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INTRODUCTION

One of the remarkable successes of quantum chromodynamics (QCD), the theory of hadrons, is the solution of the U(1) problem [1]. In a quark model with three flavours one expects as a consequence of spontaneous chiral symmetry breaking nine pseudo-Goldstone bosons. Eight of them, which are related to the breaking of the nonabelian axial $SU(3)_A$ symmetry, can be identified as pions, kaons and η. The candidate for the ninth boson, however, the isoscalar meson η', is too heavy to be the pseudo-Goldstone boson of the broken axial $U(1)_A$ symmetry.

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The solution of this U(1) problem rests on two fundamental properties of gauge theories with fermions: the Adler-Bell-Jackiw anomaly [2] of the axial U(1) current and the existence of gauge field configurations with nontrivial topology, such as instantons [3], which give rise to a new parameter Θ characterizing the QCD vacuum [4]. As recent numerical QCD lattice calculations [5] show, the deviation of the η'-mass from the naively expected pseudo-Goldstone boson mass can indeed be attributed to gauge field configurations with non-zero topological charge.

The Θ-vacuum of QCD, which solves the U(1) problem, creates a new one: the strong CP problem. The new parameter Θ in the QCD lagrangian multiplies an operator which violates P, T and CP invariance. This leads to a non-zero electric dipole moment of the neutron, and from current experimental bounds one infers $\Theta < 10^{-9}$! Why should the a priori arbitrary Θ-parameter be so small? This "strong CP problem" is, contrary to the U(1) problem, to some extent a matter of taste. It is not a discrepancy between theory and experiment, and the QCD lagrangian contains also other "unexplained" small parameters such as the current quark masses. However, the small value of the Θ-parameter becomes even more miraculous when the generation of quark masses in the Glashow-Weinberg-Salam theory is taken into account. In this case the intrinsic QCD Θ-parameter and the argument of the determinant of the complex quark mass matrix have to compensate each other to less than one part in 10^9, a cancellation which demands an explanation!

The most elegant "solution" of the strong CP problem is the Peccei-Quinn mechanism [6], where a spontaneously broken chiral U(1) symmetry yields $\overline{\Theta} = 0$. An unavoidable consequence of this mechanism is the appearance of a very light pseudo-Goldstone boson, the axion [7], which has been extensively searched for experimentally. Although the standard axion, including its recent variants, has been ruled out [8], the so called "invisible" axions remain an interesting possibility.

The main subject of these lectures are general properties of axions and recent suggestions of how to detect invisible axions. After a brief review of the strong CP problem and the Peccei-Quinn mechanism in sects. 2 and 3, we will discuss axion properties by means of an effective lagrangian approach in sect. 4. Experimental bounds on axion production and decays are reviewed in sect. 5. Sect. 6 deals briefly with the recently proposed variant axion models, and in sect. 7 we discuss the possible relevance of supersymmetry to the strong CP problem. In sect. 8

we then review the different proposals for the detection of invisible axions. Some conclusions are given in sect. 9.

QCD-VACUUM STRUCTURE AND THE STRONG CP PROBLEM

The theory of mesons, baryons and their strong interactions is quantum chromodynamics which, in the case of two quark flavours, is given by the lagrangian

$$L = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} + \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R + m_u \bar{u}_L u_R + m_d \bar{d}_L d_R + c.c., \qquad (2.1)$$

where

$$q = \binom{u}{d}, \quad q_{L,R} = \frac{1\mp\gamma_5}{2} q, \quad D_\mu = \partial_\mu - i g_s \frac{\lambda_A}{2} G^A_\mu .$$

As the current quark masses m_u and m_d are small compared to the QCD scale parameter Λ_{QCD}, the lagrangian (2.1) possesses an approximate chiral invariance $U(2)_L \times U(2)_R$ which is spontaneously broken to the vectorial subgroup $SU(2)_V \times U(1)_V$ through the condensates

$$\langle \bar{u}u \rangle_o = \langle \bar{d}d \rangle_o = O(\Lambda^3_{QCD}) . \qquad (2.2)$$

The spontaneous breaking of the approximate axial symmetry $SU(2)_A \times U(1)_A$ gives rise to pseudo-Goldstone bosons associated with the isovector and isoscalar currents $A_{a\mu}$ and $A_{s\mu}$:

$$\pi^\pm, \pi^o \quad : \quad A_{a\mu} = \frac{1}{2} \bar{q}\gamma_\mu\gamma_5\tau_a q \qquad (2.3a)$$

$$\tilde{\eta} \quad : \quad A_{s\mu} = \frac{1}{2} \bar{q}\gamma_\mu\gamma_5 q . \qquad (2.3b)$$

In the realistic case of three flavours u, d and s the isoscalar $\tilde{\eta}$ corresponds to the pseudoscalar meson η' (up to a small mixing with the η-meson). For simplicity we will ignore in the following the existence of more than two quark flavours whenever they are unimportant for the properties of axions.

The U(1) problem of QCD is the discrepancy between the current algebra prediction [9]

$$m^2_{\eta'} < 3\, m^2_\pi \qquad (2.4)$$

and the experimental observation

$$m^2_{\eta'} > 50\, m^2_\pi . \qquad (2.5)$$

The solution of this U(1) problem was found in 1976 by 't Hooft [4] who observed that the current algebra prediction is invalidated by two subtle properties of the quantum theory based on the lagrangian (2.1): the Adler-Bell-Jackiw (ABJ) anomaly [2] and the Θ-vacuum [4].

According to the ABJ anomaly the isoscalar axial current $A_{s\mu}$ is not conserved:

$$\partial^\mu A_{s\mu} = n_f \frac{g_s^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{A\mu\nu}, \qquad (2.6)$$

$$\tilde{G}^A_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} G^{A\rho\sigma},$$

where n_f is the total number of quark flavours. However, this nonconservation of the isoscalar current is not sufficient to evade the current algebra prediction (2.4). What is needed in addition is the existence of gauge field configurations for which the topological charge is different from zero although the topological charge density, which appears on the r.h.s. of eq. (2.6), is a total divergence:

$$Q[G^A_\mu] = \frac{g_s^2}{32\pi^2} \int d^4x\, G^A_{\mu\nu} \tilde{G}^{A\mu\nu} = \int d^4x\, \partial^\mu K_\mu \neq 0 . \qquad (2.7)$$

The simplest gauge fields with non zero topological charge are the well known instantons [3].

Eqs. (2.6) and (2.7) imply that the η'-mass is larger than the current algebra prediction (2.4). Within the effective lagrangian approach for pseudo-Goldstone bosons one obtains from the anomaly (2.6):

$$\Delta L_{\tilde{\eta}} = \frac{2}{f_\pi} \tilde{\eta} \frac{g_s^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{A\mu\nu} . \qquad (2.8)$$

Integrating out the gluons then yields a mass term (cf. Fig. 1):

$$\Delta L_{AN} = -\frac{1}{2}\, m^2_{AN} \tilde{\eta}^2 , \tag{2.9}$$

$$m^2_{AN} = O(\Lambda^2_{QCD}) \quad .$$

The quantitative determination of the anomaly contribution m^2_{AN} to the η'-mass is a difficult problem because it requires nonperturbative methods. In an expansion in the number of colors n_c one finds the mass formula [10,11] (f_π = 93 MeV)

$$m^2_{\eta'} + m^2_{\eta} - 2m^2_{K_o} = \frac{2n_f}{f_\pi^2} \chi_t + O(\frac{1}{n_c}) , \tag{2.10}$$

where

$$\chi_t = \frac{1}{32\pi^2} \frac{1}{V} \langle \int_V d^4x\, G^A_{\mu\nu} \tilde{G}^{A\mu\nu} \rangle \tag{2.11}$$

is the so called topological susceptibility. For $n_f = n_c = 3$ one obtains from eq. (2.10) and the known η'-, η- and K_o-masses

$$\chi_t^{EXP} \cong (180 \text{ MeV})^4 \quad . \tag{2.12}$$

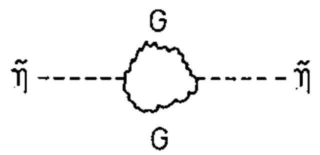

Fig. 1 Anomaly contribution to the $\tilde{\eta}$-mass

Recently χ_t has for the first time been determined numerically for the colour group SU(3) by means of lattice Monte Carlo calculations with the result [5]

$$\chi_t^{MC} = (247^{+28}_{-43} \text{ MeV})^4 , \tag{2.13}$$

where the errors are purely statistical. This result is very encouraging because it shows that topological effects are indeed large enough to explain quantitatively the η'-mass.

The existence of gauge field configurations with non zero topological charge implies that the QCD vacuum is characterized by a new quantity, the Θ-parameter. For a given value of Θ the QCD lagrangian is modified by

$$L_\Theta = -\Theta \frac{g_s^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{A\mu\nu}$$

$$= \Theta \frac{g_s^2}{8\pi^2} \vec{E}^A \cdot \vec{B}^A , \tag{2.14}$$

where $\vec{E}^A$ and $\vec{B}^A$ are the colour electric and magnetic field strengths defined in the usual way. Clearly, L_Θ violates the discrete symmetries P, T and CP.

The CP violation in strong interactions due to the Θ-term of the QCD lagrangian manifests itself in a nonvanishing electric dipole moment of the neutron. One easily estimates

$$d_N \sim \frac{e}{m_N} (\frac{m_u m_d}{m_u + m_d} \Theta) \frac{1}{\Lambda_{QCD}} \quad . \tag{2.15}$$

Here $\frac{e}{m_N}$ is the usual dipole moment factor, $\frac{m_u m_d}{m_u+m_d}\Theta$ is the phase of a chiral rotation which eliminates the Θ-term from the lagrangian at the cost of complex quark masses, and the last factor follows on dimensional grounds. With $m_u \sim \frac{m_d}{2} \sim 5$ MeV and $\Lambda_{QCD} \sim 300$ MeV one finds

$$d_N \sim 2 \cdot 10^{-16}\, \Theta \text{ e cm}, \tag{2.16}$$

which is close to the values obtained in explicit calculations:

$$d_N = \begin{cases} 2.7 \cdot 10^{-16}\, \Theta \text{ e cm}, & \text{Baluni [12]} \\ 5.2 \cdot 10^{-16}\, \Theta \text{ e cm}, & \text{Crewther et al. [13].} \end{cases} \tag{2.17}$$

The theoretical predictions (2.17) have to be contrasted with the experimental bound [14]

$$d_N < 4 \cdot 10^{-25} \text{ e cm}, \tag{2.18}$$

which implies $\Theta < 10^{-9}$! Why should Θ be so small? One might argue that the value $\Theta = 0$ is "natural" in the sense that the discrete symmetries are conserved in this case. Yet this argument is not possible anymore if QCD is considered as part of the standard model of strong and electroweak interactions where CP invariance cannot be imposed on the entire lagrangian. In this case Θ is replaced by $\overline{\Theta}$ which contains also a contribution from the quark mass matrix M:

$$\overline{\Theta} = \Theta + \arg\det M \,. \tag{2.19}$$

It is totally unclear why the two a priori unrelated parameters Θ and argdet M should compensate each other to 1 part in 10^9 - which is the strong CP problem.

Let us briefly mention that similar "strong CP problems" occur in many technicolour and preon models where Higgs bosons, quarks and leptons have a further substructure. For instance, if a composite SU(2)-doublet Higgs field ϕ contains coloured fermionic constituents and the hypercolour gauge interactions are not CP conserving, one expects an effective interaction of the form

$$\Delta L_\Theta = \frac{g_s^2}{\Lambda_c^2}\, \phi^+\phi\, G^A_{\mu\nu}\, \tilde{G}^{A\mu\nu}, \tag{2.20}$$

where Λ_c is the scale of the confining hypercolour group. After spontaneous breaking of the electroweak symmetry with $\langle\phi^+\phi\rangle_o \sim G_F^{-1}$ the effective interaction (2.20) generates the contribution to the Θ-parameter $\delta\Theta_c = 32\pi^2 \frac{v^2}{\Lambda_c^2}$. The bound $\delta\Theta_c < 10^{-9}$ then implies $\Lambda_c > 10^5$ TeV [15] which illustrates the potentially large size of CP violating effects in substructure theories with a hypercolour scale in the TeV range.

3. THE PECCEI-QUINN MECHANISM

The first step towards the Peccei-Quinn solution of the strong CP problem is to realize that in the case of at least one vanishing quark mass Θ is an irrelevant parameter. Consider, for instance, the case $m_u = o$. The QCD lagrangian (2.1) is then invariant under the chiral transformation

$$u_L \to (1-i\delta\alpha)u_L, \quad u_R \to (1+i\delta\alpha)u_R, \tag{3.1}$$

under which the lagrangian changes by

$$\begin{aligned}\delta L &= -\delta\alpha\, \partial_\mu (\bar{u}\gamma^\mu\gamma_5 u) \\ &= -\delta\alpha \frac{g_s^2}{32\pi^2}\, G^A_{\mu\nu}\, \tilde{G}^{A\mu\nu},\end{aligned} \tag{3.2}$$

where the second equality follows from the ABJ anomaly (2.6). Eq. (3.2) corresponds to a change in Θ:

$$\Theta \to \Theta' = \Theta + \delta\alpha \,. \tag{3.3}$$

Hence we can achieve $\Theta' = 0$ by means of a finite chiral rotation, i.e., we can "rotate Θ away". As the current quark masses are finite, however, the Θ-parameter cannot be rotated away in QCD.

The essential idea of the Peccei-Quinn (PQ) mechanism [6] is that the Θ-parameter can also be set to zero by means of a spontaneously broken chiral U(1) symmetry. As Weinberg and Wilczek have pointed out unavoidable consequence of this mechanism is the appearance of a very light pseudo-Goldstone boson, the axion. So far, despite remarkable experimental efforts, axions have not been discovered. Nevertheless the Peccei-Quinn mechanism appears to be the natural solution of the strong CP problem since it makes use of a chiral symmetry to which the meaning of the Θ-parameter is tied.

The simplest model, which realizes the Peccei-Quinn mechanism, contains two Higgs doublets with $SU(2)_W \times U(1)_Y$ quantum numbers

$$\phi_1 \sim (2; -\tfrac{1}{2}), \quad \phi_2 \sim (2; +\tfrac{1}{2}), \tag{3.4}$$

which couple separately to up and down quarks:

$$L_{Yuk} = \Gamma_u \bar{q}_L \phi_1 u_R + \Gamma_d \bar{q}_L \phi_2 d_R + c.c. \,. \tag{3.5}$$

The lagrangian is invariant under the chiral $U(1)_{PQ}$ transformation

$$q_L \to e^{-i\frac{\alpha}{2}} q_L \tag{3.6a}$$

$$u_R \to e^{i\frac{\alpha}{2}} u_R, \quad d_R \to e^{i\frac{\alpha}{2}} d_R, \tag{3.6b}$$

$$\phi_1 \to e^{-i\alpha}\phi_1, \quad \phi_2 \to e^{-i\alpha}\phi_2. \tag{3.6c}$$

It is clear from (3.6c) that the $U(1)_{PQ}$ invariance is lost in the minimal standard model with a single Higgs doublet where $\phi_2 = i\sigma_2\phi_1^*$. As the $U(1)_{PQ}$ symmetry acts differently on left and right handed quarks the associated current has a triangle anomaly with the colour vector current (cf. Fig. 2) and the Θ-parameter can be rotated away by means of a $U(1)_{PQ}$ transformation.

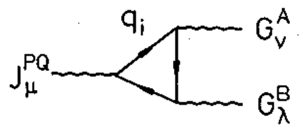

Fig. 2 Anomaly of the Peccei-Quinn current

The vacuum expectation values $(v_1^2 + v_2^2 \equiv v^2 = \frac{1}{2\sqrt{2}} G_F^{-1})$

$$\langle\phi_1\rangle_o = \begin{pmatrix} v_1 \\ o \end{pmatrix}, \quad \langle\phi_2\rangle_o = \begin{pmatrix} o \\ v_2 \end{pmatrix} \tag{3.7}$$

now break the symmetry $SU(2)_W \times U(1)_Y \times U(1)_{PQ}$ to the subgroup $U(1)_{EM}$. Hence one obtains, in addition to three massive vector bosons, a new very light pseudo-Goldstone boson, the axion, whose tiny mass $m_a = O(\frac{\Lambda^2_{QCD}}{v})$ is an effect of the anomaly. The axion mass must also vanish if at least one of the current quark masses m_q is zero, because in this case there exists an additional chiral $U(1)_A$ symmetry and hence an anomaly free $U(1)$ subgroup of $U(1)_{PQ} \times U(1)_A$ which is spontaneously broken by the vacuum expectation values (3.7). Therefore the axion mass depends on two symmetry breaking parameters, the anomaly term $m^2_{AN} \sim m^2_{\eta'} - m^2_{\pi} \sim \Lambda^2_{QCD}$, and the chiral symmetry breaking current quark masses m_q. The dependence of the masses of the pseudoscalar mesons η', $\pi°$ and a on these two symmetry breaking parameters is summarized in table I.

Table I η'-, $\pi°$- and a-masses for different explicit symmetry breakings.

Masses	$m^2_{AN} \neq o$		$m^2_{AN} = o$	
	$m_q = o$	$m_q \neq o$	$m_q = o$	$m_q \neq o$
$m^2_{\eta'}$	>o	>o	o	>o
$m^2_{\pi°}$	o	>o	o	>o
m^2_a	o	>o	o	o

4. AXION PROPERTIES

The couplings of axions to other particles, in particular the mixings with the neutral mesons $\pi°, \eta$ and η', are most easily computed by means of the effective lagrangian technique for pseudo-Goldstone bosons. Our discussion in this section follows closely the recent work of Bardeen, Peccei and Yanagida [16].

The axion field is a linear combination of the phases of the two neutral complex Higgs scalars $\phi_i°$, i=1,2:

$$\phi_i° = \frac{1}{\sqrt{2}} (v_i + \rho_i) e^{i\frac{\xi_i}{v_i}} . \tag{4.1}$$

One easily finds for the longitudinal component of the neutral Z-boson and the axion $(v = \sqrt{v_1^2 + v_2^2})$:

$$\xi = \frac{v_1}{v} \xi_1 - \frac{v_2}{v} \xi_2, \tag{4.2a}$$

$$a = \frac{v_2}{v} \xi_1 + \frac{v_1}{v} \xi_2. \tag{4.2b}$$

In order to compute the axion couplings to quarks and leptons one may set $\xi = \rho_i = 0$ and insert into the lagrangian (3.5)

$$\phi_1° = \frac{v_1}{\sqrt{2}} e^{i\frac{xa}{v}}, \quad \phi_2° = \frac{v_2}{\sqrt{2}} e^{i\frac{a}{xv}} , \tag{4.3}$$

where

$$x = \frac{v_2}{v_1} \quad . \tag{4.4}$$

If one defines the chiral $U(1)_{PQ}$ symmetry through the transformation of the axion field

$$a \to a + \alpha v, \tag{4.5}$$

there is still an ambiguity in the transformation of the quark field corresponding to an arbitrary admixture of the vectorial baryon number $U(1)_B$. The simplest definition of $U(1)_{PQ}$ acts trivial on the lefthanded quarks,

$$q_L \to q_L, \quad u_R \to e^{-i\alpha x} u_R, \quad d_R \to e^{-i\frac{\alpha}{x}} \quad , \tag{4.6}$$

yielding the current

$$J^{PQ}_\mu = v\partial_\mu a + x\, \bar{u}_R\gamma_\mu u_R + \frac{1}{x}\bar{d}_R\gamma_\mu d_R \tag{4.7}$$

with the ABJ anomaly

$$\partial^\mu J^{PQ}_\mu = n_f(x + \frac{1}{x}) \frac{g_s^2}{32^2} G^A_{\mu\nu} G^{A\mu\nu} \quad , \tag{4.8}$$

where n_f is the total number of quark-lepton families.

Inserting (4.3) into the standard model lagrangian one easily finds

$$L = L_{gauge} + \bar{q}_L i\not{\partial} q_L + \bar{q}_R i\not{\partial} q_R + \frac{1}{2}\partial_\mu a\partial^\mu a$$

$$- m_u\bar{u}_L u_R e^{i\frac{xa}{v}} - m_d\bar{d}_L d_R e^{i\frac{a}{xv}} - m_e\bar{e}_L e_R e^{i\frac{za}{v}} + c.c., \tag{4.9}$$

where

$$z = \frac{1}{x} \quad \text{or} \quad -x \quad , \tag{4.10}$$

depending on which Higgs field couples to leptons. By means of the local transformation

$$u_R \to e^{-i\frac{xa}{v}} U_R, \quad d_R \to e^{-i\frac{a}{xv}} d_R, \; e_R \to e^{-i\frac{za}{v}} e_R \tag{4.11}$$

one then obtaines the result

$$L = L_{gauge} + \bar{q}_L i\not{\partial} q_L + \bar{q}_R i\not{\partial} q_R + \frac{1}{2}\partial_\mu a\partial^\mu a$$

$$- \frac{a}{v}(xm_u\bar{u}i\gamma_5 u + \frac{1}{x} m_d\bar{d}i\gamma_5 d + zm_e\bar{e}i\gamma_5 e)$$

$$+ \frac{a}{v} n_f(x + \frac{1}{x}) \frac{g_s^2}{32\pi^2} G^A_{\mu\nu}\tilde{G}^{A\mu\nu}$$

$$+ \frac{a}{v} n_f(\frac{4}{3}x + \frac{1}{3}x + z)\frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} \quad , \tag{4.12}$$

where

$$B_{\mu\nu} = \cos\Theta_w F^\gamma_{\mu\nu} - \sin\Theta_w F^z_{\mu\nu} \quad ,$$

$$g' = \frac{e}{\cos\Theta_w} \quad ,$$

and $F^{\gamma,z}_{\mu\nu}$ are the abelian field strengths of photon and Z-boson. The lagrangian (4.12) contains all couplings of the axion to quarks, leptons, gluons, photon and Z-boson. The couplings to gauge bosons arise from the ABJ anomalies, the couplings to fermions are, as expected, pseudoscalar. All couplings scale like the inverse of the symmetry breaking vacuum expectation value v. In (4.12) n_f denotes the total number of quark-lepton generations.

The axion mass and its mixings with $\pi°$ and $\tilde{\eta}$ (η') are most easily computed from the effective chiral meson lagrangian with approximate $U(2)_L \times U(2)_R \times U(1)_{PQ}$ symmetry. The nonlinear realization of the chiral symmetry on the meson fields

$$U = e^{\frac{i}{f_\pi}(\pi^a\tau_a + \tilde{\eta}1)} \tag{4.13}$$

is defined through

$$U \to g_L U g_R^\dagger \quad , \tag{4.14}$$

where $g_L(g_R)$ denote $U(2)_L$ $(U(2)_R)$ matrices. The invariant meson lagrangian reads

$$L = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\,\partial_\mu a \partial^\mu a + \Delta L_{cs} + \Delta L_{AN}, \tag{4.15}$$

where D_μ is the $SU(2)_W \times U(1)_Y$ gauge covariant derivative, and ΔL_{cs} and ΔL_{AN} denote the small explicit symmetry breakings due to current quark masses and the ABJ anomaly:

$$\Delta L_{cs} = \frac{1}{2}\,\kappa\,\mathrm{tr}\left[UM + M^\dagger U^\dagger\right] + \frac{i}{2}\,\kappa\, a\,\mathrm{tr}\left[UMX - X^\dagger M^\dagger U^\dagger\right], \tag{4.16a}$$

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \quad X = \begin{pmatrix} x & 0 \\ 0 & \frac{1}{x} \end{pmatrix},$$

and

$$\Delta L_{AN} = -\frac{1}{2}\, m_{AN}^2 \left(\frac{\tilde\eta}{f_\pi} + \frac{1}{2}\, n_f (x + \frac{1}{x}) a\right)^2 . \tag{4.16b}$$

The form of ΔL_{cs} follows by inspection from eqs. (4.12) and (4.14); ΔL_{AN} is obtained from (4.14) and the requirement that the effective lagrangian (4.15) changes under chiral U(1) transformations by the anomaly term (2.6).

Expanding the meson field U in powers of $\frac{1}{f_\pi}$, one easily finds

$$\Delta L_{cs} = (m_u+m_d)\kappa - \frac{(m_u-m_d)\kappa}{2f_\pi^2}\left(\vec\pi^2 + \tilde\eta^2 + 2\,\frac{m_u-m_d}{m_u+m_d}\,\pi^\circ\tilde\eta\right) + \ldots$$
$$- \frac{(m_u-m_d)\kappa}{f_\pi}\,\frac{a}{v}\left(x\,\frac{m_u}{m_u+m_d}(\tilde\eta + \pi^\circ) + \frac{1}{x}\,\frac{m_d}{m_u+m_d}(\tilde\eta - \pi^\circ)\right) + \ldots \tag{4.17}$$

With $m_{AN}^2 \gg m_\pi^2$ and $v \gg f_\pi$, (4.16b) and (4.17) yield for pion and axion masses [16,17]:

$$m_\pi^2 = \frac{(m_u+m_d)\kappa}{f_\pi^2}, \tag{4.18}$$

$$m_a^2 = m_\pi^2 \left(\frac{f_\pi}{v}\right)^2 n_f^2 \left(x + \frac{1}{x}\right)^2 \frac{m_u m_d}{(m_u+m_d)^2} . \tag{4.19}$$

From (4.18) it is clear that the parameter κ plays the role of the chiral symmetry breaking condensate (2.2), $\kappa \sim \langle \bar u u\rangle_o \sim \langle \bar d d\rangle_o$. The axion mass formula (4.19) contains the factors (m_u+m_d) and $n_f^2(x + \frac{1}{x})^2$ reflecting the influence of the anomaly and chiral symmetry breaking on the axion mass, which we have qualitatively discussed in sect. 3 (cf. table I).

From (4.17) one also reads off the isotriplet and isosinglet axion mixings (cf. Fig. 3) which yield the transformation to physical states [16,17]:

$$\pi^\circ = \pi^\circ_{phys} + \xi_{a\pi}\, a_{phys}, \tag{4.20a}$$

$$\tilde\eta = \tilde\eta_{phys} + \xi_{a\eta}\, a_{phys}, \tag{4.20b}$$

$$a = a_{phys} - \xi_{a\pi}\,\pi_{phys} - \xi_{a\eta}\,\tilde\eta_{phys}, \tag{4.20c}$$

$$\xi_{a\pi} = \frac{f_\pi}{v}\,\lambda_3, \quad \xi_{a\eta} = \frac{f_\pi}{v}\,\lambda_s, \tag{4.20d}$$

$$\lambda_3 = \frac{1}{2}\left[(x - \frac{1}{x}) - n_f (x + \frac{1}{x})\,\frac{m_d-m_u}{m_d+m_a}\right], \tag{4.20e}$$

$$\lambda_s = -\frac{1}{2}\,(n_f - 1)(x + \frac{1}{x}) . \tag{4.20f}$$

The axion couplings to heavy quarks of charge $\frac{2}{3}$ (U = c, t, ...) and charge $-\frac{1}{3}$ (D = s, b, ...), leptons (ℓ = e, μ, τ, ...) and photons is given by (4.12) ($x = \frac{v_2}{v_1}$, $z = x$ or $-\frac{1}{x}$, $n_f = 3 + \ldots$):

$$L_{af} = -x\,\frac{m_u}{v}\,\bar u i\gamma_5 u\, a - \frac{1}{x}\,\frac{m_D}{v}\,\bar d i\gamma_5 d\, a - z\,\frac{m_\ell}{v}\,\bar\ell i\gamma_5 \ell\, a, \tag{4.21}$$

$$L_{a\gamma} = \frac{1}{4}\,\frac{\xi\alpha}{v}\,F_{\mu\nu}\tilde F^{\mu\nu} a = -\frac{\xi\alpha}{v}\,\vec E\cdot\vec B\, a, \tag{4.22}$$

$$\xi = \frac{n_f}{\pi}\left(\frac{4}{3}\,x + \frac{1}{3}\,x + z\right), \quad \alpha = \frac{e^2}{4\pi} .$$

Fig. 3 Isotriplet and isosinglet axion mixings.

We note that the axion mass and all of its couplings scale like $\frac{1}{v}$. Numerically, one finds from (4.19):

$$m_a = 150\ \text{keV} \left[\frac{(\sqrt{2}G_F)^{-1/2}}{v} \frac{n_f}{3} \frac{1}{2}\left(x + \frac{1}{x}\right)\right] . \tag{4.23}$$

For $m_a < 2m_e$ the axion lifetime is determined by the two photon decay mode:

$$\tau(a \to 2\gamma) \cong \tau(\pi^\circ \to 2\gamma)\left(\frac{v}{f_\pi}\right)^2\left(\frac{m_\pi}{m_a}\right)^3$$
$$\cong \left(\frac{100\ \text{keV}}{m_a}\right)^5 \ \text{sec.} \tag{4.24}$$

In the case $m_a > 2m_e$ the decay into an electron positron pair is dominant, and one obtains a much shorter lifetime:

$$\tau(a \to e^+e^-) = \frac{8\pi v}{m_e^2(m_a^2-4m_e^2)^{1/2}} \frac{1}{z^2}$$
$$\cong 3 \cdot 10^{-9}\ \text{sec}\left(\frac{1.7\ \text{MeV}}{m_a}\right)\frac{1}{z^2} . \tag{4.25}$$

This second case is of interest for "variant axion models" where due to a large value of x or $\frac{1}{x}$ the axion mass is substantially larger than the natural value of about 100 keV.

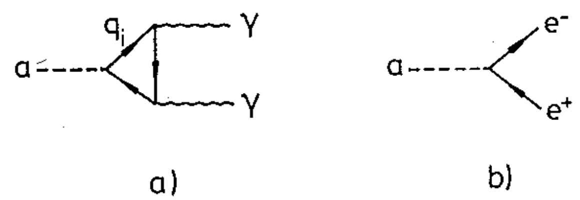

Fig. 4 Axion decays: a) $a \to 2\gamma$ (dominant for $m_a < 2m_e$),
b) $a \to e^+e^-$ (dominant for $m_a > 2m_e$)

5. EXPERIMENTAL BOUNDS ON AXIONS

Since the invention of the Peccei-Quinn mechanism more than eight years ago axions have been extensively searched for experimentally in quarkonium decays, light meson decays, beam dump experiments and nuclear deexitations. Comprehensive reviews can be found in Refs. [18,19].

Table II. Processes providing bounds on axion couplings for $m_a < 2m_e$ and/or $m_a > 2m_e$.

	$m_a < 2m_e$	$m_a > 2m_e$
meson decays		
$J/\Psi,\ \Upsilon \to a\gamma$	*	-
$K^+ \to \pi^+$ nothing $(a, \bar{\nu}\nu, \ldots)$	*	-
$\pi^+ \to \underbrace{e^+e^-}_{a}e^+\nu_e$	-	*
beam dump experiments		
$p(e^-)N \to a\,X$, $a \to \gamma\gamma, e^+e^-$	*	*
nuclear deexcitations		
$N^* \to N\,a$, $a \to \gamma\gamma,\ e^+e^-$	*	*

In table II we have listed the processes which have provided intersting constraints on axion couplings in the two cases $m_a < 2m_e$ and $m_a > 2m_e$. In the following we will discuss only the most definitive bounds which suffice to rule out axions whose $U(1)_{PQ}$ symmetry breaking scale is the Fermi scale.

(i) Quarkonium decays

The decay of heavy quarkonium into axion and photon (cf. fig. 5) has the clean signal of a monochromatic photon whose energy equals half the quarkonium mass. The branching ratio of this decay relative to the decay into μ-pairs is given by [20]

$$\frac{\Gamma((Q\bar{Q}) \to a\gamma)}{\Gamma((Q\bar{Q}) \to \mu^+\mu^-)} = \frac{G_F m_Q^2}{\sqrt{2}\pi e_Q^2 \alpha_{EM}} \cdot \begin{cases} x^2 & , \quad e_Q = \frac{2}{3} \\ \frac{1}{x^2} & , \quad e_Q = -\frac{1}{3} \end{cases} . \qquad (5.1)$$

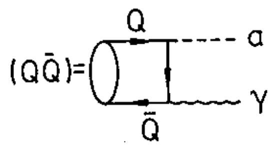

Fig. 5 Quarkonium decay into axion and photon

From the measured branching ratio into μ-pairs and eq. (5.1) one obtains the theoretical predictions

$$BR(J/\Psi \to a\gamma) = (4.9 \pm 0.8) \cdot 10^{-5}\, x^2 , \qquad (5.2a)$$

$$BR(\Upsilon \to a\gamma) = (2.7 \pm 0.7) \cdot 10^{-4} \frac{1}{x^2} , \qquad (5.2b)$$

which have to be compared with the experimental bounds [21]

$$BR(J/\Psi \to e\gamma) < 1.4 \cdot 10^{-5} \qquad (5.3a)$$

$$BR(\Upsilon \to a\gamma) < 3 \cdot 10^{-4} \qquad (5.3b)$$

(5.2) is a theoretically unambiguous prediction, and the comparison with (5.3) already rules out the standard Weinberg-Wilczek axion.

(ii) $K^+ \to \pi^+ a$

The theoretical prediction for the decay $K^+ \to \pi^+ a$ follows from the two pion decay of K^o and the isosinglet axion mixing λ_s [16]:

$$BR(K^+ \to \pi^+ a) = 2\lambda_s^2 \left(\frac{f_\pi}{v}\right)^2 \frac{p_a}{p_\pi} BR(K^o \to \pi^+\pi^-)$$
$$= 2.9 \cdot 10^{-5} \lambda_s^2 , \qquad (5.4)$$

where p_a and p_π are axion and pion momenta in the final state. In the case $m_a < 2m_e$, where the axion escapes detection, the theoretical prediction (5.4) can be compared with the KEK limit of Asano et al. [22]:

$$BR(K^+ \to \pi^+ \text{ nothing}) < 2.7 \cdot 10^{-8} . \qquad (5.5)$$

So far no bound exists for the decay chain $K^+ \to \pi^+ a (\to e^+e^-)$.

(iii) $\pi^+ \to e^+e^-e^+\nu_e$

This process provides a bound on the decay $\pi^+ \to ae^+\nu_e$ in the case $m_a > 2m_e$ where the axion can be observed through its decay into an electron-positron pair. One finds [16]

$$BR(\pi^+ \to ae^+\nu_e) = 3 \cdot 10^{-9} \lambda_3^2 . \qquad (5.6)$$

At SIN Eichler et al. [23] have recently measured the branching ratio

$$BR(\pi^+ \to e^+e^-e^+\nu_e) = (3.4 \pm 0.5) \times 10^{-9} , \qquad (5.7)$$

which agrees with standard model expectations and thereby provides the bound [23]

$$BR(\pi^+ \to ae^+\nu_e) < (1\text{-}2) \cdot 10^{-10} . \qquad (5.8)$$

(iv) Nuclear deexcitation experiments

Interesting constraints on axion couplings also follow from nuclear transition $N^* \to N\, a$. The relative magnitude with respect to hadronic and electromagnetic transitions is given by the ratio of coupling constants:

Table III. Theoretical predictions and experimental bounds for $\Gamma(N^* \to N\, a)/\Gamma(N^* \to N\gamma)$

	Theory [16]	Experiment
$^{14}N(\Delta T = 1)$ Ref. [24]	$2 \cdot 10^{-5} \lambda_3^2$	$< 4 \cdot 10^{-4}$
$^{10}B(\Delta T = 0)$ Ref. [25]	$7.9 \cdot 10^{-4} \lambda_s^2$	$< 2.6 \cdot 10^{-3}$

$$\frac{g^2_{aNN}}{g^2_{\pi NN}} \sim \left(\frac{f_\pi}{v}\right)^2 \sim 1.5 \cdot 10^{-7} , \qquad (5.9)$$

$$\frac{g^2_{aNN}}{e^2_{EM}} \sim 2.2 \cdot 10^{-4} . \qquad (5.10)$$

The ratio of the partial widths $\Gamma(N^* \to N\ a)/\Gamma(N^* \to N\gamma)$ can be rather reliably calculated. A comparison of theoretical predictions and experimental bounds is given in table III for two nuclear transitions.

The standard axion, where λ_s and λ_3 are O(1), is clearly ruled out from the K^+ and π^+ decays. Nuclear deexcitation experiments are important for variant axion models. Generally, bounds from beam dump experiments are not as stringent as the ones discussed in this section.

6. VARIANT AXION MODELS

Recent interest in axion models has partly been stimulated by heavy ion collision experiments at GSI where narrow peaks have been observed in e^+- and e^--energy spectra in coincidence. It is natural to consider the possibility that the observed e^+e^--pairs come from the decay of a new particle with a mass of 1.7 MeV given by the sum of electron and positron energies. Although the production mechanism of such a particle in heavy ion collisions has never been understood, it is an interesting question whether this hypothetical particle could be an axion.

Peccei, Wu and Yanagida [26] and Krauss and Wilczek [27] have indeed constructed "variant" axion model with an axion mass of 1.7 MeV. From

$$m_a = O(100\ \text{keV}) \cdot (x + \frac{1}{x}) \qquad (6.1)$$

one sees that either x or $\frac{1}{x}$ have to be rather large. The quarkonium bounds can be avoided by coupling c- and b-quark both to the same Higgs field, e.g. ϕ_1, and making $\frac{1}{x}$ small. From (6.1) one then finds $x \cong 70$ and hence a very short axion lifetime

$$\tau(a \to e^+e^-) \cong 6 \cdot 10^{-13}\ \text{see.} \qquad (6.2)$$

For such short-lived axions the decay $K^+ \to \pi^+$ nothing and beam dump experiments provide no constraints. One can not escape, however, the bounds from $\pi^+ \to e^+e^-\nu_e$ and nuclear deexcitation experiments. For the axion mixings with $\pi°$, η and η' one has two options [16]:
$\lambda_s = 0$, $\lambda_3 \cong 26$ or $\lambda_s \cong -27$, $\lambda_3 = 0$. In general one finds the model independent relation [16] $(\lambda_3 - \lambda_s) \cong 25^2$. This is incompatible with the experimental bounds listed in the previous section. We conclude that axions are ruled out with a $U(1)_{PQ}$ symmetry breaking scale v_{PQ} of the order of Fermi scale $G_F^{-1/2}$.

7. SUPERSYMMETRY AND THE STRONG CP PROBLEM

Supersymmetric theories are intersting with respect to the strong CP problem for a number of reasons. First of all, the supersymmetric Higgs mechanism requires at least two Higgs doublet chiral superfields with $SU(2)_W \times U(1)_Y$ quantum numbers

$$H_1 \sim (2; \frac{1}{2}) , \quad H_2 \sim (2; -\frac{1}{2}) . \qquad (7.1)$$

Hence an additional global $U(1)_{PQ}$ symmetry can occur already in the minimal supersymmetric standard model although, as we have seen in the last section, this symmetry has to be broken at a scale v_{PQ} larger than the Fermi scale $G_F^{-1/2}$. A special feature of supersymmetric theories is also that, due to non-renormalization theorems, the Θ-parameter at the Fermi scale is very small if, for some unknown reason, it vanishes at the unification or Planck mass [28].

The new particles introduced by supersymmetry can also contribute directly to the neutron electric dipole moment. The virtual exchange of scalar quarks and gluons yields, for $\mu_{\tilde{g}} \sim M_D \sim 100$ GeV and maximal phases [29] (cf. Fig. 6):

$$d_n \sim 4 \cdot 10^{-22}\ e\ \text{cm} , \qquad (7.2)$$

which exceeds the experimental bound (2.18) by three orders of magnitude! Fortunately, in a large class of supergravity models, the phases, and hence d_n, vanish at tree level.

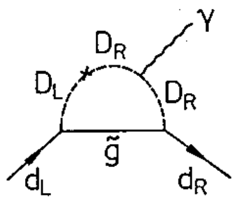

Fig. 6 Contribution to the neutron electric dipole moment from virutal gluinos and scalar quarks.

A particularly interesting aspect of supersymmetric theories is the chiral $U(1)_R$ invariance whose charge does not commute with the supersymmetry generator,

$$[R,Q_2] \neq 0 , \tag{7.3}$$

and which occurs in many supersymmetric models. Approximate R-invariance suppresses potentially large, radiatively induced operators which cause the transition $\mu \to e\gamma$ and B- and L- nonconservation. Hence one may expect [29], that in the effective low energy lagrangian R-invariance is only broken by the soft supersymmetry breaking terms:

$$\partial^\mu J^R_\mu = O(m_{3/2}) , \tag{7.4}$$

where $m_{3/2}$ is the gravitino mass. Because of the gluinos the R-current has a colour anomaly (cf. Fig. 7) and could hence serve as a PQ symmetry. Eq. (7.4) would then suggest that the axion is the pseudo-Goldstone boson of spontaneously broken R-invariance whose symmetry breaking scale is identical with the supersymmetry breaking scale $v_{SUSY} \sim (m_W m_{PL})^{1/2} \sim 10^{10}$ GeV.

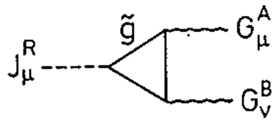

Fig. 7 Anomaly of the R-current

8. HOW TO FIND INVISIBLE AXIONS

Despite the so far fruitless experimental search for axions the Peccei-Quinn mechanism remains the most attractive solution of the strong CP problem. The experimental bounds discussed in section 5 then require that the $U(1)_{PQ}$ symmetry breaking scale is substantially larger than the Fermi-scale:

$$v_{PQ} \gg G_F^{-1/2} . \tag{8.1}$$

This means that the axion becomes (almost) massless ($m_a \sim \frac{1}{v_{PQ}}$), and that it (almost) decouples from ordinary matter ($g_a \sim \frac{1}{v_{PQ}}$). One may consider such "invisible" axions [30] to be an absurdity [31], but one may also regard them as a fascinating possibility, especially because recently various methods have been suggested by means of which invisible axions could become visible. In this section we will discuss these proposals. For recent theoretical work on invisible axions we refer the reader to the papers of ref. [32]. The interesting possibility of having truly massless Goldstone bosons ("arions"),with properties similar to those of axions, has been discussed by Anselm [33].

Constraints on the symmetry breaking scale v_{PQ} of invisible axions can be derived from astrophysical and cosmological considerations. A lower bound on v_{PQ} is obtained from the upper bound on the admisable energy loss of stars through axion emission, for instance via the Compton-type process $e\gamma \to ea$ (cf. Fig. 8). The recent analysis of Raffelt [34] yields

$$v_{PQ} > 10^7 \text{ GeV} . \tag{8.2}$$

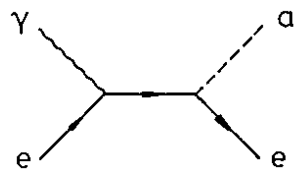

Fig. 8 Compton-type process for axion production

Less reliable is an upper bound inferred from cosmological arguments [35]

$$v_{PQ} < 10^{12}\ \text{GeV}.$$

In the following we will normalize mass and Compton wave length of invisible axions to this scale:

$$m_a \cong 10^{-5}\ \text{eV}\ \left(\frac{10^{12}\ \text{GeV}}{v_{PQ}}\right), \tag{8.3a}$$

$$\nu_a = \frac{m_a}{h} \cong 2.4\ \text{GHz}, \tag{8.3b}$$

$$\lambda\!\!\!^{-}_a = \frac{\hbar}{m_a} \cong 2\ \text{cm}. \tag{8.3c}$$

If one accepts only the lower, astrophysical bound on v_{PQ}, one obtains from $10^7\ \text{GeV} < v_{PQ} < M_{PL}$ as possible range of axion masses

$$10^{-12}\ \text{eV} < m_a < 1\ \text{eV}. \tag{8.4}$$

(i) Axion halo

It is conceivable that the dark halos of galaxies are made out of axions [35]. A method to search for axions of the Milky Way halo has been proposed by Sikivie [36] and further studied by Krauss et al. [37]. These axions, which have a number density [35]

$$\rho_a \sim \frac{10^{-24}\text{gr}}{m_a\ \text{cm}^3} \sim \frac{0.5\ \text{GeV}}{m_a\ \text{cm}^3} \sim \frac{5\cdot 10^{13}}{\text{cm}^3}\left(\frac{v_{PQ}}{10^{12}\ \text{GeV}}\right), \tag{8.5}$$

can be described by a classical field

$$a(t) \cong a_o\, e^{i\omega_o t} \tag{8.6}$$

with frequency $\omega_o \sim m_a$. The basic idea is to convert these axions via their electromagnetic coupling (cf. eq. (4.22)):

$$L = -\kappa\, \vec{E}\cdot\vec{B}\, a, \quad \kappa = \frac{\xi\alpha}{v_{PQ}}, \tag{8.7}$$

in an external magnetic field $\vec{B}_o$ into photons (cf. Fig. 9). The corresponding electric field is given by

$$\vec{\nabla}^2\vec{E} - \frac{\partial^2}{\partial t^2}\vec{E} = \kappa\, \vec{B}_o \frac{\partial^2 a}{\partial t^2}, \tag{8.8}$$

and the generated electromagnetic energy is detected by means of a cavity with volume V, quality factor Q and modes $\vec{e}_j(\vec{x})$,

$$\frac{1}{V}\int_V d^3x\, \vec{e}_i(\vec{x})\, \vec{e}_j(\vec{x}) = \delta_{ij}. \tag{8.9}$$

The energy converted from axions of the halo into electromagnetic energy of mode i is [37]

$$U_i = \kappa^2 V \eta_i^2 \int \frac{d\omega}{2\pi} \frac{|a(\omega)|^2\omega^4}{(\omega^2-\omega_i^2)^2 + \frac{\omega^4}{Q^2}}, \tag{8.10}$$

where $a(\omega)$ is the Fourier transform of $a(t)$ and η_i the overlap of the external magnetic field with mode i:

$$\eta_i = \frac{1}{V}\int_V d^3x\, \vec{B}_o\, \vec{e}_i(\vec{x}). \tag{8.11}$$

The frequency distribution $a(\omega)$ is expected to be very narrow [37],

$$Q_a = \frac{m_a}{\Delta\omega} \sim 9\cdot 10^6, \tag{8.12}$$

and in the zero-width approximation (8.6) one finds for $\omega_i = m_a$:

$$U_i = \kappa^2 V \eta_i^2 Q^2 \frac{\rho_a}{m_a}. \tag{8.13}$$

This corresponds to a detectable power $P \sim \frac{U_j\omega_o}{Q}$ ($\kappa \sim \frac{3\alpha}{v_{PQ}}$, $\eta_i \sim B_o$, $Q \sim 10^6$):

$$P \sim 8\cdot 10^{-18}\ \text{Watt}\ \left(\frac{V}{5\cdot 10^4\ \text{cm}^3}\right)\left(\frac{B_o}{8\ \text{Tesla}}\right)^2\left(\frac{10^{12}\ \text{GeV}}{v_{PQ}}\right). \tag{8.14}$$

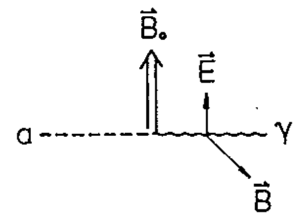

Fig. 9 Conversion of axions into photons in an external magnetic field

It appears that with a cavity of $5 \cdot 10^4$ cm³, a magnetic field of 8 Tesla and a state of the art microwave detector it is possible to sean within two years a fequency interval corresponding to a range of the symmetry breaking scale [35]

$$10^{11} \text{ GeV} < v_{PQ} < 3 \cdot 10^{12} \text{ GeV} , \quad (8.15)$$

which lies inside the range allowed by astrophysics and cosmology.

(ii) Solar axions

If invisible axions exist they will be emitted from the sun with a rather broad energy spectrum peaking at about 1 keV. The axion flux expected for $\frac{v_{PQ}}{z} = 10^7$ GeV (cf. eq. (4.12)) is shown in Fig. 10. Similar to the photoelectric effect these axions have a rather large ionization cross section for certain elements ("axioelectric effect") which can be used to detect solar axions [38]. Recent results obtained with a germanium spectrometer with energy threshold of only 4 keV are shown in Fig. 11 together with theoretical expectations. Avignone et al. obtain from this figure the remarkable lower bound

$$\frac{v_{PQ}}{z} > 0.5 \cdot 10^7 \text{ GeV} , \quad (8.16)$$

which is almost as large as the astrophysical bound (8.2).

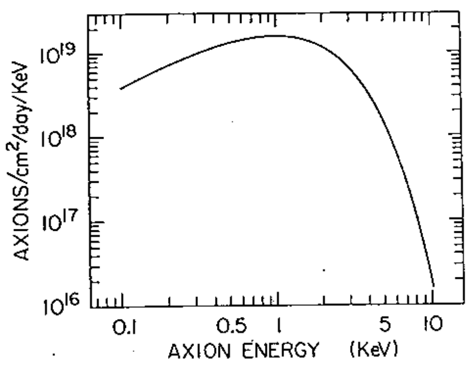

Fig. 10 Solar axion flux on earth for solar temperature T = 1 keV and bremsstrahlung production with $\frac{v_{PQ}}{z} = 10^7$ GeV. From ref. [38]

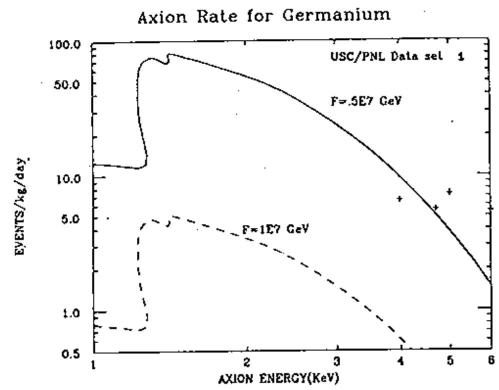

Fig. 11 Theoretical prediction for solar axion events per kg and day for Germanium with $\frac{v_{PQ}}{z} = 0.5 \cdot 10^7$ GeV (solid line) and $\frac{v_{PQ}}{z} = 10^7$ GeV (dashed line). The crosses denote recent experimental results of background count rates with $\omega \geq 4$ keV. From ref. [38]

(iii) Long-range forces

Axions may also have CP violating scalar couplings to fermions which will then lead to additional nucleon-nucleon, nucleon-electron and electron-electron forces with range $\lambda_a = \frac{\hbar}{m_a}$. Moody and Wilczek have analyzed these "new macroscopic forces" [39], especially the monopole-dipole type nucleon-electron interaction. For 10^9 GeV $< v_{PQ} < 10^{12}$ GeV all these forces are weaker than gravity with a range 0.002 cm $< \lambda_a <$ 2 cm. Such deviations from gravity are the subject of the lectures given by John Bell at his school [40].

(iv) Laser experiments

An interesting method to search for invisible axions has recently been suggested by Maiani, Petronzio and Zavattini [41]. Due to the coupling (8.7) of axions to two photons the propagation of light in an external magnetic field is modified. One easily derives the coupled classical equations of motion

$$\Box \vec{E} - \kappa \frac{\partial^2 a}{\partial t^2} \vec{B} = 0 , \quad (8.17a)$$

$$(\Box + m_a^2)a + \kappa\, \vec{E}\cdot\vec{B} = 0 \,, \tag{8.17b}$$

where, to first approximation, $\vec{B}$ can be replaced by the strong external field $\vec{B}_o$. Clearly, only the component of $\vec{E}$ parallel to $\vec{B}_o$ is modified through the interaction with the axion field whereas the orthogonal component of $\vec{E}$ propagates undisturbed (cf. Fig. 12).

Eqs. (8.17) admit plane wave solutions of the form ($\vec{B}_o = B_o \vec{e}_\parallel$, $\vec{E} = E_\parallel \vec{e}_\parallel + E_\perp \vec{e}_\perp$):

$$E_\perp = E_o \sin\alpha\, e^{-i(\omega t - \vec{k}\vec{x})}, \quad \omega^2 = \vec{k}^2 \,, \tag{8.18a}$$

$$E_\parallel = E_+ e^{-i(\omega_+ t - \vec{k}\vec{x})} + E_- e^{-i(\omega_- t - \vec{k}\vec{x})} \,, \tag{8.18b}$$

$$a = a_+ e^{-i(\omega_+ t - \vec{k}\vec{x})} + a_- e^{-i(\omega_- t - \vec{k}\vec{x})} \,, \tag{8.18c}$$

where the shifted frequencies $\omega_\pm$ are determined by

$$(\omega_\pm^2 - \omega^2 - \omega_m^2)(\omega_\pm^2 - \omega^2) - \omega_o^2\, \omega_\pm^2 = 0 \,, \tag{8.19}$$

$$\omega_m = m_a \,, \quad \omega_o = \kappa B_o \,.$$

With initial conditions (at $\vec{x} = o$)

$$E_\parallel(0) = E_o \cos\alpha \,, \tag{8.19a}$$

$$a(0) = 0 \,, \tag{8.19b}$$

which correspond to linear polarization, one finds after propagation of length $L = ct$ ($|\omega_\pm - \omega| \ll \frac{2\pi}{L}$) [41]:

$$E_\perp(t) = E_o \sin\alpha\, e^{-i\omega t} \,, \tag{8.20a}$$

$$E_\parallel(\ ) = E_o \cos\alpha\, (1-\varepsilon) e^{-i(\omega t - \delta)} \,, \tag{8.20b}$$

where

$$\varepsilon(L) = \frac{1}{8} \kappa^2 B_o^2 L^2 \,, \tag{8.20c}$$

$$\delta(L) = \frac{\kappa^2 B_o^2 m_a^2 L^3}{48\omega} = 2e \,. \tag{8.20d}$$

Eqs. (8.20) describe elliptically polarized light with ellipticity e. The major axis of the ellipse is rotated by $\delta\alpha = \frac{1}{2}\varepsilon(L) \sin 2\alpha$ with respect to the axis of linear polarization.

Experimentally feasable values for the parameters B, L and ω are [41]

$$B_o = 10 \text{ Tesla}$$

$$L = 10^5 \text{ cm}$$

$$\omega = 2.4 \text{ eV} \,. \tag{8.21}$$

Furthermore measurements of $\delta(L)$ as small as

$$\delta(L) \sim 10^{-12} \text{ rad} \tag{8.22}$$

appear possible [41]. This has to be compared with the effect of light-light scattering induced by QED vacuum polarization as described by the Euler-Heisenberg effective lagrangian [41]:

$$\delta_{QED}(L) = \frac{\alpha^2}{30\pi} \frac{L\omega B_o^2}{m_e^4} \cong 5 \cdot 10^{-12} \,, \tag{8.25}$$

which is of the order of the experimental sensitivity of 10^{-12} rad.

For the angle $\delta(L)$ due to the interaction with the axion field one finds from eqs. (8.20d), (8.21) ($\kappa = \frac{\xi\alpha}{v_{PQ}}$) :

$$\delta(L) = 2 \cdot 10^{-21} \text{ rad } \left(\frac{\xi}{3}\right)^2 \left(\frac{10^{12}\text{ GeV}}{v_{PQ}}\right)^2 \left(\frac{m_a}{10^{-5}\text{eV}}\right)^2 \left(\frac{2.4\text{ eV}}{\omega}\right) . \tag{8.26}$$

This result appears hopelessly tiny. We note, however, that the parameter ξ may be larger than 3 by an order of magnitude, and that the relation (8.3a) between axion mass and symmetry breaking scale is also model dependent. The experimentally accessible range in the m_a-v_{PQ} plane has been discussed more generally in ref. [41]. It is clear that laser experiments can probe a region which so far has not been explored in other laboratory experiments.

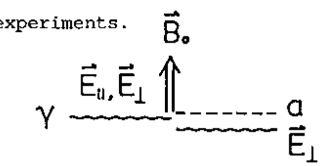

Fig. 12 Conversion of photons with parallel polarization into axions

9. CONCLUSIONS

In these lectures we have discussed some aspects of the strong CP problem. The solution of the U(1) problem in QCD through gauge field configurations with nontrivial topology and the associated Θ-vacuum gives rise to CP violation in strong interactions whose strength is governed by the new parameter Θ. This leads to a nonvanishing electric dipole moment d_n of the neutron proportional to $\overline{\Theta}$, the sum of Θ and the phase of the determinant of the quark mass matrix. The experimental upper bound on d_n implies an upper bound on $\overline{\Theta}$, $\overline{\Theta} < 10^{-9}$.

Within the Glashow-Weinberg-Salam theory it appears impossible to understand the smallness of $\overline{\Theta}$, especially the cancellation between the intrinsic QCD part Θ and the electroweak contribution from the quark mass matrix. The meaning of the parameter Θ in the standard model is tied to the absence of a global chiral symmetry with $SU(3)_c$ anomaly which would allow to "rotate Θ away". Hence the Peccei-Quinn mechanism, which rests on a spontaneously broken chiral $U(1)_{PQ}$ invariance in a minimal extension of the standard model, appears to be the natural solution of the strong CP problem.

The characteristic feature of the Peccei-Quinn mechanism is the appearance of a pseudo-Goldstone boson, the axion. However, a remarkable experimental effort has excluded the possibility that the $U(1)_{PQ}$ symmetry breaking scale v_{PQ} coincides with the scale $G_F^{-1/2}$ of electroweak symmetry breaking. Astrophysical and cosmological arguments limit the allowed range for the $U(1)_{PQ}$ symmetry breaking scale to 10^8 GeV $< v_{PQ} < 10^{12}$ GeV, corresponding to "invisible" axions of macroscopic Compton wavelength, which interact only very weakly with ordinary matter. It is intriguing that this mass range contains the preferred scale of supersymmetry breaking.

It is very remarkable that experiments are conceivable which could make such invisible axions visible. We have discussed Sikivie's axion haloscope, the search for axions from the sun, axion induced deviations from gravitational forces and laser experiments. At present the search for solar axions appears most promising, and a first experiment has already obtained a lower bound on v_{PQ} almost as large as the astrophysical bound. The other experiments are also interesting, even if they are sensitive only to smaller mass scales. For instance, the advantage of the laser experiment is that no outside source of axions is needed, because an axion beam is produced in the experiment!

The search for new interaction scales beyond the standard model of strong and electroweak interactions is an outstanding challenge in particle physics. It is likely that such new interaction scales give rise to very light, scalar or pseudoscalar particles which interact very weakly with ordinary matter. Such particles could be discovered in low energy precision experiments.

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