DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY
DESY 86-069
July 1986

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by

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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 86-069
July 1986

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Mass Issues in the Standard Model

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ABSTRACT

In these lectures I discuss mass issues both in QCD and in the standard electroweak theory. Among the topics treated are masses in the Goldstone sector of QCD, electromagnetic mass shifts and the values of quark masses. Issues connected with the Higgs mechanism and the generation of fermion masses and mixings are also examined, as well as the effect of radiative corrections for the W and Z masses. A general discussion of the U(1) problem, θ-vacua and the strong CP puzzle is given. The consequences of solving the strong CP puzzle by imposing an additional chiral symmetry in the electroweak interactions are explored and some of the phenomenology of axions is reviewed. The possibility of constructing variant axion models, motivated by recent observations at GSI, is briefly touched upon in closing.

* Lectures given at the XXV Internationale Universitätswochen für Kernphysik, Schladming, Austria, Feb. 1986. To be published in the Proceedings of the Winter School.

Introduction and Summary Plan of the Lectures

In these lectures I discuss a variety of issues in the standard SU(3)xSU(2)xU(1) model of the strong and electroweak interactions. Although the topics I will deal with are in principle quite varied, they have a common thread in that, in one way or another, they are concerned with the broad question of mass. In QCD, especially if one deals with the excitations closely connected with the spontaneous breakdown of the nearly exact global symmetries in the theory, it is possible not only to trace dynamically the origin of mass but also to compute various mass interrelations. In the electroweak theory, on the other hand, most issues connected with mass generation are beyond our present computational abilities. A notable exception, however, is provided by the masses of the intermediate vector bosons where, in fact, corrections to the lowest order predictions are computable and are on the verge of being tested experimentally. Mass issues in QCD and the electroweak theory are not, of course, disconnected since the Lagrangian masses of quarks have a weak origin. This interconnection is perhaps best illustrated by trying to solve the, so called, strong CP puzzle by imposing an additional symmetry in the electroweak sector. This issue and the concomitant axions predicted are, therefore, discussed in some detail in these lectures.

Before entering into specifics, I give a brief summary plan of the material included in these lectures, which I have divided into three parts.

Part I: Mass Issues in QCD

Ia) Symmetries and Dynamics

Ib) The Goldstone Sector of QCD

Ic) Electromagnetic Mass Shifts

Id) The Value of Quark Masses

Ie) The U(1) Problem, θ-Vacua and the Strong CP Puzzle

Mass Issues in QCD

I.a) Symmetries and Dynamics

The richness of the observed hadron spectrum is supposed to originate from QCD, the underlying theory of the strong interactions [1]. Neglecting electroweak effects the QCD Lagrangian reads:

$$L_{QCD} = - \sum_f \bar{q}_f \gamma^\mu \left[\frac{1}{i} (\partial_\mu - ig \frac{\lambda_a}{2} A_{\mu a}) + m_f \right] q_f - \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} \qquad (I.1)$$

Here f is a flavour index: f=(u,d,s,c,b,t,...) and a is a color index: a=1,...8 connected with the underlying SU(3) gauge symmetry. Although (I.1) is written in terms of quark (q_f) and gluon degrees of freedom, it is believed that these excitations are permanently confined. That is, the only observables of L_{QCD} are color singlet bound states - the hadrons. Instead of having free quarks

and gluons one presumes that it is energetically more favourable to form bound states which are color neutral (color confinement). For instance, qualitatively, one thinks of a meson as a $q\bar{q}$ color singlet bound state in which there is a color flux tube joining the quark and antiquark, leading to a linear potential $V(r) = \sigma r$. Trying to separate the $q\bar{q}$ pair costs more energy than producing a second $q\bar{q}$ pair in between, which shields away the color.

This discussion can be made slightly more quantitative in terms of the running coupling constant of QCD: $\alpha_s = g^2/4\pi$. As is well known, quantum effects make the effective coupling of the theory depend on the momentum scale with which one is probing the system [2]. For non Abelian gauge theories like QCD, provided there are not too many fermions in the theory, the running coupling constant vanishes at short distances (large q^2). This is the famous asymptotic freedom result [3]. As the distance increases (q^2 decreases) the running coupling increases and eventually the interactions among quarks and gluons becomes strong enough to lead to their binding into hadrons. An important dynamical scale is the scale Λ_0, typical of the strong binding region, where the running coupling is unity

$$\alpha_s(\Lambda_0^2) = 1 \qquad (I.2)$$

Roughly speaking, Λ_0 is the typical momentum scale which characterizes the binding of hadrons. To get an idea of this scale, let me use (rather unjustifiably) the perturbative formula for $\alpha_s(q^2)$ in QCD, which is known to two loop order [4], to estimate Λ_0. One has

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \mu^2/\Lambda^2} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln(\ln\mu^2/\Lambda^2)}{\ln\mu^2/\Lambda^2} \right\} \qquad (I.3)$$

where

$$\beta_0 = 11 - 2/3\, N_f$$

$$\beta_1 = 102 - 38/3\, N_f \qquad (I.4)$$

and N_f is the number of flavors. Using for Λ the $\overline{MS}$ scale, which is known to range from 100-300 MeV [5], one finds that Λ_0 ranges from 180 to 550 MeV for $N_f = 3$, which is the effective number of light flavors. That is, the typical dynamical scale associated with strong coupling in QCD is below 1 GeV.

These considerations allow one to classify hadrons, conveniently, into two types, depending on whether or not they possess heavy quarks. For light quark systems (q=u,d,s) and glueballs the effect of the quark mass term in (I.1) can be treated as a perturbation. The masses of these hadrons are primarily determined by the QCD dynamics. Systems involving heavy quarks (Q=c,b,t), either $Q\bar{q}$ or Qqq states or quarkonia ($Q\bar{Q}$), on the other hand, receive the principal contribution for their masses from the mass of the constituent heavy quark, since $m_Q > \Lambda_0$. Although these latter systems are very interesting, in these lectures we shall focus only on some aspects of hadrons made up of light quarks, where the QCD dynamics and symmetry properties are closely intertwined.

Let me consider therefore the QCD Lagrangian for only the u,d,s quarks and imagine, as a first approximation that m_u, m_d and m_s can be neglected. The QCD Lagrangian (I.1) in these circumstances is invariant under a large global symmetry: $U_L(3) \times U_R(3)$. This is easily seen. Let L and R stand for the helicity projections

$$\Psi_L = \frac{1}{2}(1-\gamma_5)\Psi \quad ; \quad \Psi_R = \frac{1}{2}(1+\gamma_5)\Psi \qquad (I.5)$$

Because the gluon interactions in (I.1) do not mix q_L with q_R one can write

this Lagrangian as

$$L^{QCD}_{m=0} = -\frac{1}{4} F^{\mu\nu}_a F_{a\mu\nu} - \sum_{f=u,d,s} \bar{q}_{fL}\gamma^\mu \frac{1}{i}(\partial_\mu - ig\frac{\lambda a}{2} A_{a\mu}) q_{fL}$$

$$- \sum_{f=u,d,s} \bar{q}_{fR}\gamma^\mu \frac{1}{i}(\partial_\mu - ig\frac{\lambda a}{2} A_{a\mu}) q_{fR} \qquad (I.6)$$

which is clearly invariant under

$$q_L \to e^{iT_a\alpha_{aL}} q_L \quad ; \quad q_R \to e^{iT_a\alpha_{aR}} q_R \qquad (I.7)$$

$$\text{where} \quad q_{L,R} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}_{L,R} \qquad (I.8)$$

and T_a is a U(3) generator matrix: $T_a = \frac{1}{2}\lambda_a$

The approximate $U_L(3) \times U_R(3)$ symmetry of the light quark sector of QCD, however, cannot be a symmetry which is preserved in the binding, since we do not see even an approximate sign of it in the hadronic spectrum. If $U_L(3) \times U_R(3)$ survived in the binding, one would expect to see conspicuous parity doubling in hadrons. However, there are no states $N' \sim 1/2^-$ approximately degenerate with the $1/2^+$ nucleons, the ρ and the A_1 have widely different masses ($m_\rho \sim 1/\sqrt{2}\, m_{A_1}$), etc. What happens, in reality, is that the chiral symmetries in $L^{QCD}_{m=0}$ are spontaneously broken by the QCD vacuum, so that the only surviving approximate symmetry is $U(3)_{R+L}$. This symmetry is nothing but the well known Gell-Mann-Neeman flavor SU(3) symmetry, which indeed is a good approximate symmetry of hadrons, plus $U_B(1)$ – the Abelian phase invariance corresponding to baryon number conservation.

It is possible to give plausibility arguments based on the structure of QCD, which make it natural to expect that all chiral symmetries in the QCD Lagrangian suffer a spontaneous breakdown, but all vectorial symmetries (those involving no γ_5's) remain exact. Actually, it should already be pointed out that the real symmetry of the theory, in the massless quark limit, is not $U(3)_L \times U(3)_R$ but $SU(3)_L \times SU(3)_R \times U(1)_{R+L}$. The additional axial U(1) current, $U(1)_{R-L}$ at the quantum level has an Adler, Bell, Jackiw anomaly [6]. Therefore, these transformations really cannot correspond to an actual symmetry of the theory. I shall return again to this point in sec I.e), so let me concentrate for now on the symmetries in

$$G_{qu} = SU(3)_{R-L} \times SU(3)_{R+L} \times U(1)_{R+L} \qquad (I.9)$$

and try to justify why the chiral $SU(3)_{R-L}$ symmetry, although a symmetry of L_{QCD}, is not a symmetry of the hadronic spectrum.

The argument I shall present is based on the idea of anomaly matching, originally suggested by 't Hooft [7] in another context. The key point is that the currents J^{μ}_{5i}, associated with the global $SU(3)_{R-L}$ symmetries of L_{QCD}, are chiral currents. Therefore the three point Green's function of these currents $\Gamma^{\mu\nu\lambda}_{ijk}$, shown schematically in Fig I.1, is expected to obey an anomalous Ward identity [6], even though the currents are classically conserved:

$$q_{3\lambda}\, \Gamma^{\mu\nu\lambda}_{ijk}(q_1,q_2,q_3) = A_{ijk}\, \varepsilon^{\mu\nu\alpha\beta}\, q_{1\alpha}\, q_{2\beta} \qquad (I.10)$$

$$\Gamma^{\mu\nu\lambda}_{ijk}(q_1,q_2,q_3) =$$

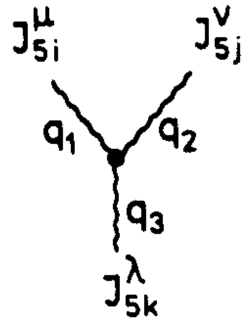


Fig I.1: Three point Green's function of chiral currents

The anomalous coefficients A_{ijk} can in fact be computed in QCD from the triangle graph of Fig I.2 and they involve a trace over three $SU(3)_{R-L}$ flavor matrices.

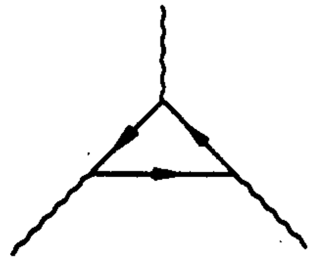


Fig. I.2: Triangle graph contributing to the anomaly A_{ijk}.

The anomalous equation (I.10) in fact implies [8] that the Green's function $\Gamma^{\mu\nu\lambda}_{ijk}$ must have a singularity in q^2 at the symmetry point $q^2 = q_1^2 = q_2^2 = q_3^2$:

$$\Gamma^{\mu\nu\lambda}_{ijk}\Big|_{\text{symm. point}} = \frac{1}{q^2} A_{ijk} \left\{ \varepsilon^{\mu\nu\alpha\beta}\, q_{1\alpha}\, q_{2\beta}\, q_3^{\lambda} + \text{cycl. perm.} \right\}$$

$$+ \text{ non singular terms} \qquad (I.11)$$

This singularity must be there irrespective of how one goes about computing $\Gamma^{\mu\nu\lambda}_{ijk}$. In particular, trying to compute this Green's function by using as intermediate states the hadronic bound states must also yield this singularity. This is the 't Hooft [7] matching condition. However, such a q^{-2} singularity can only arise if the spectrum of hadrons has in it either massless J=0 or J=1/2 bound states. (It is easy to convince oneself that J>1/2 states do not give sufficient singular behaviour [8], and that if there are no massless bound states then no q^{2} singularity arises). Let us examine these two possibilities.

If the q^{-2} singularity in $\Gamma^{\mu\nu\lambda}_{ijk}$ is produced because of the presence of massless J=0 bosons in the hadronic spectrum, as indicated in Fig I.3, then it is clear that the chiral $SU(3)_{R-L}$ symmetry is spontaneously broken. The massless bound states are nothing but the Goldstone bosons which necessarily must arise when there is such a breakdown

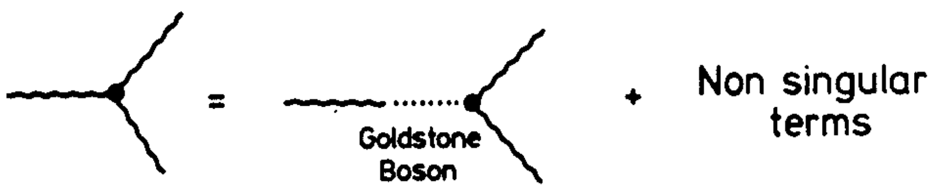


Fig I.3: Origin of q^{-2} singularity in $\Gamma^{\mu\nu\lambda}_{ijk}$, when chirality is spontaneously broken.

If, on the other hand, there are massless spin 1/2 fermions in the spectrum, as

one would naturally be lead to expect from the existence of a good chiral symmetry, then it is these bound states which produce the anomaly matching, as shown in Fig I.4.

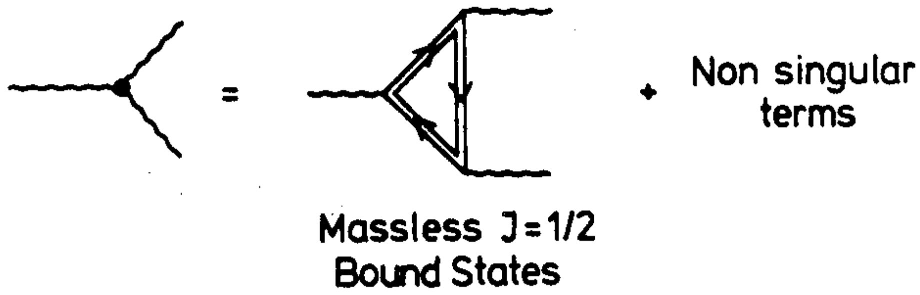


Fig I.4: Possible mechanism for anomaly matching

Because in the hadronic spectrum there appear to be no fermions which are very light with respect to the dynamical binding scale Λ_0 of QCD, it is clear that the above arguments suggest that chirality is spontaneously broken in QCD. Indeed, one then is lead to identify the octet of mesons, comprising the π, K and η, as approximate Goldstone bosons. That is, in the limit as $m_u = m_d = m_s = 0$, these states should be massless, so as to allow for anomaly matching. These plausibility arguments can be made more rigorous in a version of QCD where the number of colors $N \to \infty$ with g^2N fixed [9].

The breakdown of the chiral $SU(3)_{R-L}$ symmetry in QCD occurs dynamically because the QCD vacuum allows the formation of condensates, like $\langle 0|\bar{u}u|0\rangle$, which are not $SU(3)_{R-L}$ singlets. To guarantee that the vectorial symmetry $SU(3)_{R+L}$ is <u>not</u> broken by the presence of these non trivial condensates, it is necessary that

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = \langle 0|\bar{s}s|0\rangle \qquad (I.12)$$

Since $SU(3)_{R+L}$ appears to be a good symmetry for hadrons the above equation must obviously hold. Recently, Vafa and Witten [10] were able to give some theoretical arguments which establish that, if condensates form, they respect $SU(3)_{R+L}$ (i.e. Eq. (I.12) holds). Their results are based on two properties of vacuum expectation values, which one can establish in QCD:

i) Any bound obtained for the expectation of an operator Q in a background gauge field A is also bound for the vacuum expectation value of this operator.

ii) In the limit $m=m_u=m_d=m_s$ the expectation of the difference in quark densities in the presence of a background field vanishes. That is, for example:

$$\langle 0|(\bar{u}u - \bar{d}d)|0\rangle^A \rightarrow 0 \qquad (m_u-m_d) \rightarrow 0 \qquad (I.13)$$

Using the result i) and (I.13), plus the reasonable presumption that the limit $m\rightarrow 0$ is smooth, establishes (I.12) and the fact that $SU(3)_{R+L}$ remains unbroken in the binding (similar arguments are possible for $U(1)_{R+L}$). Although the proof of Eqs. (I.13) and (I.14) are found in Ref[10], it may be helpful to indicate here the crucial ingredient for establishing these points. Because QCD is a vector-like theory (only γ_μ-interactions at the gauge level) it is possible to prove that the measure in QCD is real and positive. Hence

$$\langle 0|Q|0\rangle = \int d\mu_A \, \langle 0|Q|0\rangle^A \qquad (I.14)$$

with $d\mu_A$ being a positive real measure. For operators like $\bar{u}u - \bar{d}d$ the expectation value in a background field just depends on (m_u-m_d) times an integral over an appropriate eigenvalue spectra. Thus in the equal mass limit, which corresponds to good $SU(3)_{R+L}$ at the Lagrangian level, the vacuum expectation values should also respect this symmetry. This is the content of Eq(I.13).

In the light quark sector, apart from the massless Goldstone bosons which arise from the $SU(3)_{R-L}$ breakdown, all the remaining bound states must have masses which are related to Λ_0, the dynamical scale of QCD. To actually compute this mass spectrum, however, requires the solution of a strong coupling problem for which, at the moment, the only technique available involves a latticizing of the theory, followed by extensive Monte Carlo simulations. I will not discuss these very interesting calculations here for two connected reasons:

i) I am not an expert in these matters and, furthermore, H. Satz in this school discusses already various aspects of lattice gauge theories.

ii) The comparison with experiment of lattice gauge theory calculations is somewhat premature, as far as the hadronic spectrum goes. The most reliable computations involve glueballs, but here we have no reliable experimental information. Systems with fermions, on the other hand, are only treated very approximatively up to now, by dropping internal fermion loops.

For the rest of this first part of my lectures, therefore, I shall concentrate only on the sector of the hadronic spectrum which is most closely related to symmetries - the Goldstone sector. As a bonus, we shall see that in this way one can learn something about quark masses, which will stand us in good stead when we shall discuss electroweak phenomena.

I.b) The Goldstone Sector of QCD

As I have mentioned in the last section, the breakdown of the approximate global $SU(3)_L \times SU(3)_R \times U(1)_{L+R}$ symmetry in QCD to $SU(3)_{L+R} \times U(1)_{L+R}$ produces an octet of Goldstone boson: $\Pi_1 = \left\{ \vec{\pi}, K^\pm, K^0, \bar{K}^0, \eta \right\}$. If $U(1)_{L-R}$ had also been a symmetry, then the η' would have had to be also an (approximate) Goldstone boson. The U(1) problem, whose resolution will be presented in I.e), was precisely connected with the difficulty of understanding how an excitation, with $m_{\eta'} \approx 960$ MeV, could be so identified.

Including mass terms for the quark fields breaks explicitly the global $SU(3)_L \times SU(3)_R$ symmetry and is the source for mass for the pion octet. To the extent that $m_q < \Lambda_0$ – something which we will have to check a posteriori – we may compute the mass generated for the Goldstone bosons by treating the quark mass terms in the Lagrangian:

$$L_{mass} = - m_u \bar{u}u - m_d \bar{d}d - m_s \bar{s}s \tag{I.15}$$

as a perturbation. In terms of covariantly normalized states

$$\langle p|p'\rangle = (2\pi)^3 2E\delta^3(p-p') \tag{I.16}$$

the expectation of the perturbing Hamiltonian

$$H_{pert} = -\int d^3x\, L_{mass} \tag{I.17}$$

is just

$$\langle p|H_{pert}|p'\rangle = \delta E\langle p|p'\rangle = \delta E(2E(2\pi)^3\delta^3(p-p'))$$

$$= \delta E^2 \int d^3x e^{i(p-p')x} \tag{I.18}$$

Obviously, therefore, for the pion octet one has

$$\delta m_\pi^2 = -\langle \pi|L_{mass}(0)|\pi\rangle \tag{I.19}$$

Using the fact that the pion octet states are Goldstone excitations, it is

possible to turn (I.19) into a formula involving vacuum expectation values. The chiral currents J^μ_{5i} , i=1...8, associated with the spontaneously broken $SU(3)_{R-L}$ symmetries, can connect the $|\pi\rangle$ states to vacuum:

$$\langle 0|J^\mu_{5i}(o)|\pi_j;k\rangle = i\delta_{ij}\, f_\pi\, k^\mu \tag{I.20}$$

The constant f_π can be determined experimentally from the decay $\pi \to \mu\nu$, because these $SU(3)_{R-L}$ currents are also the ones involved in this weak decay. One finds in this way $f_\pi \approx 93$ MeV. Because of (I.20) a Green's function containing the $SU(3)_{R-L}$ currents will have a singular contribution, connected to the possibility of the current transforming itself into a pion. Specifically let us consider the Green's function

$$M^{\mu\nu}_{ij}(k_1,k_2) = \int d^4x_1\, d^4x_2\, e^{-ik_1x_1}\, e^{+ik_2x_2} \langle 0|T(J^\mu_{5i}(x_1)\, J^\nu_{5j}(x_2) L_{mass}(0))|0\rangle \tag{I.25}$$

This Green's function has singular behaviour as k_1^2 , $k_2^2 \to 0$ because of the pion pole contribution indicated schematically in Fig. I.5. Using Eq (I.20) one finds:

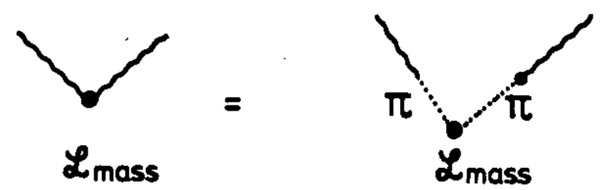


Fig I.5: Singular contributions in $M^{\mu\nu}_{ij}$

$$\left(M^{\mu\nu}_{ij} \right)_{sing} = -k_1^{\mu} \frac{f_\pi}{k_1^2} \langle \pi_i | L_{mass}\ (0) | \pi_j \rangle k_2^{\nu} \frac{f_\pi}{k_2^2} \qquad (I.22)$$

In view of the above, the scalar amplitude

$$M_{ij} = \lim_{\substack{k_1 \to 0 \\ k_2 \to 0}} k_{1\mu} M^{\mu\nu}_{ij} k_{2\nu} \qquad (I.23)$$

will isolate precisely the matrix element involved in δm_π^2 . Hence one may rewrite (I.19) as a vacuum expectation value. To wit:

$$\left(\delta m_\pi^2 \right)_{ij} = -\langle \pi_i | L_{mass}\ (o) | \pi_j \rangle$$

$$= \frac{1}{f_\pi^2} \int d^4x_1\ d^4x_2\ \partial^{x_1}_{\mu}\ \partial^{x_2}_{\nu} \langle 0 | T\left(J^{\mu}_{5i}(x_1) J^{\nu}_{5j}(x_2) L_{mass}(0) \right) | 0 \rangle \qquad (I.24)$$

This expression can be evaluated in the chiral limit, in which the $SU(3)_{R-L}$ currents are conserved, since it is already linear in the perturbation. Then the only contributions of the derivatives in (I.24) are when they act on the θ-functions of the time ordered product, resulting in equal time commutators. A simple calculation gives

$$\left(\delta m_\pi^2 \right)_{ij} = -\langle \pi_i | L_{mass}(0) | \pi_j \rangle = \frac{1}{f_\pi^2} \langle 0 | [Q_{5i}, [Q_{5j}, L_{mass}(0)]] | 0 \rangle \qquad (I.25)$$

which is Dashen's theorem [11]. Using the explicit expression for L_{mass} of Eq (I.15), and of the chiral charges:

$$Q_{5i} = \int d^3x\ J^0_{5i}(x) = \int d^3x\ \bar{q} \gamma^0 \gamma_5 \frac{\lambda_i}{2} q \qquad (I.26)$$

where λ_i is a flavor SU(3) matrix, the triple commutator in (I.25) is easily evaluated. The resulting expressions are simplified by using the Vafa Witten result [10]

$$\langle 0 | \bar{u}u | 0 \rangle = \langle 0 | \bar{d}d | 0 \rangle = \langle 0 | \bar{s}s | 0 \rangle \qquad (I.27)$$

One finds

$$M^2_{\pi^+} = M^2_{\pi^-} = -\frac{(m_u + m_d)}{f_\pi^2} \langle 0 | \bar{u}u | 0 \rangle \qquad (I.28a)$$

$$M^2_{K^+} = M^2_{K^-} = -\frac{(m_u + m_s)}{f_\pi^2} \langle 0 | \bar{u}u | 0 \rangle \qquad (I.28b)$$

$$M^2_{K^0} = M^2_{\bar{K}^0} = -\frac{(m_d + m_s)}{f_\pi^2} \langle 0 | \bar{u}u | 0 \rangle \qquad (I.28c)$$

which relates these masses to the explicit $SU(3)_{R-L}$ breaking quark masses and to the vacuum expectation value $\langle 0 | \bar{u}u | 0 \rangle$, which breaks $SU(3)_{R-L}$ spontaneously. The π^0 and the η mix due to the perturbation. However, the mixing is small. One has

$$M^2 = -\begin{bmatrix} m_u + m_d & \frac{1}{\sqrt{3}} (m_u + m_d) \\ \frac{1}{\sqrt{3}} (m_u + m_d) & \frac{1}{3} (m_u + m_d + 4m_s) \end{bmatrix} \frac{\langle 0 | \bar{u}u | 0 \rangle}{f_\pi^2} \qquad (I.29)$$

The off diagonal term causes a slight shift for the π^0 proportional to $(m_u - m_d)^2 / m_s$. We shall see soon that this is very much smaller than $(m_u + m_d)$ and indeed also much less than the electromagnetic mass shift. So we shall ignore it here and obtain

$$M^2_{\pi^0} \simeq - \frac{(m_u + m_d)}{f^2_\pi} \langle 0|\bar{u}u|0\rangle \qquad \text{(I.28d)}$$

$$M^2_{\eta} = - \frac{1}{3} \frac{(m_u + m_d + 4m_s)}{f^2_\pi} \langle 0|\bar{u}u|0\rangle \qquad \text{(I.28e)}$$

It is easy to check that the Goldstone boson masses obtained from Dashen's theorem imply the Gell-Mann-Okubo mass formula of broken $SU(3)_{L+R}$ [12]:

$$3M^2_\eta + M^2_\pi = 2M^2_{K^+} + 2M^2_{K^0} \qquad \text{(I.30)}$$

This formula holds very well experimentally. That in fact this is so is seen, alternatively, by trying to compute the ratio of the strange quark mass to the average of the u and d masses

$$\bar{m} = \frac{1}{2}(m_u + m_d) \qquad \text{(I.31)}$$

by using either the K to π or η to π mass ratios. These two methods agree well with each other, which is a reflection that the Gell-Mann-Okubo formula (I.30) is well satisfied. From the ratio

$$\frac{M^2_{K^+} + M^2_{K^0}}{M^2_{\pi^+}} = \frac{m_s + \bar{m}}{\bar{m}} \qquad \text{(I.32)}$$

one finds

$$m_s / \bar{m} = 24.2 \qquad \text{(I.33a)}$$

while from

$$\frac{M^2_\eta}{M^2_{\pi^+}} = \frac{2m_s + \bar{m}}{3\bar{m}} \qquad \text{(I.34)}$$

one finds

$$m_s / \bar{m} = 22.7 \qquad \text{(I.33b)}$$

It appears from these very large ratios that the flavor symmetry $SU(3)_{R+L}$ is very badly broken at the Lagrangian level. Yet this symmetry is not too bad a symmetry of the hadronic spectrum. This circumstance can <u>only</u> be understood if $m_s \ll \Lambda_0$ - the dynamical QCD parameter. If this is so, the spectrum of strange hadrons would still be mostly determined by the flavor independent QCD dynamics, and it would not be radically different from that of the non strange hadrons, even though $m_s \gg \bar{m}$.

It is of interest to try to determine also the ratio $(m_u - m_d) / (m_u + m_d)$, which typifies isospin breaking. However, because electromagnetism also breaks isospin, one must correct the experimentally observed meson masses for electromagnetic effects, before this mass ratio can be extracted. Let me give here the results of these calculations and leave for the next section their actual derivation.

One finds [11]:

$$\left(\delta M^2_{\pi^+}\right)_\gamma = \left(\delta M^2_{K^+}\right)_\gamma \qquad \text{(I.35)}$$

$$\left(\delta M^2_{\pi^0}\right)_\gamma = \left(\delta M^2_{K^0}\right)_\gamma = 0$$

Furthermore one can show that $(\delta M^2_{\pi+})_\gamma$ accounts quantitatively for the $\pi^+ - \pi^0$ mass difference. Using the experimental value $M_{\pi+} - M_{\pi^0} = 4.6$ MeV, the above implies that electromagnetism gives a <u>positive</u> mass shift for the K^+ relative to the K^0 :

$$M^\gamma_{K+} - M^\gamma_{K^0} \simeq \frac{M_\pi}{M_K} (M^\gamma_{\pi+} - M^\gamma_{\pi^0}) \simeq 1.3 \text{ MeV} \qquad (I.36)$$

However, experimentally, the K^0 is heavier than the K^+. In view of Eqs (I.29) this can only be understood if $m_d > m_u$. The mass difference $m_d - m_u$ can be computed from the ratio

$$\frac{M^2_{K+} + M^2_{K^0} - M^2_{\pi+} - M^2_{\pi^0}}{M^2_{K^0} - (M^2_{K+})_{QCD}} = \frac{2(m_s - \bar{m})}{m_d - m_u} = 86.8 \qquad (I.37)$$

The numerator above is insensitive to the electromagnetic mass shift. For the denominator, however, one must use for the K^+ mass $(M_{K^+})_{QCD}$, in which 1.3 MeV is subtracted away from the experimental mass, to account for the electromagnetic mass shift. Using (I.37) and our previous determinations of $m_s / \bar{m}$ gives:

$$\frac{m_d - m_u}{m_d + m_u} = 0.26 \qquad (I.38)$$

We see therefore that also isospin is badly broken at the Lagrangian level. This is, however, felt even less in the spectrum since m_u, $m_d \ll m_s < \Lambda_0$.

I.c) Electromagnetic Mass Shifts

In this section we want to establish the results (I.35) and obtain an explicit

expression for the $\pi^+ - \pi^0$ mass difference. In closing we shall also discuss the neutron-proton mass difference, which will provide another illustration of why it is necessary that m_d be greater than m_u.

The electromagnetic mass shift $(\delta M^2)_\gamma$ can be calculated from Dashen's formula (I.25), using as a perturbation L_{em} instead of L_{mass}. That is:

$$(\delta M^2_{ij})_\gamma = \frac{i}{f^2_\pi} \langle 0 | [Q_{5i}, [Q_{5j}, L_{em}(0)]] | 0 \rangle \qquad (I.39)$$

where

$$L_{em}(x) = \frac{e^2}{2} \int d^4y \, \Delta^{\mu\nu}(x-y) \, T\left(J^{em}_\mu(x) \, J^{em}_\nu(y)\right) \qquad (I.40)$$

Here $\Delta^{\mu\nu}$ is the photon propagator and

$$J^{em}_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s \qquad (I.41)$$

The commutators in Eq (I.39) are easily computed using the $SU(3)_L \times SU(3)_R$ algebra obeyed by the currents [13]:

$$[Q_{5i}, J^\mu_{5j}] = i f_{ijk} J^\mu_k$$

$$[Q_{5i}, J^\mu_j] = i f_{ijk} J^\mu_{5k} \qquad (I.42)$$

Here f_{ijk} are the SU(3) structure constants and the currents J^μ_k are the vector currents associated with the $SU(3)_{R+L}$ group. Using the above, the results (I.35) are immediate. There is no shift for π^0 and K^0 and the π^+, K^+ shifts must be the same since in J^{em}_μ the d and s quarks are treated in a totally symmetric fashion. Hence, in what follows, I shall concentrate on the π^+ electromagnetic mass shift only, which gives the $\pi^+-\pi^0$ mass difference.

For the π^+ one has

$$(\delta M^2_{\pi^+})_\gamma = \frac{1}{2f_\pi^2} \langle 0|[Q_{5,1-i2}, [Q_{5,1+i2}, L_{em}(0)]]|0\rangle \tag{I.43}$$

The commutators above can be evaluated easily by using that

$$J^\mu_{em} = J^\mu_3 + J^\mu_s \tag{I.44}$$

where J^μ_s is an isoscalar current which commutes with the chiral charges in (I.43). Simple algebra then shows that the mass shift is proportional to the difference of the time ordered product of two axial minus two vector currents. Specifically, writing a spectral decomposition for the propagators

$$i\langle 0|T\left(J^\mu_i(x)J^\nu_j(0)\right)|0\rangle = \delta_{ij} \int dM^2 \rho_V(M^2) \int \frac{d^4p\, e^{ipx}}{(2\pi)^4} \frac{(M^2\eta^{\mu\nu} + p^\mu p^\nu)}{p^2+M^2-i\epsilon} \tag{I.45a}$$

$$i\langle 0|T\left(J^\mu_{5i}(x)J^\nu_{5j}(0)\right)|0\rangle = \delta_{ij} \int dM^2 \rho_A(M^2) \int \frac{d^4p\, e^{ipx}}{(2\pi)^4} \frac{(M^2\eta^{\mu\nu} + p^\mu p^\nu)}{p^2+M^2-i\epsilon} \tag{I.45b}$$

and using for $\Delta^{\mu\nu}$ the representation

$$\Delta^{\mu\nu}(x) = \int \frac{d^4q}{(2\pi)^4} e^{iqx} \frac{\eta^{\mu\nu} + \xi q^\mu q^\nu/q^2}{q^2 - i\epsilon} \tag{I.46}$$

where ξ is a gauge parameter, one finds

$$(\delta M^2_{\pi^+})_\gamma = \frac{-ie^2}{f_\pi^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2-i\epsilon} \left[\eta^{\mu\nu} + \xi \frac{q^\mu q^\nu}{q^2} \right]$$

$$\cdot \int dM^2 \frac{(\rho_V(M^2) - \rho_A(M^2))}{q^2 + M^2 - i\epsilon} [\eta_{\mu\nu}M^2 + q_\mu q_\nu] \tag{I.47}$$

Obviously (I.47) must be gauge independent (ξ independent), which requires that the spectral functions obey the, so called, Weinberg's first sum rule [14]

$$\int dM^2 (\rho_V(M^2) - \rho_A(M^2)) = 0 \tag{I.48}$$

Using this result, Eq (I.47) simplifies to

$$(\delta M^2_{\pi^+})_\gamma = -\frac{3ie^2}{f_\pi^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2-i\epsilon} \int dM^2 \frac{M^2(\rho_V(M^2)-\rho_A(M^2))}{q^2 + M^2 - i\epsilon} \tag{I.49}$$

The above integral is logarithmically divergent, unless the second Weinberg sum rule [14]:

$$\int dM^2 M^2 (\rho_V(M^2) - \rho_A(M^2)) = 0 \tag{I.50}$$

holds. In QCD, in the chiral limit one can show that indeed (I.50) is true, so that the electromagnetic mass shift for the pions is finite and calculable. The proof of this statement [15] makes use of the operator product expansion [16] for the T product of the electromagnetic currents appearing in L_{em} to identify possibly singular pieces in this Lagrangian. In general one finds

$$T(J^\mu_{em}(x) J^{em}_\mu(0)) = C_0(x)1 + \sum_q C_q(x) m_q \bar{q}q + C_g(x) F^{\mu\nu}_a F_{a\mu\nu}$$
$$+ \text{less/singular terms} \tag{I.51}$$

That is, the divergent pieces in the T product involve operators which already appear in the QCD Lagrangian. These infinities, therefore, can be absorbed by appropriate counterterms. However, for $(\delta M_{\pi^+})_\gamma$ in the chiral limit these counterterms have <u>vanishing</u> matrix elements. So the result (I.49) must be finite, implying that the second Weinberg sum rule is valid. Obviously the operator 1 does not contribute to the $\pi^+-\pi^0$ mass difference, and when the quark masses are neglected there is no need for a mass counterterm. Furthermore, the matrix element of the gluon kinetic energy, between pion states, also vanishes in the chiral limit, because it is related to the matrix element of the trace of the energy momentum tensor θ^μ_μ, via the trace anomaly [17]

$$\theta^\mu_\mu = \frac{\beta(g)}{2g^3} F^{\mu\nu}_a F_{a\mu\nu} \qquad \text{(I.52)}$$

The matrix element of θ^μ_μ is proportional to m_π, which goes to zero in the chiral limit.

Eq (I.49) was derived originally by Das, Guralnik, Mathur, Low and Young [18]. To evaluate the integral over the spectral functions they saturated these functions with ρ, A_1 and π poles. Specifically, they assumed

$$\rho_V(M^2) = g^2_\rho\, \delta(M^2-m^2_\rho) \qquad \text{(I.53)}$$

$$\rho_A(M^2) = f^2_\pi \delta(M^2) + g^2_{A_1}\delta(M^2-m^2_{A_1})$$

The Weinberg sum rules [14] provide two interrelations between the above parameters

$$g^2_\rho = f^2_\pi + g^2_{A_1}$$

$$m^2_\rho\, g^2_\rho = m^2_{A_1}\, g^2_{A_1} \qquad \text{(I.54)}$$

Then one can either use data for m^2_ρ and $m^2_{A_1}$, or the KSFR relation [19]

$$g^2_\rho = 2f^2_\pi \qquad \text{(I.55)}$$

which implies $m^2_{A_1} = 2m^2_\rho$, to evaluate the sum rules. Using (I.54) one obtains

$$(\delta M_{\pi^+})_\gamma = \frac{3\alpha}{4\pi} \frac{m^2_\rho}{[1 - \frac{m^2_\rho}{m^2_{A_1}}]} \ln(m^2_{A_1} / m^2_\rho) \qquad \text{(I.56)}$$

The KSFR relation (I.55) gives for the pion mass difference then the value:

$$(M_{\pi^+} - M_{\pi^0}) = \frac{(\delta M^2_{\pi^+})_\gamma}{M_{\pi^+} + M_{\pi^0}} \simeq 5.20 \text{ MeV} \qquad \text{(I.57)}$$

which is close to the experimental number of 4.6 MeV. Using the ρ and A_1 masses from the Particle data booklet [20], gives a larger value for this mass difference, since $m_{A_1} > \sqrt{2}\, m_\rho$:

$$(M_{\pi^+} - M_{\pi^0}) \simeq 6 \text{ MeV} \qquad \text{(I.58)}$$

Given the crudeness of the approximation (I.53), also this result is satisfactory. It should be pointed out, however, that recent data from τ-decay have an A_1 enhancement at a mass: $m_{A_1} = \sqrt{2}\, m_\rho$ [21], which would then imply again (I.57). An evaluation of the electromagnetic mass shift (I.49), using for the spectral functions ρ_V and ρ_A values inferred from an analysis of hadronic τ decays, would be of great interest, since it would obviate partly the saturation approximation of Das et al [18]. Such an analysis is now in progress [22].

One can attempt to calculate in the same way the proton-neutron mass difference: $(m_p - m_n)_\gamma$. One finds, as is the case for the kaons and pions that this electromagnetic shift is <u>positive</u> [15]. This is an old puzzle, since experimentally the neutron is roughly 1.3 MeV heavier than the proton. However, the resolution of this puzzle is now clear: the reason for the neutron being heavier than the proton has nothing to do with electromagnetism, but is due to the fact that $m_d > m_u$! It is useful, at any rate, to try to obtain a closed formula for $(m_p - m_n)_\gamma$ - the, so called, Cottingham's formula [23].

The electromagnetic mass difference between m_p and m_n is given by (cf. Eq (I.19))

$$(m_p - m_n)_\gamma = - \frac{1}{2M_N} \left\{ \langle p|L_{em}|p\rangle - \langle n|L_{em}|n\rangle \right\} \tag{I.59}$$

$$= \frac{i\pi\alpha}{M_N} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - i\varepsilon} \left\{ i\int d^4x\, e^{iqx} \left\{ \langle p|T(J_\mu^{em}(x)J_{em}^{\mu}(0))|p\rangle \right.\right.$$

$$\left.\left. - \langle n|T(J_\mu^{em}(x)J_{em}^{\mu}(0))|n\rangle \right\}\right\}$$

This expression like $(M_{\pi^+}^2)_\gamma$, is finite in QCD in the chiral limit. In this limit, obviously, one has no mass counterterm. Furthermore, the possible singular term in (Eq I.51) which is proportional to $F_a^{\mu\nu} F_{a\mu\nu}$, being an isoscalar quantity, does <u>not</u> contribute to the proton-neutron mass difference. However, now it is no longer possible to relate this mass difference to some vacuum matrix element. Nevertheless, one can extract the matrix elements required in (I.59) directly from data on ep and en scattering. The trick to achieve this was discovered by Cottingham [23].

For these purposes let us consider the electromagnetic amplitude, between nucleon states:

$$T^{\mu\nu}(q,p) = i\int d^4x e^{iqx} \langle N;\, p|T\,(J_{em}^{\mu}(x)J_{em}^{\nu}(0))|N;p\rangle \tag{I.60}$$

Clearly, for q^2 space like $(q^2>0)$ the imaginary part of $T^{\mu\nu}$ is related to $\gamma_{virtual}$ N scattering, as shown schematically in Fig I.6. What Cottingham

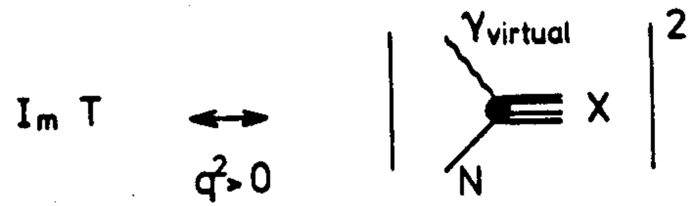


Fig I.6: Process measured by ImT for $q^2>0$

was able to show is that, in Eq (I.59), the integral over $(T_\mu^\mu)_{p-n}$ needed could be restricted to only the $q^2>0$ region. Hence, $(m_p-m_n)_\gamma$ could be directly related to measurable physics. Using the definition (I.60) one can rewrite (I.59)

$$(m_p-m_n)_\gamma = \frac{i\pi\alpha}{M_N} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2-i\varepsilon} \left\{ T_\mu^\mu(q,p) \Big|_{\text{proton}} - T_\mu^\mu(q,p) \Big|_{\text{neutron}} \right\} \tag{I.61}$$

Now T_μ^μ is a Lorentz invariant function of q_μ and p_μ and hence can only depend on q^2 and $q \cdot p$

$$T_\mu^\mu(q,p) = T_\mu^\mu(q^2, q\cdot p) = T(q^2, q^0) \tag{I.62}$$

Here the last line follows because in the mass difference $p^\mu = (\vec{0}, M_N)$. The observation of Cottingham [23] is that the singularities in (I.61) are such that the integral over q^0 can be rotated over the imaginary axis. That is

$$\int_{-\infty}^{\infty} dq^0 \int d^3\vec{q}\ \frac{1}{q^2-i\varepsilon}\ T(q^2,q^0) = \int_{-i\infty}^{i\infty} dq^0 \int d^3\vec{q}\ \frac{1}{q^2-i\varepsilon}\ T(q^2,q^0) \qquad (I.63)$$

Changing variables to $\nu = -iq^0$, one sees therefore that in (I.63) effectively

$$q^2 = \vec{q}^{\,2} - q^{02} = \vec{q}^{\,2} + \nu^2 > 0$$

Hence, indeed, the electromagnetic mass difference of protons and neutrons involves only knowledge of space like virtual photon-nucleon scattering.

Using Eq (I.63) and changing variables of integration to ν and q^2 one finds

$$(m_p-m_n)_\gamma = -\frac{\alpha}{4\pi^2 M_N} \int_0^\infty \frac{dq^2}{q^2} \int_0^{(q^2)^{1/2}} d\nu\ (q^2-\nu^2)^{1/2}$$

$$[T(q^2,i\nu)_{proton} - T(q^2,i\nu)_{neutron}] \qquad (I.64)$$

which is Cottingham's formula [23]. Because of the particular kinematical weighting in Eq (I.64), the dominant contribution in T is just the one due to elastic scattering. A recent evaluation [15], using dipole form factors for the nucleons, gives

$$(m_p-m_n)_\gamma = 0.76 \text{ MeV} \qquad (I.65)$$

Quasi inelastic and deep inelastic processes add extremely small contributions to the above [15].

In view of (I.65) and the experimentally measured proton neutron mass difference, the mass difference between protons and neutrons, to be attributed entirely to a mass difference between the down and up quarks, is

$$(m_p-m_n)_{QCD} = (m_p-m_n)_{exp} - (m_p-m_n)_\gamma \simeq -2 \text{ MeV} \qquad (I.66)$$

This very interesting matter cannot really be computed in QCD, unless one knows how to extract the expectation of $\bar{u}u$ and $\bar{d}d$ in a proton and knows the numerical value of m_u-m_d. One has, using the analog of Eq (I.19):

$$(m_p-m_n)_{QCD} = \frac{1}{2M_N}(m_u-m_d)\ \langle p|(\bar{u}u-\bar{d}d)|p\rangle \qquad (I.67)$$

In practice, one uses (I.67) and the result (I.66) to learn something about the quark densities in protons. Naively one would expect, in a valence quark approximation where $\bar{\Psi}\Psi \simeq \Psi^+\Psi$ that

$$\langle p|\bar{u}u|p\rangle = 2\ \langle p|\bar{d}d|p\rangle \qquad (I.68)$$

However, the numerical result (I.66), along with a value for the pion-nucleon σ term [15] shows that the actual number in (I.68) is much smaller.

The σ term measures the expectation of $\bar{u}u + \bar{d}d$ in a proton. To be precise [15]

$$\sigma = \frac{m_u+m_d}{4M_N}\ \langle p|\bar{u}u + \bar{d}d|p\rangle \qquad (I.69)$$

Numerically σ ranges from 30 to 60 MeV [15], with the first value arising from a fit of baryon mass formulas and the second value coming from an analysis of low energy pion nucleon scattering. (The actual discrepancy between these two numbers is not easy to understand). Using (I.66), (I.69) (with $\sigma \simeq 40$ MeV) and the value (I.38) for $\frac{m_d-m_u}{m_d+m_u}$ gives

$$\langle p|\bar{u}u|p\rangle \simeq 1.2\, \langle p|\bar{d}d|p\rangle \qquad (I.70)$$

in contrast to the naive estimate of (I.68).

I.d) The Value of the Quark Masses

The analysis of the Goldstone sector of QCD and the consideration of electromagnetic mass differences has allowed a determination of the light quark mass ratios:

$$\frac{m_s}{\bar{m}} \simeq 23.5 \quad ; \quad \frac{m_d-m_u}{m_d+m_u} \simeq 0.26 \qquad (I.71)$$

To actually get a value of the masses one needs to know, however, the matrix elements of quark bilinears. For instance, we found

$$f_\pi^2 M_{\pi^0}^2 = -(m_u + m_d)\, \langle 0|\bar{u}u|0\rangle \qquad (I.72)$$

$$(m_p - m_n)_{QCD} = \frac{(m_u-m_d)}{2M_N} \langle p|(\bar{u}u-\bar{d}d)|p\rangle \simeq -2 \text{ MeV}$$

Lattice QCD can, in principle, calculate the expectation values of the various quark bilinears and hence determine the quark masses. For example, a recent calculation of Barbour et al [24] gives

$$[-\langle 0|\bar{u}u|0\rangle]^{1/3} \simeq (220\text{-}250) \text{ MeV} \qquad (I.73)$$

where the uncertainty in the value of this expectation reflects, in part, an uncertainty in the extrapolation procedure used to extract $\langle 0|\bar{u}u|0\rangle$ from the Monte Carlo calculation. Using Eq (I.72), nevertheless, this implies that

$$m_u + m_d \simeq 15 \text{ MeV} \qquad (I.74)$$

This result, in conjunction with Eqs (I.71) gives the following values for the quark masses:

$$m_s \simeq 175 \text{ MeV}; \quad m_d \simeq 9.5 \text{ MeV}; \quad m_u \simeq 5.5 \text{ MeV} \qquad (I.75)$$

These masses, although having a considerable uncertainty, certainly are much <u>below</u> the value of the QCD dynamical scale Λ_0. ($\Lambda_0 \lesssim 1$ GeV). So these results justify, a posteriori, treating the mass term (I.15) as a perturbation in QCD.

Actually, I have been very sloppy in the above discussion. If one wants to obtain a reliable value for the quark masses, one must specify a little more carefully what one is doing. I illustrate this point by focussing, again, on Eq (I.72). Obviously, the left hand side of this equation, since it involves physically measured quantities, is independent of any renormalization scale. However, both $(m_u + m_d)$ and $\langle 0|\bar{u}u|0\rangle$ are separately dependent on how one defines them in QCD, although their product is independent of any definition. It is particularly useful to use the minimal subtraction scheme ($\overline{MS}$) of t'Hooft [25] to specify how parameters in QCD depend on a renormalization scale μ. In such a scheme the running coupling constant is μ-dependent, but is independent of the mass parameters of the quarks. These masses, however, also are dependent on μ.

More precisely, one has the following renormalization group equations [25] governing the behaviour of $g(\mu)$ and $m_q(\mu)$

$$\mu\frac{d}{d\mu}\, g(\mu) = \beta(g(\mu))$$

$$\mu\frac{d}{d\mu}\, m_q(\mu) = -\gamma(g(\mu))\, m_q(\mu) \qquad (I.76)$$

Here the β and γ functions are known up to terms of second order. One has [25]

$$\beta = -\beta_0 \frac{g^3}{(4\pi)^2} - \beta_1 \frac{g^5}{(4\pi)^4} + \ldots \qquad (I.77)$$

$$\gamma = \gamma_0 \frac{g^2}{(2\pi)^2} + \gamma_1 \frac{g^4}{(2\pi)^4}$$

where

$$\beta_0 = 11 - 2/3\, N_f \quad ; \quad \beta_1 = 102 - \frac{38}{3} N_f$$

$$\gamma_0 = 2 \quad ; \quad \gamma_1 = \frac{101}{12} - \frac{5}{18} N_f \qquad (I.78)$$

and N_f is the number of quark flavors. The solution of Eq. (I.76) for the running mass is easily seen to be

$$m_q(\mu) = m_q \left[\frac{1}{2} \ln \mu^2/\Lambda^2_{\overline{MS}} \right]^{-\frac{2\gamma_0}{\beta_0}} \left\{ 1 - \frac{2\beta_1\gamma_0}{\beta_0^3} \left(\frac{1 + \ln (\ln\mu^2/\Lambda_{\overline{MS}}^2)}{\ln \mu^2/\Lambda_{\overline{MS}}^2} \right) + \frac{8\gamma_1}{\beta_0^2 \ln\mu^2/\Lambda_{\overline{MS}}^2} + \ldots \right\} \qquad (I.79)$$

Using as an effective number of flavors $N_f = 3$, one sees that

$$\frac{2\gamma_0}{\beta_0} = \frac{4}{9} \qquad (I.80)$$

Hence the relation between the running mass $m_q(\mu)$ and the invariant mass m_q - defined in Eq. (I.79) - is, approximately

$$m_q(\mu) \simeq \frac{m_q}{(\ln \mu/\Lambda_{\overline{MS}})^{4/9}} \qquad (I.81)$$

The extraction of a quark mass from the theory, in general, will involve some scale μ. For instance, in the lattice determination of $\langle 0|\bar{u}u|0\rangle$ this quantity is evaluated at some value of the coupling g, which in turn implies a particular scale μ, for $\langle 0|\bar{u}u|0\rangle$ and the quark masses. Once $m_q(\mu)$ is obtained, however, one may use (I.81) - or more precisely (I.79) - to extract the renormalization group invariant mass m_q. It is important to note that ratios of quark masses are μ independent in the $\overline{MS}$ scheme, so that

$$\frac{m_q(\mu)}{m_{q'}(\mu)} = \frac{m_q}{m_{q'}} \qquad (I.82)$$

Hamber [26], in particular, has analyzed the lattice values for $\langle 0|\bar{u}u|0\rangle$, as a function of the coupling. Since the product $(m_u + m_d)(\mu)$ times $\langle 0|\bar{u}u|0\rangle(\mu)$ is supposed to be μ independent, one expects from Eq (I.81) that

$$\langle 0|\bar{u}u|0\rangle(\mu) \sim (\ln \mu/\Lambda_{\overline{MS}})^{4/9} \sim (g^2(\mu))^{-4/9} \qquad (I.83)$$

Fig (I.7) reproduces the values obtained by Hamber [26] for this vacuum expectation value, as a function of $\beta=6/g^2$. These values, expecially for small β, appear to deviate from the expected scaling behaviour

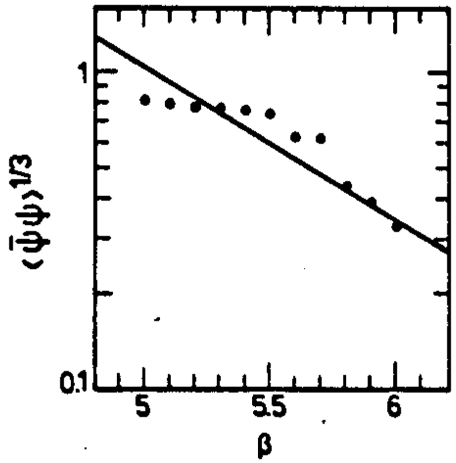


Fig I.7: Behaviour of $\langle 0|\bar{u}u|0\rangle$ as a function of $\beta=6/g^2$. From Ref [26]

Nevertheless, from the average behaviour shown, Hamber [26] extracts a value for the renormalization group invariant mass

$$m_u + m_d = (12.2 \pm 2.0)\ \text{MeV} \qquad (I.84)$$

This value is in rough agreement with the value one can infer from the analysis of Barbour et al [22]. However, this value is in some disagreement with the value obtained by Gasser and Leutwyler [15] using QCD sum rules. This method, which I shall briefly explain below, gives a value for the running mass, and the best estimate of Gasser and Leutwyler [15] is that

$$(m_d + m_u)\ (1\ \text{GeV}) = (14 \pm 2)\ \text{MeV} \qquad (I.85)$$

Given (I.85) the value of the renormalization group invariant mass follows, given a value of $\Lambda_{\overline{MS}}$. Roughly speaking

$$\hat{m}_d + \hat{m}_u \simeq (m_d + m_u)\ (1\ \text{GeV}) \left(\ln \frac{1\text{GeV}}{\Lambda_{\overline{MS}}} \right)^{4/9} \qquad (I.86)$$

Obviously, the precise value of $\hat{m}_d + \hat{m}_u$ obtained depends on the value of $\Lambda_{\overline{MS}}$ used. Hamber [26], effectively takes $\Lambda_{\overline{MS}}$ = 70 MeV but uses $N_f = 0$, since he neglects the effect of quark loops. Hence in (I.86), 4/9 → 4/11 and Hamber's result (I.84) would correspond to a running mass

$$(m_d + m_u)(1\ \text{GeV}) = (8.4 \pm 1.4)\ \text{MeV} \qquad (I.87)$$

The difference between (I.87) and (I.85) represents probably a good estimate of the real theoretical error in trying to extract a numerical value for the quark masses.

Before closing this subsection I want to indicate, in broad terms, how one can get values for $m_u + m_d$ from QCD sum rules [27]. The idea, which was pioneered by the ITEP school, is very simple. Consider the current correlation function for the divergence of two axial currents

$$\Pi(q^2) = i\int d^4x\ e^{iqx} \langle 0|T\left(\partial_\mu J^\mu_{5\ 1+i2}(x)\ \partial_\mu J^\mu_{5\ 1-i2}(0)\right)|0\rangle \qquad (I.88)$$

For large q^2 one can calculate $\Pi(q^2)$ in perturbation theory, and the leading contribution will come from the mass insertion loop of Fig I.8, which yields

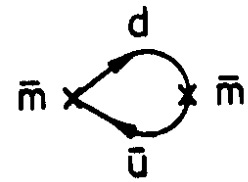


Fig I.8: Leading contribution for $\Pi(q^2)$, for q^2 large, in QCD

$$\mathrm{Im}\,\Pi(q^2) \approx \frac{3}{2\pi} q^2 \left(\bar{m}(q^2)\right)^2 + O(\alpha_s(q^2)) \tag{I.89}$$

On the other hand, at low q^2, this current correlation function should be dominated by the pion pole. Thus

$$\mathrm{Im}\,\Pi(q^2) \approx \pi\,(\sqrt{2}f_\pi)^2\, m_\pi^4\, \delta(q^2+m_\pi^2) \tag{I.90}$$

In the usual duality sense [27] the integral of the asymptotic form in the low q^2 region should average the contribution of the low energy excitations - here the pion pole. Hence one expects

$$2\pi f_\pi^2 m_\pi^4 \approx \int_0^{s_0} ds\, \frac{3}{2\pi}\, s\, \bar{m}^2(s) \tag{I.91}$$

where s_0 is the q^2 value up to which one expects this averaging to make sense and for which the pion pole approximation is reasonable. Eq (I.91) implies

$$\bar{m}(s_0) \approx \frac{2\sqrt{2}\,\pi\, f_\pi\, m_\pi^2}{\sqrt{3}\, s_0} \approx \frac{9\ \mathrm{MeV}}{(s_0\ \mathrm{GeV}^2)} \tag{I.93}$$

Obviously the analysis presented above is very rough and very dependent on the assumption one makes on s_0! One can do a much more sophisticated analysis in which s_0 is eventually self determined, by using appropriately weighted sum rules [28]. Also corrections of $O(\alpha_s)$ and those due to 3 pions can be added, as well as possible contributions of non trivial vacuum condensates, like $\langle F_a^{\mu\nu} F_{a\mu\nu}\rangle$. The result quoted in Eq (I.85), obtained by Gasser and Leutwyler [15], contains all these refinements and hence should be considered reliable. This same method can be used to study current correlation functions involving charmed and bottom quarks. By saturating these sum rules with ψ and Υ states,

respectively, one can then extract a value for the charm and bottom masses. The results obtained give masses m_c and m_b above Λ_0, showing again clearly the separation between light quarks ($m_q < \Lambda_0$) and heavy quarks ($m_q > \Lambda_0$). I reproduce in Table I.1 the running quark masses at a scale of 1 GeV, derived in this way by Gasser and Leutwyler [15]

Table I.1: Quark Masses m_q(1 GeV) from Ref [15]

m_u (1 GeV) =	5.1 ± 1.5 MeV	light quarks
m_d (1 GeV) =	8.9 ± 2.6 MeV	
m_s (1 GeV) =	175 ± 55 MeV	
m_c (1 GeV) =	1.35 ± 0.05 GeV	heavy quarks
m_b (1 GeV) =	5.9 ± 0.10 GeV	

I.e) U(1) problem, θ-vacua and the strong CP Puzzle

The natural global symmetry in QCD when $m_u = m_d = m_s = 0$ is $U(3)_L \times U(3)_R$. We have argued earlier, however, that the $U(1)_{R-L}$ symmetry is not a quantum symmetry of the theory, because of the Adler Bell Jackiw [6] anomaly. If J_5^μ is the current associated with the axial U(1) transformations then, even in the zero quark mass limit, this current does not have a zero divergence [6]:

$$\partial_\mu J_5^\mu = \frac{g^2}{32\pi^2} N_f F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \quad (I.94)$$

Here N_f is the number of flavors ($N_f = 3$ for the case under consideration) and $\tilde{F}_a^{\mu\nu}$ is the dual of the gluon field strength:

$$\tilde{F}_a^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \quad (I.95)$$

Because $U(1)_{R-L}$ is not a real symmetry of QCD, the presence of the condensates $\langle 0|\bar{u}u|0\rangle \neq 0$, which violates also $U(1)_{R-L}$ invariance, does not lead to an associated Goldstone boson. Hence for the η' there is no mass formula equivalent to (I.28).

The U(1) problem arose because, before the discovery of the θ-vacua, it appeared that the existence of the anomaly (Eq I.94) was not sufficient ground to avoid the appearance of a Goldstone boson in the $U(1)_{R-L}$ channel. If such a Goldstone boson really existed, then one could show that, upon introducing non zero quark masses, one would have the bound

$$m_{\eta'} \leq 3 m_\pi \quad (I.96)$$

which is a phenomenological disaster. 't Hooft's [29] solved this problem by

pointing out that the vacuum state of QCD is more complicated (θ-vacuum) and that using the correct vacuum then the existence of the anomaly is sufficient to prevent Goldstone bosons to appear in the $U(1)_{R-L}$ channel. It turns out, however, that having a θ-vacuum engenders another problem: the strong CP puzzle. So the physics involved is more complicated and, at the same time, more interesting!

I want to begin my discussion of this topic by recalling the reason why it appeared that having an ABJ anomaly was not enough to prevent one from having a Goldstone boson in the $U(1)_{R-L}$ channel. Although J_5^μ is not conserved, one can always define another current $\tilde{J}_5^\mu$ which in fact is conserved:

$$\partial_\mu \tilde{J}_5^\mu = 0 \quad (I.96)$$

This current is easily constructed by means of Bardeen's identity [30]. Defining

$$K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\alpha\beta\gamma} A_{a\alpha} \left\{ F_{a\beta\gamma} - \frac{1}{3} f_{abc} A_{b\beta} A_{c\gamma} \right\} \quad (I.97)$$

one can show that

$$\partial_\mu K^\mu = \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \quad (I.98)$$

Clearly, therefore, the current $\tilde{J}_5^\mu$ is just

$$\tilde{J}_5^\mu = J_5^\mu - N_f K^\mu \quad (I.99)$$

The charge associated with the $U(1)_{R-L}$ transformations is connected to the $\tilde{J}_5^\mu$ current, since this charge should be time independent:

$$\tilde{Q}_5 = \int d^3x\, \tilde{J}_5^0 \quad ; \quad \dot{\tilde{Q}}_5 = 0 \tag{I.100}$$

Because the vacuum expectation value $\langle 0|\bar{u}u|0\rangle \neq 0$ violates $\tilde{Q}_5$, one expects a zero mass excitation coupled to $\tilde{J}_5^\mu$. However, since K^μ is gauge dependent, the question is whether this excitation is really a physical Goldstone boson. That is, one needs to check whether this zero mass state also is coupled to the gauge invariant current J_5^μ. The U(1) problem is that it appears that such a zero mass state also couples to J_5^μ.

To see this point, and also to understand where the θ-vacua will eventually become useful, let me consider an operator Ω which:

i) has non zero $U(1)_{R-L}$ quantum numbers. Therefore

$$[\tilde{Q}_5\,,\,\Omega] = -\chi_\Omega\,\Omega \qquad \chi_\Omega \neq 0 \tag{I.101}$$

and

ii) has non zero vacuum expectation value

$$\langle 0|\Omega|0\rangle \neq 0 \tag{I.102}$$

Using the above properties, it is easy to show that there must be a zero mass excitation coupled to $\tilde{J}_5^\mu$. Consider

$$\int d^4x\, \partial_\mu \langle 0|T(\tilde{J}_5^\mu(x)\Omega)|0\rangle = -\chi_\Omega \langle 0|\Omega|0\rangle \tag{I.103}$$

Since the vacuum expectation value and χ_Ω do not vanish, by assumption, the only way this total derivative can give a non zero contribution is if there are zero mass excitations coupled to the $\tilde{J}_5^\mu$ current.

Using Eq (I.99) one can rewrite Eq (I.103) in terms of the gauge invariant current J_5^μ. Since the anomaly condition (I.94) corresponds to an operator insertion, it follows that

$$\int d^4x\, \partial_\mu\langle 0|T(J_5^\mu(x)\Omega)|0\rangle = \Big\{ -\chi_\Omega \langle 0|\Omega|0\rangle + \frac{g^2}{32\pi^2} N_f \int d^4x\, \langle 0|T(F_a^{\mu\nu}\tilde{F}_{a\mu\nu}(x)\Omega)|0\rangle \Big\} \tag{I.104}$$

One sees that, <u>unless there is a cancellation on the right side of Eq (104)</u>, the existence of operators of non zero chirality, with non vanishing vacuum expectation values, implies that J_5^μ must connect to a zero mass excitation. In QCD, one has precisely such operators – for instance $\Omega = \bar{u}_L u_R$ with $\chi_\Omega = 1$ and $\langle 0|\Omega|0\rangle \neq 0$. So, even in the presence of the Adler Bell Jackiw anomaly, it appears that there should be $U(1)_{R-L}$ Goldstone bosons coupled to the symmetry current J_5^μ.

't Hooft's resolution of this connundrum [29] was to show that in the correct vacuum of QCD – the so called θ-vacuum – it always follows that the RHS of Eq (I.104) vanishes:

$$\chi_\Omega\langle\theta|\Omega|\theta\rangle = \frac{g^2}{32\pi^2} N_f \int d^4x\, \langle\theta|T(F_a^{\mu\nu}\tilde{F}_{a\mu\nu}(x)\,\Omega)|\theta\rangle \tag{I.105}$$

Thus, even though $\tilde{J}_5^\mu$ couples singularily, no such q^{-2} singularity is present in the physical J_5^μ current. To be a bit more precise, Eq (I.105) only holds for operators Ω which are $SU(3)_{R+L}$ singlets. For operators Ω which can connect with the $SU(3)_{R-L}$ Goldstone bosons, one has to proceed more carefully [31]. However, it is still true that J_5^μ has no couplings to a massless excitation.

To understand why Eq (I.105) holds, I need to explain some properties of the QCD vacuum. Naively speaking, the vacuum state of QCD is the state where all gauge fields vanish, or more correctly, where the vector potential is a pure gauge. To discuss these questions it is particularly convenient to adopt the $A_a^o = 0$ gauge [32]. In this gauge only A_a^i is nonvanishing and these fields are time independent. If A_a^i is a pure gauge field one has

$$A^i = \frac{1}{2} \lambda_a A_a^i = \frac{i}{g} w \, \partial^i w^{-1} \qquad (I.106)$$

For simplicity lets restrict ourselves to an SU(2) subgroup of the color SU(3) group. Then the matrices w can be simply written as

$$w(r) = \exp i(\vec{\tau} \cdot \hat{r}) F(r) \qquad (I.107)$$

The physical demand that the gauge fields vanish at spatial infinity implies that

$$w \to 1 \quad \text{as} \quad r \to \infty \qquad (I.108)$$

This condition allows one to classify the distinct pure gauge fields depending on the behaviour of the function F(r) as $r \to \infty$. Namely one has

$$F_n(r) \underset{r \to \infty}{\to} 2\pi n \qquad (I.109)$$

where the integer n classifies distinct vacua of QCD. Since, however, a gauge transformation can change $w_n \to w_{n+1}$, it is clear that the correct QCD vacuum must be a superposition of all these vacua. This is the θ-vacuum:

$$|\theta\rangle = \sum_n e^{-in\theta} |n\rangle \qquad (I.110)$$

Because the condition (I.108) allows a mapping of the three Sphere $S_3 \to SU(2)$, the integer n in Eq (I.109) is related to the winding number of this

transformation [33]. Hence it can be expressed as an integral over the pure gauge fields, which is just the Jacobian of the transformation. One has [33]

$$n = \frac{ig^3}{24\pi^2} \int d^3x \, \mathrm{Tr} \, \epsilon_{ijk} A_n^i A_n^j A_n^k \qquad (I.111)$$

where

$$A_n^i = \frac{i}{g} w_n \, \partial^i w_n^{-1} \qquad (I.112)$$

with

$$w_n = \exp i \, \vec{\tau} \cdot \hat{r} F_n(r) \qquad (I.113)$$

If U is the operator which changes a pure gauge field A_n^i into a field A_{n+1}^i :

$$U A_n^i U^{-1} = A_{n+1}^i \qquad (I.114)$$

then it is clear that U leaves the θ-vacuum invariant.

$$U|\theta\rangle = \Sigma \, e^{-in\theta} U|n\rangle = \Sigma \, e^{-in\theta}|n+1\rangle = e^{i\theta}|\theta\rangle \qquad (I.115)$$

This is the property we wanted.

One is normally interested in the vacuum amplitude, the transition amplitude from the vacuum state at $t=-\infty$ to that at $t=+\infty$. In terms of the θ-vacua one has

$$\langle\theta_+|\theta_-\rangle = \sum_{n,m} e^{in\theta} \langle n_+|m_-\rangle e^{-im\theta}$$

$$= \sum_{\nu} e^{i\nu\theta} \left\{ \sum_n \langle (n+\nu)_+|n_-\rangle \right\} \qquad (I.116)$$

This amplitude, therefore, is the sum of amplitudes of fixed difference ν between the n-vacua at $t=\pm\infty$, each weighed by a phase $e^{i\nu\theta}$. Since the vacuum amplitude is expressible as a Feynman path integral, where one integrates over all possible field configurations weighed by the action $e^{iS[A]}$, Eq (I.116) tells us that in QCD this amplitude is a sum of distinct integrals characterized by the integer difference ν. For each of these distinct pieces, the effective QCD action is just

$$S_\nu^{eff} = S[A] + \nu\theta \qquad (I.117)$$

The integer $\nu = n_+ - n_-$ can be related to the density which appears in the ABJ anomaly. Consider for these purposes the integral of $F\tilde{F}$ over all spacetime. Using Bardeen's identity (I.98) one has

$$\frac{g^2}{32\pi^2} \int d^4x\, F_a^{\mu\nu} \tilde{F}_{a\mu\nu} = \int d\sigma_\mu K^\mu \qquad (I.118)$$

However, in the $A_a^0 = 0$ gauge, only K^0 is non vanishing. Hence the RHS of (I.118) reads

$$RHS = \int d\sigma_0 K^0 = \int d^3x\, K^0 \Big|_{t=-\infty}^{t=+\infty} = n_+ - n_- = \nu \qquad (I.119)$$

where to obtain this result I have used that at $t = \pm\infty$ one has only pure gauge fields, along with the definition (I.97) and the formula (I.111).

The existence of the θ-vacua implies that the QCD Lagrangian is augmented by a θ-dependent term. From (I.117) and (I.119) one has

$$L_{QCD}^{eff} = L_{QCD} + \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu} \qquad (I.120)$$

This extra term, unless $\theta=0$ (or $\theta=\pi$) violates P and T invariance and therefore CP invariance, and is the origin of the strong CP problem. However, as promised, the existence of the θ-vacua does solve the U(1) problem. Because of Eq (I.120), the insertion of a $g^2/32\pi^2 \int d^4x\, F_a^{\mu\nu} \tilde{F}_{a\mu\nu}$ term in a Green's function is equivalent to acting with $\frac{1}{i}\frac{\partial}{\partial\theta}$ on the Green's function. Hence Eq (I.104), with the vacuum state being $|\theta\rangle$, is simply given by

$$\int d^4x\, \partial_\mu \langle\theta|T(J_5^\mu(x)\Omega)|\theta\rangle = (N_f \frac{1}{i}\frac{\partial}{\partial\theta} - \chi_\Omega)\langle\theta|\Omega|\theta\rangle \qquad (I.121)$$

We see that this expression vanishes identically <u>provided</u> that the non vanishing vacuum expectation value of Ω has a particular θ dependence. 't Hooft [29] realized that this dependence obtains precisely for operators Ω which break $U(1)_{R-L}$ due to instantons [34]. In the presence of an instanton field of index ν (i.e. $g^2/32\pi^2 \int d^4x F_a^{\mu\nu} \tilde{F}_{a\mu\nu} = \nu$) the operator which breaks chirality for each flavor has $\chi_{\Omega_f} = \nu$, so that $\chi_\Omega = N_f\nu$. Thus Eq (I.121) is trivially satisfied. Having found an instance where indeed the RHS of Eq (I.121) vanishes identically, it is not difficult to imagine that the way the U(1) problem is solved is that for any operator Ω, with non zero vacuum expectation value, the θ-phase precisely matches the chirality, so that the equation

$$(N_f \frac{1}{i}\frac{\partial}{\partial\theta} - \chi_\Omega)\langle\theta|\Omega|\theta\rangle = 0 \qquad (I.122)$$

always holds. [If Ω can connect to the $SU(3)_{R-L}$ Goldstone bosons (I.122) needs modifications [31]].

The presence of the θ term in the effective Lagrangian of QCD will give rise to CP violating effects, most notably a nonvanishing elecric dipole moment for the neutron. Since there is a very strong experimental bound on the neutron electric dipole moment, this bound will imply a bound on θ. As we shall see, the resulting bound on θ is so strong that it becomes difficult to believe that the QCD vacuum angle is not fixed, somehow, by some other physics. This is the strong CP puzzle.

To calculate the electric dipole moment of the neutron due to the θ term in (I.120), it proves particularly convenient to make use of the ABJ anomaly to transform the $\theta F\tilde{F}$ term into a γ_5-dependent mass term for the quarks (in what follows we shall restrict ourselves only to u and d quarks, since these are the ones that are relevant.) The important point is that <u>a chiral transformation can alter θ [35]</u>. Indeed, if one were really in a world of <u>massless</u> quarks then the physics would be θ-independent, since one could pass from a given θ-vacuum to another by a chiral transformation, which is a symmetry of the action. When quark masses are included, one can still alter θ by a chiral transformation, but now the action is no longer invariant. By this trick, however, one can transform the $\theta F\tilde{F}$ term in L^{eff}_{QCD} into a γ_5-dependent quark mass term. The chiral charge $\tilde{Q}_5$ can change when one performs a gauge transformation U, which changes the pure gauge field A^1_n to A^1_{n+1} (Eq I.114). Using Eq (I.99) one has that

$$\tilde{Q}_5 = \int d^3x\, \tilde{J}^0_5 = \int d^3x J^0_5 - N_f \int d^3x K^0 \qquad (I.123)$$

Since for pure gauge fields A^1_n

$$\int d^3x K^0 = n$$

it follows that

$$U\, \tilde{Q}_5\, U^{-1} = \tilde{Q}_5 - N_f \qquad (I.124)$$

This result [35] implies that the $|\theta\rangle$ vacua can be altered by a chiral

rotation. It is easy to see that

$$e^{-i\tilde{Q}_5\alpha}|\theta\rangle = |\theta - N_f\,\alpha\rangle \qquad (I.125)$$

That is, rotating each of the flavors chirally by an angle α changes the θ-vacuum to a new vacuum with $\theta \to \theta - N_f\alpha$.

To get rid of the θ-term in the effective QCD Lagrangian of Eq (I.120) we must make an overall chiral rotation of the u and d quarks of an amount θ. If $m_u = m_d$ this rotation is obviously by $\theta/2$ for each flavor. For $m_u \neq m_d$ one can argue that, for small θ [36], the chiral rotations to be performed are [37]

$$u \to e^{i\gamma_5 \frac{\phi_u}{2}}\, u$$

$$d \to e^{i\gamma_5 \frac{\phi_d}{2}}\, d \qquad (I.126)$$

where

$$\phi_u = \frac{m_d}{m_d+m_u}\,\theta \qquad (I.127)$$

$$\phi_d = \frac{m_u}{m_d+m_u}\,\theta$$

Eq (I.127) is rather easily arrived at. In the θ-vacuum, the expectation values of $\bar{u}u$ and $\bar{d}d$ are θ-dependent. Indeed one has

$$\langle\theta|\, \bar{u}_L\, u_R\, |\theta\rangle = c\, e^{-i\phi_u(\theta)}$$

$$\langle\theta|\, \bar{d}_L\, d_R\, |\theta\rangle = c\, e^{-i\phi_d(\theta)} \qquad (I.128)$$

where the constant c above is the same for u and d quarks because of the Vafa Witten [10] result. Note that the phases ϕ_u and ϕ_d must appear in (I.128) since after the transformation (I.126), which gets rid of θ, we want the expectation values of the transformed quark fields to have no θ dependence. We know already that ϕ_u and ϕ_d add up to θ [36], so one needs only a further constraint. This is provided by Eq (I.122) using for Ω the mass breaking operator

$$L_{mass} = -m_u \bar{u}u - m_d \bar{d}d \qquad (I.129)$$

A simple calculation yields the result (I.127). Note that Eq (I.128) is an example of a famous theorem of Dashen [38], which states that the orientation of operators in the vacuum depends on the breaking and aligns itself so as to be a minimum with respect to rotations of the unbroken symmetry.

Performing the rotation (I.126) gets rid of the CP violating $\theta F\tilde{F}$ term in L^{eff}_{QCD} but one now induces a CP violating quark mass term:

$$L_{CPviol} \simeq \frac{m_u m_d}{(m_u+m_d)} \theta \, [\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d] \qquad (I.130)$$

where I have used that θ is presumably small. Note that (I.130) has two interesting features:

1. There is no CP violation if either m_d or m_u vanishes. This is easy to understand, because in the limit in which any quark is massless, one is free to perform chiral transformations on this quark, and thereby get rid of θ and any possible CP violation.

2. If $m_d >> m_u$ the expression (I.130) becomes essentially undependent of m_d. Thus our neglect of heavy quarks (s, c, etc) in the analysis appears justified.

The neutron electric dipole moment has been estimated, starting from Eq (I.130) by Baluni [37] and by Crewther, di Vecchia, Veneziano and Witten [39]. What one needs to calculate is the following matrix element, between nucleons:

$$d_n \bar{N} \sigma_{\mu\nu} k^\nu \gamma_5 N = \langle N|T\,(J^{em}_\mu \, i\int d^4x \, L_{CPviol})\,|N\rangle \qquad (I.131)$$

Baluni calculated the above by using a bag model, while Crewther et al [39] used chiral perturbation theory. They obtained

$$d_n = \begin{cases} 2.7\times10^{-16}\ \theta \text{ ecm} & \text{Baluni} \\ 5.2\times10^{-16}\ \theta \text{ ecm} & \text{Crewther et al} \end{cases} \qquad (I.132)$$

Comparing these results with the present bound on d_n [40]

$$d_n^{exp} \leq 4\times10^{-25} \text{ e cm} \qquad (I.133)$$

one sees that θ must be extremely small

$$\theta \lesssim 10^{-9} \qquad (I.134)$$

Why should this be so is the strong CP puzzle. As we shall see in Part III of these lectures a possible resolution of the puzzle is to invoke the existence of a real chiral symmetry [41] even though $m_q \neq 0$! To understand this strange statement it is necessary that we look at the meaning of mass when the electroweak interactions are included. It is to this topic that I now turn.

Part II: Mass Issues in the Electroweak Theory

In contrast to QCD, where mass arises through the dynamical binding of quarks and gluons into hadrons, all masses in the electroweak theory result from the spontaneous breakdown of the gauge symmetry. The gauge fields of the SU(2)xU(1) electroweak theory [42] are naturally massless and they only acquire mass when SU(2)xU(1) breaks down to $U(1)_{em}$. This breakdown leaves the photon massless but the $W^{\pm}$ and the Z^{0} acquire a mass, by the Higgs mechanism [43]. Furthermore, because the fermions - quarks and leptons - have asymmetrical chiral assignments under SU(2)xU(1) (required by the V-A nature of the charged current weak interactions) they also cannot have any Lagrangian mass term. Specifically, since all left handed fermions are part of SU(2) doublets and all right handed fermions are SU(2) singlets:

$$\psi_L \sim 2 \; ; \qquad \psi_R \sim 1 \tag{II.1}$$

it follows that all mass terms

$$L_{mass} = - m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \tag{II.2}$$

are forbidden by SU(2)xU(1). After the break down SU(2)xU(1) → $U(1)_{em}$, however, an effective mass term can arise, since Eq (II.2) does not violate $U(1)_{em}$.

IIa) The Higgs Mechanism

I want to briefly review here how gauge fields actually can acquire a mass through the, so called, Higgs mechanism [43]. Consider a theory which is locally invariant under a group of transformations G and imagine that G suffers a spontaneous breakdown to a subgroup H. Then, necessarily, the gauge fields associated with the symmetry transformations in the coset space G/H acquire a mass. This is simple to demonstrate at the Lagrangian level since:

i) The spontaneous breakdown of G→H always requires that some scalar field have vacuum expectation value: $\langle\phi\rangle \neq 0$

ii) If G is locally realized then the scalar field's kinetic energy must be expressed in terms of covariant derivates

$$L_{kinetic} = - \frac{1}{2} (D_\mu\phi)(D^\mu\phi) \tag{II.3}$$

Points i) and ii) can be seen to imply that the gauge fields connected with the broken transformations acquire a mass.

The classic example is the Abelian U(1) theory [43] where the local invariance

$$\phi(x) \to e^{i\alpha(x)} \phi(x)$$

$$A_\mu(x) \to A_\mu(x) + \frac{1}{g} \partial_\mu \alpha(x) \tag{II.4}$$

is respected by the Lagrangian

$$L = - (D_\mu\phi)(D^\mu\phi) - V(\phi^+\phi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{II.5}$$

with the covariant derivative being:

$$D_\mu\phi = (\partial_\mu - i g A_\mu) \phi \tag{II.6}$$

If this U(1) symmetry is to break down, the potential $V(\phi^+\phi)$ must have an asymmetric minimum. A natural choice for this to happen is

$$V = \lambda(\phi^+\phi - \frac{1}{2} v^2)^2 \tag{II.7}$$

The classical minimum $\phi = \frac{1}{\sqrt{2}} v$ corresponds, quantum mechanically, to the field

ϕ having non zero vacuum expectation value:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} v \neq 0 \qquad \text{(II.8)}$$

It follows immediately from (II.8) and the form of the ϕ kinetic energy term (Points i) and ii) above) that the A^μ field acquires a mass. The kinetic energy term contains a "seagull" contribution

$$L_{seagull} = - g^2 \phi^+ \phi A^\mu A_\mu \qquad \text{(II.9)}$$

which, because ϕ has a non zero vacuum expectation value, generates a mass for A^μ:

$$L_{mass} = - g^2 (\frac{1}{2} v^2) A^\mu A_\mu \qquad \text{(II.10)}$$

This is shown pictorially in Fig (II.1).

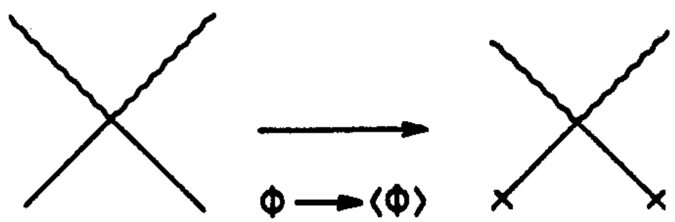


Fig. II.1: The seagull term becoming a mass term

In the electroweak theory one induces the breakdown of $SU(2)xU(1) \rightarrow U(1)_{em}$ precisely in this way. One introduces - in an ad hoc manner - a complex Higgs doublet

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \qquad \text{(II.11)}$$

with a potential which forces it to have a non vanishing vacuum expectation value:

$$V(\phi) = \lambda(\phi^+\phi - \frac{1}{2} v^2)^2 \qquad \text{(II.12)}$$

Some remarks are in order:

i) The above is clearly the simplest possibility to break $SU(2)xU(1)_{em}$, since to accomplish this one needs at least a doublet of SU(2) and this field must have non trivial U(1) properties (and therefore be complex).

ii) The vacuum expectation value $\langle\phi\rangle \neq 0$ will always leave some U(1) unbroken. The choice of the unbroken $U(1) \equiv U_{em}(1)$ and

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \qquad \text{(II.13)}$$

identifies the charges of ϕ as those given in (II.11). However, there is no loss of generality implied by these choices. With more than one doublet, however, it is necessary to require that some U(1) be left unbroken and then to identify further that U(1) with $U(1)_{em}$. Without a particular vacuum alignment there is no reason to suppose that SU(2)xU(1) would not break down totally.

iii) The potential $V(\phi)$ in (II.12) has actually a larger symmetry than SU(2)xU(1). To see this let us write

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad \text{(II.14)}$$

Then

$$V(\phi) = \lambda(\phi^{\dagger}\phi - \frac{v^2}{2})^2 = \frac{\lambda}{4}(\phi_1^2 - v^2)^2 \tag{II.15}$$

which is, obviously, O(4) symmetric. The breakdown (II.13) in terms of this new notation implies

$$\langle\phi_1\rangle = v \tag{II.16}$$

which corresponds to O(4) → O(3). Thus after the breakdown there is also a larger symmetry in V than $U(1)_{em}$ [custodial O(3) symmetry].

The presence of this custodial symmetry in the standard electroweak theory has been checked experimentally, since this extra symmetry implies a special relationship between the masses of the W and Z bosons. If g_2 and g' are the coupling constant of the SU(2) and U(1) gauge interactions, respectively, then one finds

$$M_Z^2 = \left(\frac{g_2^2 + g'^2}{g_2^2}\right) M_W^2 \equiv \frac{1}{\cos^2\theta_W} M_W^2 \tag{II.17}$$

That is, in the absence of the U(1) interactions the Z and $W^{\pm}$ bosons would have the same mass. Including the U(1) interactions, there is in general mixing between W_3 and the U(1) gauge boson Y - typified by the mixing angle θ_W - and the relation between M_W and M_Z is that of Eq (II.17). If it were not for this mixing the O(3) ~ SU(2) custodial symmetry would have given equal mass for the triplet of W bosons ($W^{\pm}$,Z).

It is perhaps useful to derive Eq (II.17) explicitly. The electromagnetic charge Q is a combination of the U(1) charge Y and T_3 . Writing

$$Q = T_3 + Y \tag{II.18}$$

one sees that ϕ has Y = - 1/2. Thus the covariant derivative of the Higgs field ϕ, to preserve the local SU(2)xU(1) invariance, must read

$$D_\mu \phi = [\, \partial_\mu - i g_2 \frac{\tau_a}{2} W_{a\mu} + i \frac{g'}{2} 1\, Y_\mu \,]\, \phi \tag{II.19}$$

The corresponding seagull term for the SU(2)xU(1) theory is then (II.20)

$$\mathcal{L}_{seagull} = - \phi^{+} (g_2 \frac{\tau_a}{2} W_{a\mu} - \frac{g'}{2} 1\, Y_\mu) \cdot (g_2 \frac{\tau_b}{2} W_b^{\mu} - \frac{g'}{2} 1 Y^{\mu})\, \phi$$

Using (II.13) gives for the gauge mass term the expression (II.21)

$$\mathcal{L}_{mass} = - \frac{1}{2} v^2 \left\{ \frac{g_2^2}{4} W_1^{\mu} W_{1\mu} + \frac{g_2^2}{4} W_2^{\mu} W_{2\mu} + \frac{1}{4} (g_2 W_3^{\mu} - g' Y^{\mu}) (g_2 W_{3\mu} - g' Y_\mu) \right\}$$

Neglecting the U(1) mixing (g' = 0) one sees how the custodial symmetry gives the same mass for all the fields W_i^{μ} . In real life g' ≠ 0 and so one must diagonalize the 2x2 matrix involving W_3 and Y. Clearly there will be one massive state - the Z^{o} - and one massless excitation which is the photon. Writing

$$\begin{pmatrix} W_3 \\ Y \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix} \tag{II.22}$$

and

$$W^{\pm} = \frac{1}{\sqrt{2}} (W_1 \mp i W_2) \tag{II.23}$$

one finds

$$M_W^2 = \left(\frac{1}{2} g_2 v\right)^2 ; \quad M_Z = \frac{M_W}{\cos\theta_W} ; \quad M_A = 0 \tag{II.24}$$

with

$$\tan\theta_W = g'/g_2 \tag{II.25}$$

To make connection with experiment, it is convenient to rewrite the interaction of the gauge fields with the currents in the theory entirely in terms of the physical gauge fields $W^\pm$, Z^0 and A. If J_Y^μ is the U(1) current and J_i^μ is the SU(2) current then one has

$$L_{int} = g' J_Y^\mu Y_\mu + g_2 J_i^\mu W_{\mu i} \tag{II.26}$$

Replacing J_Y^μ by

$$J_Y^\mu = J_{em}^\mu - J_3^\mu \tag{II.27}$$

and using the conventional definition of the charged weak currents

$$J_\pm^\mu = 2 (J_1^\mu \mp i J_2^\mu) \tag{II.28}$$

along with Eqs (II.22) and (II.23), a little algebra gives

$$L_{int} = \frac{g_2}{2\sqrt{2}} [W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu}]$$

$$+ \left\{ (g_2 \cos\theta_W + g' \sin\theta_W) J_3^\mu - g' \sin\theta_W J_{em}^\mu \right\} Z_\mu$$

$$+ \left\{ g' \cos\theta_W J_{em}^\mu + (g' \cos\theta_W - g_2 \sin\theta_W) J_3^\mu \right\} A_\mu \tag{II.29}$$

I note that, in view of Eq (II.25), the last term in the last line vanishes.

Thus the photon field, as expected, couples only to the electromagnetic current and one infers that the electromagnetic charge is given by

$$e = g' \cos\theta_W = g_2 \sin\theta_W \tag{II.30}$$

Using Eq (II.30) one can further simplify (II.29), obtaining for the weak interactions

$$L_{weak} = \frac{e}{2\sqrt{2} \sin\theta_W} [W_+^\mu J_{-\mu} + W_-^\mu J_{+\mu}] + \frac{e}{2\cos\theta_W \sin\theta_W} Z^\mu J_\mu^{NC} \tag{II.31}$$

where the neutral current J_{NC}^μ is just

$$J_{NC}^\mu = 2 [J_3^\mu - \sin^2\theta_W J_{em}^\mu] \tag{II.32}$$

For processes where the momentum transfer $q^2 \ll M_W^2$, M_Z^2, the effect of W or Z exchange is to produce a contact 4-Fermi interaction:

$$L_{eff} = \left(\frac{e}{2\sqrt{2} \sin^2\theta_W} \right)^2 \frac{1}{M_W^2} J_+^\mu J_{-\mu} + \frac{1}{2} \left(\frac{e}{2\cos\theta_W \sin\theta_W} \right)^2 \frac{1}{M_Z^2} J_{NC}^\mu J_\mu^{NC} \tag{II.33}$$

where the factor of 1/2, from 2^{nd} order perturbation theory, is absent in the first term since there are two crossed terms.

The currents $J_\pm^\mu$ are so normalized that the charged current interaction in (II.33) is precisely that of the usual Fermi theory [44]. Hence one identifies the Fermi constant as

$$\frac{G_F}{\sqrt{2}} = \frac{e}{8\sin^2\theta_W M_W^2} = \frac{g_2^2}{8M_W^2} = \frac{1}{2v^2} \tag{II.34}$$

This equation gives a prediction for M_W in terms of the electric charge e, the Fermi constant G_F and the weak mixing angle θ_W - the Weinberg angle. It also relates G_F directly to the scale of the breakdown v, appearing in the Higgs potential (II.12). Using $G_F \sim 10^{-5}$ GeV^{-2} one obtains for v - the Fermi scale:

$$v = (\sqrt{2}\, G_F)^{-1/2} \approx 250 \text{ GeV} \tag{II.35}$$

in terms of G_F, the effective Lagrangian (II.33), takes the simple form

$$L_{eff} = \frac{G_F}{\sqrt{2}} \left\{ J_+^\mu J_{-\mu} + \left[\frac{M_W^2}{\cos^2 \theta_W M_Z^2} \right] J_{NC}^\mu J_\mu^{NC} \right\} \tag{II.36}$$

The quantity in the square bracket above, which is usually denoted by

$$\rho = \frac{M_W^2}{\cos^2\theta_W M_Z^2} \tag{II.37}$$

is fixed to be unity in the electroweak theory, if the breakdown is induced by a Higgs doublet. Putting it more physically the custodial symmetry of $V(\phi)$, which gives $\rho = 1$, predicts equal strength for charged current and neutral current weak interactions. This prediction has been verified experimentally to very good accuracy. For instance, Kim et al [45], in their global fit of neutral current interactions, find

$$\rho = 0.999 \pm 0.025 \tag{II.38}$$

Since the neutral current J_{NC}^μ depends on $\sin^2\theta_W$ (c.f. Eq II.32) this parameter can be also extracted from an experimental study of neutral current interactions. Using again the analysis of Kim et al [45], one finds from low energy neutrino scattering experiments that, without including radiative corrections,

$$\sin^2 \theta_W = 0.234 \pm 0.011 \tag{II.39}$$

This value of $\sin^2 \theta_W$ allows one to predict the values for the W and Z masses, using Eqs (II.34) and (II.17). One has

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2 \theta_W} \quad ; \quad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \tag{II.40}$$

where $\alpha = e^2/4\pi \approx 1/137$. Numerically these equations give

$$M_W \approx 77.5 \text{ GeV}$$

$$M_W \approx 88.5 \text{ GeV} \tag{II.41}$$

The discovery at the CERN collider of excitations with all the characteristics of the W [46] and Z [47] bosons, and with masses in good agreement with Eq (II.41), is one of the great triumphs of the electroweak theory. The most recent compilation of the UA_1 and UA_2 results [48] gives the following experimental values for these masses:

$$\begin{aligned} M_W' &= 83.1 {}^{+1.3}_{-0.8} \pm 3 \text{ GeV } UA_1 \\ M_W &= 81.2 \pm 1.1 \pm 1.3 \text{ GeV } UA_2 \\ M_Z &= 93.0 \pm 1.6 \pm 3 \text{ GeV } UA_1 \\ M_Z &= 92.5 \pm 1.3 \pm 1.5 \text{ GeV } UA_2 \end{aligned} \tag{II.42}$$

IIb) Radiative Corrections to M_W and M_Z

The collider results, although agreeing within three standard deviations with the predictions of Eq (II.41), are a bit high. The result (II.41), however, is expected to be modified by radiative corrections. Since SU(2)xU(1) is a renormalizable theory one can compute these corrections. Clearly an important issue is whether the radiative corrections move the values of M_W and M_Z upwards or downwards from the numerical values given in (II.41). It turns out that including the effect of $o(\alpha)$ corrections actually <u>increases</u> M_W and M_Z by about 5 GeV from the values quoted in (II.41). Thus the agreement with the collider experiments is made even better, although the magnitude of the errors, both in $\sin^2\theta_W$ and in M_W and M_Z, preclude a definitive test for the moment.

Recall that to compute M_W and M_Z to lowest order we had to know the values of the three fundamental parameters in the theory: $\{ g_2, g' \text{ and } v \}$, which we eventually traded into the experimentally measured quantities $\{ \alpha, \sin^2\theta_W \text{ and } G_F \}$. To compute corrections to M_W and M_Z we need to specify <u>precisely</u> how $\{ \alpha, \sin^2\theta_W \text{ and } G_F \}$ are extracted from experiment. Different specifications are related to each other by terms of $O(\alpha)$. I illustrate this with an example. To lowest order in the weak interactions the ratio of the cross section for $\nu_\mu e \to \nu_\mu e$ and $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ scattering is only a function of the (lowest order) Weinberg angle θ_W^o:

$$\frac{\sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)} = \frac{3-12\sin^2\theta_W^o+16\sin^4\theta_W^o}{1-4\sin^2\theta_W^o+16\sin^4\theta_W^o} \qquad \text{(II.43)}$$

With doublet Higgs breaking the ratio of M_W to M_Z is also related to $\sin^2\theta_W^o$ (C.f. Eq II.17):

$$\frac{M_W^2}{M_Z^2} = 1 - \sin^2\theta_W^o \qquad \text{(II.44)}$$

Both $\sigma(\nu_\mu e \to \nu_\mu e) / \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$ and M_W^2 / M_Z^2, when calculated to higher order, get corrections of $O(\alpha)$ which serve to <u>redefine</u> the Weinberg angle. Obviously one can <u>define</u> the Weinberg angle, including $O(\alpha)$ corrections, by the relation

$$\sin^2\theta_W = 1 - M_W^2 / M_Z^2 \qquad \text{(II.45)}$$

or <u>define</u> it via the experimentally measured cross section ratio using Eq (II.43) with $\sin^2\theta_W^o$ replaced by $(\sin^2\theta_W)_\sigma$. Because SU(2)xU(1) is a renormalizable theory, either definition is acceptable and one can relate these two distinct definitions of the measured Weinberg's angle via a formula which, schematically, reads

$$(\sin^2\theta_W)_\sigma = \sin^2\theta_W [1 + O(\alpha)] \qquad \text{(II.46)}$$

where the $O(\alpha)$ terms are finite and calculable.

For the purposes of computing the radiative corrections to M_W and M_Z, it proves extremely convenient [49] to define $\sin^2\theta_W$, including $O(\alpha)$ corrections, via the mass ratio of Eq (II.45). The other two physical parameters, the Fermi constant and α, are defined in terms of μ decay and the static value of the electron's electromagnetic coupling at zero momentum transfer. More specifically, one defines a Fermi constant, G_μ, from the muon lifetime removing certain $O(\alpha)$ photonic corrections [50]:

$$\frac{1}{\tau_\mu} = G_\mu^2 \left[\left(1 - \frac{8m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right) \frac{m_\mu^5}{192\pi^3} \right] P_\gamma \qquad \text{(II.47)}$$

where

$$P_\gamma = 1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left[1 + \frac{2\alpha}{3\pi}\ln\frac{m_\mu}{m_e}\right] \qquad (II.48)$$

Experimentally then one finds

$$G_\mu = (1.16637 \pm 0.00002)\times 10^{-5}\ \mathrm{GeV}^{-2} \qquad (II.49)$$

while for the fine structure constant one has

$$\alpha^{-1} = 137.03604 \pm 0.00011 \qquad (II.50)$$

Including radiative corrections, the lowest order formula for M_W, given in Eq (II.40) is replaced by

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_\mu \sin^2\theta_W}\left[\frac{1}{1-\Delta r}\right] \qquad (II.51)$$

while, by definition,

$$M_Z = \frac{M_W}{\cos\theta_W} \qquad (II.52)$$

The correction Δr has been theoretically computed [51] and it is weakly dependent on the two unknown parameters in the electroweak model, which are the mass of the top quark, m_t, and the mass of the Higgs boson, M_H, (this is the excitation in the doublet ϕ which is not eaten by the Higgs mechanism), provided that $m_H \lesssim$ TeV, $m_t \lesssim M_W$. Taking M_H = 100 GeV, m_t = 40 GeV one finds [51]

$$\Delta r = 0.07 \qquad (II.53)$$

If M_H were 1 TeV this would give an addition to $\Delta r \approx 0.01$. If m_t were 100 GeV this would shift $\Delta r \approx -0.01$. So this gives one an idea of what is the theoretical uncertainty, at the moment, of the prediction (II.53).

Marciano [52] has given a qualitative argument for understanding the magnitude of Δr. With the definition of $\sin^2\theta_W$ adopted the largest radiative correction in the formula for M_W should come from the running of the electromagnetic coupling constant (G_F does not run). So if α were replaced by $\alpha(M_W)$ one should have no large corrections left. Indeed

$$\alpha(M_W) \approx \frac{1}{128} \approx \frac{\alpha}{1-\Delta r} \qquad (II.54)$$

Having calculated Δr, to predict M_W one still needs information on $\sin^2\theta_W$. This parameter can be extracted from deep inelastic neutrino scattering after applying radiative corrections, using for $\sin^2\theta_W$ the definition (II.45). These calculations have been carried out by Marciano and Sirlin [53] and Wheater and Llewellyn Smith [54] who, using the world average analysis of Kim et al [45], arrived at a value

$$\sin^2\theta_W = 0.217 \pm 0.014 \qquad (II.55)$$

Note that this radiatively corrected value is considerably smaller than the lowest order value given in (II.39). Most of this shift can be understood because $\sin^2\theta_W$ defined in Eq (II.45) via the W and Z masses is a parameter whose natural scale is M_W, while the neutrino scattering experiments considered are low q^2 experiments. Thus there is a rather large logarithmic shift - analogous to that displayed in Eq (II.54) for α. The error quoted in (II.55) includes some theoretical uncertainty connected with the analysis, since various experiments were averaged. More recent data from individual experiments quote somewhat smaller errors and a larger central value for $\sin^2\theta_W$:

$$\sin^2\theta_W = 0.227 \pm 0.008 \pm 0.009 \qquad \text{(II.56a)}$$

CDHS, Ref [55]

$$\sin^2\theta_W = 0.242 \pm 0.011 \pm 0.005 \qquad \text{(II.56b)}$$

CCCFRR, Ref [56]

If one uses the value of $\sin^2\theta_W$ given in Eq (II.55), and the computed value for Δr, one arrives at the following predictions for M_W and M_Z

$$M_W = 83.0 \,^{+2.9}_{-2.7} \text{ GeV}$$

$$M_Z = 93.8 \,^{+2.4}_{-2.2} \text{ GeV} \qquad \text{(II.57)}$$

These values agree remarkably well with the values obtained by the UA_1 and UA_2 collaborations (c.f. Eq II.42). Averaging these results one has, experimentally

$$M_W = 82.1 \pm 1.7 \text{ GeV}$$

$$M_Z = 93.0 \pm 1.7 \text{ GeV} \qquad \text{(II.58)}$$

However, as mentioned earlier, the errors are still too large, both in (II.58) and in $\sin^2\theta_W$, to allow a real test of the radiative corrections. For instance, rewriting the formula for Δr entirely in terms of the W and Z masses one has

$$\Delta r = 1 - \frac{\pi\alpha}{\sqrt{2}\, G_\mu M_W^2 \,[1 - M_W^2 / M_Z^2]} \simeq 0.07 \qquad \text{(II.59)}$$

However, the errors in (II.58) imply in (II.59) an uncertainty: $\delta(\Delta r) \simeq 0.07$, which is of the size of the effect one wants to test. A real test of the radiative corrections in the electroweak theory will have to await SLC and LEP. In these e^+e^- collisions by carefully measuring the Z^0 mass and $\sin^2\theta_W$, via the forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$, one should be able to test Δr to the 1% level.

II.c) Technicolor and SU(2)xSU(2)

The calculation of M_W and M_Z - including the radiative corrections - was done by assuming that the breakdown of SU(2)xU(1) is effected by an elementary Higgs field acquiring a vacuum expectation value. Although this seems to give satisfactory results, the whole Higgs sector is somewhat arbitrarily introduced into the theory, precisely to give mass to the gauge bosons. One may well ask if it is not possible to generate mass in a more elegant way? In this respect there is a useful example from condensed matter physics. Before the discovery of the BCS theory of superconductivity there was quite a successful phenomenological theory - the Ginzburg Landau theory - where a scalar field with non vanishing expectation value $\langle\phi\rangle$ played the role of the order parameter. In the more fundamental BCS theory this order parameter was replaced by $\langle\psi\downarrow\psi\uparrow\rangle$, the expectation value of a Cooper pair.

Technicolor [57] is an attempt to perform the same replacement for the Higgs sector. Thus instead of having a Higgs field ϕ and an asymmetric potential V, one imagines here that there is a deeper underlying theory, whose condensates - the analogues of the Cooper pairs - are responsible for the breakdown of SU(2)xU(1). The original Technicolor scenario of Susskind and Weinberg [57] is very easy to understand, since it is just a scaled up version of what really happens in QCD. I want to briefly examine it here because, apart from its own intrinsic interest, it makes use of some of the material which was discussed in Part I of these lectures.

The crucial observation made by Susskind [57] was that if one considered QCD plus the electroweak theory, without any elementary Higgs fields, one still gets a breakdown of $SU(2)\times U(1)\to U(1)_{em}$. Although the W and Z masses one obtains this way are phenomenologically very badly off, theoretically this phenomenon is very interesting. To analyze this, let me concentrate only on two flavor QCD. Apart from color, the u and d quarks carry non trivial $SU(2)\times U(1)$ quantum numbers. Namely under $SU(2)\times U(1)$:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2, 1/6)$$

$$u_R \sim (1, 2/3) \qquad \text{(II.60)}$$

$$d_R \sim (1, -1/3)$$

As we discussed in Part I, in QCD there exist non trivial condensates of u and d quarks. One has

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{u}_R u_L + \bar{u}_L u_R|0\rangle \neq 0$$

$$\langle 0|\bar{d}d|0\rangle = \langle 0|\bar{d}_R d_L + \bar{d}_L d_R|0\rangle \neq 0 \qquad \text{(II.61)}$$

with the u and d condensates being equal by the Vafa Witten arguments [10]. These condensates, besides breaking the global $SU(2)_{R-L}$ symmetry of QCD (in the absence of the electroweak interactions), actually break $SU(2)\times U(1)$. Indeed it is easy to check that because of (II.61), $SU(2)\times U(1)$ breaks down precisely to $U(1)_{em}$. Hence it follows that the W and Z bosons must acquire some mass, purely from the nonvanishing condensates in (II.61)!

For the $W^{\pm}$ and Z^0 to acquire a mass they must absorb some zero mass scalar excitation connected with the breakdown. Since the π-mesons are the Goldstone bosons which arise from having the condensate (II.61) in QCD, it must be that it is these bosons which are eaten by the $W^{\pm}$ and the Z^0. Since the pions are real particles, obviously this scenario is unphysical! Furthermore, since the scale of the condensates in (II.61) is in the 100 MeV range (c.f. Eq I.73), the masses that the W and Z can pick up by this mechanism must also lie in this range, which is again phenomenologically irrelevant. Nevertheless, let me work through what is going on since it is a nice theoretical laboratory.

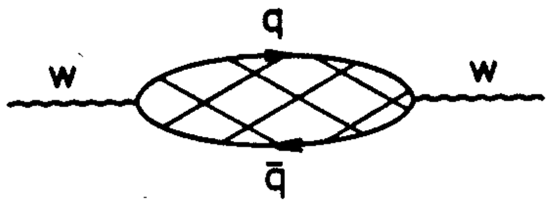
How does one really compute the induced masses for the W and Z bosons, due to (II.61)? Clearly, now it is not just the case of looking at the seagull terms, since here there is no elementary Higgs field, which gets a vacuum expectation value. However, that mass is generated for the W and Z bosons ought to be apparent by looking at their propagators. Including the effect of vacuum polarization, the gauge field propagator is proportional to:

$$\text{Gauge propagator} \sim \frac{1}{q^2[1+\pi(q^2)]} \qquad \text{(II.62)}$$

where $\pi(q^2)$ is the vacuum polarization contribution and the factor of q^2 in (II.62) is there because of gauge invariance. So if $\pi(q^2)$ is singular - which it will be if there are Goldstone boson excitations in the theory - one sees that one can generate a non trivial mass for the gauge field. In particular one has for this mass the expression:

$$M^2 = q^2\pi(q^2)\Big|_{\text{sing}} \qquad \text{(II.63)}$$

I have indicated pictorially in Fig II.2 the reason why the $W^{\pm}$ and the Z^0 get a mass from pure QCD. The singular π meson contributions to the gauge field propagators lead to a non trivial mass shift.



Pion pole

Fig II.2 Mass generation for the W's in QCD

To compute M_W and M_Z specifically we need to know the coupling of the Hypercharge current J_Y^μ and of the SU(2) current J_i^μ to the pions. This is easily ascertained by extracting from these currents the piece which corresponds to the isospin current J_{5i}^μ . For the hypercharge current of the u and d quarks one has

$$J_Y^\mu = \frac{1}{6} (\bar{u}\bar{d})_L \gamma^\mu \binom{u}{d}_L + 2/3\ \bar{u}_R \gamma^\mu u_R - 1/3\ \bar{d}_R \gamma^\mu d_R$$

$$= \frac{5}{12} \bar{u} \gamma^\mu u - \frac{1}{12} \bar{d} \gamma^\mu d + \frac{1}{2} [\bar{u} \frac{\gamma^\mu \gamma_5}{2} u - \bar{d} \frac{\gamma^\mu \gamma_5}{2} d] \qquad (II.64)$$

and we recognize the last term as $\frac{1}{2} J_{53}^\mu$. Similarly for the SU(2) current one has

$$J_i^\mu = (\bar{u}\bar{d})_L \gamma^\mu \frac{\tau_i}{2} \binom{u}{d}_L$$

$$= \frac{1}{4} (\bar{u}\bar{d}) \gamma^\mu \tau_i \binom{u}{d} - \frac{1}{2} [(\bar{u}\bar{d}) \gamma^\mu \gamma_5 \frac{\tau_i}{2} \binom{u}{d}] \qquad (II.65)$$

and we see that the last term is just $-\frac{1}{2} J_{5i}^\mu$. It follows therefore that

$$\langle 0 | J_Y^\mu | \pi_j \rangle = i \frac{f_\pi}{2} k^\mu \delta_{j3}$$

$$\langle 0 | J_i^\mu | \pi_j \rangle = - i \frac{f_\pi}{2} k^\mu \delta_{ij} \qquad (II.66)$$

Note that the electromagnetic current J_{em}^μ has no direct coupling to π_3 , since $U(1)_{em}$ is not spontaneously broken:

$$\langle 0 | J_{em}^\mu | \pi_3 \rangle = \langle 0 | J_3^\mu + J_Y^\mu | \pi_3 \rangle = 0 \qquad (II.67)$$

Because of this, when we compute the vacuum polarization contribution for the W and Z bosons due to the pion intermediate state, we can drop altogether the J_{em}^μ contribution from the neutral current. Effectively

$$L_{int} = \frac{e}{\sin\theta_W} J_i^\mu W_{\mu i} + \frac{e}{\cos\theta_W \sin\theta_W} Z^\mu (J_{\mu 3} - \sin^2\theta_W J_\mu^{em})$$

$$\simeq \frac{e}{\sin\theta_W} [J_i^\mu W_{\mu i} + \frac{Z^\mu}{\cos\theta_W} J_{\mu 3}] \qquad (II.68)$$

where i=1,2 in the above.

Using (II.68) and the matrix elements (II.66) it is now immediate to compute the W and Z masses from the formula (II.63). One has

$$M_W^2 = \left[\frac{e}{\sin\theta_W} \cdot \left(- \frac{f_\pi}{2} \right) \right]^2 = \left(\frac{g_2 f_\pi}{2} \right)^2 \qquad (II.69a)$$

$$M_Z^2 = \left[\frac{e}{\sin\theta_W \cos\theta_W} \left(- \frac{f_\pi}{2} \right) \right]^2 = \left(\frac{g_2 f_\pi}{2\cos\theta_W} \right)^2 \qquad (II.69b)$$

I note that these formulas are the same as those obtained in the elementary Higgs case (Eq II.24), except that the Fermi scale $v \simeq 250$ GeV is replaced by $f_\pi \simeq 93$ MeV, which is a typical scale connected to QCD. Although the magnitude of M_W and M_Z are obviously wrong, their ratio is OK. This should come as no surprise, since the formula $M_W = M_Z \cos\theta_W$ arose in the Higgs case because of a custodial O(3)~SU(2) symmetry. In QCD this same custodial SU(2) symmetry is present, since the condensates (II.61) break $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$. This forces f_π be the same for all 3 pions and thus gives Eqs (II.69).

Having gone through this discussion, it is clear how one must go about to replace the elementary Higgs sector in the electroweak theory. One must imagine that there is an underlying QCD-like theory with two species of fermions $\begin{pmatrix} T_u \\ T_d \end{pmatrix}$ which have the same transformation properties under SU(2)xU(1) as the u and d quarks have. Condensation of these techniquarks [57]

$$\langle \bar{T}_u T_u \rangle = \langle \bar{T}_d T_d \rangle \neq 0$$

due to the strong forces present in this new underlying theory, then will cause a spontaneous breakdown of $SU(2) \times U(1) \to U(1)_{em}$. Provided that the dynamical scale of Technicolor is much greater than that of QCD, then the W and Z bosons will acquire their correct phenomenological masses. Roughly one requires

$$(\Lambda_o)_{Technicolor} \sim \left(\frac{v}{f_\pi} \right) (\Lambda_o)_{QCD} \simeq 1 \text{ TeV} \qquad (II.70)$$

Technicolor is a marvelous idea to give the W and Z masses. however, it has no mechanism built in to give, at the same time, quark and leptons masses. For these one need some techniquark-quark/lepton transitions. However, extensions of Technicolor to incorporate interactions which eventually lead to quark and lepton masses [ETC interactions [58]] have lead to theories which are rather baroque and which have various phenomenological difficulties, like flavor changing neutral currents. Thus I shall not further pursue this topic here, but I direct the interested reader to Ref [59].

IId) Fermion Masses and Mixings

One of the advantages of having an elementary Higgs field in the electroweak theory is the possibility that it affords to generate quark and lepton masses, once SU(2)xU(1) has broken down. The Higgs field ϕ, which transforms under SU(2)xU(1) as

$$\phi \sim (2,-1/2) \qquad (II.71)$$

and its charge conjugate

$$\tilde{\phi} = i \tau_2 \phi^* \sim (2,1/2) \qquad (II.72)$$

can be coupled to the quarks and leptons in an SU(2)xU(1) invariant way. Let me denote the quarks and leptons of different families as

$$Q_{Li} = \left\{ \begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L , \dots \right\}$$

$$u_{Ri} = \left\{ u_R , c_R , t_R , \dots \right\}$$

$$d_{Ri} = \{ d_R , s_R , b_R , \dots \}$$

$$L_{Li} = \{ ({}^{\nu_e}_{e})_L , ({}^{\nu_\mu}_{\mu})_L , ({}^{\nu_\tau}_{\tau})_L , \dots \}$$

$$l_{Ri} = \{ e_R , \mu_R , \tau_R , \dots \}$$

then the most general interaction of these fields with the Higgs doublets ϕ and $\tilde{\phi}$ is

$$L_{Yukawa} = - \Gamma^u_{ij} \bar{Q}_{Li} \phi u_{Ri} + \Gamma^d_{ij} \bar{Q}_{Li} \tilde{\phi} d_{Rj}$$

$$+ \Gamma^l_{ij} \bar{L}_{Li} \tilde{\phi} l_{Ri} + h.c. \qquad (II.74)$$

The coupling constants Γ^f_{ij} $f = (u,d,l)$ in (II.74) are arbitrary. The spontaneous breakdown of SU(2)xU(1), characterized by the vacuum expectation values

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \binom{v}{0} ; \qquad \langle\tilde{\phi}\rangle = - \frac{1}{\sqrt{2}} \binom{0}{v} \qquad (II.75)$$

will induce masses for the quarks and leptons, because of the Yukawa interactions in (II.74). Since the couplings Γ_{ij} are in general not diagonal, one gets mass matrices .

$$M^f_{ij} = \frac{1}{\sqrt{2}} \Gamma^f_{ij} v \qquad (II.76)$$

There is a series of remarks to be made:

i) The eigenvalues of M^f_{ij} are the quarks and lepton masses. These are

obtained by diagonalizing (II.76) by a biunitary transformation

$$(U^f_L)^+ M^f U^f_R = (M^f)_{diag} \qquad (II.77)$$

ii) Going to a diagonal basis for M^f induces mixing among the elementary fermions. This is reflected experimentally in the appearance of mixing angles in the charged currents of quarks. The relevant mixing matrix there is just the Cabibbo-Kobayashi-Maskawa matrix [60]

$$V_{quarks} = U^{+u}_L U^d_L \qquad (II.78)$$

The analogous mixing matrix for leptons

$$V_{leptons} = U^{+\nu}_L U^l_L \qquad (II.79)$$

in unity because, if the neutrinos are massless, one can always choose $U^\nu_L = (U^e_L)^{-1}$. For neutral currents, one also gets no mixing since $U_L^{+f} U^f_L = 1$.

iii) The only physical component of the doublet ϕ is a scalar field H, which does not get absorbed in the Higgs mechanism. In a unitary representation, one can write

$$\phi = e^{i\vec{\tau}\cdot\vec{\xi}} \frac{1}{\sqrt{2}} \binom{v+H}{0} \qquad (II.80)$$

with the fields $\vec{\xi}$ being the ones that are absorbed by the W's. Because diagonalizing the mass matrices M^f also diagonalizes the Yukawa couplings Γ^f , it follows that the coupling of the physical Higgs field to the fermions in the theory is just

$$L_{Hff} = -\frac{m_f}{v}\bar{f} f H \qquad (II.81)$$

iv) Since the Yukawa couplings are arbitrary in the electroweak theory, the mass matrices M^f_{ij} and their eigenvalues - the quark and lepton masses - are arbitrary. Hence it is not possible to determine interesting ratios like $m_\mu/m_e \simeq 200$, $m_d/m_u \simeq 2$, etc, without going <u>beyond</u> SU(2)xU(1). Although the standard electroweak theory allows these masses to appear after the SU(2)xU(1) breakdown, we learn nothing about their interrelationships. In this sense, the electroweak theory is deeply disappointing.

IIe) Solving the strong CP puzzle via a PQ symmetry

Understanding the structure of the electroweak theory a bit more makes the strong CP puzzle, discussed in the first part of these lectures, somewhat worse. However, it also opens the way towards a possible resolution of this puzzle. First let me discuss why this problem is worse. We saw in the last section that although quark and leptons can get masses, there is no way to fix the value of these masses a priori. In fact, what one obtains from Eq (II.74) are really mass matrices which, because the Γ^f_{ij} are arbitrary, are arbitrary. The diagonalization of these mass matrices by the biunitary transformations (II.77), in general, will involve some $U(1)_{R-L}$ transformations for quarks. Because of the ABJ anomaly [6], however, these transformations will induce an additional $F\tilde{F}$ term in the effective QCD Lagrangian. Hence, including the effect of the electroweak interactions (particularly the Yukawa couplings (II.74)), the CP violating term in the strong sector is seen to be:

$$L_{CPviolating} = \frac{g^2}{32\pi^2}(\theta + \text{Arg det } M) F^{\mu\nu}_a \tilde{F}_{a\mu\nu} \qquad (II.82)$$

where the Arg det M contribution is the extra piece originating from diagonalizing the quark mass matrix M, originating from (II.74). The strong CP

puzzle therefore is why is the combination of QCD and electroweak parameters

$$\bar{\theta} = \theta + \text{Arg det } M \qquad (II.83)$$

so small ($\bar{\theta} \lesssim 10^{-9}$ from the bound on the neutron electric dipole moment)

The electroweak theory, however, suggest a possible solution to the strong CP puzzle. Quark masses are no longer static numbers but they arise because of the spontaneous breakdown of SU(2)xU(1), so that $m_q \sim \langle\phi\rangle$. Thus it is not impossible to conceive that even though quarks have mass, the <u>total</u> Lagrangian of the theory may indeed have a chiral symmetry [41]. We saw in Part I that if one really had a chiral symmetry - for example if some quark had zero mass - then the θ angle plays no physical role, since by a chiral transformation it can be rotated to zero. One can show that this also happens even when this chiral symmetry eventually is spontaneously broken. Specifically, H.R. Quinn and I [41] showed that, if the electroweak theory was invariant under an extra chiral U(1) symmetry $\left(U(1)_{PQ}\right)$, then the phase associated with the quark mass matrix was related to the θ parameter in such a way that $\bar{\theta}$ vanished identically.

This possibility to get rid <u>automatically</u> of the strong CP puzzle appears to be very nice. Let us examine, therefore, what needs to be done to implement this extra $U(1)_{PQ}$ in the electroweak theory. Since gauge interactions are invariant under chiral transformations, the only place that needs attention are the Yukawa couplings of the Higgs field ϕ to fermions. For simplicity, consider Eq (II.74) for the case of one family only. One has

$$L_{Yukawa} = -\Gamma^u (\overline{ud})_L \phi u_R + \Gamma^d (\overline{ud})_L \tilde{\phi} d_R + \text{h.c.} \qquad (II.84)$$

Clearly the first term above is invariant under a $U(1)_{R-L}$ transformation

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \to e^{-\frac{i\alpha}{2}} \begin{pmatrix} u \\ d \end{pmatrix}_L \; ; \qquad \begin{matrix} u_R \to e^{i\alpha/2}\, u_R \\ d_R \to e^{i\alpha/2}\, d_R \end{matrix} \tag{II.85}$$

provided the Higgs field ϕ also rotates under this transformation

$$\phi \to e^{-i\alpha}\, \phi \tag{II.86}$$

However, for the second term in (II.84) to be invariant it is necessary that $\tilde{\phi}$ also rotate as

$$\tilde{\phi} \to e^{-i\alpha}\, \tilde{\phi} \tag{II.87}$$

This last transformation with $\tilde{\phi} = i\tau_2\, \phi^*$, contradicts the assumed behaviour of ϕ. Hence the usual Yukawa Lagrangian, with just one Higgs field ϕ is not invariant under a global $U(1)_{R-L}$ transformation. One can build in this invariance very easily, however, by introducing a second Higgs doublet and requiring that this doublet take the role of $\tilde{\phi}$. That is, the Yukawa interaction

$$L_{Yukawa} = - \Gamma_u\, (\bar{u}\bar{d})_L\, \Phi_1\, u_R + \Gamma_d\, (\bar{u}\bar{d})_L\, \Phi_2\, d_R + h.c. \tag{II.88}$$

with

$$\Phi_1 \sim (2,\, -1/2)$$

$$\Phi_2 \sim (2,\, 1/2) \tag{II.89}$$

has an additional $U(1)_{PQ}$ invariance, in which

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \to e^{-i\alpha/2} \begin{pmatrix} u \\ d \end{pmatrix}_L \; ; \qquad \begin{matrix} u_R \to e^{i\alpha/2}\, u_R \\ d_R \to e^{i\alpha/2}\, d_R \end{matrix}$$

$$\Phi_1 \to e^{-i\alpha}\, \Phi_1 \; ; \quad \Phi_2 \to e^{-i\alpha}\, \Phi_2 \tag{II.90}$$

To solve the strong CP problem, therefore, it appears to be sufficient to introduce two distinct doublets in the electroweak theory. Of course, since Φ_1 and Φ_2 now have another phase invariance, besides that of $U(1)_Y$, it is also necessary to make sure that the potential $V(\Phi_1\, ,\, \Phi_2)$ respects (II.90). There is, however, a price to pay. Because we want SU(2)xU(1) to break down, the Higgs field Φ_1 and Φ_2 must acquire a vacuum expectation value. These vacuum expectation values not only break SU(2)xU(1) but they break also the extra global $U(1)_{PQ}$ symmetry. This breakdown produces a Goldstone boson, the axion [61]. Hence if this is the way in which the strong CP puzzle is solved, one must find some evidence for this excitation. I take up this topic in the third, and last, part of my lectures.

Part III: Old and New Axion Models

IIIa) Axions and their Properties

We have seen that with two Higgs doublets Φ_1 and Φ_2, it is easy to impose an extra $U(1)_{PQ}$ global symmetry in the Yukawa interactions. If this symmetry is really an invariance of the whole Lagrangian, then the strong CP problem is automatically solved. However, since $\langle\Phi_i\rangle \neq 0$ (i=1,2) there appears a Goldstone boson in the electroweak theory - the axion [61]. Actually, when one considers the effect of the strong interactions, since the $U(1)_{PQ}$ symmetry also has a strong anomaly, it is no longer clear whether the axion should be really massless or not. After all, we argued in Part I that the η' is not a Goldstone boson, even though the Lagrangian symmetry of QCD, in the $m_u = m_d = m_s = 0$ limit, contains a $U(1)_{R-L}$ symmetry, which is spontaneously broken. It turns out that the axion is actually massless in the chiral limit, in which all quark masses vanish. Turning on the quark masses (i.e. really turning on the Yukawa couplings) gives the axion a small mass of order

$$m_a \simeq m_\pi f_\pi / v \qquad \text{(III.1)}$$

Before deriving these results in detail, let me explain them qualitatively. It is rather easy to understand why in the chiral limit there is a massless excitation coupled to the $U(1)_{PQ}$ current. Even though the $U(1)_{PQ}$ current and the current J_5^μ, associated with the $U(1)_{R-L}$ transformation, both have an ABJ anomaly [6] there is a linear combination of these currents which is anomaly free. Because of the spontaneous breakdown of the global U(1) symmetries, there must exist a massless excitation coupled to this anomaly free combination of currents. This is the axion. When the Yukawa couplings are included this current obtains a non vanishing divergence, which then induces a small mass for the axion. Since quark masses are proportional to the Yukawa

couplings, one understands the structural appearance of (III.1). It just reflects the fact that $m_a \to 0$ as the Yukawa couplings are neglected.

Because $V(\Phi_1, \Phi_2)$ is $SU(2)xU(1)xU(1)_{PQ}$ symmetric, when spontaneous breakdown occurs due to the vacuum expectation values

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix} ; \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} \qquad \text{(III.2)}$$

there will appear <u>two</u> zero mass field combinations in the zero charge sector. If one writes for the complex zero charge fields the decomposition

$$\phi_i^0 = \frac{1}{\sqrt{2}}(v_i + \rho_i + i\sigma_i) \qquad \text{(III.3)}$$

then it is easy to see that it is the two fields σ_i which are massless. One linear combination of these fields is eaten by the Z^0, via the usual Higgs mechanism, while the orthogonal combination is the axion. Since Φ_1 and Φ_2 have opposite hypercharge, the field which gets absorbed by the Z^0 is clearly

$$\xi_Z = \frac{1}{\sqrt{v_1^2+v_2^2}}(v_1\sigma_1 - v_2\sigma_2) \qquad \text{(III.4)}$$

so that the axion is

$$a = \frac{1}{\sqrt{v_1^2+v_2^2}}(v_2\sigma_1 + v_1\sigma_2) \qquad \text{(III.5)}$$

It is easy to check that, with two Higgs doublets, the parameter which plays the role of the Fermi scale (c.f. Eq II.35) is

$$v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_F)^{-1/2} \simeq 250 \text{ GeV} \qquad \text{(III.6)}$$

Thus the only free parameter in the model is the ratio of vacuum expectation values

$$x = v_2/v_1 \qquad \text{(III.7)}$$

The chiral current which contains the axion a, but not ξ_z, is linear combination of the current generated by the $U(1)_{PQ}$ transformations

$$\phi_i \to e^{-i\alpha}\phi_i \qquad \begin{pmatrix} u \\ d \end{pmatrix} \to e^{+i\gamma_5 \frac{\alpha}{2}} \begin{pmatrix} u \\ d \end{pmatrix} \qquad \text{(III.8)}$$

and that generated by the hypercharge transformations. Namely

$$J^\mu_a = \left(\frac{v_1^2 + v_2^2}{2v_1 v_2}\right) J^\mu_{PQ} + \left(\frac{v_2^2 - v_1^2}{v_1 v_2}\right) J^\mu_Y \qquad \text{(III.9)}$$

Retaining only the axial pieces of the above (the vector pieces are conserved and irrelevant for the considerations that follow) one easily obtains

$$J^\mu_a = - v\partial^\mu a + x \sum_i \bar{u}_i \frac{\gamma_\mu \gamma_5}{2} u_i + \frac{1}{x} \sum_i \bar{d}_i \frac{\gamma_\mu \gamma_5}{2} d_i + z \sum_i \bar{l}_i \frac{\gamma_\mu \gamma_5}{2} l_i \qquad \text{(III.10)}$$

The parameter z appearing in (III.10) depends on whether in the Yukawa couplings we couple the leptons to ϕ_2 or to $\tilde{\phi}_1$. In the former case the PQ assignment for the leptons is like that for d quarks and $z = \frac{1}{x}$, while for the latter case the assignment is like that for u quarks, except that $L \leftrightarrow R$, so that $z = -x$. It is also easy to check that the parameters $\left\{ x, \frac{1}{x}, z \right\}$ characterize the coupling of the axion to charge 2/3 quarks, charge -1/3 quarks, and leptons, respectively. Using the Yukawa interaction (II.74) with $\phi \to \phi_1$, $\tilde{\phi} \to \phi_2$ for charge -1/3 quarks and $\phi \to \phi_2$ or $\tilde{\phi}_1$ for leptons, one easily derives that

$$L_{eff} = \frac{m_f}{v} \bar{f}\, i\, \gamma_5\, f\, a \cdot \left\{ \begin{matrix} x \\ \frac{1}{x} \\ z \end{matrix} \right\} \qquad \text{(III.11)}$$

In the absence of QCD, the axion current J^μ_a is conserved. In the presence of the strong interactions, however, this current has an ABJ anomaly[6]. From the structure of (III.10) it is easy to see that

$$\partial_\mu J^\mu_a = \frac{1}{2} (x + \frac{1}{x}) (2N_f) \frac{g^2}{32\pi^2} F^{\mu\nu}_a \tilde{F}_{a\mu\nu} \qquad \text{(III.12)}$$

where N_f is the number of quark flavors. To be able to compute the axion mass, we need to construct a current, related to (III.10), which contains the axion but has no anomaly. There are an infinity of such currents but, as Bardeen and Tye [62] have argued, the current we want is one which has a soft divergence (i.e. a divergence which vanishes in the chiral limit). For this current it will be easy to apply Dashen's theorem [11] and thereby obtain a formula for the axion mass.

Since $m_s \gg m_d$, m_u, let us forget altogether for the moment of the strange quarks. It is easy to see then that the current [62]

$$\tilde{J}^\mu_a = J^\mu_a - (x + \frac{1}{x}) (\frac{N_f}{2}) \left\{ \frac{m_d}{m_u+m_d} \bar{u} \frac{\gamma^\mu \gamma_5}{2} u + \frac{m_u}{m_u+m_d} \bar{d} \frac{\gamma^\mu \gamma_5}{2} d \right\} \qquad \text{(III.13)}$$

has the properties enunciated above. First of all, the divergence of the second term in (III.13) has an anomaly which precisely cancels the anomaly in J^μ_a. Secondly, it is straightforward to compute $\partial_\mu \tilde{J}^\mu_a$ and verify that it indeed vanishes as $m_u \to 0$ and/or $m_d \to 0$. Specifically, one finds

$$\partial_\mu \tilde{J}^\mu_a = - (x + \frac{1}{x}) (\frac{N_f}{2}) \frac{m_u m_d}{(m_u+m_d)} (\bar{u}\, i\, \gamma_5\, u + \bar{d}\, i\, \gamma_5\, d) \qquad \text{(III.14)}$$

Using (III.14) the mass of the axion is readily computed. What we need is a variant expression of Dashen's theorem, Eq (I.25).

Let us recall the expression for the pion mass from Dashen's theorem. For the π^o, specifically, one has

$$M^2_{\pi^o} = \frac{1}{f^2_\pi} \langle 0|[Q_{53} , [Q_{53} , L_{mass}]]|0\rangle \qquad \text{(III.15)}$$

This expression can be rewritten in a more succint way, since L_{mass} is the perturbation that breaks the chiral symmetry. It follows therefore that

$$[Q_{53} , L_{mass}] = - i \, \partial_\mu J^\mu_{53} \qquad \text{(III.16)}$$

Hence, an alternative expression for Dashen's theorem is

$$M^2_{\pi^o} = -i \frac{1}{f^2_\pi} \langle 0|[Q_{53} , \partial_\mu J^\mu_{53}]|0\rangle \qquad \text{(III.17)}$$

If the quark masses m_u and m_d are non vanishing it is easy to see that

$$\partial_\mu J^\mu_{53} = m_u \bar{u} \, i \, \gamma_5 \, u - m_d \bar{d} \, i \, \gamma_5 \, d \qquad \text{(III.18)}$$

so that evaluating the commutator in (III.18) yields again the familiar formula for the π^o mass

$$M^2_{\pi^o} = - \frac{(m_u+m_d)}{f^2_\pi} \langle 0|\bar{u}u|0\rangle \qquad \text{(III.19)}$$

We can apply precisely the same formula to calculate the axion mass with the

following replacements: $Q_{53} \leftrightarrow \tilde{Q}_a$; $\partial_\mu J^\mu_{53} \leftrightarrow \partial_\mu \tilde{J}^\mu_a$ and $f_\pi \leftrightarrow v$. Thus

$$M^2_a = -i \frac{1}{v^2} \langle 0|[\tilde{Q}_a , \partial_\mu \tilde{J}^\mu_a]|0\rangle \qquad \text{(III.20)}$$

Using (III.14) for the divergence of the axion current and evaluating the commutator yields, finally

$$M^2_a = \frac{-\left(\frac{N_f}{2} \right)^2 (x + \frac{1}{x})^2}{v^2} \frac{m_u m_d}{(m_u+m_d)} \langle 0|\bar{u}u|0\rangle \qquad \text{(III.21)}$$

This expression can be simplified by using Eq (III.19) for the π^o mass, giving

$$M_a = \frac{M_\pi f_\pi}{v} \frac{N_f}{2} (x + \frac{1}{x}) \left(\frac{m_u m_d}{(m_u+m_d)^2} \right)^{1/2} \qquad \text{(III.22)}$$

Numerically, using the values for the ratio of m_u/m_d obtained in Part I, this gives for the axion a mass:

$$M_a \simeq 12.5 \, N_f \, (x + \frac{1}{x}) \text{ KeV} \qquad \text{(III.23)}$$

For six flavors the above implies that M_a must be greater than 150 KeV. If x or $\frac{1}{x}$ is not large, the axion mass will be below the e^+e^- threshold ($M_a < 2 \, m_e$). In this circumstance the only possible decay channel for the axion will be into two photons and the axion will almost be stable. Roughly speaking, one expects that:

$$\tau(a \to \gamma\gamma) \sim \tau(\pi^o \to \gamma\gamma) \left(\frac{v}{f_\pi}\right)^2 \left(\frac{M_\pi}{M_a}\right)^3 \qquad \text{(III.24)}$$

which gives indeed a very long lifetime for the axion. A more precise calculation using the electromagnetic anomaly, has been done by Bardeen and Tye [62], with the result

$$\tau(a \to \gamma\gamma) \approx \left(\frac{100\ \mathrm{KeV}}{M_a}\right)^5 \mathrm{sec} \tag{III.25}$$

If x or 1/x is sufficiently large so that $m_a > 2\ m_e$, then the axion can have a relatively fast decay into e^+e^- pairs. A simple computation gives for the lifetime

$$\tau(a \to e^+e^-) = \frac{8\pi\ v^2\ z^{-2}}{m_e^2\ (M_a^2 - 4m_e^2)^{1/2}} \tag{III.26}$$

For instance, if one had a 1.7 MeV axion - a possibility which we shall discuss very soon - (III.26) gives

$$\tau(a \to e^+e^-) \approx 3\mathrm{x}10^{-9}\ z^{-2}\ \mathrm{sec} \qquad (M_a \approx 1.7\ \mathrm{MeV}) \tag{III.27}$$

The precise value for this lifetime depends on what z really is. Recall that z could be either $\frac{1}{x}$ or -x. Further, from (III.23) M_a gets large either by having x or $\frac{1}{x}$ big. So, depending on the model, the decay $a \to e^+e^-$ is either very fast (z>>1) or is rather comparable to the $\gamma\gamma$ rate ($\tau(a\to\gamma\gamma) \approx 10^{-6}$ sec).

Because the axion is so weakly coupled (c.f. Eq III.11) and, if $M_a < 2\ m_e$, is so long lived, it is very difficult to rule out experimentally. Nevertheless, as I will discuss in what follows, <u>the standard axion</u>, described above, <u>is ruled out by experiment</u>. This means that the strong CP puzzle remains an open

problem.

III.b) The Death of the Standard Axion

There are basically four different classes of experiments that have a bearing on light standard axions ($M_a < 2\ m_e$). I want to briefly review these results, since I will later try to construct variant axion models which can escape these constraints.

1) Quarkonia decays

A heavy quark-antiquark system, like the ψ or the Υ, can decay via axion emission. In particular the ratio of $Q\bar{Q} \to a\gamma$ to $\mu^+\mu^-$ decay is easy to estimate [63], since the details of the quarkonia wavefunction cancel out, as shown pictorially below:

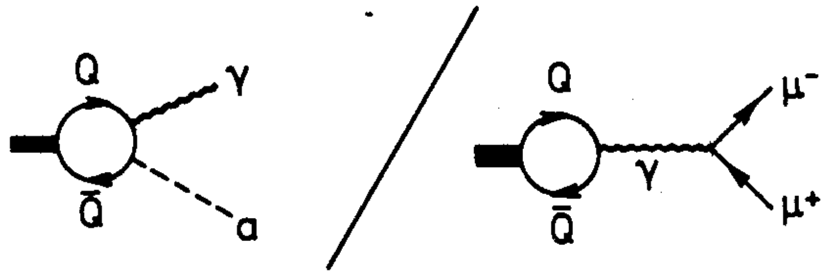


Fig III.1 Graphs needed to compute the ratio R

One finds for this ratio the expression [63]

$$R = \frac{\Gamma(Q\bar{Q} \to a\gamma)}{\Gamma(Q\bar{Q} \to \mu^+\mu^-)} = \frac{G_F\, m_Q^2}{\sqrt{2}\,\pi\,\alpha} \begin{Bmatrix} x^2 \\ x^{-2} \end{Bmatrix} \qquad \text{(III.28)}$$

where the top line applies for ψ decay and the bottom line for Υ decay, since the axion couples with x to charmed quarks but with 1/x to bottom quarks. (c.f. Eq III.11). Using the values $m_c \simeq 1.4$ GeV and $m_b = 4.9$ GeV, which are the values of the quark masses at the momentum scale of the respective quarkonia, along with the measured values for the μ pair branching ratios, leads to the predictions

$$B(\psi \to \gamma a) = (4.9 \pm 0.8)\times 10^{-5}\, x^2$$

$$B(\Upsilon \to \gamma a) = (2.7 \pm 0.7)\times 10^{-4}\, x^{-2} \qquad \text{(III.29)}$$

Experimentally axions have been searched for in both these processes but have not been found, leading to the bounds [64]

$$B(\psi \to \gamma a) < 1.4\times 10^{-5}$$

$$B(\Upsilon \to \gamma a) < 3\times 10^{-4} \qquad \text{(III.30)}$$

One sees that a combination of these bounds excludes standard axions for any value of x. However, this result is marginal and one has to worry about the accuracy of the predictions (III.28). For instance, it is known that R is subject to a rather large QCD correction [65]. At any rate, it is clear from these results that values of x or $\frac{1}{x}$ much larger than unity cannot be tolerated. This point will be important below.

ii) $K^+ \to \pi^+ a$

This process is quite difficult to estimate theoretically since the K-π transition involves a non leptonic weak interaction. There is, however, a one loop dsa vertex, (see Fig III.2) which one can calculate rather readily, and which should provide a good guesstimate for the rate [66]. A direct calculation

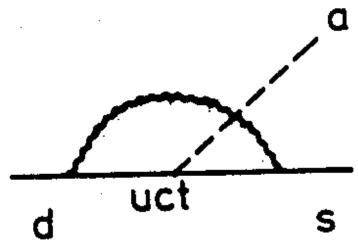


Fig III.2 One loop dsa vertex

yields the branching ratio

$$B(K^+ \to \pi^+ a) \simeq 10^{-6}\, x^2\, A(m_c, m_t) \qquad \text{(III.31)}$$

where the function $A(m_c, m_t)$ is explicitly given in Ref [66] and is of order 1 to 10. There is a very strong bound on this branching ratio, coming from KEK [67].

$$B(K^+ \to \pi^+ a) < 3.8\times 10^{-8} \qquad \text{(III.32)}$$

which, in view of (III.31), implies that x must be small ($x \lesssim 10^{-1}$). However, if x is this small then the $\Upsilon \to a\gamma$ bound can be used to rule out the standard axion.

iii) Beam dump experiments

There have been a variety of beam dump experiments done to search for axions [68]. These experiments also did not see a signal and they put a bound on the product of the cross section for producing the axion at the dump, times the cross section for detecting the axion away from the dump. Typically the bounds obtained on $\sigma_{prod} \cdot \sigma_{det}$ are about a factor of 100 below the expectation from the standard axion model (with $x \approx 1$) [68]. Thus also these experiments serve to rule the axion out. However, the calculations of σ_{prod} and σ_{det} have considerable uncertainty and these bounds on axions are rather soft. It should be pointed out, nevertheless, that if x or x^{-1} is large, then the theoretical predictions for $\sigma_{prod} \cdot \sigma_{det}$ are increased by a factor of x^4, or x^{-4}. Hence, in these circumstances, the beam dump bounds are very strong indeed. The only way these bounds could be avoided is if the axion decayed so rapidly that after been produced at the dump it never got to the target region. I shall make use of this caveat soon!

IV Nuclear deexcitation experiments

Light axions, if they existed, could cause an excited nuclear level to decay part of the time by axion emission. Thus a variety of nuclear deexcitation experiments have been performed to look for axions [68]. Typically these experiments look for the two photon decay mode of the emitted axions. However, since the lifetime for $a \to \gamma\gamma$ is so long, it is necessary to have a very large axion rate to hope for a measurable signal. Hence these experiments use reactors or highly radioactive sources, which have very large rates of nuclear deexcitation. The results of these searches [68] have also been negative, with the bounds obtained for the 2γ rates being several orders of magnitude below the predictions of the standard axion model. One may worry that the computation of the nuclear axion transition may be unreliable. However, the axion being a

0^- state acts essentially as a "magnetic" photon and one can estimate rather reliably (say within a factor of two or so) the axion to photon deexcitation rate [69]. Thus these nuclear bounds on the standard axion should be taken seriously.

Of course, if $M_a > 2\,m_e$, many of the nuclear bounds which concentrated on the slow 2γ signal would be ineffective. There was, however, an experiment done by Calaprice et al [70], which specifically looked for nuclear axion decays, followed by $a \to e^+e^-$. The transition looked for were the 15.1 MeV $1^+1 \to 0^+0$ and the 12.7 MeV $1^+0 \to 0^+0$ decays in ^{12}C. This experiment rules out standard axions of $M_a > 2\,m_e$, provided the lifetime of the axion was longer than about 5×10^{-11} sec [70].

III c) GSI positrons and a 1.7 MeV axion?

The above axion necrology should be sufficient to convince even the most fervent admirer that standard axions do not exist! So the strong CP puzzle remains a puzzle, although perhaps it could still be solved via a $U(1)_{PQ}$ symmetry which breaks down at a scale much above the Fermi scale [71]. In this case all couplings of the axions are much reduced and the axion is rendered essentially invisible. It would take me too far away from the scope of these lectures to discuss the intriguing astrophysical and cosmological properties of invisible axions [72]. Rather I prefer to discuss some recent experimental observations at GSI which have revived (albeit only temporarily!) the hope that perhaps some "visible" axions exist.

What is being studied at GSI are collisions between two heavy ions; in particular the production of positrons in these collisions. If $Z_1 + Z_2$ is sufficiently large, then when the two ions get close together one has a quasi Coulombic system where the ground state has an energy below $-2\,m_e$. In these circumstances, one expects that it will be energetically favorable to emit spontaneously positrons during the collision [73]. This spontaneous positron emission has indeed been observed at GSI. However, the positron spectrum seen exhibits a narrow peak at $T_{e^+} \sim 350$ KeV, above a continuum spectrum whose shape and magnitude is in agreement with theoretical calculations [75]. Furthermore, this sharp peak appears at roughly the same energy for a variety of values of $Z_1 + Z_2 > 178$. This last property argues against the possibility that the origin of the positron line is due to the formation of a quasimolecular state [76], where the two ions stick temporarily together, since then one would expect the peak energy T_{e^+} to vary very rapidly with $Z_1 + Z_2$. Very recently, to make the matter more intriguing, the EPOS collaboration at GSI found a correlated signal between positrons with energy around the peak energy and electrons being produced with also $T_{e^-} \sim 350$ KeV [77].

If one takes these observations at face value, there is a "natural" particle physics interpretation of the GSI data. What is being produced at GSI is a particle with mass around 1.7 MeV, which then decays into e^+e^- pairs. Provided this particle is produced nearly at rest, both the electron and the positrons from its decay will carry a kinetic energy of roughly 350 KeV. Presto: the positron peak and the e^+-e^- correlation! However, the real question is whether this "explanation" is tenable. Apart from the very difficult issue of trying to produce a 1.7 MeV object essentially at rest in the heavy ion collision, one must ask whether it is reasonable to suppose that there exists an elementary particle (with $M \sim 3\, m_e$!) which has a theoretical basis for being so light and which could have escaped experimental detection up to now? The only elementary excitation I know which has some theoretical reason to have a mass in this range is the axion (c.f. Eq III.23). However, as discussed in detail in the last section, the standard axion is dead! Nevertheless, motivated by the GSI observations, T.T. Wu, T. Yanagida and I [78] and, independently, L. Krauss and F. Wilczek [79], invented a variant of the standard

axion model [41] which avoided the bounds incurred by the standard axion. This new axion could then perhaps be adduced to explain the GSI phenomena, although it should be said that what physics allows its production nearly at rest in the heavy ion collision was not terribly clear to us, when we proposed this variant axion.

What are the ingredients of these new axion models? If one takes the axion mass to be that inferred from GSI, $M_a \simeq 1.7$ MeV, and uses the standard axion mass formula (III.23), for six flavors, one deduces that x or x^{-1} is of the order of 20. Such a large value is totally ruled out by either the $\psi \to a\gamma$ or the $\Upsilon \to a\gamma$ bound. Clearly, to be sensible, the new axion model must avoid this quarkonium stranglehold. Furthermore, with x or x^{-1} being so big, the beam dump bounds discussed in the last section, are now very strong bounds, <u>unless</u> the decay $a \to e^+e^-$ is very fast. This means that it is necessary that the parameter z for the new axion model be large, so that the lifetime of the axion given in Eq (III.27) can be short enough. Remarkably, it is rather simple to construct a simple variation of the original model of Ref [41] which allows one to avoid the above experimental hurdles [78][79].

Specifically, variant axion models have the following general structure [78][79]:

i) To avoid the quarkonium stranglehold one must couple b_R and c_R to the same Higgs field, so that the Yukawa interactions for these quarks read

$$L^{b,c}_{Yukawa} = -\Gamma^b_i \bar{Q}_{Li}\, \Phi_2\, b_R - \Gamma^c_i \bar{Q}_{Li}\, \tilde{\Phi}_2\, c_R + h.c. \qquad (III.33)$$

With x large, these couplings then imply that both $\Upsilon \to a\gamma$ and $\psi \to a\gamma$ are now suppressed by x^{-2} .

ii) To avoid Higgs induced flavor changing neutral currents in the charge -1/3 sector, which are dangerous since the K-$\bar{K}$ system is very well constrained, one must couple also d_R and s_R to the Φ_2 Higgs field. In this case, as we showed in Part II, diagonalization of the quark mass matrix will also diagonalize the Higgs couplings.

iii) To implement a PQ symmetry, at least one of the charge 2/3 quark fields must be coupled to the Φ_1 Higgs field. In the most simple model one only couples u_R to Φ_1 [78][79]. Because the charmed quark is coupled to a different Higgs field (Φ_2), it is now not possible to automatically avoid Higgs induced charm changing neutral currents. How big these effects are, however, is a model dependent question. In fact, Krauss and Wilczek [79], by complicating the model sufficiently, were able to completely suppress this effect.

iv) To get a sufficiently rapid decay of an axion into e^+e^- pairs, it is necessary to couple e_R to Φ_1, so that $z = -x$ and is large. However, one cannot couple then μ_R also to Φ_1, because then one would obtain too big a contribution to (g-2) due to axion exchange [78][79].

We see that these variant axion models are very similar to the standard axion model. What has happened is that only certain pairs of quarks have a PQ symmetry: those for which the charge -1/3 and charge +2/3 right-handed fields couple to different Higgs fields. The simplest such variant model is the one in which only the u and d quarks have a PQ symmetry [78][79]. For this model we can use all the formulas we derived earlier for the standard axion, with now the "effective" number of flavors N_f being 2. This means, in particular, that the axion mass is now given by the formula

$$M_a \simeq 12.5\, N_f \left(x+\frac{1}{x}\right) \text{ KeV} = 25\left(x+\frac{1}{x}\right) \text{ KeV} \qquad \text{(III.34)}$$

If we want $M_a \simeq 1.7$ MeV, we see that x must really be large

$$x \simeq 70 \qquad \text{(III.35)}$$

In view of point iii) above, $z \simeq -70$ and the variant axion lifetime is indeed very short:

$$\tau(a \to e^+e^-) \simeq 6\times10^{-13} \text{ sec} \qquad \text{(III.36)}$$

The above variant axion model clearly avoids the quarkonia bounds by construction. Furthermore, because the axion decay into e^+e^- pairs is so prompt, even though x is very large, the axion beam dump bounds are not relevant. (However, new beam dump experiments, looking near the dump, could be very effective to constrain these kind of axions). For the same reason, the experiment of Calaprice et al [70] on the ^{12}C decays also cannot rule these axions out. It would appear that the estimate given in Eq (III.31) for the decay $K^+ \to \pi^+ a$, on the other hand, would be fatal for variant axions, since the branching ratio is proportional to x^2 and x is very large. However, the large numerical value obtained in Eq (III.31) is due to the c-quark contribution in Fig III.2, which in the model we are examining is suppressed by x^{-2}. The u quark contribution in Fig III.2 is indeed enhanced by x^2 for variant axions, but it contributes an insignificant amount numerically, being suppressed by a factor of $(m_u/m_c)^4$. This does not mean necessarily that the process $K^+ \to \pi^+ a$ might not be dangerous for variant axions, since there are other contributions than those of Fig III.2 to the decay amplitude. These are, however, more difficult to estimate reliably (see below). So at first sight these variant axion models appear viable [78] and, in view of the GSI data, they hold considerable interest.

III d) A postscript on variant axions

The enthusiasm for variant axions expressed in the last sentence correctly mirrors the state of affairs at the time these lectures were given. Indeed this is where, more or less, I ended my lectures. Alas, since then a number of developments have taken place which effectively rule out the simple model discuss above and any variations thereof. Even though I obviously did not lecture on these matters. I think it worthwhile to make a few brief comments here on these new developments. Two recent experiments have specifically looked for an axion of mass near 1.7 MeV which has a fast decay into e^+e^- pairs:

i) A Caltech group studied the nuclear deexcitation of the 9.17 MeV 2^+1 state of ^{14}N to its 1^+0 ground state [80]. This state can decay by direct pair conversion. However the direct e^+e^- pairs are peaked at zero relative angle, while the pairs originating from axion decay would peak at relative angles around $15\text{-}20^{\circ}$. No excess large angle pairs were found, allowing Savage et al [80] to set the 90% confidence limit bound on the rate of axion to photon decay for this state given below:

$$\frac{\Gamma_a}{\Gamma_\gamma} < 4\times10^{-4} \qquad \text{(III.37)}$$

ii) At SIN, the recently obtained data on the allowed, but rare, π decay process: $\pi^+ \to e^+e^-e^+\nu_e$ [81], was examined for possible traces of an e^+e^- enhancement at $M_{e^+e^-} = M_a \simeq 1.7$ MeV. No such enhancement was found, allowing the SINDRUM group to set a very strong bound for the process $\pi^+ \to ae^+\nu_e$ of [82]

$$B(\pi^+ \to ae^+\nu_e) \lesssim (1\text{-}2)\times 10^{-10} \qquad \text{(III.38)}$$

(The exact bound depends a bit on the $a \to e^+e^-$ lifetime).

In addition, Calaprice et al [83] reanalyzed the old, but extensive, internal pair conversion experiments of Warburton et al [84], to look for possible traces of a 1.7 MeV axion which decays rapidly into e^+e^- pairs. For our purposes, the most interesting outcome of this reanalysis [83] concerns a decay involving an isoscalar transition in Boron. What Calaprice et al [83] found is that:

iii) for the decay of the 3.58 MeV $2^+,0$ state of ^{10}B to the 0.72 MeV $1^+,0$ state, at the 1σ level, the axion to photon rate is bounded by

$$\frac{\Gamma_a}{\Gamma_\gamma} < 7.5\times 10^{-5} \qquad \text{(III.39)}$$

Bardeen, Yanagida and I [85] have computed the prediction for variant axions for these cases. The first two decays depend on the isovector content of the axion, while the bound in iii) measures the isoscalar content of the axion (In general, the axion has <u>not</u> a definite isospin). The simplest variant axion model, discussed in the last section, turns out to have an axion which is purely isovector. So (iii) is irrelevant. However the bounds i) and particularly ii) rule this model out. One can make more fancy variant axion models, which can avoid the isovector bounds i) and ii). It turns out, however, that in this case the isoscalar bound iii) is strong enough, to rule these models out too. Hence, variant axions appear also to be excluded experimentally.

I do not want here to repeat the details of the analysis done in Ref [85], but I will give for completeness, the principal results we obtained. What one needs to know is the isoscalar and isovector content of the axion. This is easily gathered by focusing on the current $\tilde{J}^{\mu}_a$ and extracting from it the amount of isoscalar and isovector axial currents that it has. That is, one writes

$$\tilde{J}^{\mu}_a = - v\, \partial^{\mu} a + \lambda_s \left(\frac{1}{2} \bar{u} \gamma^{\mu} \gamma_5 u + \frac{1}{2} \bar{d} \gamma^{\mu} \gamma_5 d \right)$$

$$+ \lambda_3 \left(\frac{1}{2} \bar{u} \gamma^{\mu} \gamma_5 u - \frac{1}{2} \bar{d} \gamma^{\mu} \gamma_5 d \right) + \text{heavy quark pieces} \qquad \text{(III.40)}$$

For variant axion models one finds for λ_s and λ_3 [85]

$$\lambda_s = \frac{1}{2} \left[(\tilde{z} + \frac{1}{x}) - N\, (x + \frac{1}{x}) \right]$$

$$\lambda_3 = \frac{1}{2} \left[(\tilde{z} - \frac{1}{x}) - N\, (x + \frac{1}{x}) \frac{(m_d - m_u)}{(m_d + m_u)} \right] \qquad \text{(III.41)}$$

Here $\tilde{z} = x$ if u_R couples to Φ_1, but $\tilde{z} = -\frac{1}{x}$ if it couples to $\tilde{\Phi}_2$, while N is the number of quark doublets which have an effective PQ symmetry. For the simple model discussed in the last section, one has $\tilde{z} = x = 70$ and $N = 1$, so that

$$\lambda_s = 0$$

$$\lambda_3 \simeq \frac{3}{8} x \simeq 26 \qquad \text{(III.42)}$$

where we have used Eq (I.38) for the quark mass ratio. Note, however, that if $N = 4$ and $\tilde{z} = x \simeq 18$ (to fit $M_a \simeq 1.7$ MeV), then λ_3 essentially vanishes. In this case λ_s is then big and one has

$$\lambda_s \simeq - \frac{3}{2} x \simeq - 27 \qquad \text{(III.43)}$$

$$\lambda_3 \simeq 0$$

The numerical coincidence of Eqs (III.42) and (III.43) is not accidental but a general property of variant axions [85]. One finds:

$$(\lambda_s - \lambda_3)^2 \simeq \left(\frac{v}{f_\pi}\right)^2 \left(\frac{M_a}{M_\pi}\right)^2 \left(\frac{m_u}{m_d}\right) \simeq (25)^2 \qquad \text{(III.44)}$$

The rates for the experimental processes described above in i) -iii) depend specifically on the values of λ_3 and λ_s . We found [85]

i) ^{14}N : $\frac{\Gamma_a}{\Gamma_\gamma} = 2 \times 10^{-5}\, (\lambda_3)^2$ (III.45)

ii) $B(\pi^+ \to ae^+\nu_e) = 3 \times 10^{-9}\, (\lambda_3)^2$ (III.46)

iii) ^{10}B : $\frac{\Gamma_a}{\Gamma_\gamma} \simeq 2.7 \times 10^{-4}\, (\lambda_s)^2$ (III.47)

It is clear that the experimental bounds, particularly for the cases ii) and iii), put extremely strong constraints on both λ_3 and λ_s which are incompatible with the general constraint (III.44).

Acknowledgements:

I am very grateful to Profs H. Latal and H. Mitter for the splendid hospitality shown to me at Schladming. I would like to thank in addition W.A. Bardeen, J. Solà, T.T. Wu and T. Yanagida for their insights of some of the material presented in these lectures and for very enjoyable collaborations.

References:

1.) For a useful introduction, see for example C. Quigg, Gauge Theories of the Strong Weak and Electromagnetic Interactions (Benjamin, Reading, Mass, 1983)

2.) For a discussion, see for example, J.D. Bjorken in the Proceedings of the 1979 SLAC Summer Institute in Particle Physics, ed by A. Mosher, SLAC report SLAC-224 January 1980

3.) D.J. Gross and F. Wilczek, Phys Rev Lett 30 (1973) 1343;
H. D. Politzer, Phys Rev Lett 30 (1973) 1346

4.) W. Caswell, Phys. Rev. Lett 33 (1974) 244;
D.R.T. Jones, Nucl Phys B75 (1974) 531

5.) For a recent review, see F. Sciulli in the Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energy, ed by M. Konuma and K. Takahashi, ISLEPH485, Kyoto, Japan

6.) S.L. Adler, Phys Rev 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cimento 60A (1969) 49

7.) G't Hooft in Proceedings of the 1979 Cargese Summer School, ed by G. 't Hooft et al (Plenum Press, N.Y. 1980)

8.) S. Coleman and B. Grossman, Nucl Phys B203 (1982) 205

9.) S. Coleman and E. Witten, Phys Rev Lett 45 (1980) 100

10.) C. Vafa and E. Witten, Nucl Phys B234 (1984) 173

11.) R. Dashen, Phys Rev 183 (1969) 1245

12.) M. Gell Mann in "The Eightfold Way" ed by M. Gell Mann and Y. Ne'eman (W.A. Benjamin, New York 1964);
S. Okubo, Prog Theo Phys 27 (1962) 949

13.) M. Gell Mann, Physics 1 (1964) 63

14.) S. Weinberg, Phys Rev Lett 18 (1967) 507

15.) J. Gasser and H. Leutwyler, Phys Rept 87C (1982) 77

16.) K. Wilson, Phys Rev 179 (1969) 1499;
W. Zimmermann in Lectures on Elementary Particle Physics and Quantum Field theory (MIT Press, Cambridge, 1970)

17.) R. Crewther, Phys Rev Lett 28 (1972) 1421;
M.S. Chanowitz and J. Ellis, Phys Lett 40B (1972) 397;
S.L. Adler, J.C. Collins and A. Duncan, Phys Rev D15 (1977) 1712

18.) T. Das, G. Guralnik, V. Mathur, F. Low and J. Young, Phys Rev Lett 18 (1967) 759

19.) K. Karawabayashi and M. Suzuki, Phys Rev Lett 16 (1966) 225;
Riazuddin and Fayazuddin, Phys Rev 147 (1966) 1071

20.) Review of Particle Properties, Rev Mod Phys 56 (1984) 1

21.) W. Ruckstuhl et al, Phys Rev Lett 56 (1986) 2132;
H. Albrecht et al, DESY 86-060

22.) R.D. Peccei and J. Solá, paper in preparation

23.) W.N. Cottingham, Ann. of Phys 25 (1963) 424;
for a review, see also, A. Zee, Phys Rept 3C (1972) 129

24.) I. Barbour et al, Phys Lett 158B (1985) 61

25.) G 't Hooft, Nucl Phys B61 (1973) 455

26.) H. Hamber, Phys Rev D31 (1983) 586

27.) A.I. Vainshtein et al, Sov J Nucl Phys 27 (1978) 274;
M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl Phys B147 (1979) 385, ibid 448, ibid 519

28.) S. Narison and E. de Rafael, Phys Lett 103B (1981) 75;
M.A. Shifman et al Ref (27)

29.) G 't Hooft, Phys Rev lett 37 (1976) 8; Phys Rev D14 (1976) 3432

30.) W.A. Bardeen, Nucl Phys B75 (1974) 246

31.) For a discussion of this point see for example,
R.D. Peccei in Particle Physics 1980 ed by I. Andric, I. Dadic and M. Zovko (North Holland, Amsterdam 1981)

32.) C.G. Callan, R.F. Dashen and D.J. Gross, Phys Lett 63B (1976) 334

33.) See, for example, R. Crewther, Proceedings of the XVII Internationale Universitätswochen für Kernphysik, Schladming 1978

34.) A.A. Belavin, A.M. Polyakow, A.S. Schwartz and Yu. S. Tyupkin, Phys Lett 59B (1975) 85

35.) R. Jackiw and C. Rebbi, Phys Rev Lett 37 (1976) 172

36.) The formulas here given are only valid for θ small. In effect the phases ϕ_u and ϕ_d are periodic in θ. For more details see Ref (31)

37.) V. Baluni, Phys rev D19 (1976) 2227

38.) R. Dashen, Phys Rev D3 (1971) 1879

39.) R. Crewther, P. di Vecchia, G. Veneziano and E. Witten, Phys Lett 89B (1979) 123

40.) I.S. Altarev et al, Phys Lett 102B (1981) 13

41.) R.D. Peccei and H.R. Quinn, Phys Rev Lett 38 (1977) 1440;
Phys Rev D16 (1977) 1791

42.) S.L. Glashow, Nucl Phys 22 (1961) 579;
A. Salam in Elementary Particle Theory, ed by N. Svartholm (Almquist and Wiksell, Stockholm 1969); S. Weinberg, Phys Rev Lett 19 (1967) 1264

43.) P.W. Higgs, Phys Lett 12 (1964) 132;
R. Brout and F. Englert, Phys Rev Lett 13 (1964) 321

44.) See for example the monograph of C. Quigg, Ref (1)

45.) J. Kim et al, Rev Mod Phys 53 (1980) 211

46.) G. Arnison et al, Phys Lett 122B (1983) 103;
M. Banner et al, Phys Lett 122B (1983) 476

47.) G. Arnison et al, Phys Lett 129B (1983) 273
P. Bagnaia et al, Phys Lett 129B (1983) 130

48.) L. di Lella in the Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energy, ed by M. Konuma and K. Takahashi, ISLEPH 85, Kyoto, Japan

49.) A. Sirlin, Phys Rev D22 (1980) 971

50.) A. Sirlin, Phys Rev D29 (1984) 89

51.) W. Marciano and A. Sirlin, Phys Rev D22 (1980) 2695, ibid D29 (1984) 945;
M. Consoli, L. Maiani and S. Lo Presti, Nucl Phys B223 (1983) 474

52.) W. Marciano, Phys Rev Lett 20 (1979) 274

53.) W. Marciano and A. Sirlin, Nucl Phys B189 (1981) 442

54.) C.H. Llewellyn Smith and J. Wheater, Phys Lett 105B (1981) 486

55.) H. Abramowicz et al, Zeit für Phys C28 (1985) 51

56.) P.G. Reutens et al, Phys Lett 152B (1985) 404

57.) L. Susskind, Phys Rev D20 (1979) 2619;
S. Weinberg, Phys Rev D16 (1976) 974; ibid D19 (1979) 1277

58.) S. Dimopolous and L. Susskind, Nucl Phys B155 (1979) 237
E. Eichten and K.D. Lane, Phys Lett 90B (1980) 125

59.) E. Fahri and L. Susskind, Phys Rept 74C (1981) 277
R. Kaul, Rev Mod Phys 55 (1983) 449

60.) M. Kobayashi and T. Maskawa, Prog Theo Phys 49 (1973) 652

61.) S. Weinberg, Phys Rev Lett 40 (1978) 223;

62.) W.A. Bardeen and S. - H.H. Tye, Phys Lett 74B (1978) 229

63.) F. Wilczek, Phys Rev Lett 39 (1977) 1304

64.) For a review of these bounds, see S. Yamada in the Preceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energy, ed by D.G. Cassel and D.L. Kreinick (Cornell 1983)

65.) M. Visotoski, Phys Lett 97B (1980) 159

66.) M.B. Wise, Phys Lett 103B (1981) 121;
J.M. Frere, M.B. Gavela and J.A.M. Vermaseren, Phys Lett 103B (1981) 159

68.) For a review see A. Zehnder, Proceedings of the 1982 Gif-sur-Yvette

Summer School

69.) T.W. Donnelly et al, Phys Rev D18 (1978) 1607

70.) F.P. Calaprice et al, Phys Rev D20 (1979) 2708

71.) J.E. Kim, Phys Rev Lett 43 (1979) 103;
M. Dine, W. Fischler and M. Srednicki, Phys Lett 104B (1981) 199

72.) For recent reviews see, for example, P. Sikivie
Lectures at the 1982 Gif sur Yvette School or F. Wilczek,
Lectures at the 1983 Erice Summer School

73.) W. Pieper and W. Greiner, Z. Phys. 218 (1969) 237;
S.S. Gershtein and Y.B. Zeldovich, Lett Nuovo, Cim. I (1969) 835

74.) J. Schweppe et al, Phys Rev Lett 51 (1983) 2261;
M. Clemente et al, Phys Lett 137B (1984) 41;
T. Cowan et al, Phys Rev Lett 54 (1985) 761

75.) S. Reinhardt, B. Müller and W. Greiner
Phys Rev A24 (1981) 103

76.) S. Reinhardt et al Z. Phys A303 (1981) 173

77.) T. Cowan et al, Phys Rev Lett 56 (1986) 446

78.) R.D. Peccei, T.T. Wu and T. Yanagida, Phys Lett 172B (1986) 435

79.) L. Krauss and F. Wilczek, Phys Lett 173B (1986) 189

80.) M.J. Savage et al. Caltech Preprint 1986

81.) S. Egli et al, SIN Preprint PR-86-08

82.) R. Eichler et al, SIN Preprint PR-86-07

83.) F.P. Calaprice et al, Internal Princeton Preprint 1986

84.) E.K. Warburton et al, Phys Rev B133 (1964) 42

85.) W.A. Bardeen, R.D. Peccei and T. Yanagida, DESY Preprint, DESY 86-054