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Short-Distance Effects in the K^0-$\bar{K}^0$ Mixing in the Standard Model *)

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From the accelerator experiments in the 1950s up to the CP experiments of the present time the neutral kaon complex has remained one of the most interesting systems in nature. The breakthroughs which it has provided in our knowledge of fundamental physics have been crucial in establishing the standard SU(3)xSU(2)xU(1) model (SM) of electroweak and strong interactions. More recently there have been claims that the existing CP-violation measurements represent a serious threat to the standard model. In this paper I would like to illustrate that such announcements are premature in view of the degree of accuracy in theoretical predictions. Therefore, I would stick to the theoretically more tractable short-distance contributions and then I shall gradually show that, on the one hand, their present treatment is incomplete and, on the other hand, they have to be supplemented at various stages (which I intend to explicate) by some sort of long-distance (LD) calculation.

Since this meeting is intended to be a school, it might be appropriate to start with a short history. Strange particles are produced in a large number in strong interactions, and thus it has been natural to introduce the strong-interaction (and strangeness) eigenstates

$$|K^0\rangle \ (S=-1) \ \text{and} \ |\bar{K}^0\rangle \ (S=+1) \ .$$

However, it has been discovered that these states oscillate (mix) owing to the weak interaction. The resulting removal of degeneracy

$$\Delta m_K \simeq 3.5 \times 10^{-15}\,\text{GeV} \tag{1}$$

represents the most precisely measured quantity in particle physics. The corresponding weak-interaction (and CP-symmetry) eigenstates

$$|K_1\rangle = (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2} \quad (CP=+) \quad \text{and} \quad |K_2\rangle = (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2} \ (CP=-)$$

are still not the last word in the story. The small impurity discovered in these states,

$$\varepsilon = 2.27 \times 10^{-3} \tag{2}$$

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represents the only measured CP-violating effect. Thus, our present knowledge stops at short- and long-lived eigenstates,

$$|K_S\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + \varepsilon_S|K_2\rangle) \quad , \quad |K_L\rangle = \frac{1}{\sqrt{2}}(|K_2\rangle + \varepsilon_L|K_1\rangle)$$

and we take $\varepsilon_S = \varepsilon_L = \varepsilon$, assuming CPT invariance.

Although the derivation of hadronic states is still beyond our capabilities, we can hope to get a better understanding of the inner machinery of the $\Delta S=2$ mixing, i.e. to reproduce the values in (1) and (2) theoretically. Our hope resides in the SM. From the outset there are two possible contributions within the SM, namely, the short-distance /SD/ contribution $\Delta S=2$ (Fig. 1) and the long-distance /LD/ dispersive contribution $(\Delta S=1)^2$ (Fig. 2). Accordingly,

$$\Delta m_K = \Delta m_K^{SD} + \Delta m_K^{LD} \tag{3}$$

and

$$\varepsilon = \varepsilon^{SD} + \varepsilon^{LD} \quad . \tag{4}$$

In general, the calculation of the dispersive LD effects is plagued with large uncertainties. We would like to distinguish the SD from the LD contribution or at least to know their relative size for a given physical quantity. For the mass difference Δm_K the potential significance of the LD effects has been pointed out by Wolfenstein /1/ and Hill /2/, and our ignorance in its precise knowledge has been phrased /1/ as a D-parameter

$$\Delta m_K^{LD} \doteq D\, \Delta m_K \quad . \tag{5}$$

The situation with the ε-parameter is considerably better /3/,

$$\varepsilon^{LD} < 0.2\, \varepsilon \quad , \tag{6}$$

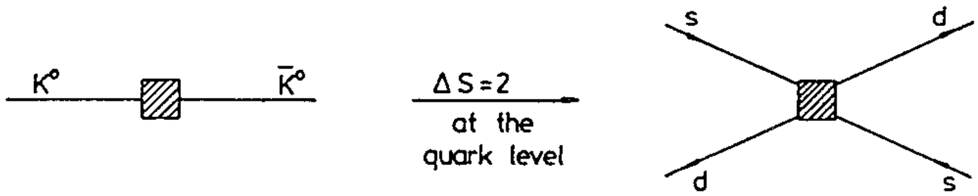

Fig. 1

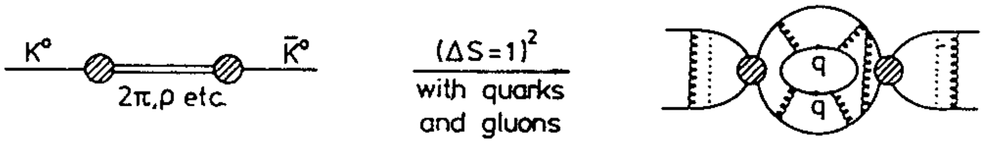

Fig 2

indicating that ε is SD dominated. Then, this motivates us to explore the SD contributions to ε in more detail. Since ε and Δm are, respectively, the imaginary and real parts of the same K^0 to $\bar{K}^0$ amplitude, we simultaneously obtain the improvement of the SD contribution to Δm. However, in view of (5), this improvement will only be of marginal importance.

The simplest SD diagram for the $\Delta S=2$ transition of interest is the standard box /4/ (Fig. 3). In principle, this diagram can give rise to both Fig. 1 and Fig. 2. Here we restrict ourselves to its contribution described by a four-quark effective hamiltonian

$$H^{\Delta S=2}_{Box} = \frac{G_F^2}{16\pi^2}\Big[(\eta_1)\, m_c^2 \lambda_c^2 + (\eta_2)\, m_t^2 \lambda_t^2 + (\eta_3)\frac{2\lambda_c\lambda_t m_c^2}{1-m_c^2/m_t^2}\ln\frac{m_t^2}{m_c^2}\Big]\, O^{\Delta S=2}, \tag{7}$$

where $\lambda_i = V_{id}V^*_{is}$ are the Kobayashi-Maskawa factors constrained by the relation $\lambda_u + \lambda_c + \lambda_t = 0$, and

$$O^{\Delta S=2} = \bar{d}^i \gamma^\mu (1-\gamma_5) s^i\, \bar{d}^j \gamma_\mu (1-\gamma_5) s^j \quad . \tag{8}$$

The real and imaginary parts of (7) account for Δm_K and ε, respectively, whereby one has to calculate the K^0 to $\bar{K}^0$ transition amplitude. Conventionally, this is done by the vacuum saturation approximation (VSA)

$$\langle K^0 | O^{\Delta S=2} | \bar{K}^0\rangle \xrightarrow{VSA} \frac{16}{3} F_K^2 m_K^2 \;;\; F_K \simeq 0.113\,\mathrm{GeV} . \tag{9}$$

Here we face the <u>second</u> LD uncertainty, phrased as a B-value

$$B = \frac{\langle K^0 | O^{\Delta S=2} | \bar{K}_0\rangle}{\langle K^0 | O^{\Delta S=2} | \bar{K}^0\rangle_{VSA}} . \tag{10}$$

This represents the gap in tracing the transition from the quark to the hadronic states. In particular, this uncertainty reflects for the mass

Fig. 3

difference as

$$\Delta m_K^{Box} = B \times (\Delta m_K^{Box})_{VSA} \quad . \qquad (11)$$

In the standard box, the role of the QCD part of the SM is not explicit. The QCD enters when taking into account the hard-gluon (SD) corrections to $H^{\Delta S=2}_{Box}$ /5/, namely the three terms in (7) corrected by the factors

$$\eta_1 \simeq 0.7 \quad , \quad \eta_2 \simeq 0.6 \quad , \quad \eta_3 \simeq 0.5 \quad . \qquad (12)$$

The role of QCD is more explicit in considering a topologically different class of diagrams, the double penguin-like (DPL) diagrams of Fig. 4. Here one employs the induced flavour-changing (penguin) vertices, known by a "magical" property of reducing the power like GIM cancellation to the logarithmic one /6/. The role of double-penguin (DP) diagrams for Δm_K has been controversial for some time /7,8/. The solution /9/ is that the particular DP diagram displayed in Fig. 4 represents only a percentage of the standard box and thus is completely negligible. The larger class of DPL diagrams /10/ might give rise to $\sim$ 10% of the standard box, but it is still immaterial in view of the LD contribution in (3). Note that, in the ratio, the two SD contributions in (3)

$$\Delta m_K^{SD} = \Delta m_K^{Box} + \Delta m_K^{DPL} \qquad (13)$$

are free of the LD uncertainty of the B-factor given in (10) and (11). This is due to the fact that both contributions in (13) result in the same local four-quark operator.

However, the DPL diagrams turn out to be more relevant for the ε-parameter in (2) and (4) /10,11/. This is due to the negligible ε^{LD} from eq. (6). Accordingly,

$$\varepsilon \simeq \varepsilon^{SD} = \varepsilon^{Box} + \varepsilon^{DPL} \quad . \qquad (14)$$

The numerical value of the RHS of (14) depends on the parameters involved, the top quark mass m_t and the infrared (IR) cut-off μ. The latter has been introduced by

$$\alpha_s(\mu^2) = 1 \quad , \qquad (15)$$

in order to have a sensible perturbative QCD calculation $(\alpha_s(p^2) < 1)$. At this point we face the third uncertainty of the LD type, related to the IR cut-off μ. This cut-off indicates the border of the non-perturbative region and the LD effects in the form of bound states. Since the effects of the latter are not well controlled by the present QCD techniques, we have to check the stability of our results with respect to the variation

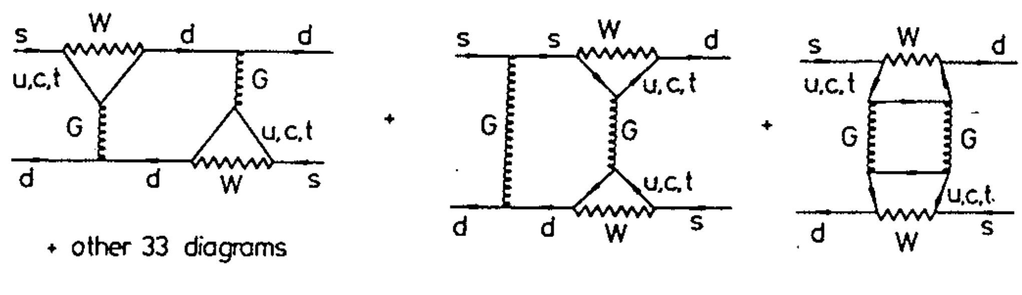

Fig. 4

in μ /11/. For m_t = 45 GeV and μ = 0.7 GeV, ε^{DPL} represents -25% of the QCD-corrected standard-box value. Varying μ in the range (0.3, 1.2) GeV gives ε^{DPL} in the interval (-15%, -40%).

Apparently, the result for ε with the DPL added, (eq. (14)), seems to be even more off the experimental value (2) than the standard box alone. The mere fact that the DPL contributions are not negligible is not surprising in view of previous experience that the extra powers of α_s might be compensated when penguin vertices are inserted in a higher loop /12/. Since our DPL class of diagrams represents just another contribution, which in no sense makes the final prediction for ε , it merely illustrates the need of more accurate calculations. First of all, this refers to the need of going beyond the leading logarithm. For example, the QCD corrections to the standard box quoted in (12) represent the result of leading logarithms summed by the renormalization-group method to all orders in α_s. There is no such simple treatment of non-leading terms. As a result, the calculations become more tedious and resemble the calculation for the g-2 (compare the lecture presented at this School by V. Hughes) where one has to procede order by order in the coupling. The point is that, in these cases, the level of accuracy required from theoretical predictions is dictated by the precise measurements performed so far. Before reaching the appropriate calculational accuracy, one is not able to infer about the crisis of the SM with respect to CP phenomenology.

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