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Abstract

Sextupole distributions for various values of phase advance per FODO cell (ϕ_c = 60°...90°) in the HERA electron ring are investigated. The chromatic corrections reduce the changes of the beta-functions to less than 5 % and the changes of tunes to less than 0.01 in an energy range between -1 % and 1 %. The nonlinear acceptance accomodates at least 13 σ for a 35 GeV electron beam in the whole range of ϕ_c. The scaling of the dynamic aperture with ϕ_c is found to be in good agreement with a simple analytical estimate.

1. Introduction

In a previous paper[1] chromatic corrections and the dynamic aperture in the HERA electron ring have been presented and discussed for a lattice with a betatron phase advance of $\phi_C = 60°$ per FODO cell. In the present report, we wish to extend the considerations to lattices with stronger focussing up to 90° per FODO cell.

The need for the flexibility to change the focussing and thus the emittance of the electron beam results from the fact that the size of the proton beam at the interaction point (I.P.) is not very well-known. For instance, the proton emittance depends on the dilution during pre-acceleration, beam transfer and intra-beam scattering. The lower limit of β at the I.P. cannot be precisely determined before the proton ring is in operation. In order to achieve optimum luminosity, the size of the electron beam at the I.P. should therefore be adjustable over a sufficient range.

Stronger focussing in the arcs of the electron ring results in a rapid increase in the sextupole strengths needed to compensate the chromaticity. This in turn may result in a severe reduction of the nonlinear acceptance. We will show in this report, that the good chromatic and nonlinear properties of the $\phi_C = 60°$ optics presented in ref. 1 can essentially be preserved over the entire range of phase advance up to 90° per cell.

In the next section, we discuss the sextupole correction schemes for the 8 different values of ϕ_C investigated. In section 3 the results of particle tracking calculations are presented. In section 4 the results are discussed and some final conclusions on the nonlinear acceptance as a function of phase advance per cell are drawn.

2. Sextupole Correction Schemes for Different Phase Advances per FODO cell

Our investigations start with the setting up of the linear optics of the ring for each value of ϕ_c in the regular arc cells. The optics matching is done in such a way that the phase advance through the West quadrant is an integer in both planes. This results in a quasi-threefold symmetry for the optics. In our previous report[1] we have shown that this procedure is advantageous with respect to chromatic and nonlinear distortions. The chromatic corrections are then made individually for the West quadrant and the other quadrants in the ring.

In order to compensate the chromatic distortions of the linear optics generated in a section* extending from $s = s_0$ to $s = s_0 + L$, the sextupole strengths m(s) must be adjusted in such a way that the integrals

$$I_n = \int_{s_0}^{s_0+L} ds\ \beta(s)\ [k(s) - m(s)\ D(s)]\ e^{in\phi(s)} \qquad (n = 0,2) \qquad (2.1)$$

vanish for both the x- and z-planes. For n = 0, the integral defines the mean sextupole strength necessary to compensate the linear chromaticity. For n = 2, it describes the compensation by sextupole families of different strengths of the off-energy beat of the β-function and its derivative . The differences between sextupole strengths can be kept small if the phase advance between different families is close to $\Delta\phi = (n + 1/4)\pi$. The existence of such "orthogonal" families in one of the criteria for a "good" sextupole distribution.

Another boundary condition is that the nonlinear perturbations introduced by the sextupoles should be kept as small as possible. A reasonable measure of the nonlinear perturbations is the amplitude of the resonance driving terms which are defined in lowest order by integrals of the form

$$d_{30n0} \propto \oint ds\ \beta_x^{3/2}(s)\ m(s)\ e^{in\phi_x(s)} \qquad (n = 1,3) \qquad (2.2)$$

or

$$d_{121n} \propto \oint ds\ \beta_x^{1/2}(s)\ m(s)\ e^{i\phi_x(s)\ \pm\ n\,\phi_z(s)} \qquad (n = 0,2) \qquad (2.3)$$

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* Here, a section is either an octant extending from the I.P. to the centre of the arc or the West quadrant.

We consider now a section of the ring which consists of N identical subperiods (e.g. N FODO cells with identical sextupole strengths and periodic β- functions). The contribution of this section to the integrals (2.2) and (2.3) can then be written as

$$\mathbf{d}_{30n0} = \mathbf{d}^{s}_{30n0} \sum_{\ell=0}^{N-1} e^{i\ell n\phi_x^s}$$

$$= \frac{1 - e^{iNn\phi_x^s}}{1 - e^{in\phi_x^s}} \mathbf{d}^{s}_{30n0} \qquad (n = 1,3) \qquad (2.4)$$

$$\mathbf{d}_{121n} = \mathbf{d}^{s}_{121n} \sum_{\ell=0}^{N-1} e^{i\ell\phi_x^s \pm n N\phi_z^s}$$

$$= \mathbf{d}^{s}_{121n} \frac{1 - e^{iN\phi_x^s \pm nN\phi_z^s}}{1 - e^{i\phi_x^s \pm n\phi_z^s}} \qquad (n = 0,2) \qquad (2.5)$$

where $\mathbf{d}^s$ denotes the integrals over the subperiod and ϕ^s is the phase advance between subperiods. For the simplest case in which a subperiod is a FODO cell with $\phi_x^s = \phi_z^s = \phi_c$ we obtain from (2.4) and (2.5) that all first order driving terms vanish if

$$\phi_c = \frac{M}{N} 2\pi \qquad (2.6)$$

where N is the number of cells and M is an integer. In the HERA electron ring there are 25 cells per octant and we have investigated sextupole distributions with N = 23, 24 and 25 leading to the selection of ϕ_c's listed in Table 2.1. There are two exceptions from the selection rule (2.6), namely the optics with 67.5°/cell and 88.5°/cell. The reasons for including these cases are the following:

For the 67.5° optics we have M/N = 3/16 and a full arc with 3 x 16 = 48 cells is needed to cancel the first order perturbations. This is not a problem for the arcs between IP's South and East and between East and North, but the other two arcs are interrupted by the insertion of the West quadrant with different sextupole strengths. However, one can check that due to the integer phase advance of the West quadrant:

1. the West quadrant itself does not contribute to first order perturbations if we assume perfect periodicity in the arcs

2. the two "half arcs" adjacent to the West quadrant are connected by a unit transfer matrix and are therefore equivalent to a complete 48 cell section.

ϕ_C/deg.	M/N	# of families	structure of subperiod
60	1/6	6	A-B-C
67.5	3/16	10	A-B-C-B-D-B-E-B
72	1/5	8	A-B-C-D-B
75	5/24	10	A-B-B-C-B-B-D-B-B-E-B-B
78.3	5/23	6	A-C-A-B-A-B-C-A-C-A-B-A- -B-C-A-C-A-B-A-B-C-A-C
86.4	6/25	10	C-E-C-E-C-E-C-E-B-B-B-B-A- -D-A-D-A-D-A-D-A-B-B-B-B
88.5	≃ 1/4	4	A-B
90	1/4	4	A-B

Table 2.1 Sextupole distributions for various values of phase advance per FODO cell.

We now consider the case where more than one sextupole family per plane is used for compensating the off-energy β-beat. The FODO cells are then no longer identical and the periodicity of the arcs is changed. However, if there still exist two or more identical subsections of phase advance $\phi_C \neq 2n\pi$ within a subperiod of integer phase advance $M\phi_C$ the same arguments as used above apply and the cancellation of first order driving terms can be retained.

Let us explain this for the example of the $\phi_C = 75°$ optics. One octant consists of two identical subsections of 12 cells each. Within the subperiod we define a sequence with 5 sextupole families for each plane according to:

-A-B-B-C-B-B-D-B-B-E-B-B-

For contributions of each sextupole to the integrals (2.1) the relative phase between them is essential. This illustrated in Fig. 2.1, where the individual sextupoles are represented by unit vectors in a complex plane with a phase separation of $2\phi_c$.

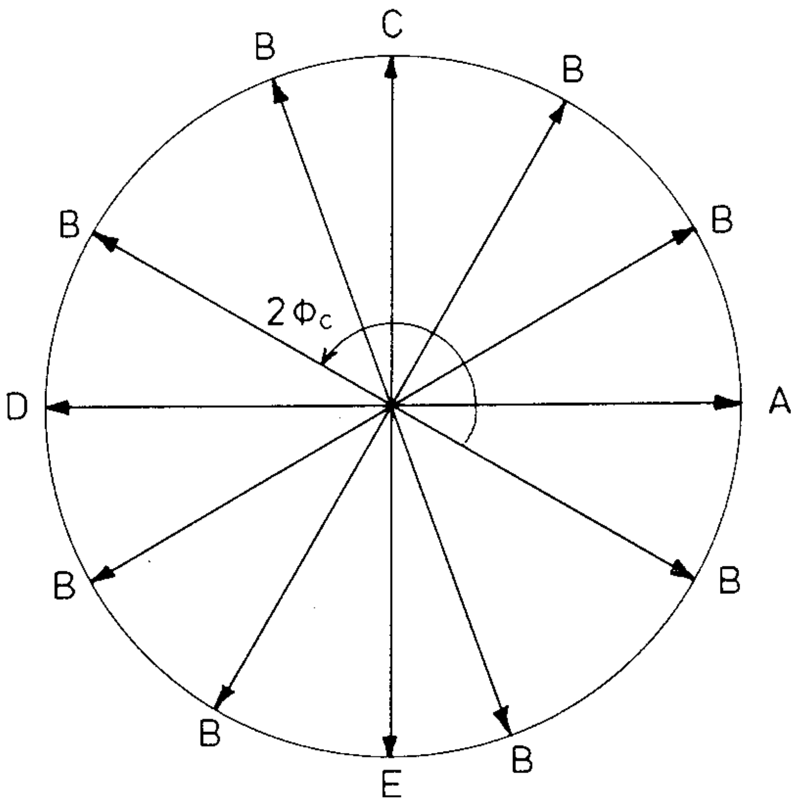


Fig. 2.1 Contributions of sextupole families to the "β-beat" integral, eq. (2.1) (schematically)

From Fig. 2.1 one concludes that family B does not contribute to the β-beat whereas the differences between families A, D and C, E give orthogonal contributions. Thus, with the constraints $m_A + m_D = 2m_B$, $m_c + m_E = 2m_B$ we have three orthogonal "knobs" (m_B, $m_A - m_D$ and $m_c - m_E$) with which to compensate the chromaticity and the sine- and cosine-like part of the β-beat independently. This is a very convenient feature of this sextupole distribution.

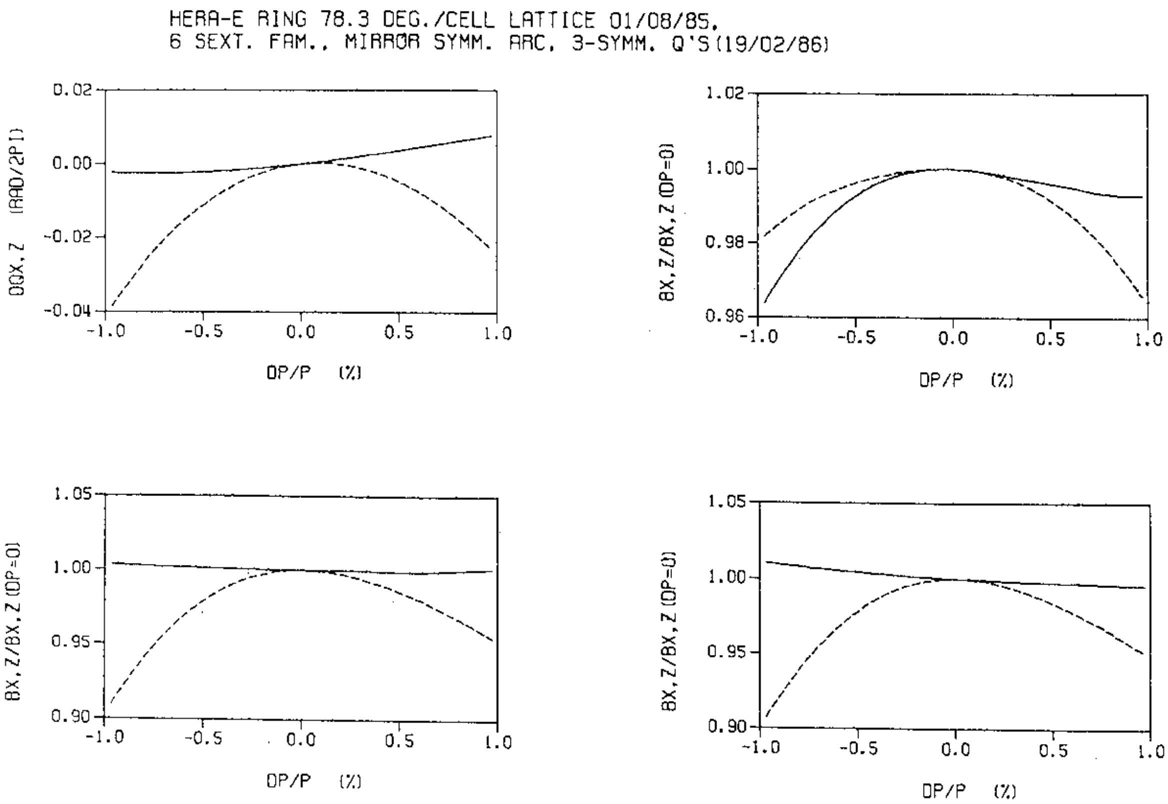
Subperiods within an octant (or a quadrant) can also be defined for $\phi_C = 60^o$, $\phi_C = 67.5^o$, $\phi_C = 72^o$ and $\phi_C = 90^o$ (see table 2.1 for the structure of all sextupole distributions). The 67.5^o distribution has the same features as the one for 75^o. For $\phi_C = 60^o$ only three sextupole families per plane are useful and one cannot define orthogonal knobs for chromaticity and β-beat compensation. For $\phi_C = 90^o$ only the sine-like part of the β-beat with respect to the sextupole lattice can be compensated. In this case, the phase advance between the arc and the I.P. was adjusted properly in order to be able to compensate the β- beat generated in the low-β-insertion (which is purely cosine-like with respect to the I.P.).

The 72^o-case has the disadvantage that the subsections of 5 cells themselves have integer phase advance. Then the first order perturbations caused by the differences of sextupole strengths add up coherently and we expect a bad non-linear acceptance for this optics.

For $\phi_C = 78.3^o$ and $\phi_C = 86.4^o$ no complete subperiods with integer phase advance within an octant can be defined and the first order driving terms do not cancel exactly if more than two sextupole families are introduced. However, for these optics there is no coherent build-up of perturbations (c.f the 72^o case) and with the distributions shown in table 2.1 the cancellation of nonlinear perturbations still works quite well.

Good chromatic properties are obtained with our sextupole distributions for all phase advances. The remaining variation of β-functions at the I.P.'s is kept below about ± 5 % and the tune variation below ± 0.01 for momentum deviations of $-1\ \% \leqslant \Delta p/p \leqslant 1\ \%$.

Fig. 2.2 shows the energy dependence for the 78.3^o optics as an example.



<u>Fig. 2.2</u> Variation of tunes (a) and β-functions at interaction points East (b), North (c) and South (d) with momentum deviation for the 78.3° per FODO cell optics. (continuous line: x-plane, broken line: z-plane)

3. Tracking Results

The dynamic aperture for the different optical solutions has been investigated by particle tracking using a modified version of the RACETRACK[2)] code. The procedure and our definition of dynamic aperture is described in detail in ref. 1) and here we confine ourselves to a presentation of the results.

For each of the optics an optimum working point with maximum dynamic aperture is determined by scanning the horizontal and vertical tunes. At these working points, tracking calculations are carried out both for constant momentum deviation and for synchrotron oscillations. The parameters used for tracking are listed in Table 3.1.

The results for the maximum stable phase space volume are quoted in Table 3.2. There is a strong decrease of dynamic aperture with increasing phase advance per cell. This is of course expected because of the increase of sextupole strength needed to compensate the chromaticity (see next chapter). The 72°/cell case with 8 sextupole families represents a remarkable exception from the monotonic decrease of $A_x + A_z$ with ϕ_c.

Number of turns	1000
Number of particles	1
Number of amplitude iterations	8
Range of momentum amplitudes	0.8 %
Initial ratio of emittances (for $\phi_x=\sigma_z=0$) $\frac{\varepsilon_z(\phi_x=\phi_z=0)}{\varepsilon_x(\phi_x=\phi_z=0)}$	10 %
Harmonic number	10600
Synchronous phase	45°
Accelerating voltage	200 MV

Table 3.1 Parameters used for all particle tracking runs.

The small acceptance (in contrast to the value for the same optics with 2 sextupole families) confirms the statement of chapter 2 that for $\phi_C = 72^o$ first order nonlinear resonances are strongly driven if sextupole families with different strengths are introduced. We conclude here that this phase advance should not be used for the HERA electron ring.

The results show that the acceptance is reduced in the presence of synchrotron oscillations by a factor of about 0.7...0.5. This reduction is mainly due to the tune variation caused by nonlinear chromaticity and by the off-energy β-beat which increases the peak amplitudes of the particle in the nonlinear fields. There is no strong evidence that satellite resonances play an important role. However, in some cases a dip shows up in the dependence of acceptance on synchrotron oscillation amplitude (Fig. 3.1). At this amplitude ($\Delta\hat{p}/p$ = 0.7 %) the synchrotron frequency Q_S is shifted such that $Q_X - 3|Q_S|$ is an integer. Thus, we interpret these dips as a third order satellite resonance. It has to be noted, however, that a correct investigation of synchro-betatron resonances requires fully symplectic 6 dimensional tracking. This will be done in the future by using the recently developed 6-D version of the RACETRACK code[3)].

We should add the remark here that for the latter cases we do not quote the "on-resonance" values of $A_X + A_Z$ in Table 3.2 but rather take the mean value of points adjacent to the resonance in the $A_X + A_Z$ vs. $\Delta\hat{p}/p$ curves.

From the tune scans performed for each optical solution we notice that resonances driven by terms quadratic or cubic in sextupole strengths become more and more important with increasing ϕ_C. In particular, the 90^o optics shows very strong 4th and 5th integer resonances, see Fig. 3.2. It is interesting to note that the influence of 5th integer resonances is reduced when changing ϕ_C to 88.5^o. This results in a larger dynamic aperture in a certain range of tunes, but on the other hand first order driving terms do not cancel and this results in a rather strong 3rd integer resonance which is almost absent in the 90^o case.

The existence of 3rd integer resonances even for sextupole distributions which fulfill the criteria of driving term cancellation described in section 2 is due to the fact that the periodicity of the optics is destroyed in the matching sections at the end of the arcs, as was already pointed out in ref. 1.

Optics	ϕ_c/deg.	Q_x	Q_z	A_x+A_z ($\frac{\Delta p}{p}$ = 0) [πmmmrad]	A_x+A_z ($\frac{\Delta\hat{p}}{p}$ = 6 %) [πmmmrad]
HE60Q3S6[1]	60.0	47.16	47.28	53	37
HE67Q3S10	67.5	51.16	51.28	36	24
HE72Q3S2[2]	72.0	54.16	54.28	24	14
HE72Q3S8	72.0	54.16	54.28	3.4	
HE75Q3S10	75.0	55.14	55.28	19.0	10.2
HE78Q3S6	78.3	57.79	57.89	18.0	12.9
HE86Q3S10	86.4	59.18	59.23	11.2	5.5
HE88Q3S4	88.5	63.11	62.39	10.4	7.7
HE90Q3S4	90.0	63.11	62.31	7.0	5.5

Table 3.2 Stable phase space areas obtained from tracking calculations. The total acceptance (A_x + A_z) is quoted for on-energy particles and for $\Delta\hat{p}/p$ = 0.6 % synchrotron oscillation amplitude.
1) Results taken from ref. 1
2) 2-family sextupole distribution

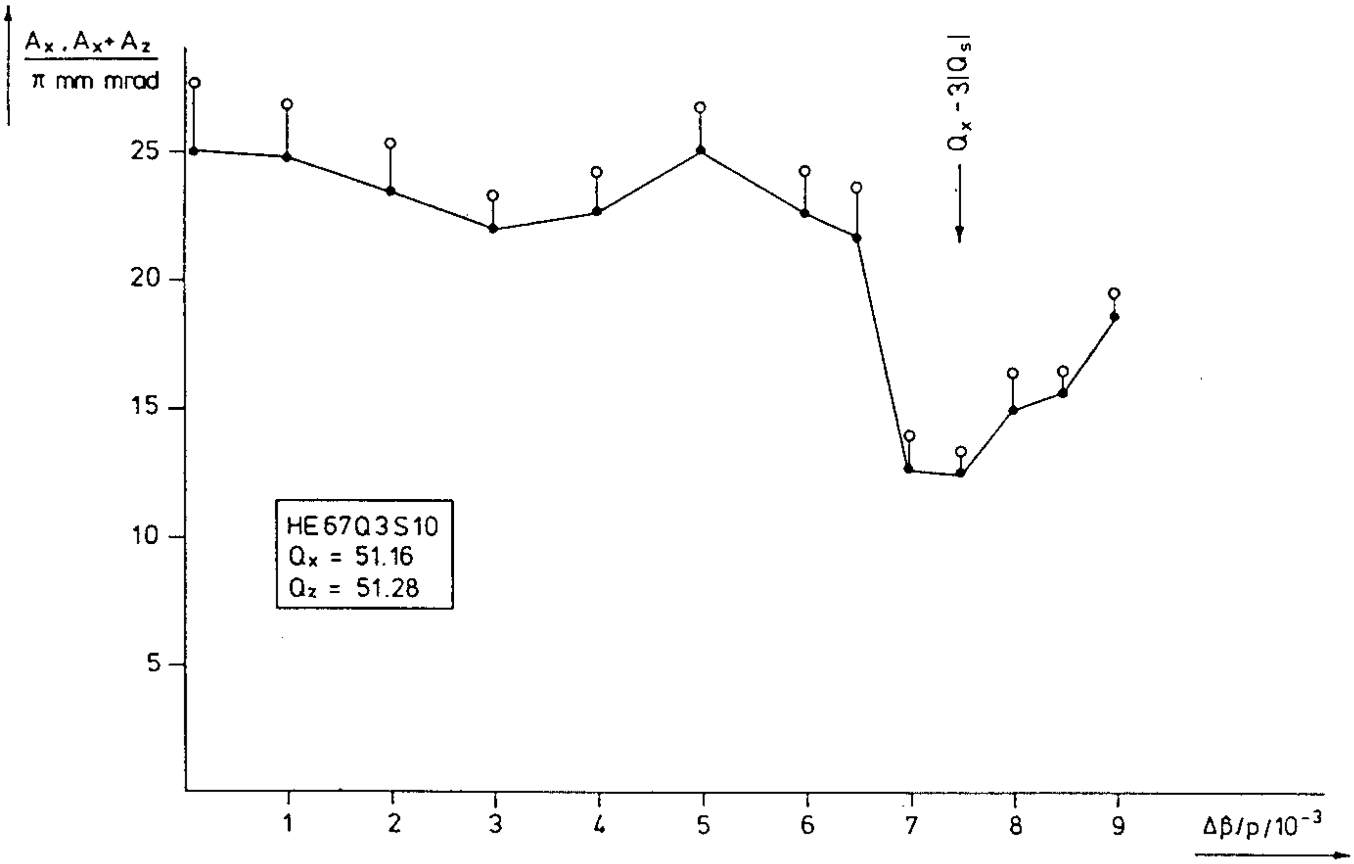
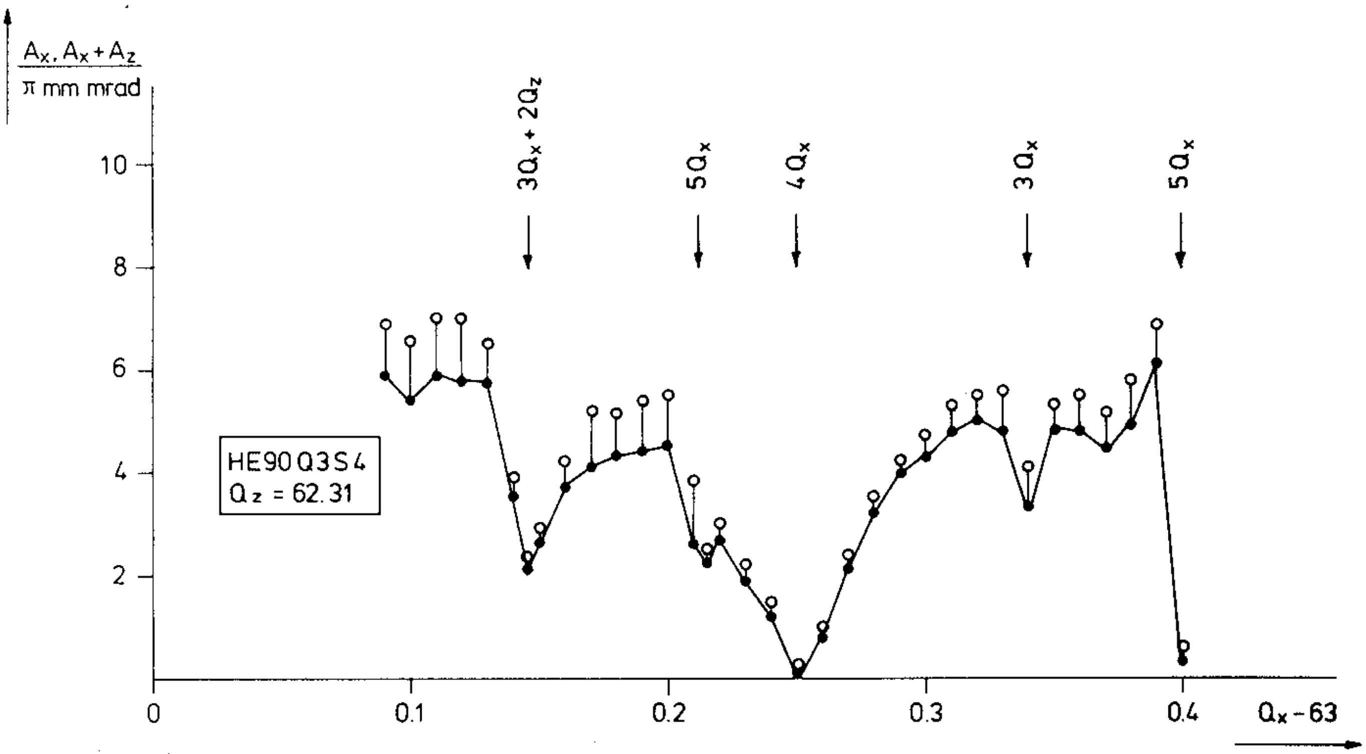
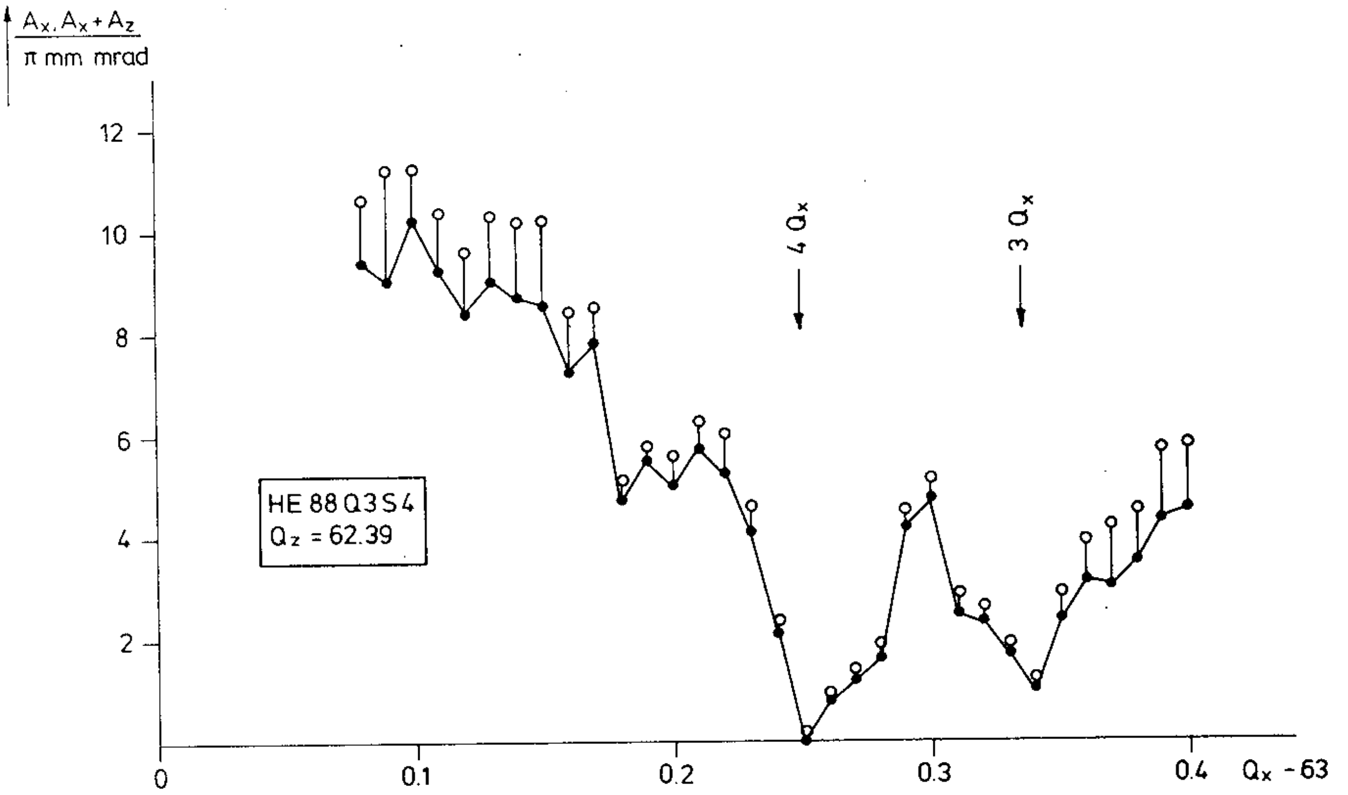


Fig. 3.1 Dependence of the nonlinear acceptance on energy oscillation amplitude for the 67.5° optics.





<u>Fig. 3.2</u> Nonlinear acceptance vs. horizontal tune for the 90°/cell and 88.5° optics.

4. Discussion

The results presented in the preceding chapter show a rapid decrease of nonlinear acceptance with phase advance per cell. However, since the emittance also shrinks with increasing ϕ_c, the dynamic aperture divided by the equilibrium beam size (thus giving the number of standard deviations, n_σ, available for the beam) is a more relevant parameter than the absolute value of the acceptance. Fig. 4.1 shows the results for

$$n_\sigma(\phi_c) = (A_x(\phi_c)/\varepsilon_0(\phi_c))^{1/2} \tag{4.1}$$

for each of the optical solutions with $\Delta p/p = 0$ ($\varepsilon_0(\phi_c)$ is the equilibrium emittance for given phase advance). The space available for the beam decreases slowly from $n_\sigma = 26$ (at $\phi_c = 60°$) to $n_\sigma = 17$ (at $\phi_c = 90°$). Since we expect a value of $n_\sigma = 10$ to be sufficient for a beam storage of several hours, our first conclusion is that the acceptance of the electron ring is sufficiently large over the whole range of phase advance.

A reasonable interpretation of the scaling of nonlinear acceptance with phase advance per cell can be obtained by an analytical estimate of n_σ vs. ϕ_c, based on simple arguments. In order to judge the "quality" of a sextupole distribution, we first need to know what is the effect of the change in sextupole strength for a given distribution and optical structure.

The starting point is that one expects the onset of instability to occur if the relative perturbation $\Delta\varepsilon/\varepsilon$ of the Courant-Snyder invariant reaches a certain critical value. Theory then predicts a scaling behaviour like

$$n_\sigma \sim [\sqrt{\varepsilon_0(\phi_c)}\ \bar{m}(\phi_c)]^{-1} \tag{4.2}$$

where $\bar{m}(\phi_c)$ is the mean sextupole strength for given phase advance ϕ_c (see Appendix). From the equation of motion we therefore get a simple scaling law which relates change of the sextupole strength with change of the dynamic aperture.

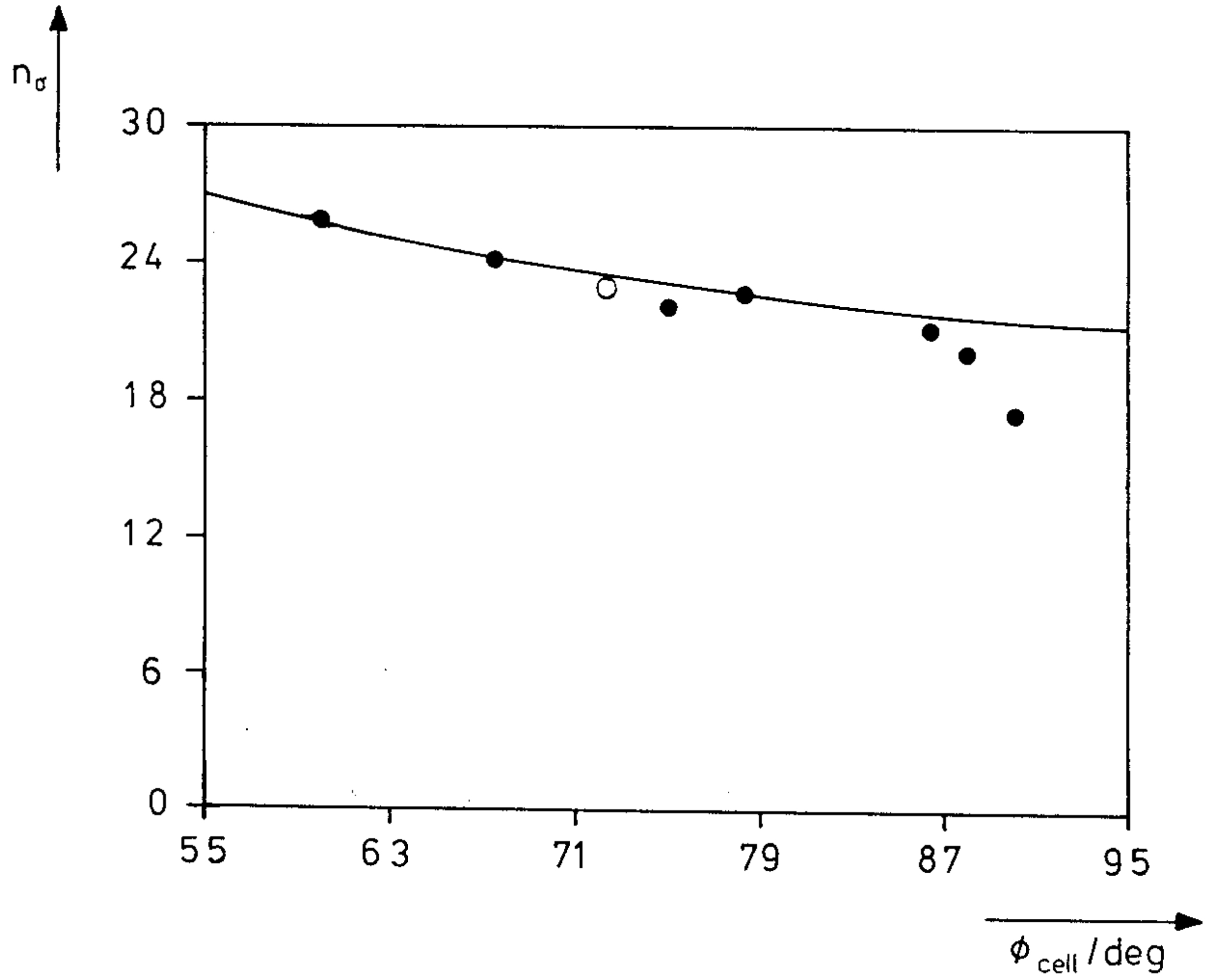


Fig. 4.1 Dynamic aperture divided by equilibrium beam size (at E = 35 GeV), vs. phase advance per FODO cell.
Dots: tracking results
Curve: analytical scaling law (see text)
Open circle: 2-family-distribution (see chapter 3).

The sextupole strengths m_H and m_V needed to compensate the horizontal and vertical chromaticity, respectively, are obtained from

$$m_H = \frac{\hat{\beta}\xi_x + \check{\beta}\xi_z}{\hat{D}(\hat{\beta}^2 - \check{\beta}^2)} \quad , \quad m_V = \frac{\check{\beta}\xi_x + \hat{\beta}\xi_z}{\check{D}(\hat{\beta}^2 - \check{\beta}^2)} \qquad (4.3)$$

where $\hat{\beta}$, $\check{\beta}$ and $\hat{D}$, $\check{D}$ denote the maxima and minima of the periodic beta- and dispersion function, respectively. They are given by (see e.g. K. Steffen in ref. 4):

$$\hat{\beta} = \frac{L}{\sin\phi_c/2}\left(\frac{1 + \sin\phi_c/2}{1 - \sin\phi_c/2}\right)^{1/2} \quad , \quad \check{\beta} = \frac{L}{\sin\phi_c/2}\left(\frac{1 - \sin\phi_c/2}{1 + \sin\phi_c/2}\right)^{1/2}$$

$$\hat{D} = \frac{L^2}{\rho\sin^2\phi_c/2}\left(1 + \frac{1}{2}\sin\phi_c/2\right) \quad , \quad \check{D} = \frac{L^2}{\rho\sin^2\phi_c/2}\left(1 - \frac{1}{2}\sin\phi_c/2\right) \qquad (4.4)$$

where L is the half cell length and ρ the bending radius of the dipole magnet.

The total chromaticities consist of the sum over n FODO cells and the contribution from the straight sections:

$$\xi = n\xi_c + \xi_{ss} = \frac{n}{\pi}\tan\frac{\phi_c}{2} + \xi_{ss} \qquad (4.5)$$

For HERA, we have

$$\xi_{ss,x} \simeq \xi_{ss,z} \simeq 0.2\ n \qquad (4.6)$$

Combining eqs. (4.5) to (4.8) yields, after some algebra:

$$m_{H/V} = \frac{n\rho^2\sin^3\phi_c/2}{4\pi L^3(1 \pm 1/2\sin\phi_c/2)}\,[1 + 1.26\cot\phi_c/2] \qquad (4.7)$$

The equilibrium emittance ε_0 for a machine with a perfectly periodic FODO structure scales like[5]:

$$\varepsilon_0(\phi_C) \sim (1 - \frac{3}{4} \sin^2\phi_C/2 + \frac{1}{60} \sin^4\phi_C/2)/(\sin^2\phi_C/2 \sin\phi_C) \qquad (4.8)$$

Inserting this expression and the mean sextupole strength from eq. (4.7) into eq. (4.2) yields the desired scaling law for n_σ vs. ϕ_C. This curve is shown in Fig. 4.1 together with the tracking results. The points obtained from the tracking simulation lie close to the expected behaviour, indicating that the "quality" of all sextupole distributions is comparable. The largest deviation occurs for $\phi_C = 90°$. We believe that this is a hint to the fact that for this particular rational value of ϕ_C the build-up of higher order perturbations is worse than for the other cases. This interpretation is supported by the results for $\phi_C = 88.5°$ (see chapter 3) which show weaker higher order resonances and a better nonlinear acceptance.

5. Conclusion

We have shown that for the HERA electron ring good chromatic correction schemes exist for a large range of phase advance ϕ_C per arc cell. The use of lowest order resonance compensation in a periodic FODO structure proves to yield satisfactory nonlinear acceptance especially for lower and intermediate values of ϕ_C. For the particular case of $\phi_C = 90°$ the concept is less advantageous because higher order sextupole-driven resonances become more important.

With a dynamic aperture of > 13 standard deviations (for $\Delta\hat{p}/p = 5\,\sigma_E$ at 35 GeV) over the whole range of $\phi_C = 60°...90°$, we conclude that the electron ring will not be operated at the chromaticity limit.

Acknowledgement

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APPENDIX I

Scaling Law for the Dynamic Aperture with Sextupoles

Given a lattice with a certain distribution of sextupole magnets: How does the acceptance A or the dynamic aperture change if the strengths of the sextupoles are changed by the same relative amount? This question can be answered by looking at the equation of motion. In the following this is demonstrated for horizontal betatron oscillations but it can be easily demonstrated for 2 degrees of freedom, too.

If one writes the equation of motion in presence of sextupoles

$$x'' + \kappa(s)x = \frac{m}{2}(s)x^2$$

κ quadrupole strength
m sextupole strength
s coordinate along closed orbit
x horizontal betatron amplitude.

in action and angle variables ϵ, ϕ using the solutions of the linear equation of motion

$$x(s) = \sqrt{2\epsilon}\,\sqrt{\beta}\,\cos(\varphi(s) + \phi)$$

β envelope function
$\varphi(s)$ betatron phase advance function

one finds

$$\epsilon' = \sum_{\ell=1,3} \binom{3}{\frac{3-\ell}{2}} \cdot \frac{1}{3}\left(\frac{\beta}{2}\right)^{3/2} m\cdot\ell\,\epsilon^{3/2}\sin(\ell\varphi(s) + \ell\phi)$$

$$\phi' = \sum_{\ell=1,3} \binom{3}{\frac{3-\ell}{2}} \cdot \frac{1}{3}\left(\frac{\beta}{2}\right)^{3/2} m\,\frac{3}{2}\epsilon^{1/2}\cos(\ell\varphi(s) + \ell\phi)$$

thus

$$\epsilon'/\epsilon = f(\phi,\ m\epsilon^{1/2},\ s)$$

$$\phi' = g(\phi,\ m\epsilon^{1/2},\ s)$$

This means that if m is changed but $m\sqrt{\epsilon_i}$ is kept constant (where ϵ_i is the initial value of ϵ) $\delta\epsilon/\epsilon$ must be the same for all s and all values of ϕ. Thus, the phase space trajectories have the same shape and only the measure of ϵ changes. It also means that the acceptance changes only as the measure of ϵ changes. Expressing the acceptance for an electron beam in terms of the equilibrium emittance ϵ_o at one standard deviation of the Gaussian distributed beam and the number of standard deviations n_σ we find the simple scaling law

$$n_\sigma \sim \frac{1}{m\sqrt{\epsilon_o}}$$