frac{g_d}{2m_d}\right) f_\pi \tag{28}$$

which yields the estimate given in Eq (2). For an axion, however, this logic is not correct since the current A_3^μ <u>does</u> <u>not</u> connect the axion state to vacuum (c.f. Eq (17), or the equivalent statement that the offdiagonal sigma term, Eq (14), vanishes). In fact, in the chiral limit, one can show that the matrix element (25) vanishes altogether.

To prove the above statement we consider Eq (25) in the limits in which $p_a^\mu \to 0$ and $p_\pi^\mu \to 0$. One has by a direct application of Dashen's theorem

$$\langle a | V_-^{\mu} | \pi^+ \rangle \underset{P_a \to 0}{=} \frac{1}{f} \langle 0 | [\tilde{Q}, V_-^{\mu}] | \pi^+ \rangle = P_\pi^\mu \left(f_+(m_\pi^2) - f_-(m_\pi^2)\right) \tag{29a}$$

$$\langle a | V_-^{\mu} | \pi^+ \rangle \underset{P_\pi \to 0}{=} \frac{1}{f_\pi} \langle a | [V_-^{\mu}, Q_+^5] | 0 \rangle = P_a^\mu \left(f_+(m_a^2) + f_-(m_a^2)\right) \tag{29b}$$

Using Eq (17) it is easy to show that both (29a) and (29b) vanish:

$$\langle 0 | [\tilde{Q}, V_-^{\mu}] | \pi^+ \rangle = \langle 0 | [\tilde{J}^\mu, Q_-] | \pi^+ \rangle = \langle 0 | \tilde{J}^\mu | \pi^0 \rangle = 0 \tag{30}$$

$$\langle a | [V_-^{\mu}, Q_+^5] | 0 \rangle = 2 \langle a | A_3^\mu | 0 \rangle = 0 \ .$$

Whence it follows that

$$f_+(m_\pi^2) = + f_-(m_\pi^2) \tag{31}$$

$$f_+(m_a^2) = - f_-(m_a^2) \ .$$

Writing for the form factors the expansion

$$f_\pm(q^2) = f_\pm(0) + \lambda_\pm q^2 + \cdots \tag{32}$$

these equations imply, if the q^2 dependence is mild, [F4]

$$f_+(0) = f_-(0) = 0$$

$$\lambda_+ \simeq \lambda_-$$

instead of the estimate (28). There is a simple way to understand these results by focussing on Eq (24), which describes the current $\mathcal{J}^\mu$ in terms of the <u>physical</u> axion field. We see that what appears in this current is not A_3^μ but $A_3^\mu - f_\pi \partial^\mu \pi^0_{phys}$. So, effectively, the axion couples to the divergence of the axial current <u>with the pion contribution removed</u>. It is for this reason that the matrix element $\langle a | V_-^\mu | \pi^+ \rangle$ is unrelated to that of $\langle \pi^0 | V_-^\mu | \pi^+ \rangle$ and vanishes at zero momentum transfer.

Although our results tell us that $f_+(0) = 0$, they do not provide a value for λ_+, which now determines the rate for $\pi^+ \to a e^+ \nu$. A most naive estimate, but probably one which should provide the relevant order of magnitude, is that λ_+ should be proportional to the amount of A_3^μ in $\mathcal{J}^\mu$, divided by f, and be characterized by a typical hadronic scale for V_-^μ - say the ρ-mass squared. Thus it is

$$\lambda_+ \sim \frac{\lambda}{f} \frac{f_\pi}{m_\rho^2} = \frac{m_u x}{m_u + m_d} \frac{f_\pi}{f} \frac{1}{m_\rho^2} \ . \tag{33}$$

Hence for the $B(\pi^+ \to a e^+ \nu)$ we expect a value of the order of 10^{-9} - which may be in the interesting experimental range [8].

Let us turn now to the $K^+ \to \pi^+ a$ decay. The discussion above should already make it apparent that this process <u>cannot</u> be related to the $K^+ \to \pi^+\pi^0$ decay, since one subtracts away the π^0 piece from A_3^μ. It is convenient to write the amplitude for the weak decay as

$$A(K^+ \to \pi^+ a) = \langle \pi^+ a | H_W(0) | K^+ \rangle = C + B_a p_a \cdot k + B_\pi p_\pi \cdot k \tag{34}$$

and to try to calculate the coefficients C, B_a and B_π by going to the various chiral limits $|F_5|$. Let us consider first the limit in which $p_\pi \to 0$. By the usual Dashen's formula one has

$$C + B_a p_a \cdot k = \frac{1}{f_\pi} \langle a | [Q^5_+, H_W(0)] | K^+ \rangle \tag{35}$$

Since $H_W(0)$ involves (V-A) currents $|F_6|$, the above commutator is also given by

$$C + B_a p_a \cdot k = -\frac{1}{f_\pi} \langle a | [Q_+, H_W(0)] | K^+ \rangle \tag{36}$$

Letting $k \to 0$, it is easy to show that C vanishes. Basically one goes through the same steps as above to turn a chiral charge into an ordinary charge and overuses the fact that this charge annihilates both the vacuum and the axion state. So we have identified

$$B_a p_a \cdot k = \frac{1}{f_\pi} \langle a | H_W(0) | K^0 \rangle \quad . \tag{37}$$

To calculate $B_\pi p_\pi \cdot k$ we go to the $p_a \to 0$ limit. Then one has, since $C = 0$,

$$B_\pi p_\pi \cdot k = \frac{1}{f} \langle \pi^+ | [\tilde{Q}, H_W(0)] | K^+ \rangle . \tag{38}$$

To calculate this matrix element we make use of Eq (24) for $\tilde{J}^\mu$ and write therefore

$$B_\pi p_\pi \cdot k = \frac{1}{f} \langle \pi^+ | \Big[\int d^3x\, \partial^0 a_{phys} + \lambda \big(Q_3^5 - f_\pi \int d^3x\, \partial^0 \pi^0_{phys} \big), H_W \Big] | K^+ \rangle . \tag{39}$$

It is natural to presume that the pure axion piece of the above commutators vanishes - since H_W contains only quark currents. The usual estimate of $B_\pi p_\pi \cdot k$ (and indeed of the above amplitude A) consists of retaining in the second term only the λQ_3^5 term [14]. This is what gives, for instance, the Suzuki result (2). For an axion, however, what is needed is the commutator of $Q_3^5 - f_\pi \int d^3x$

$\partial^0 \pi^0_{phys}$ with H_W. To the extent that the axial charge Q_3^5 is dominated by the π^0 contribution this commutator vanishes. Hence one expects

$$B_\pi p_\pi \cdot k \simeq 0 \tag{40}$$

and

$$A(K^+ \to \pi^+ a) \simeq B_a p_a \cdot k = \frac{1}{f_\pi} \langle a | H_W | K^0 \rangle . \tag{41}$$

Since the physical axion should not really strongly couple to quark currents which then turn into mesons, this matrix element should be well below the (incorrect) a-π^0 mixing result of Eq (4). However, we are not really able to give a convincing estimate of the matrix element (41). Of course if one believes in a soft K^+ analysis, as indicated above, then A vanishes. So perhaps a not unreasonable estimate for the rate of $K^+ \to \pi^+ a$ is the value in Eq (4) multipled by m_K^4 / Λ^4 where Λ is some appropriate scale for the problem ($\Lambda \gtrsim$ GeV ?). Branching ratios $B(K^+ \to \pi^+ a)$ in the range 10^{-6} to 10^{-7} are thus <u>not</u> inconceivable, but this is just a guess estimate.

We want to conclude this note by briefly considering the coupling of axions to nuclears. The above discussion may have given the impression that since one must remove the pion contribution from $\tilde{J}^\mu$ nothing is really left! For nuclears this is <u>not</u> so, because one is dealing with two different form factors. Let us write, in general

$$\langle N | \tilde{J}_\mu | N \rangle = \bar{U}(p') \Big[i \gamma_\mu \gamma_5 \tilde{G}_A + i (p-p')^\mu \gamma_5 \tilde{G}_p \Big] \frac{\tau_3}{2} U(p) \tag{42}$$

where we have anticipated that only the isovector contribution is important. Since for the A_3^μ current one has - retaining only the nearby π^0 contribution -

$$\langle N | A_3^\mu | N \rangle = \bar{U}(p') \Big[i \gamma_\mu \gamma_5 G_A + i (p-p')^\mu \gamma_5 \frac{2 g_{\pi NN} f_\pi}{(p-p')^2 + m_\pi^2} \Big] \frac{\tau_3}{2} U(p) . \tag{43}$$

The contribution in $\tilde{J}^\mu$ involving $A_3^\mu - f_\pi \partial^\mu \pi^0_{phys}$ will <u>remove</u> the π^0 pole in the $(p-p')^\mu$ contribution but still will retain the G_A piece. It follows from Eq (24), therefore that

$$\tilde{G}_A = \lambda G_A \tag{44}$$

and that the contribution of $\tilde{G}_p$ will be dominated by the axion pole only

$$\tilde{G}_P = \frac{2 g_{aNN} f}{(P-P')^2 + m_a^2} \qquad (45)$$

Using the fact that the divergence of $\mathcal{J}^\mu$ is dominated by the axion pole, this gives, in the usual Goldberger-Treiman [15] way,

$$g_{aNN} = \lambda \frac{M}{f} G_A \qquad (46)$$

This is precisely the result given by Donnelly et al [16] for the case in which the effective number of generations is N = 1.

Footnotes

[F0] A similar suggestion has been put forth by L.M. Krauss and F. Wilczek [17].

[F1] We understand that an actual measurement of this branching ratio will soon be forthcoming from SIN [8].

[F2] We have written in (7) only the axial piece of J^S_μ. The actual current contains also some conserved vector pieces, which are of no interest in what follows.

[F3] The vanishing of the sigma terms requires that

$$\langle 0| \bar{u}\, e^{i\gamma_5 \frac{x a}{f}}\, u\, |0\rangle = \langle 0| \bar{d}\, e^{i\gamma_5 \frac{a}{xf}}\, d\, |0\rangle$$

which replaces the familiar equation: $\langle 0| \bar{u}u |0\rangle = \langle 0| \bar{d}d |0\rangle$ which holds in the absence of an axion field.

[F4] The result $f_+(0) = 0$ can also be obtained directly by looking at the matrix element of the charge $Q_- = \int d^3x\, V^0_-$, between an axion and a pion, in the infinite momentum frame and using the fact that the axion and the π^0 are orthagonal states.

[F5] Of course, for physical states, the amplitude is just a sum of these terms

$$A = C + \frac{1}{2} B_a (m_K^2 + m_\pi^2 - m_a^2) + \frac{1}{2} B_\pi (m_K^2 + m_a^2 - m_\pi^2)$$

[F6] We neglect here possible Penguin contributions in $H_W(0)$. A direct $\bar{d}sa$ [13] Penguin contribution has been shown in Ref [1] to be unimportant, since the contributions of heavy quarks are suppressed in the new axion model.

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