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ABSTRACT

We present a calculation of the process $e^+e^- \to \nu\bar{\nu}\gamma\gamma$ where both photons are hard. As a radiative correction to $e^+e^- \to \nu\bar{\nu}\gamma$, this process gives a large, positive contribution which cancels partially the large, negative corrections coming from the virtual one loop Q.E.D. and soft corrections.

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Introduction

The study of the final state $\nu\bar{\nu}\gamma$ in e^+e^- interactions has been proposed long time ago [1] as a direct and clean method for determining the number of light neutrino types. Although the cross section is small, it is measurable for c.m. energies above the Z° threshold and it is more sensitive to the number of light neutrinos than the decay width of the Z° boson. Indeed, the existence of an extra light neutrino will increase the total Z° width by $\sim 6\,\%$ while the cross section of the process $e^+e^- \to \nu\bar{\nu}\gamma$ will increase by $\sim 33\,\%$.

In this paper we want to discuss the process $e^+e^- \to \nu\bar{\nu}\gamma\gamma$ as the hard photon correction to the process $e^+e^- \to \nu\bar{\nu}\gamma$ in the spirit of complementing the existing calculations [2] of the one loop virtual Q.E.D. corrections (i.e. attachement of a single photon line) to the Z° channel in the tree diagram and the soft photon corrections (i.e. $e^+e^- \to \nu\bar{\nu}\gamma\gamma$ where one photon is soft). Moreover, among the non-Q.E.D. corrections, we would like to comment on the effect of the Z° self-energy which gives a non-negligible contribution at energies close to the Z° pole.

This paper consists of three sections. In section I we explain briefly the technique used to evaluate the transition amplitude. In section II we comment on the Phase Space used for the Monte Carlo evaluation of the differential and the total cross sections. Last, section III contains a discussion of the results and an estimation of the total radiative corrections for the process $e^+e^- \to \nu\bar{\nu}\gamma$ within a reasonable experimental set-up.

I - The Transition Amplitude

In order g^4, the process

$$e^-(p_1) + e^+(p_2) \to \nu(p_3) + \bar{\nu}(p_4) + \gamma(p_5) + \gamma(p_6) \qquad (1)$$

is mediated by a Z boson in the s-channel and by a W in the t-channel. For energies close to the Z° mass, the contribution of the W t-channel is very small and can be neglected. We are therefore left with the 6 Feynman diagrams given in figure 1. Clearly, they form a gauge invariant subset and we have grouped them in two topologically different classes (A and B in figure 1) such that by an overall permutation of particles, a diagram of one class can be transformed into any other one belonging to the same class but can not be obtained from any one of the other class. It is therefore sufficient to calculate the transition amplitude for one characteristic diagram of each group.

We have evaluated the transition amplitude for this process using the helicity amplitudes technique. This method provides a fast and simple way of evaluating the Feynman diagrams in a suitable way for numerical calculations and is described in great detail in the existing literature [3 – 5].

With the basic definitions given in reference [5], one can express the amplitude for the diagram A.1 as

$$i\,M_{A1} = \frac{i\,e^4}{\sin 2\vartheta_w} \cdot T_{A1} \cdot G(p_1,p_5,p_6) \cdot F(p_1,p_5) \cdot D(q^2) \tag{2}$$

where

$$\begin{aligned} T_{A1}(p_i,\lambda_i) = &[\bar{v}(p_2,\lambda_2)\,\gamma^\mu\,(C_L\,P_L + C_R\,P_R)\,(b_1(\not p_1 + m) + b_5\,\not p_5 + b_6\,\not p_6)\;\not\varepsilon^*(p_6,\lambda_6)\cdot \\ &(b_1(\not p_1 + m) + b_5\,\not p_5)\;\not\varepsilon^*(p_5,\lambda_5)\,u(p_1,\lambda_1)] \cdot [\bar{u}(p_3,\lambda_3)\,\gamma_\mu\,P_L\,v(p_4,\lambda_4)] \end{aligned} \tag{3}$$

$$G(p_i,p_j,p_k) = \frac{1}{2\,(b_i b_j(p_i\cdot p_j) + b_i b_k(p_i\cdot p_k) + b_j b_k(p_j\cdot p_k))}$$

$$F(p_i,p_j) = \frac{1}{2\,b_i b_j(p_i\cdot p_j)}$$

$$D(q^2) = \frac{1}{q^2 - M_{Z^\circ}^2 + i\,M_{Z^\circ}\,\Gamma_{Z^\circ}} \qquad \text{with} \qquad q^2 = 2\,(p_3\cdot p_4) \tag{4}$$

and b_i a dichotomic variable [3] equal to +1 (-1) if the momentum p_i corresponds to an ingoing (outgoing) particle. In a similar way, one can express the amplitude for the diagram B.1 as

$$i\,M_{B1} = \frac{-i\,e^4}{\sin 2\vartheta_w} \cdot T_{B1} \cdot F(p_1,p_5) \cdot F(p_2,p_6) \cdot D(q^2) \tag{5}$$

where

$$\begin{aligned} T_{B1}(p_i,\lambda_i) = &[\bar{v}(p_2,\lambda_2)\;\not\varepsilon^*(p_6,\lambda_6)\,(b_2(\not p_2 + m) + b_6\,\not p_6)\,\gamma^\mu\,(C_L\,P_L + C_R\,P_R)\cdot \\ &(b_1(\not p_1 + m) + b_5\,\not p_5)\;\not\varepsilon^*(p_5,\lambda_5)\,u(p_1,\lambda_1)] \cdot [\bar{u}(p_3,\lambda_3)\,\gamma_\mu\,P_L\,v(p_4,\lambda_4)] \end{aligned} \tag{6}$$

If we introduce the $Z(p_i,\lambda_i)$ functions defined as the product of two bilineals with the following notation

$$\begin{aligned} &Z(p_i,\lambda_i;p_j,\lambda_j;p_k,\lambda_k;p_l,\lambda_l;C_L,C_R,C'_L,C'_R) \equiv \\ &\qquad [\bar{u}(p_i,\lambda_i)\,\Gamma^\mu\,u(p_j,\lambda_j)][\bar{u}(p_k,\lambda_k)\,\Gamma'_\mu\,u(p_l,\lambda_l)] \end{aligned} \tag{7}$$

where C_L (C_R) and C'_L (C'_R) are the left (right) components of the Γ^μ and Γ'_μ couplings, and the polarization vector of the photon as

$$\varepsilon^\mu(k,\lambda) = N\,\bar{u}(k,\lambda)\,\gamma^\mu\,u(p,\lambda) \tag{8}$$

with p any four-momentum occurring in the process and N the normalization factor, we can write T_{A1} and T_{B1} as

$$\begin{aligned} T_{A1} = \;& Z(p_2,\lambda_2;p_1,\lambda';p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_1\cdot \\ &\cdot(Z(p_1,\lambda';p_5,\lambda;p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_5\cdot Z_1 + \\ &+ Z(p_1,\lambda';p_1,\lambda;p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_1\cdot Z_2) + \\ &+ Z(p_2,\lambda_2;p_5,\lambda';p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_5\cdot \\ &\cdot(Z(p_5,\lambda';p_5,\lambda;p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_5\cdot Z_1 + \\ &+ Z(p_5,\lambda';p_1,\lambda;p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_1\cdot Z_2) + \\ &+ Z(p_2,\lambda_2;p_6,\lambda';p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_6\cdot \\ &\cdot(Z(p_6,\lambda';p_5,\lambda;p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_5\cdot Z_1 + \\ &+ Z(p_6,\lambda';p_1,\lambda;p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_1\cdot Z_2) \\ T_{B1} = \;& Z(p_2,\lambda_2;p_6,\lambda';p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_6\cdot \\ &\cdot(Z(p_6,\lambda';p_1,\lambda;p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_1\cdot Z_2 + \\ &+ Z(p_6,\lambda';p_5,\lambda;p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_5\cdot Z_1) + \\ &+ Z(p_2,\lambda_2;p_2,\lambda';p,\lambda_6;p_6,\lambda_6;1,1,1,1)\cdot b_2\cdot \\ &\cdot(Z(p_2,\lambda';p_1,\lambda;p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_1\cdot Z_2 + \\ &+ Z(p_2,\lambda';p_5,\lambda;p_3,\lambda_3;p_4,\lambda_4;C_L,C_R,1,0)\cdot b_5\cdot Z_1) \end{aligned} \tag{9}$$

with

$$\begin{aligned} Z_1 &= Z(p_5,\lambda;p_1,\lambda_1;p,\lambda_5;p_5,\lambda_5;1,1,1,1) \\ Z_2 &= Z(p_1,\lambda;p_1,\lambda_1;p,\lambda_5;p_5,\lambda_5;1,1,1,1) \end{aligned} \tag{10}$$

and where a summation over λ and λ' is understood.

Given a set of four-momenta and a particular helicity configuration, one can easily evaluate the Z functions obtaining an explicit value for the transition amplitude of diagrams A.1 and B.1 and consequently, with the appropriate permutation of indices and conjugations, the corresponding ones for the remaining diagrams. Then, the total transition probability, averaged over initial spins, is given by

$$\frac{1}{4}\sum_{spins}|i\,M_T|^2 = \frac{N_\nu}{8}\sum_{\lambda}|\sum_{j} i\,M_j|^2 \qquad (11)$$

where the indices j and λ run over all contributing diagrams and helicity configurations respectively, N_ν stands for the number of light neutinos and the extra factor 2 in the denominator comes from the symmetrization of the final state photons.

II - The Phase Space

The differential cross section for the process (1) is given by

$$d^8\sigma = \frac{1}{4\,(p_1\cdot p_2)\,(2\pi)^8}\cdot\frac{1}{4}\sum_{spins}|i\,M_T|^2\cdot dR_4 \qquad (12)$$

where dR_4 is the four-body Phase Space differential element. Since we are interested in having under control the two final state photons, we express the phase space as a function of the polar angle of the photons $(\vartheta_5,\vartheta_6)$ with the incoming electron defining the Z-axis, their azimuthal angles (ϕ_5,ϕ_6) and their energies (E_5,E_6) together with the solid angle of one of the remaining particles (Ω_3 for instance). Then, one has that

$$d\,R_4 = W_{PS}\cdot d\Omega_5\,dx_5\,d\Omega_6\,dx_6\,d\Omega_3 \qquad (13)$$

where

$$W_{PS} = \frac{E_b^4}{2^4}\,x_5\,x_6\,\frac{2 + x_5x_6(1-C_{56}) - 2(x_5+x_6)}{((2-x_5-x_6) + x_5C_{53} + x_6C_{63})^2} \qquad (14)$$

with

$$x_i = \frac{E_i}{E_b} \qquad \text{and} \qquad C_{ij} = \cos\,(\widehat{p_i,p_j}) \qquad (15)$$

In order to have a fast convergence for the numerical integration and an efficient generation of events, we have absorbed the peak for the photonic angular and energy distributions rewriting the Phase Space as a function of new integration variables $\eta_i \in [0,1]$ such that the original variables are expressed in terms of these new ones by the inverse transformations [5]

$$x_5 = x_{min}\cdot\left(\frac{x_{max}}{x_{min}}\right)^{\eta_1}$$

$$\cos\vartheta_5 = \frac{\alpha-1}{\beta\,(\alpha+1)} \qquad \text{with} \qquad \alpha = \left(\frac{1+\beta}{1-\beta}\right)^{2\eta_2-1} \qquad (16)$$

and similarly for x_6 and $\cos\vartheta_6$ where $\beta = \sqrt{1-(\frac{m_e}{E_b})^2}$.

After an event (i.e. a set of 4 four-vectors) has been generated considering all the kinematical restrictions among the variables, the transition probability given by eq. 11 is evaluated. This probability, together with the Phase Space weight (W_{PS}) gives the total weight for the event. The cyclic repetition of this procedure leads to the evaluation of the total and differential cross sections.

III - Results

In order to study the contribution of the process $e^+e^- \to \nu\bar{\nu}\gamma\gamma$ to the reaction $e^+e^- \to \nu\bar{\nu}\gamma$ we have defined an hypothetical experimental set-up such that only one photon is detected under the following conditions:

- minimum photon energy of $E_\gamma^{cut} = 1GeV$
- tagging angle for photons of $\vartheta_\gamma^{cut} = 20°$
- maximum angle of indistinguishibility for two nearby photons of 1°

with $x_\gamma^{min} = .005$ the minimum energy for hard photon emission.[1] As experimental inputs we have taken the fine structure constant $\alpha = 1/137.036$ and the fermion and boson masses. In particular, for the intermediate vector bosons we have used $M_W = 82\,GeV$ and $M_{Z^\circ} = 93\,GeV$. In this way, our results can be directly combined with the calculations of the virtual corrections done in the on-shell renormalization scheme. Moreover, we have taken $\Gamma_{Z^\circ} = 2.7\,GeV$ and $N_\nu = 3$. Under these conditions, the cross section of $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$ is shown in figure 2 as a function of the c.m. energy(dashed line) together with the lowest order cross section (continuous line). As expected, the correction due to hard photon bremsstrahlung is large and positive. For $\sqrt{s} = 100\,GeV$ we have estimated it to be $\delta_{hard}^{\gamma} \simeq +37\%$. This contribution cancells partially the large negative corrections coming from the virtual one loop Q.E.D. radiative corrections and soft corrections. For $\sqrt{s} = 100\,GeV$ we have estimated them (based on the results of reference [2]) to be $\delta_{virt.}^{QED} + \delta_{soft} \simeq -48\%$.

Among the non-Q.E.D. corrections, one would expect a big contribution only from the self energy of the Z° due to the appearance of $\log(M_Z{}^2/m_f{}^2)$ terms. Based on the

[1]One has to be aware that two photons with $\vartheta \in [\vartheta_\gamma^{cut}, 180° - \vartheta_\gamma^{cut}]$ and within a cone of 1° should be accepted if $E_{\gamma_1} < E_\gamma^{min}$ and $E_{\gamma_2} \in [E_\gamma^{cut}, E_\gamma^{cut} - E_{\gamma_1}]$ since $E_{\gamma_1} + E_{\gamma_2} \geq E_\gamma^{cut}$. Under the previously referred conditions, this gives a very small contribution to the observed cross section.

calculations of reference [6], we have estimated this contribution to be[2] $\delta^{Z^\circ}_{self} \simeq +15\%$ for $\sqrt{s} = 100\,GeV$. Adding this contribution, we can conclude that the radiative corrections to the total observed cross section of the process $e^+e^- \to \nu\overline{\nu}\gamma$ under the conditions we have choosen are small. In particular, for a c.m. energy of $100\,GeV$ and the previously defined experimental set-up, they are about 10% to 15% of the expected increase in the cross section due to the existence of an extra light neutrino. In spite of that, figure 3 shows that the contribution from the double hard photon emission tends to shift the photon energy spectrum to lower values and so will do with the P_t spectrum (figure 4) making less efficient the separation of the signal from the QED background (references 5 and 7). Last, the differential cross section with respect to the photon polar angle is shown in figure 5.

In summary, we have calculated the cross section for $e^+e^- \to \nu\overline{\nu}\gamma(\gamma)$,where only one of the photons is detected, as a correction to the neutrino counting process $e^+e^- \to \nu\overline{\nu}\gamma$. We have used the helicity amplitudes formalism developped in [3–5] and we have neglected the contribution from the diagrams containing a W propagator. This is expected to be a good approximation for center of mass energies close to the Z° mass, which is precisely the range of energies where the signal from $e^+e^- \to \nu\overline{\nu}\gamma$ is larger compared to the radiative Bhabha scattering background [5, 7]. The obtained cross section gives a large, positive contribution to the neutrino counting process and is such that, when added to the Z° self energy , the soft photon and the QED corrections, leads to a total change in the cross section of a few percent. Nevertheless, the photonic energy and P_t differential cross sections are different to the ones obtained at the tree level making the separation from $e^+e^- \to e^+e^-\gamma$ less efficient.

[2]The quark masses (in GeV) used in this evaluation are [6] $m_u = m_d = .032$, $m_s = .15$, $m_c = 1.5$, $m_b = 4.5$, $m_t = 35.$ and for the Higgs boson $M_H = 100$.

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Figure Captions

Fig. 1 Feynman diagrams contributing to $e^+e^- \to \nu\bar{\nu}\gamma\gamma$ in the Z° s-channel to order g^4.

Fig. 2 Cross sections for the processes $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$ (dashed line) and $e^+e^- \to \nu\bar{\nu}\gamma$ (full line) as function of $\sqrt{s}$

Fig. 3 Differential cross sections with respect to the photon energy for the processes $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$ (dashed line) and $e^+e^- \to \nu\bar{\nu}\gamma$ (full line)

Fig. 4 Differential cross sections with respect to the photon P_t for the processes $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$ (dashed line) and $e^+e^- \to \nu\bar{\nu}\gamma$ (full line)

Fig. 5 Differential cross sections with respect to the photon polar angle for the processes $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$ (dashed line) and $e^+e^- \to \nu\bar{\nu}\gamma$ (full line)

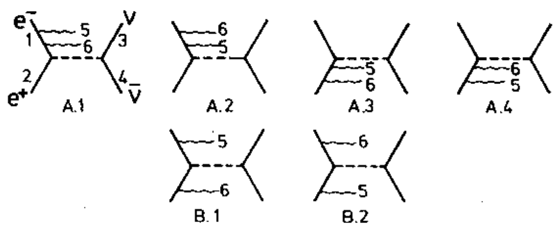

Fig. 1

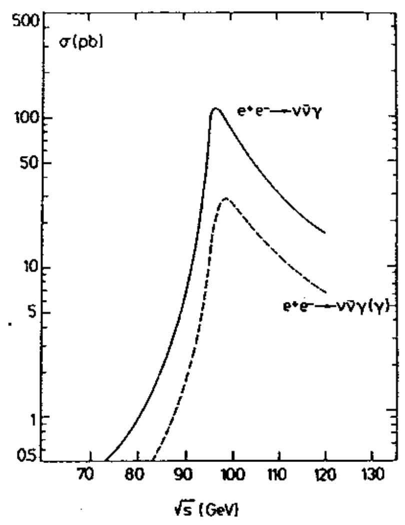

Fig. 2

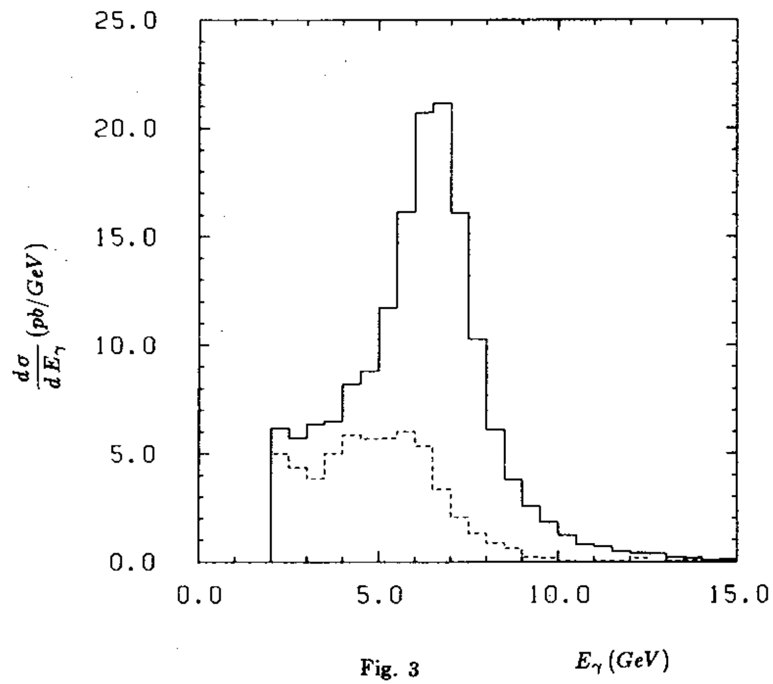

Fig. 3

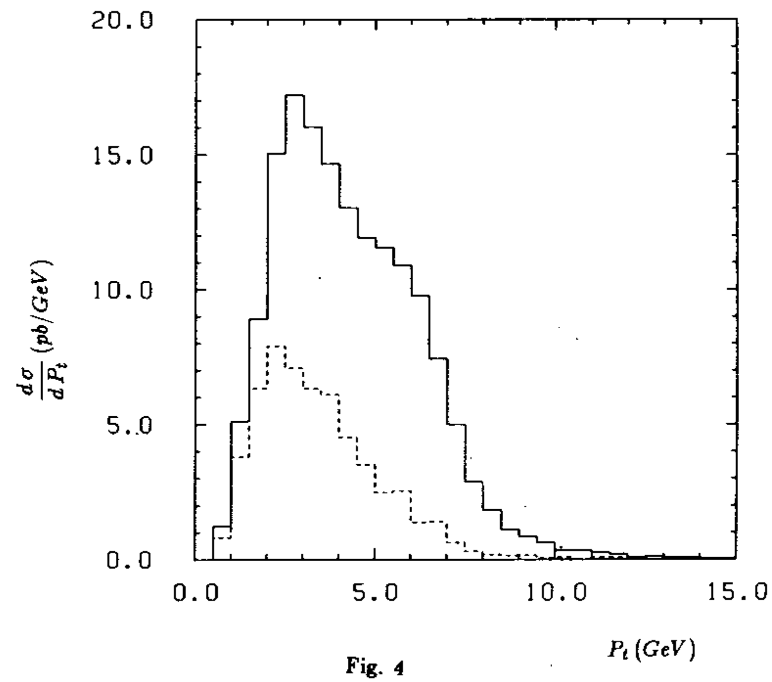

Fig. 4

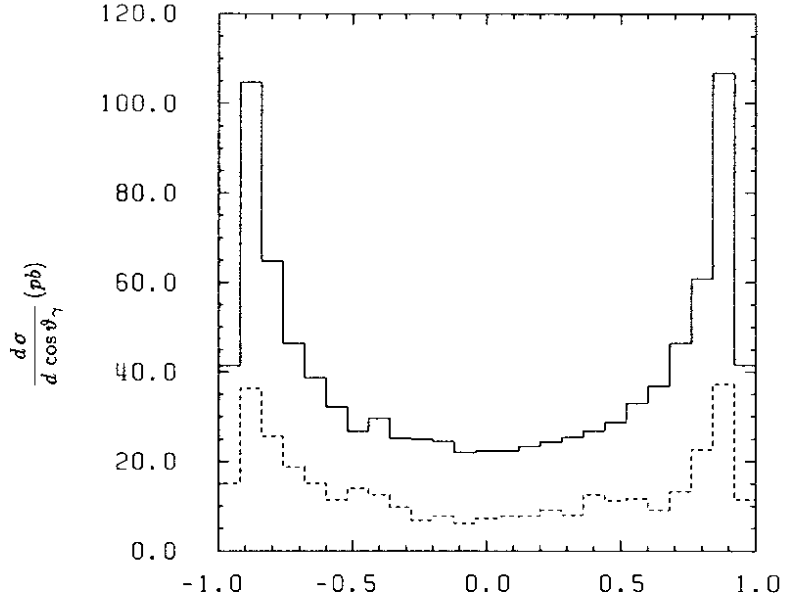
120.0
100.0
80.0
60.0
40.0
20.0
0.0
-1.0
-0.5
0.0
0.5
1.0
$\frac{d\sigma}{d\cos\vartheta_\gamma}$ (pb)
$\cos\vartheta_\gamma$

Fig. 5