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R. D. Kohaupt

Deutsches Elektronen-Synchrotron DESY, Hamburg

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Damping of Multi-Bunch Oscillations by a Narrow-Band Feedback System Using Frequency Splitting Between Bunches

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Introduction

In order to damp multi-bunch oscillations in a storage ring, the single bunches must either be damped individually by a multi-channel feedback system or by a system producing a broad-band impedance which damps the different multi-bunch modes. In principle, these two systems are, of course, equivalent; they only differ in the technical realization. Common to both is the high time resolution or - equivalently - the high bandwidth of all components, especially of the deflecting or accelerating devices. In the case of multi-bunch instabilities, growth rates, maximum acceptable amplitudes and bandwidth determine the power required for the system. In the HERA rings and their PETRA injector the bunch-to-bunch distance is about 100 nsec and thus, in the multi-bunch operation of these machines the bandwidth required is about 10 MHz. This bandwidth is within standard techniques far as digital signal processing is concerned[1]). Great effort, however, is needed to develop the devices (kickers, resonators etc.) acting on the beam.

Therefore, the question arises whether such a high bandwidth is really necessary in a multi-bunch machine.

In this article it is shown that, indeed, the bandwidth necessary for damping multi-bunch oscillations can be drastically reduced.

This becomes possible by splitting the individual bunch frequencies in combination with a feedback system that only acts on the barycentric mode of the beam. The bandwidth of this system is determined by the band of bunch frequencies, which is "narrow". Without frequency splitting, of course, only the barycentric mode will be damped. All the other modes (delivering no signal at the pick-up) will not be affected. When imposing a frequency splitting, each configuration of motion will contribute to the barycentric motion at least after a time of the order of inverse frequency differences.

Therefore, the combination of frequency splitting and barycentric mode damping "transforms" the frequency differences between individual bunches into damping rates such that all possible oscillations are damped.

The basic idea

A.) The two bunch model

We consider the oscillations (transverse or longitudinal) of two bunches. The coordinates of these bunches are x_1 and x_2 respectively; the frequencies are Ω_1 and Ω_2. A barycentric feedback system acts on the "beam" with a "gain" constant D:

$$\begin{aligned} \ddot{x}_1 + \Omega_1^2 x &= - iD(x_1 + x_2) \\ \ddot{x}_2 + \Omega_2^2 x &= - iD(x_1 + x_2) \end{aligned} \quad , D > 0 \qquad (1)$$

We define

$$\Omega_2 = \Omega_1 + \delta\Omega \qquad , \delta\Omega > 0 \qquad (2)$$

and

$$x_{1,2} = \hat{x}_{1,2}\, e^{i\Omega t} \qquad (3)$$

To first approximation we put

$$\Omega_1^2 - \Omega^2 \approx 2\Omega_1(\Omega_1 - \Omega) = \Delta_1$$

$$\Omega_2^2 - \Omega^2 \approx 2\Omega_2(\Omega_2 - \Omega) = 2\Omega_1(\Omega_1 - \Omega) + 2\Omega_1\delta\Omega = \Delta_1 + \delta \qquad (4)$$

Equs. (1), (2), (3) together with (4) lead to the characteristic equation for Δ_1 :

$$(\Delta_1 - iD)([\Delta_1 - iD] + \delta) = - D^2 \qquad (5)$$

with the solution

$$\Delta_1 = 2\Omega_1(\Omega_1 - \Omega) = iD - \frac{1}{2}\delta \pm \frac{1}{2}\sqrt{\delta^2 - 4D^2} \qquad (6)$$

For $\delta = 0$ (no frequency splitting) we find two values for Δ_1 namely

$$\begin{aligned} \Delta_1 &= 0 && \text{undamped mode} \\ \Delta_1 &= 2iD && \text{damped (barycentric) mode} \end{aligned} \qquad (7)$$

as expected.

For $\delta \neq 0$, however, and $D < \frac{1}{2}\delta$, the quare root on the r.h.s. of (6) remains real so that both solutions (modes) are damped.

The maximum possible damping rate is determined by

$$D = \frac{1}{2}\delta \qquad (8)$$

Since the frequency band of the feedback term

$$F(t) = D(x_1 + x_2) \qquad (9)$$

is limited by the frequency "spread" $\delta\Omega$, the bunch system is damped by a feedback loop which is "narrow" as compared to the bandwidth of a "fast" feedback loop that acts on each bunch separately.

B.) The multi-bunch case

In the multi-bunch case we introduce the bunch coordinates x_j, where j runs from 1 to N, N being the number of bunches. We assume that the frequencies of the single bunches can be written as

$$\Omega_j = \Omega_o + j \cdot \Delta\omega \quad , \quad j = 1, \ldots, N \quad , \quad \Delta\omega > 0 \qquad (10)$$

Here $\Delta\omega$ denotes the frequency difference between adjacent bunches. The total spread of the circular frequency within the beam is then given by

$$\delta\Omega = N \cdot \Delta\omega \qquad (11)$$

Instead of (1) we obtain in the multi-bunch case

$$\ddot{x}_k + \Omega_k^2 x_k = - iD \sum_{j=1}^{N} x_j \quad , \quad k = 1, \ldots, N \qquad (12)$$

Putting

$$x_j = \xi_j\, e^{i\Omega j t} \qquad (13)$$

we keep only first order terms in D and $\delta\Omega$ and rewrite (12) as

$$\dot{\xi}_k + A\xi_k = - A \sum_{j\neq k}^{N} \xi_k\, e^{i\Delta\omega(j-k)t} \qquad (14)$$

with

$$A = \frac{D}{2\Omega_o} \qquad (15)$$

Let us consider what happens in a time interval Δt with

$$\Delta t < 1/A$$

Within Δt we can assume the ξ_j to be nearly constant so that we can integrate (14) to obtain

$$\frac{\Delta\ \ k}{\Delta t} + A\xi_k = -A \sum_{j\neq k}^{N} \xi_k \frac{e^{i\Delta\omega\Delta t(j-k)}}{i\Delta\omega\Delta t(j-k)} , \quad k = 1, \ldots, N \qquad (16)$$

This equation describes damping for each k due to the second term on the l.h.s. of (16) and an "excitation" through the other bunches due to the r.h.s of (16). However, if $\Delta\omega \cdot \Delta t$ is sufficiently large, or if

$$\frac{A}{\Delta\omega} \ll 1 \qquad (17)$$

the influence of the other bunches remains small and we expect damping for each individual k. This can be seen assuming identical absolute values for the $_k$. In this case, the r.h.s. of (16) is of the order

$$A \cdot |\xi_k| \frac{A}{\Delta\omega} (1 + \ln N) . \qquad (18)$$

Therefore if

$$\frac{A}{\Delta\omega} (1 + \ln N) < 1 \qquad (19)$$

we find damping for all k.

The properties of (16) considered so far are similar to the properties of those equations used in formulating stochastic cooling[2]. Due to the frequency differences (mixing!), each bunch is influenced only by "its own" signal since the influence of the other bunches averages out within a time of the order $1/\Delta\omega$. This time also sets a limit on the maximum A.

In the next section, the possible damping rates will be derived making use of a formal approach, and we will obtain relations similar to (19).

Derivation of damping rates

Before we derive the damping rates for the general N-bunch case, it should be emphasized, that the description of bunch oscillations by equ. (12) simplifies the real motion, especially in the transverse case where strong focussing is applied.

However, by choosing appropriate coordinates, the bunch oscillations can also in case of strong focussing by formally described in terms of harmonic oscillators, as shown in ref. (3).

I) Signal at the pick-up

The signal produced by N bunches of the beam at a local pick-up station can be written as

$$S(t) = \sum_{j=1}^{N} \sum_{p=-\infty}^{+\infty} x_j(t-j\tfrac{T}{N})\, e^{ip\omega_0(t-j\frac{T}{N})}\, \rho(p) \qquad (20)$$

where T is the revolution time and ω_0 is the circular revolution frequency; the quantity $\rho(p)$ is related to the Fourier component of the bunched beam current. The terms $j\frac{T}{N}$ in (20) account for the delay due to the different times of arrival. If a frequency band around an integer multiple of $N\omega_0$ is filtered out, the "demodulation" leads to the "low-frequency" signal

$$S_L(t) = \varkappa \sum_{j=1}^{N} x_j(t-j\tfrac{T}{N}) \qquad (21)$$

where $\varkappa$ is a constant determined by the pick-up system and by the process just described. The bandwidth of S_L is defined by the frequency spread among the N bunches.

Now we introduce a filter "F" by

$$f_L(t) = F[S_L(t)] \qquad (22)$$

which in frequency space has the property

$$F[e^{i\omega t}] = -iD(\omega)\, e^{i\omega t} \qquad (23)$$

where $D(\omega)$ will be specified later.

II) Signal at the acting device and the equations of motion

If the k-th bunch arrives at a local device acting on the beam at $t' + k\frac{T}{N}$ the "force" on this bunch will be written as

$$F_k = K f_L(t' - \tau + k\frac{T}{N}) \, , \tag{24}$$

where K accounts for the transfer constant between f_L and F_k, and τ denotes the total time delay of the system.

From (24) we obtain an equation similar as (12), namely

$$\ddot{x}_k + \Omega_k^2 x_k = K f_L(t' - \tau + k\frac{T}{N}) \quad , \quad k = 1, \dots N \tag{25}$$

In order to solve (25) we look for eigensolutions of the form

$$\vec{x} = \vec{\xi} e^{i\omega t'} \tag{26}$$

Taking

$$\vec{\xi} = \lambda \begin{pmatrix} \vdots \\ \frac{1}{\Omega_k^2 - \omega^2} e^{i\omega k \frac{T}{N}} \\ \vdots \end{pmatrix} \tag{27}$$

with an arbitrary constant λ, equ. (25) can be solved. Together with equs. (21), (22) and (23) we find for k = 1, ..., N

$$\lambda e^{i\omega k\frac{T}{N}} = -i\lambda \varkappa K D e^{-i\omega\tau} \sum_{j=1}^{N} \frac{e^{i\omega k\frac{T}{N}}}{\Omega_j^2 - \omega^2} e^{-ij\omega\frac{T}{N}} e^{ij\omega\frac{T}{N}} \tag{28}$$

or

$$1 = -i\varkappa K D e^{-i\omega\tau} \sum_{j=1}^{N} \frac{1}{\Omega_j^2 - \omega^2} \quad , \tag{29}$$

an equation which determines the eigenfrequencies ω.

To first order in all quantities of the order $\delta\omega$, $\frac{\varkappa K D}{2\Omega_o}$ we rewrite (29) as

$$1 = -i \frac{\varkappa K D(\Omega_c)}{2\Omega_o} e^{i\Omega_c\tau} \sum_{j=1}^{N} \frac{1}{\Omega_j - \omega} \, ; \, \Omega_c = \Omega_o + \frac{N}{2}\Delta\omega \tag{30}$$

Finally if the filter is arranged so that

$$-i \frac{\varkappa K D(\Omega_c)}{2\Omega_o} e^{-i\Omega_c\tau} = -iA \quad , \quad A > 0 \tag{31}$$

we arrive at

$$1 = -iA \sum_{j=1}^{N} \frac{1}{\Omega_j - \omega} \tag{32}$$

The difference in phase shift due to the frequency spread has been neglected. This, however, sets a limit to the total delay or to $\Delta\omega$.

Since we assume A to be sufficiently small, the N possible eigenfrequencies are located around the unperturbed frequencies

$$\omega_\ell \approx \Omega_\ell \quad , \quad \ell = 1, \dots, N \tag{33}$$

For a certain r we therefore obtain

$$1 = -iA \sum_{j=1}^{N} \frac{1}{\Omega_r - \omega_r + \Delta\omega(j-r)}$$

$$= -iA/\Delta\omega \cdot \sum_{j=1}^{N} \frac{1}{\frac{\Omega_r - \omega_r}{\Delta\omega} + (j-r)} \tag{34}$$

or

$$1 = -i\frac{A}{\Omega_r - \omega_r} - i\frac{A}{\Delta\omega} \sum_{j\neq r} \frac{1}{\alpha + (j-r)} \tag{35}$$

with

$$\alpha = \frac{\Omega_r - \omega_r}{\Delta\omega} \tag{36}$$

If α is sufficiently small as compared to 1,

$$\alpha \ll 1 \tag{37}$$

from (36) follows

$$1 = -i\frac{A}{\Omega_r - \omega_r} - i\frac{A}{\Delta\omega} G_r(N) \tag{38}$$

and

$$G_r(N) = \sum_{j\neq r} \frac{1}{j-r} \tag{39}$$

Solving for $\Omega_r - \omega_r$ yields

$$\Omega_r - \omega_r = -iA \frac{1}{1 + i\frac{A}{\Delta\omega} G_r(N)} \tag{40}$$

The damping rates are determined by the imaginary part of (40). Therefore, we derive from (40)

$$\delta_r = \operatorname{Im}\omega_r = \frac{A}{1 + \frac{A^2}{\Delta\omega^2} G_r^2(N)} \tag{41}$$

The maximum possible damping rate follows for

$$\frac{A}{\Delta\omega}\,|G_r(N)| = 1 \tag{42}$$

and therefore we finally obtain

$$\delta_r = \frac{1}{2}\,\frac{\Delta\omega}{|G_r(N)|} \tag{43}$$

Since the individual contributions to the sum (39) have the same sign only for $r = 1$ or $r = N$, the maximum value for $|G_r(N)|$ is given by

$$\mathrm{Max}\,|G_r(N)| = S(N) = \sum_{j=1}^{N-1} \frac{1}{j} \tag{44}$$

If we choose A to be determined as

$$A = \frac{\Delta\omega}{S(N)} \tag{45}$$

equ. (43) leads to

$$\delta_r = \frac{1}{2}\,\frac{\Delta\omega}{S(N)} \tag{46}$$

and this result is conistent with the initial assumption (37) because of

$$S(N) \gg 1 \quad \text{for} \quad N \gg 1 \tag{47}$$

Under the condition (10), (45) the eigenstates ξ_r belonging to the different N different solutions ω_r of equ. (40) form a linearly independent base.

Therefore due to the damping (46) all possible configurations of motion are damped.

It should be pointed out, that this is true only if all bunch frequencies are different as guaranteed by (10).

Frequency splitting

The standard method[4,5] to split the frequencies between the different bunches in the transverse direction is the use of an rf-quadrupole operating on an appropriate low harmonic of the revolution frequency, which leads to a sinusoidal modulation of the betatron tune. In the longitudinal direction the frequency splitting can be simply done by operating part of the rf transmitters on a frequency one harmonic below or above the fundamental frequency of the rf-system. The superposition of the voltages results in a sinusoidal modulation of the synchrotron frequency. As pointed out in the preceeding sections, damping of all possible modes of oscillation is only possible if the frequencies of different bunches are different. For a sinusoidal modulation of tunes even in the case where the period of modulation is one revolution, there are pairs of bunches which have the same frequency.

This difficulty can be avoided filling only those buckets that are located in the "linear" range of the modulation.

Fig. 1 shows the frequency f as a function of the longitudinal position s; L is the circumference. If the bunches are filled either into section I or II, all bunch frequencies will be different. This means, of course, that only one half of the ring can be occupied with bunches. However, also the second half can be filled if the feedback system acts on each half separately.

The maximum frequency difference between adjacent bunches can be utilized if we restrict the filling to the "linear" part of the sinusoidal modulation. In this case, we can apply relation (46) to evaluate the damping rates.

For $N > 5$ the sum (44) can be approximated by

$$S(N) = C + \ln(N-1) \quad ; \quad C \approx 0.577 \tag{48}$$

with sufficient accuracy[6], so that the achievable damping rates can be written as

$$\delta_r \approx \pi\,\frac{\delta f}{N[C + \ln(N-1)]} \tag{49}$$

where δf is the total frequency spread within the beam.

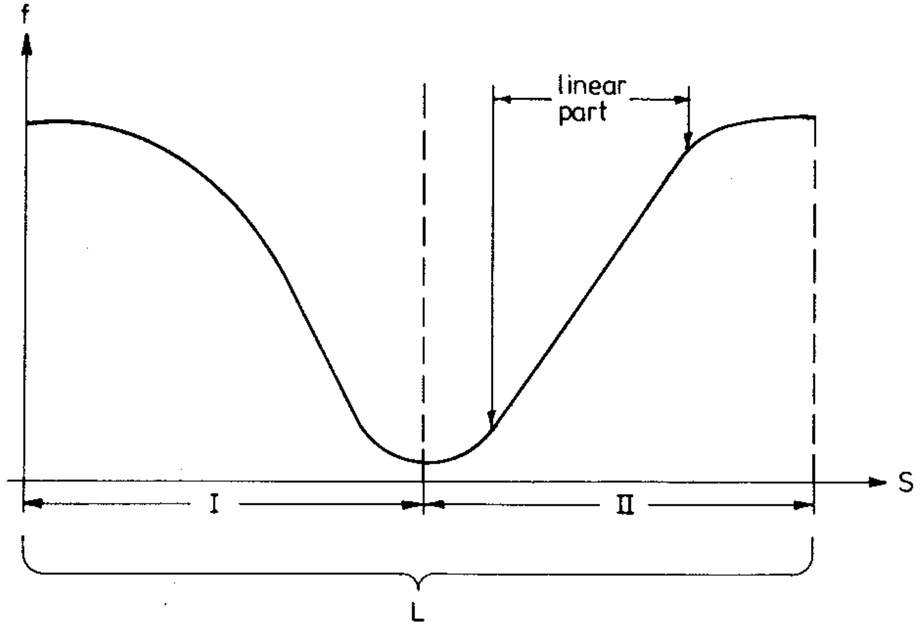

Fig. 1 Bunch frequency as a function of the longitudinal position.

Numerical estimates

The tolerable range of transverse tune for injection and storage in PETRA in the case of small single bunch currents is

$$\Delta f_{x,z} \approx \pm 5 \text{ kHz}$$

and the tolerable range of the longitudinal tune is about

$$f_s = 6 \text{ kHz} \ldots 12 \text{ kHz}$$

Thus we obtain from (49)

N	$1/\delta_{x,z}$/msec	$1/\delta_s$/msec
10	0.9	1.4
20	2.2	3.7
30	3.8	6.2

The Landau damping time for electrons in transverse and longitudinal direction was measured to be around 20 msec in an injection optics.

For HERA the damping rates are considerably reduced due to the high number of bunches. However, one could apply the described damping system on a quarter of the ring, which means $N \approx 50$. In this case, we obtain in the transverse direction for $\delta Q_{x,y} \approx 0.1$

$$\frac{\delta_r}{\omega_0} \approx 2 \cdot 10^{-4}$$

The Landau tune spread in HERA with chromatic corrections is about 10^{-5} [7].

Experimental study

The damping system proposed in this article is planned to be experimentally studied in PETRA.

Use can be made of the existing longitudinal pick-up and signal processing system which can be operated in such a way as to produce a signal proportional to the sum of the longitudinal displacements of single bunches (barycentric mode pick-up system). After processing, this signal can be used to modulate the phase output of the klystrons in one of the main transmitters. An other transmitter will be detuned such that it runs one harmonic above the fundamental frequency of the rf system and provides the splitting of synchrotron frequencies.

The bunches will be injected into that part of the ring where the modulation of synchrotron frequencies has nearly constant slope.

In this state the longitudinal damping time of single bunches can be measured in the following way:

Each bunch can be excited on its individual synchrotron frequency. If this excitation is interrupted, the decay of synchrotron oscillations picked up from the corresponding bunch can be observed.

Since the possible damping rates are significantly smaller than $\Delta\omega$ due to the factor $1/S(N)$, the damping behaviour of single bunches can then be investigated by the same method when the feedback system is turned on, especially as a function of the number of bunches N.

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References

1) D. Boussard and G. Lambert, IEEE Trans. Nucl. Sci. NS-30, No.4, 1983

2) F.J. Sacherer, CERN-ISR-TH/78-11, 1981

3) R.D. Kohaupt, DESY 80/22, 1980

4) R.D. Kohaupt, Proc. of 4th ALL. Union Conf. of Charged Particle Accelerators, 1975

5) D. Boussard and J. Gareyte, Proc. of 8th Int. Conf. on High Energy Accelerators, CERN, 1977

6) L.S. Gradstem and I.M. Myshik, "Tables", Verlag Harri Deutsch, Frankfurt/ Main

7) R. Brinkmann and F. Willeke, DESY 86-079, 1986