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ABSTRACT

The dimension-four gluon condensate $\langle \alpha_s G^2 \rangle$ and the dimension-six four-quark condensate are related to experimental data on $\sigma(e^+e^- \to$ hadrons, $I = 1)$ in the framework of Gauss-Weierstrass and finite energy QCD sum rules. Stable eigenvalue solutions for these vacuum condensates, consistent with duality, are obtained. Results from this determination confirm earlier conjectures calling for a substantial increase in the standard value of $\langle \alpha_s G^2 \rangle$, as well as previous claims casting doubt on the validity of the vacuum saturation approximation for estimating the four-quark condensate.

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1. INTRODUCTION

Following the introduction of the QCD sum rules by Shifman, Vainshtein and Zakharov (SVZ) /1/ considerable effort has been devoted to the development and application of this technique to a wide variety of problems in hadronic physics (for reviews see e.g. /2/-/7/). One starts by considering the two-point function

$$\Pi(q^2) = i \int d^4x \, e^{iqx} \langle 0 | T(J(x) J^{+}(0)) | 0 \rangle , \quad (1)$$

where J(x) is a composite local operator with specific quantum numbers, and constructed from the fundamental quark and gluon fields of QCD. The underlying assumption of the SVZ programme is the validity of the operator product expansion (OPE) in Eq. (1) even in the presence of non-perturbative effects. In other words, when the OPE is used in (1) the vacuum state is identified with the physical vacuum. The contributions to the unit operator in the OPE, i.e. the purely perturbative effects, are in principle calculable in the deep Euclidean region $Q^2 \equiv -q^2 \gg \Lambda^2_{QCD}$ to any order in the strong interaction running coupling constant $\alpha_s(Q^2)$. Corrections to asymptotic freedom due to long distance dynamics are parametrized by a series in inverse powers of Q^2. The Wilson coefficients in this expansion can be calculated in perturbation theory but the corresponding vacuum expectation values of quark and gluon fields, the vacuum condensates, unfortunately cannot be computed within QCD. These condensates remain then as parameters to be determined e.g. by relating them to experiment through some dispersion relation (QCD sum rules). In principle, numerical simulations of QCD on a lattice could be used to fix the value of these condensates but results are still not precise enough /8/. In any case, once fixed in some way the vacuum condensates should be the same in all sum rules where they may appear, and thus the method retains considerable predictive power.

To be more specific let us consider the case where J(x) in Eq. (1) is identified with the vector-isovector current, i.e. the current with the quantum numbers of the ρ-meson

$$V^{\mu}(x) = \frac{1}{2}\left[\bar{u}(x)\gamma^{\mu}u(x) - \bar{d}(x)\gamma^{\mu}d(x)\right]. \tag{2}$$

Equation (1) can then be written as

$$\begin{aligned}\Pi^{\mu\nu}(q^2) &= i\int d^4x\, e^{iqx}\langle 0|T(V^{\mu}(x)\, V^{\nu}(0))|0\rangle \\ &= -\left(g^{\mu\nu}q^2 - q^{\mu}q^{\nu}\right)\Pi(q^2)\end{aligned} \tag{3}$$

The function $\Pi(q^2)$ in (3) has been calculated in perturbation theory, in the $\overline{MS}$ renormalization scheme /9/, up to three loops /10/ and including the leading non-perturbative corrections /1/, with the result

$$\begin{aligned}8\pi^2\,\Pi(q^2) = &-\ln\frac{Q^2}{\nu^2} + \frac{5}{3} - 3\,\frac{m_u^2(\nu^2) + m_d^2(\nu^2)}{Q^2} \\ &- \frac{\alpha_s(\nu^2)}{\pi}\ln\frac{Q^2}{\nu^2} + \left[\frac{\alpha_s(\nu^2)}{\pi}\right]^2\left[-\frac{\beta_1}{4}\ln^2\frac{Q^2}{\nu^2}\right. \\ &\left.- F_3\ln\frac{Q^2}{\nu^2}\right] + \frac{C_4\langle O_4\rangle}{Q^4} + \frac{C_6\langle O_6\rangle}{Q^6} \\ &+ O\left(\frac{m_q^4}{Q^4}\right) + O\left[\left(\frac{\alpha_s}{\pi}\right)^3\right] + O\left(\frac{1}{Q^8}\right),\end{aligned} \tag{4}$$

where $\beta_1 = -29/6$, and $F_3 = 1.756...$ for three colours and two flavours. The non-perturbative term of dimension-four, a combination of the gluon and the quark condensates, is given by

$$C_4\langle O_4\rangle = \frac{\pi}{3}\langle\alpha_s G^2\rangle + 4\pi^2\left[m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right]. \tag{5}$$

There are various sources of the $C_6\langle O_6\rangle$ term from the dimension-six quark-gluon, three-gluon, and four-quark condensates. The first is suppressed by factors of m_q^2 /1/, while the second gives no contribution in the chiral limit /11/, thus leaving us with the four-quark condensate which in the vacuum saturation approximation is given by /1/

$$C_6\langle O_6\rangle\Big|_{SVZ} = -\frac{896}{81}\pi^3\alpha_s\langle\bar{q}q\rangle^2. \tag{6}$$

After the early estimate of $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ by SVZ /1/ many authors /12/ have reconsidered this issue using a variety of theoretical tools and experimental information. The original SVZ values, which we shall refer to as the "standard values", are as follows

$$\frac{\pi}{3}\langle\alpha_s G^2\rangle \simeq 0.04\ \text{GeV}^4, \tag{7}$$

$$C_6\langle O_6\rangle \simeq -0.06\ \text{GeV}^6. \tag{8}$$

Notice that the light-quark condensate term in (5) can be accurately estimated from a current algebra-PCAC low energy theorem. The most recent value, which accounts for small but calculable corrections to π-PCAC, is /13/

$$4\pi^2\left[m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle\right] = -0.0064\ \text{GeV}^4. \tag{9}$$

From our introductory discussion it should be clear that an accurate knowledge of the numerical value of the vacuum condensates is a matter of the utmost importance, as they provide information on physical quantities such as quark and hadron masses and coupling constants, symmetry breaking, etc. Unfortunately, in spite of all previous efforts there is still some controversy about the standard values of the condensates, to wit. (i) The possibility that the gluon condensate (7) has been underestimated, as conjectured by Bell and Bertlmann in the framework of potential models /12.b-d/, and by the authors of /12.g-i/ in two-dimensional QCD. Also, a 33 % increase in the value (7) has been advocated in /6/, while the charmonium analysis of /12.o/ suggested an underestimation of (7) by a factor 1.5-2. A recent reexamination of QCD sum rules for S-wave charmonium /12.j/ indicates an even larger departure from the standard value, i.e. a factor of 4-5. (ii) The validity of the vacuum saturation approximation which allows for the four-quark vacuum condensate to be written as in Eq. (6). In fact, several authors /12.n/, /14/ have noticed that this approximation leads to inconsistencies.

The above issues have been studied recently /7/, /15/ by relating $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ to available experimental data on $\sigma(e^+e^- \to$ hadrons, $I = 1)$ in the framework of the Gauss-Weierstrass sum rules proposed by Bertlmann, Launer, and de Rafael (BLR) /12.q/. A very important feature of these QCD sum rules is that they provide the necessary criteria to formulate quantitatively the old notion of local duality /16/ through the so called "heat evolution test". They also lead to Finite Energy Sum Rules (FESR) allowing in the process for an unambiguous calculation of radiative corrections. Up to second order in α_s, $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ obey uncoupled eigenvalue equations which relate them to different moments of the e^+e^- data (for a generalization to higher orders in α_s See /17/). A study has also been made of the stability of the eigenvalue solutions to the FESR according to the criteria first proposed by Pich and de Rafael /18/.

In this talk I wish to discuss the main results from this investigation /15/; preliminary results were already reported by Bertlmann /7/ at Seewinkel. The talk is organized as follows. In Section 2 I summarize the various versions of QCD sum rules and compare the advantages and disadvantages of Laplace transform and FESR. In Section 3 I discuss several parametrizations and fits to the e^+e^- data, and in Section 4 I present the solutions to the eigenvalue problem posed by the FESR, and the stability tests. Because of time limitations I shall not be discussing here the heat evolution tests and the Laplace transform analysis. The interested reader should consult Ref. /15/ for full details.

2. THE VARIOUS VERSIONS OF QCD SUM RULES

Once the OPE is performed in Eq. (1), the next step consists in relating the resulting QCD expression for $\Pi(q^2)$ to physical quantities such as hadronic masses, widths, cross sections, etc. This may be achieved e.g. by writing a dispersion relation

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty ds \, \frac{\mathrm{Im}\,\Pi(s)}{s-q^2} \quad + \text{subtraction constants,} \tag{10}$$

defined up to a finite number $\underline{n}$ of subtractions. One can dispose of these subtractions by taking the $\underline{n}$th derivative of $\Pi(q^2)$ which yields the power moments or Hilbert sum rules

$$M_n(Q^2) \equiv \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2) = \int_0^\infty \frac{ds}{(s+Q^2)^{n+1}} \, \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s), \tag{11}$$

where $Q^2 \equiv -q^2 > 0$, q^2 spacelike. These sum rules may be regarded as a global duality relation in the sense that the weighted average of the hadronic spectral function $\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$, for sufficiently large Q^2, should match the theoretical QCD expression for $M_n(Q^2)$. One should keep in mind, though, that in practice the amount of available QCD and hadronic information is limited. The former is restricted to the first few leading power corrections in the OPE, while the latter is usually limited to the ground state resonance or at most its first radial excitation.

By taking an appropriate limit in (11) it is possible to obtain exponentially weighted moments or Laplace transform sum rules, as first proposed by SVZ /1/, viz

$$M(\sigma) = \int_0^\infty ds\, e^{-s\sigma} \frac{1}{\pi} \mathrm{Im}\, \Pi(s) . \qquad (12)$$

In this limiting process a transmutation of the variable $1/Q^2$ into σ has occurred, so that now the non-perturbative effects appear as power corrections in the new short distance variable σ. The Laplace sum rules may be regarded as an improvement over the Hilbert sum rules in the sense that because of the exponential factor the r.h.s. of (12), at moderate values of σ, is now more sensitive to the low energy behaviour of the spectral function. At the same time, higher dimensional non-perturbative contributions to the OPE become factorially suppressed, a welcomed feature. For instance, in the case of the vector-isovector current (2), the QCD Laplace transform of the associated two-point function (3)-(4) reads /19/

$$\begin{aligned} 8\pi^2 \sigma M(\sigma) = 1 &- 3\left[\hat{m}_u^2 + \hat{m}_d^2\right] \frac{\sigma}{\left[\frac{1}{2}\ln(1/\sigma\Lambda^2)\right]^{-4/\beta_1}} \\ &+ \frac{2}{\beta_1 \ln(\sigma\Lambda^2)} + \left[\frac{2}{\beta_1 \ln(\sigma\Lambda^2)}\right]^2 \times \Big[F_3 \\ &- \frac{1}{2}\beta_1 \gamma_E - \frac{\beta_2}{\beta_1} \ln \ln\left(\frac{1}{\sigma\Lambda^2}\right) \Big] \\ &+ C_4 \langle O_4 \rangle \sigma^2 + C_6 \langle O_6 \rangle \frac{\sigma^3}{2!} \\ &+ O(m_q^4 \sigma^2) + O\left[\left(\frac{\alpha_s}{\pi}\right)^3\right] + O(\sigma^4) , \end{aligned} \qquad (13)$$

where β_1 and F_3 were defined after Eq. (4), $\beta_2 = -115/12$ for three colours and two flavours, $\hat{m}_u$ and $\hat{m}_d$ are the invariant quark masses, Λ is the $\overline{\mathrm{MS}}$ QCD scale, γ_E is the Euler constant, $C_4\langle O_4\rangle$ is given in Eq. (5), and $C_6\langle O_6\rangle$ (in the vacuum saturation approximation) is given in Eq. (6).

Concerning the r.h.s. of the sum rules, either (11) or (12), for consistency reasons the low energy hadronic parametrization of the spectral function should smoothly evolve, at higher energies, into a continuum which can be computed perturbatively in QCD. In practical applications it is customary to assume a step function type onset of asymptotic freedom at some energy threshold $\sqrt{s_0}$. Unfortunately, the exact location of this threshold is not determined by the sum rules (11) or (12) and must then be fixed by hand. To this extent, the Hilbert and Laplace sum rules do not provide by themselves a quantitative formulation of <u>local duality</u>. In any case, once s_0 is fixed by means of some criterion, predictions for resonance parameters <u>expected</u> to be dual to a given QCD information (or viceversa) follow from the Laplace sum rules when the following criterion is fulfilled /1/. There should exist some region or "window" in the short distance expansion parameter σ such that ordinary perturbative QCD remains valid, and at the same time only the leading power corrections are required. A consistency check of the expectation that the resonance parameters so determined are dual to the input QCD information (or viceversa) should then be performed. Lacking a quantitative formulation of local duality, a complementary analysis in a different framework may then be necessary. One such framework is that of the Gaussian QCD sum rules which I discuss next.

Recently BLR /12.a/ introduced a new type of QCD sum rules based on the Gauss-Weierstrass transform of spectral functions, viz

$$G(\hat{s}, \tau) = \frac{1}{\sqrt{4\pi\tau}} \int_0^\infty ds\, \exp\left[-\frac{(s-\hat{s})^2}{4\tau}\right] \frac{1}{\pi} \mathrm{Im}\, \Pi(s), \qquad (14)$$

i.e. the convolution of the spectral function with a Gaussian centered at an arbitrary point $\hat{s}$ with a finite-width resolution $\sqrt{4\tau}$. The Gauss-Weierstrass transform calculated via QCD is dual to the hadronic spectral function $\frac{1}{\pi}$ Im II(s) in the sense that the more one knows about QCD the sharper one can take the Gaussians (i.e. τ smaller) and the more accurately the calculated $G(\hat{s},\tau)$ should approximate the physical spectrum. If the QCD bound state problem would be completely solved then one could take $\tau = 0$. In this hypothetical case

$$G(\hat{s},0) = \frac{1}{\pi} \text{ Im II}(\hat{s}) \tag{15}$$

and one would have strict local duality. In practice, however, due to the limited amount of QCD information τ must be kept finite, typically $\sqrt{\tau} \simeq 0.5\text{-}1 \text{ GeV}^2$. Notice that the Gaussian resolution $\sqrt{4\tau}$ can be chosen analogously to the short distance parameter σ in the Laplace sum rules (12) to quantitatively tune the duality interval, i.e. the resolution with which the physical spectrum is to be sampled at the various points $\hat{s}$. At the same time, there is a very interesting and useful analogy between Gaussian sum rules and the theory of the heat equation. This analogy is based on the observation that $G(s,\tau)$ obeys the partial differential equation

$$\frac{\partial^2 G(s,\tau)}{(\partial s)^2} = \frac{\partial G(s,\tau)}{\partial \tau} \tag{16}$$

which is the one-dimensional heat equation if one reinterprets s as a "position" variable and τ as a "time" variable. In this analogy the hadronic spectral function $\frac{1}{\pi}$ Im II(s) represents the initial heat distribution in a semi-infinite rod $0 \leq s \leq \infty$ and $G(s,\tau)$ measures the evolution in "time" of the heat distribution in this rod. This provides a very convenient framework to check the consistency between a given phenomenological ansatz (or data) for the spectral function and a specific choice of vacuum condensate parameters in the SVZ parametrization of non-perturbative QCD corrections. In fact, after a "time" τ

sufficiently large so that the uncalculated QCD corrections become relatively small, the predicted QCD heat distribution should match the evolution of the phenomenological ansatz (or data). This is the heat evolution test proposed by BLR /12.q/, and which serves as a quantitative formulation of the idea of local duality.

Returning to the heat equation (16), there are two general solutions to the problem of the temperature distribution in the semi-infinite rod corresponding to the following two choices of boundary conditions /20/

$$G(0,\tau) = 0 , \qquad (\tau > 0), \tag{17}$$

$$\left.\frac{\partial G(s,\tau)}{\partial s}\right|_{s=0} = 0 , \qquad (\tau > 0) \tag{18}$$

subject to the initial condition (15). Denoting these two solutions by $U^{(-)}(s,\tau)$ and $U^{(+)}(s,\tau)$, respectively, it follows that

$$U^{(-)}(s,\tau) = G(s,\tau) - G(-s,\tau), \tag{19}$$

$$U^{(+)}(s,\tau) = G(s,\tau) + G(-s,\tau). \tag{20}$$

General rules for the construction of $G(s,\tau)$ in QCD are given in the original paper by BLR /12.q/ to which the reader is referred for more details. In particular, the calculation of the Gauss-Weierstrass transforms corresponding to the two-point function associated to the vector-isovector current, Eq. (4), is discussed in full detail in /12.q/.

An interesting outcome of the heat equation analogy is the fact that QCD Finite Energy Sum Rules can be shown to follow from /12.q/

(a) the principle of conservation of the total heat, i.e. for all τ-values

$$\int_{-\infty}^{\infty} ds\, G(s,\tau) = \frac{1}{\pi}\int_{0}^{\infty} ds\, \mathrm{Im}\, \Pi(s) \tag{21}$$

and

(b) the asymptotic freedom property of QCD, which for large values of s and finite τ requires that

$$\lim_{s \gg \tau} G(s,\tau) \longrightarrow \frac{1}{\pi} \mathrm{Im}\, \Pi(s)\Big|_{\mathrm{pert.\,QCD}} \tag{22}$$

The advantage of this approach to FESR is that it is possible to show explicitly the way perturbative corrections can be incorporated; and also, it gives an insight on the effect of non-leading non-perturbative corrections. A derivation of FESR from the Gaussians was first discussed by BLR /12.a/ (see also /13.b/, /15/, /17/; to second order in α_s the FESR for the vector-isovector channel are

$$C_{4N+2}\langle O_{4N+2}\rangle = 8\pi^2 \int_0^{s_0} ds\, s^{2N} \frac{1}{\pi}\mathrm{Im}\,\Pi(s) - \frac{s_0^{2N+1}}{2N+1}\left[1+F_{4N+2}(s_0)\right] \tag{23}$$

$$-C_{4N+4}\langle O_{4N+4}\rangle = 8\pi^2 \int_0^{s_0} ds\, s^{2N+1} \frac{1}{\pi}\mathrm{Im}\,\Pi(s) - \frac{s_0^{2N+2}}{2N+2}\left[1+F_{4N+4}(s_0)\right] \tag{24}$$

where N = 0,1,2,..., and the radiative corrections $F_p(s_0)$, with p = 4N + 2 or p = 4N + 4, are given by

$$F_p(s_0) = \frac{2}{-\beta_1 \ln s_0/\Lambda^2} + \left(F_3 - \frac{\beta_1}{p} - \frac{\beta_2}{\beta_1}\ln\ln\frac{s_0}{\Lambda^2}\right) \times \left(\frac{2}{-\beta_1 \ln s_0/\Lambda^2}\right)^2 + O\left[\left(\frac{1}{\ln s_0/\Lambda^2}\right)^3\right]. \tag{25}$$

The cutoff S_0 in the FESR (23)-(24) corresponds to the threshold for asymptotic freedom, which in this framework is not fixed by hand but rather it is predicted as an eigenvalue solution. In fact, notice that Eq. (23) with N=0 corresponds to the leading quark mass insertion treated as an effective $C_2\langle O_2\rangle$ contribution, i.e.

$$C_2\langle O_2\rangle = -3\, \frac{\hat{m}_u^2 + \hat{m}_d^2}{\left(\frac{1}{2}\ln s_0/\Lambda^2\right)^{-4/\beta_1}} \tag{26}$$

Since this term is negligible, the threshold of the perturbative QCD continuum is predicted as a solution to the eigenvalue equation

$$1 + F_2(s_0) = \frac{8\pi^2}{s_0}\int_0^{s_0} ds\, \frac{1}{\pi}\mathrm{Im}\,\Pi(s)\,. \tag{27}$$

At this point we face an important issue, discovered recently by Pich and de Rafael /18/, which refers to the stability of the eigenvalue solutions to the FESR. In general, when using FESR to make predictions for physical quantities, e.g. masses, widths, etc., there should exist some (hopefully) wide region of values of S_0 for which the eigenvalue solutions to the FESR exhibit some stability against changes in the values of the QCD parameters, and viceversa. In short, this principle may be stated as: "Trust FESR only if they are stable in

S_o; only then there exists duality". This principle, overlooked in past applications /21/, was first used in /18/ in connection with the calculation of the B-parameter governing the short distance contribution to the K^o-$\bar{K}^o$ off-diagonal matrix element. Subsequent applications include an improved determination of light quark masses and subtraction constants /13.b/, and estimates of the masses and widths of scalar /22.a/ and tensor gluonium /22.b/. For the case under discussion here, this principle implies that e.g. the l.h.s. of (27) should coincide with the r.h.s. in a "duality region" which should be wider than just a single crossover point corresponding to the eigenvalue solution for S_o. Furthermore, the values of the various condensates $C_4\langle O_4\rangle$, $C_6\langle O_6\rangle$, etc. determined from the FESR (23)-(24) should be stable against changes in S_o within this "duality region". One would expect that the more accurate the parametrization of the spectral function, the broader the "duality region". For instance, it is easy to see that a zero-width parametrization of the hadronic spectral function does not yield stable eigenvalue solutions. In fact, the l.h.s. of (27) is a slowly varying function of S_o approaching unity logarithmically, while the r.h.s. behaves as $1/S_o$ in the zero-width approximation.

In closing this section I wish to summarize the salient features of the Laplace and the Finite Energy QCD sum rules.

(i) Because of the exponential weight in the Laplace transform (12), these sum rules weigh more the low energy region being then more sensitive to the ground state resonance. An accurate knowledge of the hadronic spectral function may not be necessary. In contrast, the FESR weigh the high energy region calling then for a much more accurate hadronic parametrization.

(ii) In spite of the Laplace factorial suppression of higher dimensional condensates it may be hard to avoid correlations, as e.g. when trying to determine simultaneously both $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ from (13). This correlation is absent

in the FESR, at least to second order in α_s (for a generalization to higher orders see /17/). In this case $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ obey uncoupled eigenvalue equations.

(iii) The threshold of asymptotic freedom S_o is an adjustable parameter not fixed by the Laplace sum rules. In contrast, S_o can be determined within FESR as an eigenvalue solution. However, the dependence of physical quantities (or condensates) on S_o is power-like in the FESR while it is exponentially damped in Laplace sum rules.

(iv) Once S_o is fixed with some criterion, predictions from Laplace sum rules follow from the philosophy that there should exist some "window" in σ such that ordinary perturbative QCD remains valid and, at the same time, only the leading power corrections are required /1/. Lacking a quantitative formulation of duality it is necessary to check "a posteriori" whether physical quantities are dual to QCD. A convenient framework is that of the Gaussian sum rules and the heat evolution test /12.q/. On the other hand, predictions from FESR, including S_o, emerge as solutions to a system of eigenvalue equations. This is not enough, however, as instabilities may occur. The following principle /18/ should be implemented: "Trust FESR only if they are stable in S_o: only then there exists duality".

The lesson to be learned from the above comparison is that Laplace transform and Finite Energy sum rules are <u>complementary</u> tools to study QCD. A careful analysis of a given problem should be based on both approaches, together with heat evolution tests.

3. PARAMETRIZATION OF THE HADRONIC SPECTRAL FUNCTION

Since data on $\sigma(e^+e^- \rightarrow$ hadrons, I=1) is available /23/, the most obvious procedure would be to extract the hadronic spectral function directly from a fit to the experimental cross sections using the relation

$$\frac{1}{\pi} \operatorname{Im} \Pi(s) = \frac{1}{\pi} \, s \, \sigma_{I=1}(s) \; . \qquad (28)$$

However, in order to compare with other analyses as well as to study the sensitivity of the condensate predictions to the hadronic parametrization, we shall also use a variety of resonance model parametrizations. In increasing order of expected accuracy these are:

MODEL A: Single ρ-meson resonance in zero-width, normalized according to the Vector Meson Dominance Model /24/, i.e.

$$\frac{1}{\pi} \operatorname{Im} \Pi_\rho(s) = \frac{M_\rho^2}{4\gamma_\rho^2} \, \delta(s - M_\rho^2) \qquad (29)$$

where M_ρ = 770 MeV /25/, and γ_ρ^2 = 6.06 $\pm$ 0.33 as calculated from the experimental electronic width of the ρ-meson /25/.

MODEL B: Single ρ-meson resonance in finite-width (Breit-Wigner) normalized as in Model A, i.e.

$$\frac{1}{\pi} \operatorname{Im} \Pi_\rho(s) = \frac{M_\rho^2}{4\gamma_\rho^2} \frac{M_\rho \Gamma_\rho}{\pi} \frac{1}{(s-M_\rho^2)^2 + M_\rho^2 \Gamma_\rho^2} \qquad (30)$$

with Γ_ρ = 153 MeV /25/.

MODEL C: Two-resonance ($\rho + \rho'$) model in finite-width (Breit-Wigner), normalized according to Extended Vector Meson Dominance /16.a/, /16.b/, i.e.

$$\frac{1}{\pi} \operatorname{Im} \Pi_{\rho+\rho'}(s) = \sum_{i=\rho,\rho'} \frac{M_i^2}{4\gamma_i^2} \frac{M_i \Gamma_i}{\pi} \frac{1}{(s-M_i^2)^2 + M_i^2 \Gamma_i^2} \qquad (31)$$

with $M_{\rho'}$ = 1590 MeV, $\Gamma_{\rho'}$ = 260 MeV /25/, and $\gamma_{\rho'}^2 \simeq$ 23 /16.a/, /16.b/.

DATA FIT: Available data /23/ on the ratio

$$R_{I=1} \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons}, I=1)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \qquad (32)$$

below $\sqrt{s}$ = 2 Gev was fitted using

$$\left.\frac{1}{\pi} \operatorname{Im} \Pi(s)\right|_{FIT} = \frac{1}{48\pi^2} \left(1 - \frac{4\mu_\pi^2}{s}\right)^{3/2} |A(s)|^2 , \qquad (33)$$

where the complex amplitude A(s) was taken as a linear combination of a two-parameter (M_ρ, Γ_ρ) Gounaris-Sakurai type of pion form factor and a four-parameter (complex) resonance amplitude for the $\rho'(1600)$. A fit to the available 71 data points with this six-parameter expression using MINUIT gave a total χ-squared of χ^2 = 103. As may be seen from Fig. 1 this fit reproduces quite well the data, especially the dip at $\sqrt{s} \simeq$ 1.1 GeV.

PERTURBATIVE QCD CONTINUUM: To all of the above resonance models one should add the perturbative QCD continuum

$$\left.\frac{1}{\pi} \operatorname{Im} \Pi(s)\right|_{Pert.\,QCD} = \frac{1}{8\pi^2} \, \theta(s - s_0) \, [1 + F_p(s_0)] \; ; \qquad (34)$$

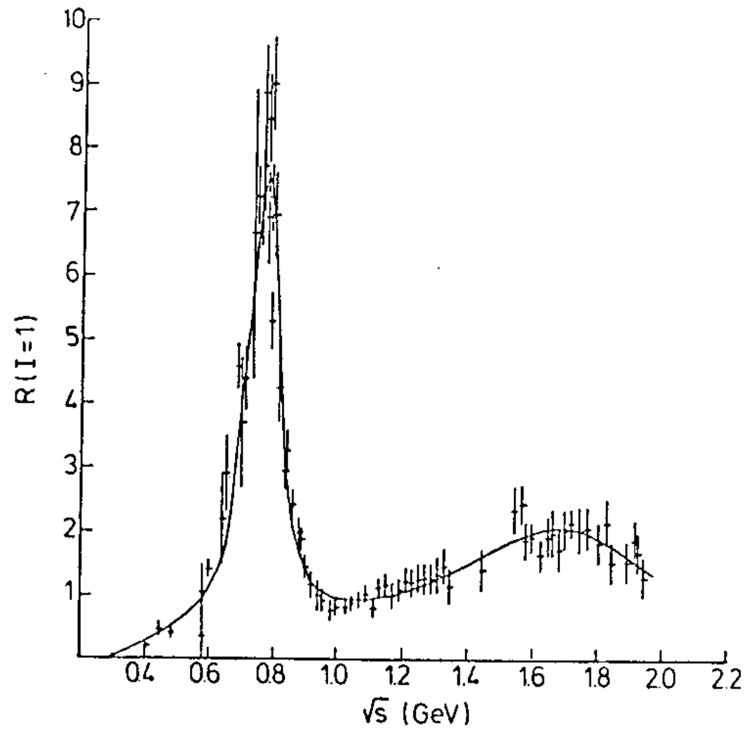

Fig. 1. Best chi-squared fit to the data on R(I=1).

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where S_o is the threshold of asymptotic freedom, and the radiative corrections $F_p(S_o)$ were defined in (25).

4. DETERMINATION OF THE VACUUM CONDENSATES

I shall discuss now the eigenvalue solutions to the FESR (23)-(24) for S_o, $C_4 \langle O_4 \rangle$ and $C_6 \langle O_6 \rangle$ using the various parametrizations of the e^+e^- data introduced in Section 3.

Figures 2 and 3 show the variation of the (theoretical) l.h.s. and of the (hadronic)

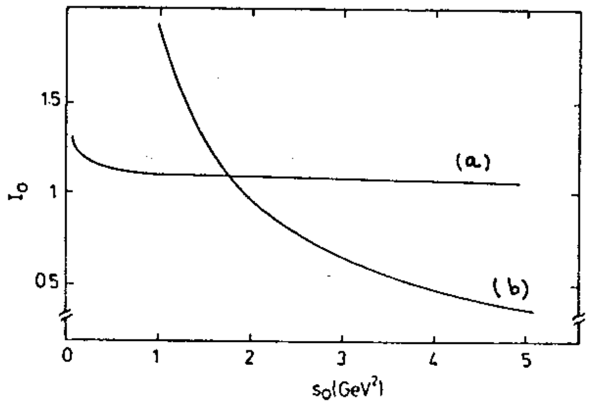

Fig. 2. Curve (a) is the l.h.s. of (27) and curve (b) the r.h.s. for Model A, Eq. (29).

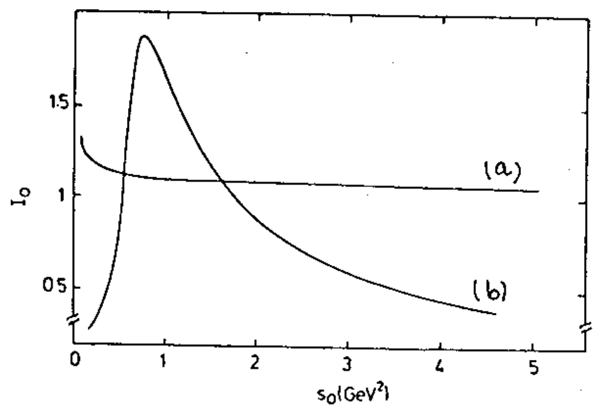

Fig. 3. Same as Fig. 2 but for Model B, Eq. (30).

r.h.s. of Eq. (27) as a function of S_0 for models A, Eq. (29), and B, Eq. (30), respectively. The crossovers between both curves determine the eigenvalue solutions for S_0.

It should be clear from these figures that the eigenvalue problem is unstable. The "duality region", i.e. the S_0-interval within which the l.h.s. and the r.h.s. of (27) coincide, reduces to a single point. The source of this instability lies in the model for the spectral function. In fact, both the zero-width and the single finite-width resonance parametrizations are unable to account for the structure present in the data above 1 GeV (see Fig. 1). As this hadronic information is lacking from the spectral function, the r.h.s. of (27) falls too fast with S_0. This instability is compounded in the eigenvalue equations (23)-(24) fixing $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ as they involve higher weights of the spectral function. This may be appreciated from Figs. 4 and 5 which show the variation of the condensates as a function of S_0 for models A and B, respectively.

Reading off the S_0-eigenvalues from Figs. 2-3 and using them in Fig. 4-5 leads to the condensate values listed in Table 1. However, these results could hardly represent a prediction, viz.a small variation of S_0 around its eigenvalue leads to a huge variation in the value of the condensates. The situation improves when the finite-width two-resonance $(\rho+\rho')$ Model C, Eq. (31), is used. In fact, as seen from Fig. 6 there is now a "duality region" for $1.9\ \mathrm{GeV}^2 \lesssim S_0 \lesssim 3.2\ \mathrm{GeV}^2$. The variation of $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ as a function of S_0 is shown in Fig. 7. Although the condensates still exhibit a pronounced sensitivity to S_0, the addition of a second resonance to the spectral function has a strong qualitative impact: $C_4\langle O_4\rangle$ and $-C_6\langle O_6\rangle$ are no longer monotonically increasing functions of S_0.

The stability of the eigenvalue problem improves considerably when the actual fit to the data, Eq. (33), is used for the calculation of the hadronic

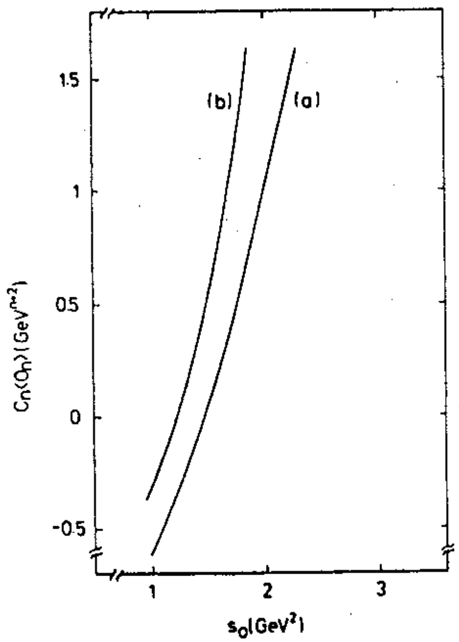

Fig. 4. Curves (a) and (b) show the variation of $C_4\langle O_4\rangle$ and $-C_6\langle O_6\rangle$, respectively, for Model A, Eq. (29).

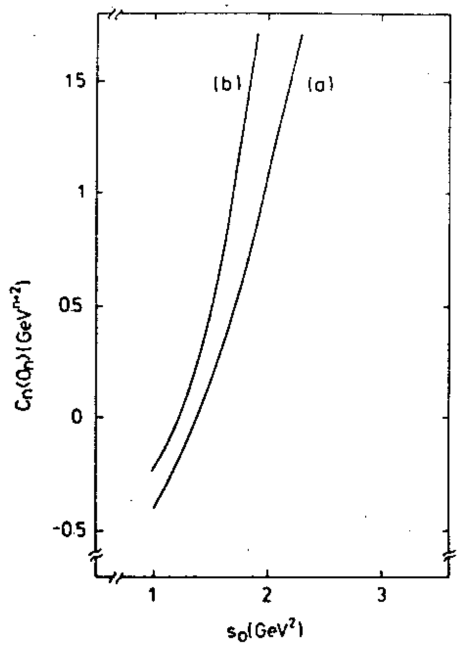

Fig. 5. Same as Fig. 4 but for Model B, Eq. (30).

side of the FESR. As seen from Fig. 8 there is now a wide "duality region" for $1.7\ \mathrm{GeV}^2 \lesssim S_0 \lesssim 4.5\ \mathrm{GeV}^2$ which leads to a smoother variation of the condensates as shown in Fig. 9.

To remain on the safe side and optimize the predictions one may narrow down the "duality region" to the conservative range $1.7\ \mathrm{GeV}^2 \lesssim S_0 \lesssim 2.5\ \mathrm{GeV}^2$. Reading off the values of the condensates within this region leads to the numbers listed in Table 1.

An alternative stability test may be performed by taking ratios of FESR, e.g. the ratio between the FESR for $C_6\langle O_6\rangle$ and that for $C_4\langle O_4\rangle$, i.e.

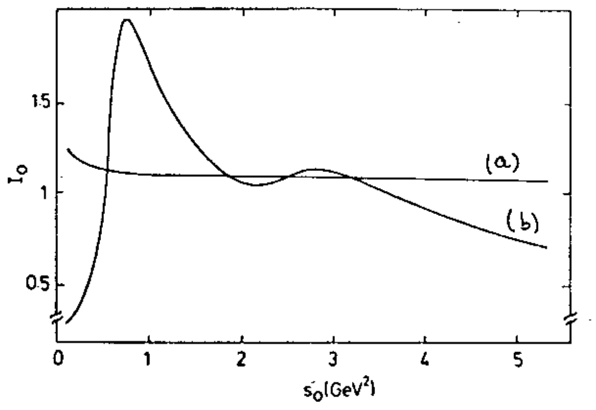

Fig. 6. Same as Fig. 2 but for Model C, Eq. (31).

TABLE 1. Eigenvalue solutions to the FESR for various parametrizations of the hadronic spectral function

MODEL	$C_4\langle O_4\rangle$ (GeV^4)	$C_6\langle O_6\rangle$ (GeV^6)	S_0 (GeV^2)	STABILITY
STANDARD VALUES /1/	0.034	-0.06	-	-
MODEL A	0.56	-1.33	1.8	unstable
MODEL B	0.31	-0.71	1.6	unstable
MODEL C	0.40-0.65	-(0.84-1.45)	1.9-2.5	improved stability
DATA FIT	0.084-0.20	-(0.3-0.5)	1.7-2.5	stable

$$\langle s\rangle \equiv \frac{C_6\langle O_6\rangle + \frac{S_0^3}{3}\left[1+F_6(S_0)\right]}{-C_4\langle O_4\rangle + \frac{S_0^2}{2}\left[1+F_4(S_0)\right]} = \frac{\int_0^{S_0} ds\; s^2 \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)}{\int_0^{S_0} ds\; s\,\frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)}\,. \qquad (35)$$

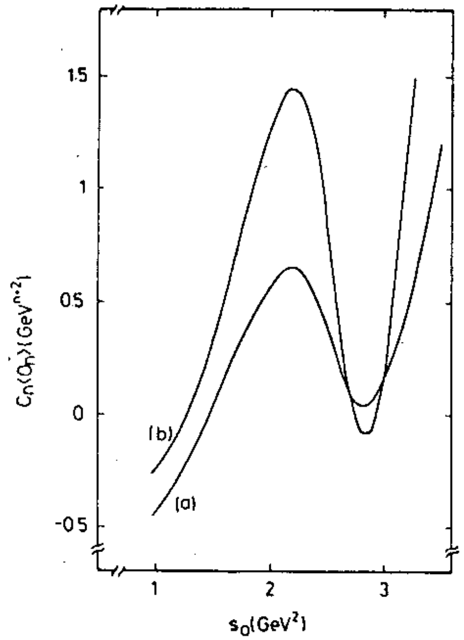

Fig. 7. Same as Fig. 4 but for Model C, Eq. (31).

For values of S_0, $C_4\langle O_4\rangle$ and $C_6\langle O_6\rangle$ within the "duality region" the QCD l.h.s. of (35) should coincide with the hadronic (experimental) r.h.s. For large S_0 the theoretical (QCD) ratio $\langle s\rangle$ behaves as

$$\lim_{S_0\to\infty} \langle s\rangle\Big|_{QCD} = \frac{2}{3} S_0 \qquad (36)$$

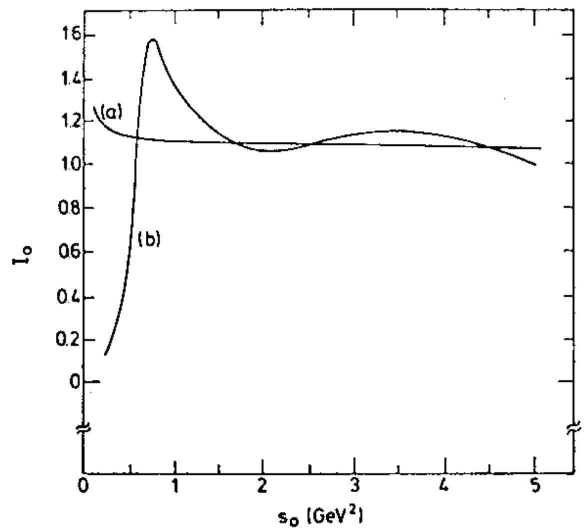

Fig. 8. Same as Fig. 2 but for the best chi-squared fit to the data.

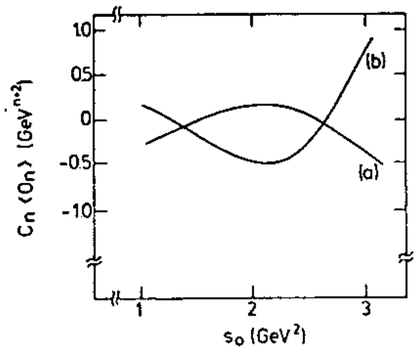

Fig. 9. Same as Fig. 4 but for the best chi-squared fit to the data.

Given the numerical values of $C_4 \langle O_4 \rangle$ and $C_6 \langle O_6 \rangle$ this asymptotic limit is reached relatively fast. Clearly, the zero-width parametrization of the spectral function, model A, gives only a constant ratio: $\langle s \rangle = M_\rho^2$, while the single finite-width resonance model B leads to an $\langle s \rangle$ which does not grow fast enough to match Eq. (36). The reason for this mismatch can of course be traced back to the lack of sufficient hadronic information in these model spectral functions above $s \gtrsim 1$ Gev^2. In fact, when the actual fit to the data is used to compute $\langle s \rangle$ one finds a nice agreement between both sides of (35), inside the duality region, as shown in Fig. 10.

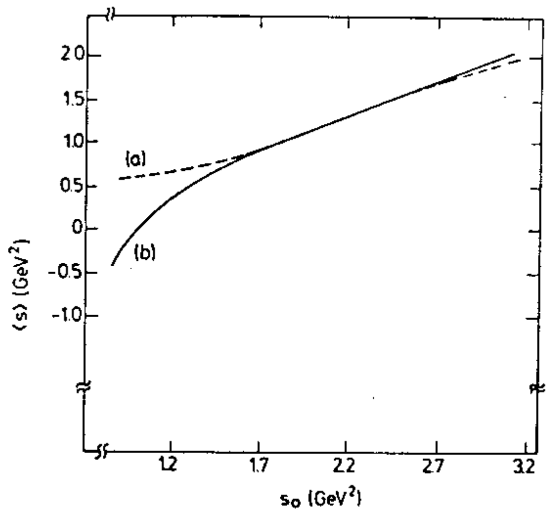

Fig. 10. Curves (a) and (b) are the l.h.s. and the r.h.s. of Eq. (35), respectively, for the best chi-squared fit to the data.

In closing, I wish to stress the highly non-trivial nature of the above stability tests, first proposed by Pich and de Rafael /18/. Notice that if we had not have beforehand experimental information these stability tests would have told us the correct qualitative shape of the e^+e^- cross sections, viz a prominent resonance peak at $M_\rho \simeq$ 700-800 MeV of relatively narrow width $\Gamma \simeq$ 100-200 MeV, followed by at least another resonance at $M \simeq$ 1.5 - 1.6 Gev with a broader width $\Gamma \simeq$ 200-300 MeV. For instance, an analysis of the pseudoscalar channel along these lines /13.b/ provides important constraints on the π' parameters. Also, stability tests for tensor gluonium reveal the presence of at least two bound states in this channel /22.b/.

5. SUMMARY

The availability of experimental data on $\sigma(e^+e^- \to$ hadrons, I=1) singles out the vector-isovector channel as an important source of information on the values of the gluon and the four-quark vacuum condensates. Finite Energy QCD-sum rules are an ideal framework for a correlation-free determination of these condensates as they obey, to second order in α_s, uncoupled eigenvalue equations. However, as these sum rules weigh the high energy region great care must be exercised in parametrizing the hadronic spectral function. In fact, zero-width and single finite-width resonance models are not accurate enough and lead to unstable eigenvalues. Only an optimal chi-squared fit to the actual data yields a stable eigenvalue problem. Performing heat evolution tests it is possible to show that the values of the condensates which satisfy this stable eigenvalue problem are quantitatively dual to the hadronic data.

After taking into account the two-quark vacuum condensate contribution (9) to $C_4\langle O_4\rangle$, Eq. (5), we predict

$$\frac{\pi}{3}\langle\alpha_s G^2\rangle \simeq 0.09 - 0.2 \text{ Gev}^4, \qquad (37)$$

which confirms earlier conjectures /12.b-d/, /12.g-j/, /6/ calling for a substantial increase in the standard value (7). For the dimension-six four-quark vacuum condensate we predict

$$C_6\langle O_6\rangle \simeq -(0.3-0.5) \text{ GeV}^6, \qquad (38)$$

which supports previous claims /12.n/, /14/ casting doubt on the validity of the vacuum saturation approximation (6).

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