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The lattice-regularized standard Higgs model *

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Abstract

Some recent non-perturbative investigations of the lattice-regularized $SU(2)$ Higgs model with a scalar doublet field are reviewed.

The lattice regularization makes possible to define and study quantum field theories non-perturbatively. The main emphasis in recent years' numerical Monte Carlo investigations was put on quantum chromodynamics or on related gauge models with only fermionic matter fields. The obvious motivation is the basic property of asymptotic freedom, which implies strongly interacting infrared physics and hence an immediate need for non-perturbative methods. The necessity of non-perturbative investigations in the standard $SU(2) \otimes U(1)$ electroweak model is not so obvious. Although the $SU(2)$ gauge coupling is also asymptotically free, the Higgs mechanism cuts off the low energy growth of the coupling at a scale where it is still rather weak. The $U(1)$ gauge coupling and the scalar self-coupling in the Higgs sector are not asymptotically free, therefore have a tendency to be weak in the infrared. The electromagnetic $U(1)$ coupling is, indeed, known to be weak at low energies. However, the Higgs sector is not yet known phenomenologically, therefore it can, in principle, be a source of non-perturbative effects. The question of the large cut-off behaviour of non-asymptotically free couplings is always non-perturbative. These two latter points make, in my opinion, the non-perturbative study of the electroweak sector interesting (perhaps also unavoidable).

For the question of the large cut-off (or small lattice spacing) behaviour numerical Monte Carlo calculations are not very well suited: not even the best existing computer is able to treat correlation lengths much more than 10 lattice units. A reliable study of four-dimensional gauge field systems with correlation lengths in the order of 100 lattice spacings seems to be still far away. Nevertheless, there are interesting unsolved problems in the standard electroweak model which can be studies with present computers:

- The simplest model for a strongly interacting Higgs sector is the standard Higgs model [1] even if it implies a relatively low cut-off.
- The phase transition between the confining- and Higgs-phase is, both at zero and non-zero temperatures, presumably a non-perturbative phenomenon.

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- The previous two points imply that in the standard Higgs sector there is an upper and a lower limit for the Higgs to W-mass ratio $R_{HW} \equiv m_H/m_W$. The precise value of R_{HW}^{max} and R_{HW}^{min} can presumably be obtained only by combining analytical and numerical work in the lattice regularization framework.
- There is a confining ("composite model") interpretation of the standard electroweak model [2], which is not yet ruled out by experiment. This would be a genuine strongly interacting theory, in many respects very similar to QCD (in some other aspects, however, different from it).

EXPANSIONS AT THE PARAMETER SPACE BOUNDARY. Because of the limited applicability of numerical methods, analytic expansions are very important to support and supplement the numerical work. The general strategy of analytic expansions is to reduce the number of coupling parameters by sending some of them to the boundary of the coupling parameter space. The small parameter in the expansion is the distance from the boundary in some appropriately chosen metric. In the standard Higgs model there are three couplings: the scalar self-coupling λ, the gauge coupling g (or $\beta \equiv 4g^{-2}$) and the hopping parameter κ standing in lattice regularization for the mass parameter of the scalar field. Possible expansions in the standard Higgs model are:

(a) strong gauge coupling expansion (SGCE) at $\beta = 0$ [3];
(b) hopping parameter expansion (HPE) at $\kappa = 0$;
(c) strong self-coupling expansion (SSCE) at $\lambda = \infty$ [4];
(d) weak gauge coupling expansion (WGCE) at $\beta = \infty$ [5];
(e) inverse hopping parameter expansion (IHPE) at $\kappa = \infty$;
(f) weak self-coupling expansion (WSCE) at $\lambda = 0$.

The latter two (IHPE and WSCE) were, to my knowledge, not yet studied in the standard Higgs model. The difficulty with WSCE is that the Higgs phase is singular for $\lambda \to 0$. At the intersections one can also define double- or triple-expansions. (For instance, combined WGCE and SSCE at the line $\lambda = \beta = \infty$.)

Here we shall discuss in some detail WGCE. In this case the expectation values at an arbitrary point (λ, β, κ) of the bare parameter space are expressed in terms of a series of expectation values at the point $(\lambda, \beta = \infty, \kappa_0)$ with vanishing gauge coupling. This is achieved by performing the integration over the gauge field variables in perturbation theory, thereby explicitly displaying the dependence on the gauge field propagators and vertices. In Ref. [5] the generating function of connected expectation values of some gauge invariant composite fields was considered in WGCE. For the study of renormalization and large cut-off behaviour it is better to expand the gauge dependent generating functions in renormalizable gauges. In order to write down the WGCE master formula for the gauge dependent Green's functions, it is useful to introduce a shorthand notation for index repetitions:

$$(f_.)_\nu^n \equiv f_{\nu_1} f_{\nu_2} \cdots f_{\nu_n} \qquad \sum_{[\nu]_n} (f_.)_\nu^n \equiv \sum_{\nu_1 \cdots \nu_n} f_{\nu_1} f_{\nu_2} \cdots f_{\nu_n} \tag{1}$$

In the master formula the gauge field expectation values $< \cdots >_{\alpha g}^c$ have to be expressed by gauge field propagators and vertices in the same way as in pure gauge perturbation theory. (α

is the gauge parameter.) The occuring expectation values in the ϕ^4 model at $g^2 = 0$ contain, in addition to the original scalar fields σ_{0x} and π_{rx}, $(r = 1, 2, 3)$, also the bilinear composite fields

$$s_{x\mu} = 2(\sigma_{0x}\sigma_{0x+\hat{\mu}} + \pi_{rx}\pi_{rx+\hat{\mu}})$$
$$u_{rx\mu} = 2i(\pi_{rx}\sigma_{0x+\hat{\mu}} - \sigma_{0x}\pi_{rx+\hat{\mu}} + \epsilon_{rst}\pi_{sx}\pi_{tx+\hat{\mu}}) \tag{2}$$

The derivatives of the generating function $W[h, i]$ with respect to $h \equiv (h_{0x}, h_{rx})$ and $i_{rx\mu}$ give the connected Green's functions of the scalar fields (σ_{0x}, π_{rx}) and, respectively, of the gauge field $A_{rx\mu}$. Let us first introduce the notations

$$C[i]^{mn}_{[rx\mu]_m[y\nu]_n} \equiv \frac{1}{2^{m+3n}m!n!}(i_{\cdot} + i\kappa u_{\cdot})^m_{rx\mu}(1 - \kappa s_{\cdot})^n_{y\nu}$$
$$A^{(\alpha)mn}_{[rx\mu]_m[y\nu]_n} \equiv \left\langle (A_{\cdot})^m_{rx\mu}(A_{s\cdot}A_{s\cdot})^n_{y\nu}\right\rangle^c_{\alpha g} \tag{3}$$

The result for the generating function of connected Green's functions W is in this notation (applying the trick (1) twice):

$$W[h, i]^{\alpha}_{\lambda\beta\kappa} = \sum_{LMN} \sum_{[X]_L[RY]_M[Z\lambda]_N} \sum_K \sum_{[m]_K[n]_K} \sum_{[[rx\mu]_m[y\nu]_n]_K} \frac{(h_{0\cdot})^L_X (h_{\cdot})^M_{RY}(\kappa - \kappa_0)^N}{K!L!M!N!}\cdot$$
$$\cdot \left(g^{m+2n}A^{(\alpha)mn}_{\cdot\cdot}\right)^K_{[rx\mu]_m[y\nu]_n} \left\langle (\sigma_{0\cdot})^L_X(\pi_{\cdot})^M_{RY}(s_{\cdot})^N_{Z\lambda}\left(C[i]^{mn}_{\cdot\cdot}\right)^K_{[rx\mu]_m[y\nu]_n}\right\rangle^c_{\lambda\kappa_0} \tag{4}$$

In the connected ϕ^4 expectation value $< \cdots >^c_{\lambda\kappa_0}$ the contents of the parentheses have to be considered as single entities in the definition of connectedness. The same applies also to $(A_{sy\nu}A_{sy\nu})$ in the connected gauge field expectation value of Eq. (3). The most useful way to apply WGCE is to choose the expansion point (λ, κ_0) in the vicinity of the critical line of the ϕ^4 model. In this case it is possible to obtain enough information about the behaviour of the ϕ^4 Green's functions by assuming the triviality of the continuum limit in ϕ^4 [6,7]. By combining the hopping parameter expansion ("high temperature expansion" in the terminology of statistical physics) and the Callan-Symanzik renormalization group equations one can, for instance, determine the curves of constant physics of ϕ^4 everywhere in the bare parameter space [7]. The knowledge of the non-perturbative dynamics of pure ϕ^4 is an input in WGCE. In what follows, we shall assume the triviality of the continuum limit in ϕ^4, but if this would finally turn out to be wrong, WGCE could still be a useful expansion in order to determine the perturbation due to a weak $SU(2)$ gauge coupling.

CURVES OF CONSTANT PHYSICS. Before going to the Higgs model, let us first formulate how to obtain differential equations for the curves of constant physics in the general case. Let us consider a lattice quantum field theory with n bare couplings $g_1, g_2, \ldots, g_n$. In order to define the CCP's one has to keep $(n-1)$ independent physical quantities $F_2, F_3, \ldots, F_n$ constant (we are assuming here that the number of relevant couplings is n):

$$F_j(g_1, \ldots, g_n) = F_{j0} = const. \qquad (j = 2, \ldots, n) \tag{5}$$

The CCP's are characterized by the constant values F_{j0}. The points of a singled out CCP can be parametrized, for instance, by the first bare coupling g_1: $g_j = g_j(g_1)$ $(j = 2, \ldots, n)$. In this case we have

$$\frac{dg_j(g_1)}{dg_1} = \frac{\det^{[1,j]}_{n-1}\left(\frac{\partial F}{\partial g}\right)}{\det^{[1,1]}_{n-1}\left(\frac{\partial F}{\partial g}\right)} \tag{6}$$

Here $\det_{n-1}^{[i,k]}(\frac{\partial F}{\partial g})$ denotes the $(n-1)\times(n-1)$ subdeterminant of the $n\times n$ derivative matrix $\frac{\partial F}{\partial g}$ belonging to the matrix element $\frac{\partial F_i}{\partial g_k}$.

Another possibility is to parametrize the points of a CCP by the value of some reference physical quantity F_1. (In practical cases F_1 is usually some physical mass in lattice units.) In this case the differential equations for $g_i(F_1)$ $(i=1,\ldots,n)$ are:

$$\frac{dg_i(F_1)}{dF_1}=\frac{\det_{n-1}^{[1,i]}\left(\frac{\partial F}{\partial g}\right)}{\det_n\left(\frac{\partial F}{\partial g}\right)} \tag{7}$$

where $\det_n(\cdots)$ is the $n\times n$ determinant of the derivative matrix.

Sometimes it is also useful to consider curves in subspaces of the bare parameter space which belong to constant values of an appropriately smaller number of physical quantities. These "curves of partially constant physics" (CPCP's) are defined by fixing $(n-k)$ physical quantities $F_2,\ldots,F_{n-k+1}$ and $(k-1)$ bare parameters $g_{n-k+2},\ldots,g_n$. The differential equations for CPCP's have the same form as Eqs. (6-7). For simplicity, let us consider here only the case with $n=3$ bare parameters (as we have in the standard Higgs model) and look at the plane with constant bare coupling g_3. Keeping the value of some physical quantity $F_2(g_1,g_2,g_3)=F_{20}$ fixed and parametrizing the points of the curve by the reference quantity F_1, the differential equation for the function $g_2(F_1)$ is:

$$\frac{dg_2(F_1)}{dF_1}=\left(\frac{-\frac{\partial F_2}{\partial g_1}}{\frac{\partial F_1}{\partial g_1}\frac{\partial F_2}{\partial g_2}-\frac{\partial F_1}{\partial g_2}\frac{\partial F_2}{\partial g_1}}\right)_{g_2=g_2(F_1,g_2,g_3)} \tag{8}$$

As an example in the standard Higgs model, one can take $g_1=\kappa$, $g_2=\lambda$, $g_3=g^2$ and $F_1=\mu_W$ (the W-mass), $F_2=R_{HW}\equiv m_H/m_W$ (the ratio of Higgs- to W-mass). In this case Eq. (8) gives the curves with constant Higgs- to W-mass ratio in the $g^2=const.$ planes.

As discussed before, in WGCE the expectation values at a point (λ,g^2,κ) with weak bare gauge coupling and arbitrary bare scalar self-coupling are given in WGCE by explicit gauge propagators and vertices and by scalar blobs representing expectation values in the pure ϕ^4 model at $g^2=0$ and (λ,κ_0). We are assuming that the continuum limit in ϕ^4 is trivial therefore, if the point (λ,κ_0) is close to the critical line, the renormalized ϕ^4 coupling λ_r in the pure ϕ^4 model is small and the renormalized ϕ^4 Green's functions can be well approximated by a low order perturbative expansion in λ_r. In WGCE we need unrenormalized $g^2=0$ expectation values, which are obtained from the renormalized ones by multiplying with the wave function renormalization factor of the scalar field Z_r and with the multiplicative renormalization factors of the composite fields Z_s, Z_u.

Since the $g^2=0$ expectation values in WGCE are given in terms of the pure ϕ^4 renormalized coupling λ_r, it is natural to parametrize the points of the bare parameter space, instead of (λ,g^2,κ), by (λ_r,g^2,κ). (In this case the ϕ^4 Z-factors have to be considered also as functions of λ_r and κ: $Z_{r,s,u}=Z_{r,s,u}(\lambda_r,\kappa)$.) In this way the problem of determining the CCP's for small bare gauge coupling is reduced by WGCE to the problem of finding the CCP's, with $\lambda_r=const.$, in the ϕ^4 model at $g^2=0$. The renormalization is also decomposed in two steps: after going to the renormalized variables at $g^2=0$, λ_r is considered as one of the bare parameters for WGCE in the Higgs model. The renormalized quantities of the Higgs model are introduced in WGCE in the same way as in ordinary perturbation theory. (In what follows only 1-loop graphs will be considered. The renormalization of WGCE up to higher loops have to be investigated in the future.) The CCP's in the Higgs model can be defined by the requirement that the renormalized ϕ^4 coupling λ_R and renormalized gauge coupling squared g_R^2 be constant. (Note that capital R denotes renormalized quantities in the Higgs model, whereas small r is reserved for the renormalized quantities at $g^2 = 0$.) As a parameter along the CCP's, one can take the renormalized ϕ-mass squared μ_R^2 (or $\tau \equiv \log \mu_R^{-1}$). The differential equations corresponding to Eq. (7) are, in the large cut-off (small μ_R^2) limit up to leading order:

$$\frac{d\lambda_r(\tau)}{d\tau} = -\frac{9}{16\pi^2}\lambda_r g^2 + \cdots \qquad \frac{dg^2(\tau)}{d\tau} = -\frac{43}{48\pi^2}g^4 + \cdots \tag{9}$$

The dots stand here for higher orders in λ_r and g^2. In what follows the higher order terms will be neglected, although an estimate of their importance can only be given after a more detailed study of multiloop renormalization of WGCE. The solution of the asymptotic equations is:

$$g^2(\tau) = \left[g_0^{-2} + \frac{43}{48\pi^2}(\tau - \tau_0)\right]^{-1} \qquad \lambda_r(\tau) = \lambda_{r0}\left[1 + \frac{43 g_0^2}{48\pi^2}(\tau - \tau_0)\right]^{-\frac{27}{43}} \tag{10}$$

Here g_0^2 and λ_{r0} are the initial values at $\tau = \tau_0$.

Since both $g^2(\tau)$ and $\lambda_r(\tau)$ tend to zero for $\tau \to \infty$, WGCE is an asymptotically free expansion. This does not, however, mean that for $\tau \to \infty$ a non-trivial continuum limit exists. The reason is that on the (τ, λ_r) plane not every point is possible. The triviality of the continuum limit of ϕ^4 implies that the CCP's in ϕ^4 with $\lambda_r = const.$ are ending at $\lambda = \infty$ near the critical point $\kappa_{cr}(\lambda = \infty)$ for some finite cut-off [7]. This means that on the (τ, λ_r) plane there is a limiting curve and the allowed points are below this (at smaller values of τ and λ_r). To obtain the exact shape of the limiting curve is a non-perturbative problem in the four-component $O(4)$-symmetric ϕ^4 model. In order to have a rough guess about its qualitative behaviour, one can take for large τ the position of the "Landau-pole" in one-loop perturbation theory:

$$\lambda_r(\tau)_{max} \simeq \frac{\pi^2}{6\tau} \tag{11}$$

The intersection of the curve $\lambda_r(\tau)$ in Eq. (10) with $\lambda_r(\tau)_{max}$ determines the maximal cut-off τ_{max} which belongs to the Higgs model CCP given by $(g^2(\tau), \lambda_r(\tau))$. If the maximal cut-off is required to be the Planck mass ($\tau \simeq 20$), this crude estimate for a CCP with gauge coupling $g_R^2 = 0.5$, roughly equal to the physical value in the standard electroweak model, gives λ_R about a factor of 2 larger than a one-loop perturbative calculation [8] would give. In order to obtain this "Planck mass cut-off upper limit" more precisely for λ_R (or for the Higgs mass to W-mass ratio $R_{HW} \equiv m_H/m_W$), we need a careful non-perturbative study of the 4-component ϕ^4 model to pin down the limiting curve $\lambda_r(\tau)_{max}$. In addition, a non-perturbative investigation in the Higgs model itself is also necessary for obtaining the non-asymptotic form of $\lambda_r(\tau)$.

The above asymptotic estimates determine the qualitative behaviour of the CCP's for small g^2 in the (λ, g^2) plane (assuming that the higher order contributions can be neglected, indeed). For a schematic picture see Fig. 1. Since every CCP with non-zero λ_R and g_R^2 is ending at a finite cut-off in the $\lambda = \infty$ plane near the phase transition line, the only possibility to reach an infinite cut-off is to put $\lambda_R = g_R = 0$. Therefore, the continuum limit in the Higgs model on the critical line at $g^2 = 0$ is trivial, both in the confining- and

Higgs-phase. The triviality of the $g^2 = 0$ continuum limit in the Higgs phase was recently concluded also in Ref. [11] on the basis of perturbation theory near the Gaussian fixed point ($\lambda = 0, g^2 = 0, \kappa = 1/8$). Perturbation theory is, however, not applicable for large bare self-coupling λ, therefore Ref. [11] did not exclude the possibility of a non-trivial fixed point in the combined gauge-scalar system at $g^2 = 0$ and large λ. In any case, the non-trivial λ-independent continuum limit conjectured in Ref. [9] is not possible. The behaviour of the CCP's near $\beta = \infty$ is different from the picture suggested there. The only open possibility for a search of a non-trivial continuum limit in the standard Higgs model is to go inside the bare parameter space to points where also the gauge coupling is non-perturbative. However, even if such a fixed point would exist, it would not necessarily be adequate for the description of the standard electroweak physics.

The framework of WGCE is obviously more general than the specific case of the standard $SU(2)$ Higgs model. It would certainly be interesting to consider in the future more general Higgs models, too. In particular, as one can see from Eq. (10), there is an interesting class of models, where the Callan-Symanzik β-function coefficients are such that the power of the squared brackets in $\lambda_r(\tau)$ is, instead of $\frac{-27}{43}$, equal to -1. In this case the leading asymptotic behaviour of $\lambda_r(\tau)$ coincides with the asymptotics of the limiting curve in Eq. (11). The question of a possible non-trivial continuum limit at $\lambda = \infty$ is then decided on the next-to-leading order level. In any case, even if the strict continuum limit would turn out to be trivial, such models are interesting, because they can easily allow for very large cut-off's in a wide range of physical situations. A simple example of a model with a τ^{-1} leading λ_r-behaviour is an $SU(2)$ Higgs model with 1 scalar doublet and 4 vector-like spin-$\frac{1}{2}$ fermion doublets. Namely, in this case the coefficient of the g^4 term in Eq. (9) is equal to $-27/(48\pi^2)$. Because of the vector-like fermions, Yukawa-couplings are forbidden. In cases with Yukawa-couplings and chiral fermions (as in the standard model) the appropriate lattice formulation has to be constructed first, and similar questions can be asked only afterwards.

MONTE CARLO CALCULATIONS AT WEAK GAUGE COUPLING: The SU(2) coupling in the standard electroweak sector is weak: at the W-mass scale we have $g_R^2 \simeq 0.5$ (or $\alpha_{SU(2)} \equiv g_R^2/(4\pi) \simeq 0.04$). Since the Monte Carlo calculations are done in a region where the W-mass in lattice units am_W is of the order 1, this implies for the bare gauge coupling a value of about $\beta \simeq 8$. A numerical Monte Carlo calculation in this β region is feasible and it can yield interesting information about the non-perturbative behaviour in the strong self-coupling regime [12]. Such calculations are complementary to the information one can obtain from WGCE. The shape of the regions where Monte Carlo calculations are interesting and where WGCE can give a good approximation is schematically shown in Fig. 2, on a $\lambda = const.$ plane. The figure is optimistic in the sense that the MC and WGCE regions touch. In reality there may be some no-man's-land inbetween, where the correlation lengths are too large for a numerical investigation but not large enough to make the couplings small enough for a low order WGCE. The phase transition line shown in Fig. 2 is given in the small g^2 region by WGCE as

$$\kappa_{cr}(\lambda, g^2) = \kappa_{cr}(\lambda, 0)\{1 + g^2 0.04358\ldots\} \tag{12}$$

The Monte Carlo calculation at $\beta = 8$ was restricted in Ref. [12] to two points at $\lambda = 1.0$. To explore the λ-dependence in the large bare self-coupling region, the same calculation has been repeated recently on 12^4 lattice in a few other points at $\beta = 8$ [13]. The points were selected in such a way that the link expectation value L was nearly constant. This is known from previous studies [9] to minimize the λ-dependence for a given β. A sample of the numerical results is given in Table I. (For the notations see [12].)

Table I.

Sample numerical results in the standard Higgs model at $\beta = 8.0$ on 12^4 lattice.

λ	κ	L	P	am_W	am_H	$\alpha_{SU(2)}^{(R=3)}$	R_{HW}
0.1	0.177	0.3816(1)	0.09592(1)	0.208(10)	0.96(5)	0.0511(15)	4.6(5)
1.0	0.280	0.3695(1)	0.09599(1)	0.190(10)	1.21(5)	0.0521(8)	6.4(8)
∞	0.370	0.36919(4)	0.09600(1)	0.198(12)	1.49(6)	0.0525(15)	7.5(8)

As it can be seen from the table, the renormalized gauge coupling is, within errors, the same in the three points. The Higgs- to W-mass ratio R_{HW} is monotonously increasing. The value of R_{HW} at $\lambda = \infty$ can be considered as a first rough estimate for an upper limit [14] of the Higgs mass:

$$R_{HW}^{max} \simeq 7.5 \pm 0.8 \tag{13}$$

Of course, our 12^4 lattice is not big enough for $\lambda = \infty$, we need many other points to check the λ- and β-dependence of R_{HW} etc. Nevertheless, R_{HW}^{max} can be determined in the future to a good precision, with reasonable amount of computer time on existing computers.

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Figure captions

Fig. 1. The qualitative picture of CCP's in the standard Higgs model projected on the (g^2, λ) plane. The small g^2 behaviour is the result of WGCE. The extension to larger g^2 is a guess supported by some approximate numerical Monte Carlo calculations at $\lambda = \infty$, $\beta = 2 - 3$ [9,10]. Note that in reality there is a two-parameter family of CCP's, but here only a one-parameter subset is shown for simplicity.

Fig. 2. The schematic lay-out of the regions where interesting Monte Carlo calculations can be done (MC) and where WGCE can be expected to give a good approximation (WGCE). The uninteresting region of dominant lattice artifacts is denoted by LA. The confining-Higgs phase transition is at the dashed line. The whole picture is for $\lambda = const.$.

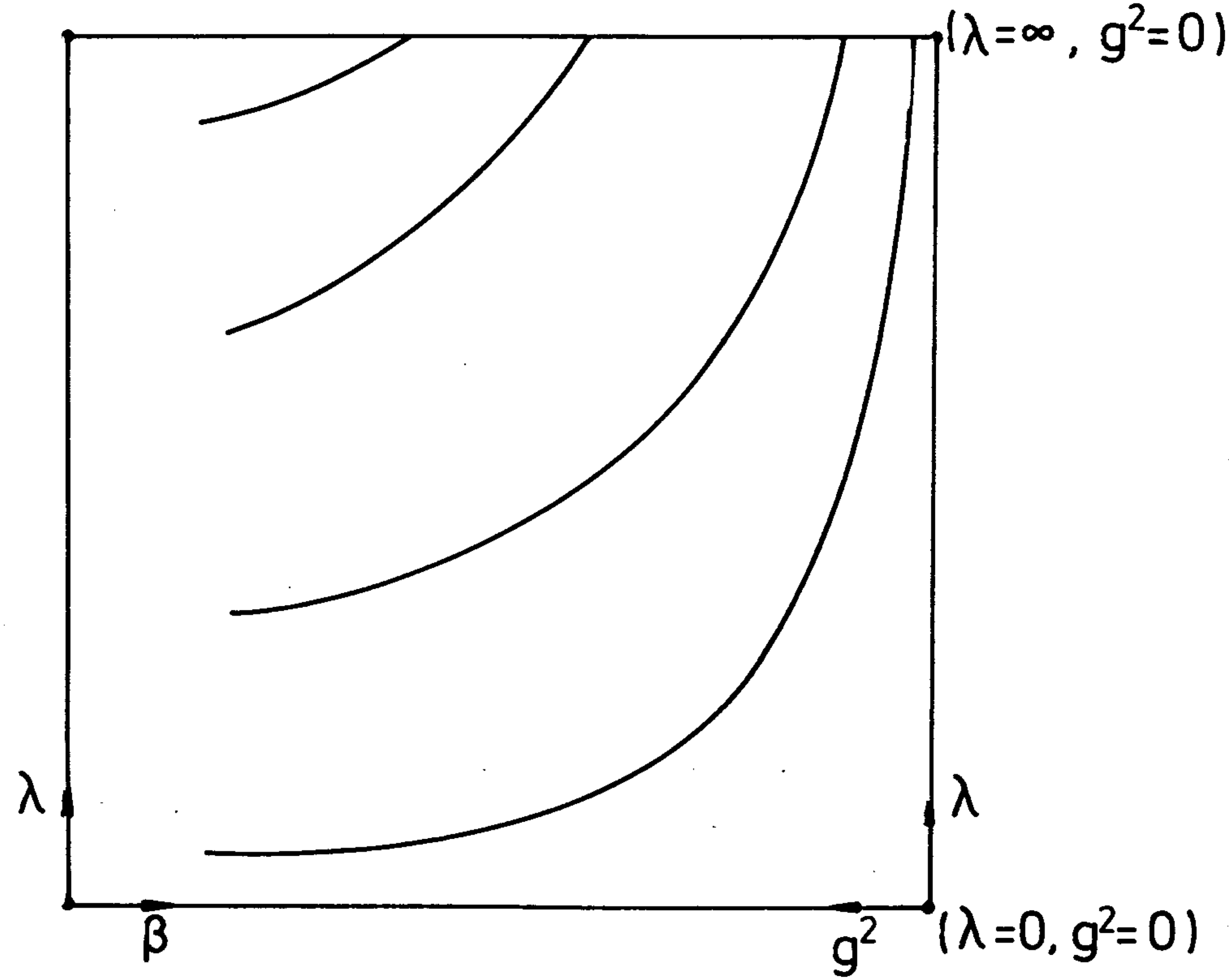

Fig. 1.

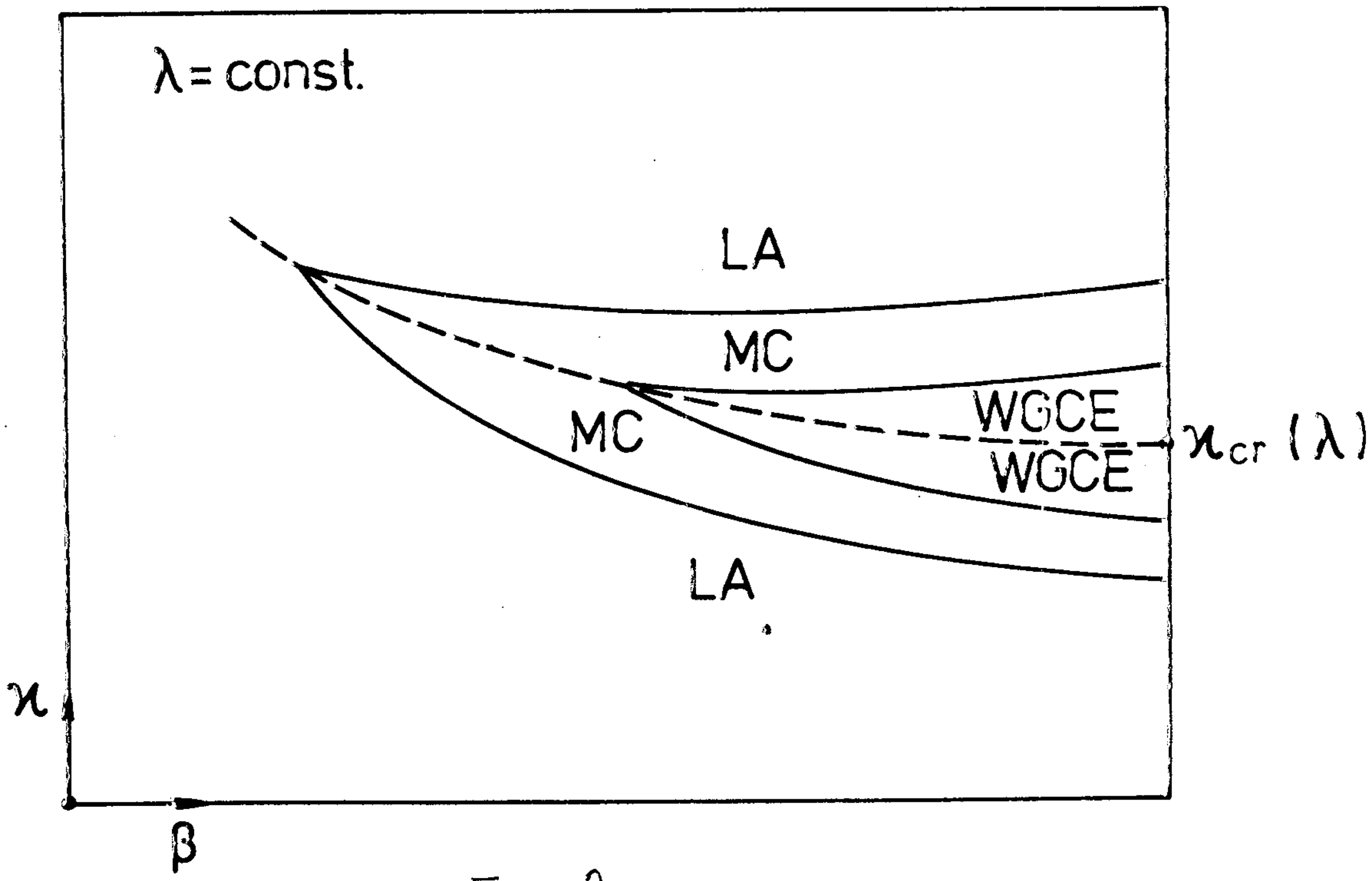

Fig. 2.