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On Leading Logarithm Behaviour of Jet Cross Sections in e^+e^- Annihilation

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Abstract:
We report an unexpected leading logarithmic behaviour of the two-jet cross section in e^+e^--annihilation in order α_s^2.

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1. Introduction

In 1977 Sterman and Weinberg /1/ calculated the $O(\alpha_s)$ two-jet cross section in e^+e^--annihilation. Two years later Smilga /2/ claimed that the leading and next-to-leading logarithm of the Sterman-Weinberg formula should exponentiate to give the leading and next-to-leading two-jet cross section in any order. This is reminiscent of the behaviour of certain cross sections and form factors in QED.

In a recent paper /3/ we have calculated the full two-jet cross section in $O(\alpha_s^2)$. We were able to confirm Smilga's conjecture to a certain degree. Namely the QED-part of the cross section exponentiates as predicted. The pure QCD-part which according to the conjecture should only contribute a next-to-leading logarithm gets in addition a leading contribution - although with a rather small coefficient. It is the purpose of this letter to examine the origin of its appearance.

One remark is in order here: If we naively integrate the four parton diagrams of the process $e^+(p_+) + e^-(p_-) \rightarrow \gamma(q) \rightarrow q(p_1) + \bar{q}(p_2) + g(p_3) + g(p_4)$ over the two-jet regions of phase space we confirm Smilga's conjecture /4/. However, this is no longer true, if we want the two-jet cross section to be adjusted to the three- and four-jet cross sections as used in experimental analyses. By this we mean that the two-jet cross section together with the three- and four-jet cross sections should add up to yield the total cross section as calculated by Celmaster and Gonsalves /5/.

2. Deviation from Exponentation

The Sterman-Weinberg formula in $O(\alpha_s)$ reads

$$\sigma^{SW}_{2\text{-jet}} = \sigma_0 \left[1 + \frac{\alpha_s}{2\pi} C_F \left(-2\ln^2 y - 3\ln y - 1 + \frac{\pi^2}{3} + O(y)\right)\right] \quad (1)$$

Here $\sigma_0 = 4\pi\alpha^2 N_c (\sum_f Q_f^2)/(3q^2)$ is the lowest order cross section, $C_F = 4/3$, $N_c = 3$ and y is the invariant mass cut used to define a jet /3/. Exponentiating the leading and next-to-leading logarithm of (1) gives

$$\sigma^{exp}_{2\text{-jet}} = \sigma_0 \exp\left\{-\frac{\alpha_s(yq^2)}{2\pi} C_F \left(2\ln^2 y + 3\ln y\right)\right\} \quad (2)$$

Expanding this to $O(\alpha_s^2)$ we get

$$\sigma_{2\text{-jet}} = \sigma_0 \left\{1 - \frac{\alpha_s(q^2)}{2\pi}\left(2\ln^2 y + 3\ln y\right) + \frac{\alpha_s(q^2)}{2\pi} C_F \left[C_F\left(2\ln^4 y + 6\ln^3 y\right) + \left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\ln^3 y\right]\right\} \quad (3)$$

In (2) the running coupling has been introduced at the scale yq^2 to produce the desired N_c- and n_f-contributions in (3) (n_f = number of flavours). We call the N_c-contribution "pure QCD", because in the QED-limit $N_c = 0$, $C_F = 1$. Eq. (2) is in accordance with Smilga's result /2/. In contrast to (3) the explicit calculation of the $O(\alpha_s^2)$ two-jet cross section /3/ gives a term $-\frac{1}{12}\sigma_0\left(\frac{\alpha_s}{2\pi}\right)^2 C_F N_c \ln^4 y$ to be added to the terms on the right hand side of (3). It is the purpose of this letter to trace its origin.

3. Origin of the Additional N_c-Term

The origin of the additional leading logarithmic contribution lies in a term

$$b_0 := \frac{y_{12}}{2\, y_{13}\, y_{24}\, y_{34}} \quad (4)$$

where $y_{ij} = 2p_i p_j/q^2$. This term is the most singular contribution from the four-parton diagrams to the N_c-term. If integrated over the two-jet region

$$P_c := (y_{134} < y \text{ or } y_{234} < y) + (y_{13} < y,\ y_{24} < y,\ y_{134} > y,\ y_{234} > y) \quad (5)$$

$$(\text{with } y_{ijk} := y_{ij} + y_{ik} + y_{jk})$$

the additional term $\sim N_c \ln^4 y$ does <u>not</u> appear. In other words: Using $P_o b_o$ one verifies Smilga's conjecture /2/. This is the "singular approach" of ref. 3 (see also /4/). However, the three- plus four-jet region used to calculate three- and four-jet cross sections /6/ is not the complement to (5). Instead it is defined as the region, where at most one of the y_{ij} is smaller than y. So what one does is the following:

As b_o appears in the symmetrical combination b_o + (1-2) + (3-4) + (1-2, 3-4) (here (1-2) etc. refers to interchange of momenta $p_1 \leftrightarrow p_2$), one rewrites it as

$$b_0 + (1\text{-}2) + (3\text{-}4) + (1\text{-}2,3\text{-}4) = \mathcal{B}_0 + (1\text{-}2) + (3\text{-}4) + (1\text{-}2,3\text{-}4) \quad (6)$$

with $B_o = B_{34} + B_{13}$ and

$$\mathcal{B}_{34} = \frac{y_{12}}{2\,y_{34}(y_{13}+y_{34})(y_{24}+y_{34})} \tag{7}$$

$$\mathcal{B}_{13} = \frac{y_{12}}{y_{13}(y_{13}+y_{34})(y_{13}+y_{24})} + \frac{y_{12}}{(y_{13}+y_{34})(y_{24}+y_{34})(y_{13}+y_{24})} \tag{8}$$

(In fact one has $2b_o = B_o + B_o(1\text{-}2,\ 3\text{-}4)$.) The partial fractioning in (7) and (8) has the advantage that in the three- plus four-jet region every term has a singularity for at most one y_{ij} going to zero. For example, B_{34} has a singularity only for $y_{34} \to 0$. Therefore it is natural to define the three-jet region as ($y_{34} < y$, $y_{134} > y$, $y_{234} > y$) and the two-jet region as being only $P_{34} := (y_{134} < y$ or $y_{234} < y)$ in contrast to (5). (P_{34} is the natural two-jet region for B_{34} also from the standpoint of differential three-jet cross sections. There one defines effective three-particle variables $y_{\mathrm{I\ III}} = y_{134}$, $y_{\mathrm{II\ III}} = y_{234}$ for e^+e^- going into jets I = 1, II = 2, III = 3+4.) In the remaining phase space B_{34} is finite and can be integrated numerically. For B_{13}, on the other hand, we must define the two-jet region as $P_{13} := (y_{134} < y) + (y_{13} < y,\ y_{24} < y,\ y_{134} > y)$. This follows from the fact that for the y_{13} pole term the three-jet region is defined as ($y_{13} < y$, $y_{24} > y$, $y_{134} > y$) for calculating the three-jet cross section /6/. In the following we shall prove that the difference $P_{13}B_{13} + P_{34}B_{34} - P_o b_o$ provides for the additional term $\sim N_c \ln^4 y$.

Let us first introduce the notation

$$P_1 := (y_{13} < y,\ y_{24} < y,\ y_{134} > y,\ y_{234} > y) = P_o - P_{34}\ . \tag{9}$$

One can rewrite $P_{13}B_{13} + P_{34}B_{34}$ as

$$P_{13}\mathcal{B}_{13} + P_{34}\mathcal{B}_{34} = P_o\,(\mathcal{B}_{13} + \mathcal{B}_{34}) - P_1\mathcal{B}_{34}$$
$$- \left[(y_{134} > y,\ y_{234} < y) - (y_{13} < y,\ y_{24} < y,\ y_{134} > y,\ y_{234} < y)\right]\mathcal{B}_{13} \tag{10}$$

This is just a trivial redistribution of the different contributions. Because of the symmetry properties of P_o and B_o one has $P_o(B_{13}+B_{34}) = P_o b_o$. Furthermore $y_{234} < y$ implies $y_{24} < y$. Therefore ($y_{13} < y$, $y_{24} < y$, $y_{134} > y$, $y_{234} < y$) = ($y_{13} < y$, $y_{134} > y$, $y_{234} < y$). With this one gets from (10)

$$P_{34}\mathcal{B}_{34} + P_{13}\mathcal{B}_{13} - P_o b_o = -P_1\mathcal{B}_{34} - \left[(y_{134} > y,\ y_{234} < y)\right.$$
$$\left. - (y_{13} < y,\ y_{134} > y,\ y_{234} < y)\right]\mathcal{B}_{13} \tag{11}$$
$$= -P_1\mathcal{B}_{34} - (y_{13} > y,\ y_{134} > y,\ y_{234} < y)\,\mathcal{B}_{13}$$
$$= -P_1\mathcal{B}_{34} - (y_{13} > y,\ y_{234} < y)\,\mathcal{B}_{13}$$

The second term on the right hand side of (11) is finite. It has been calculated numerically as part of the partial fractioned four-jet cross section in /6/. It does not contribute any leading or next-to-leading logarithms. Therefore we have to look only at P_1B_{34}. In full length P_1 is given as /3/

$$P_1 = \int_0^y dy_{13}\, y_{13}^{-\varepsilon} \int_y^1 dy_{134}\, y_{134}^{-1-\varepsilon}(1-y_{134})^{1-\varepsilon} \left\{ \int_0^{y(1-\frac{y_{13}}{y_{134}})} dy_{24} \int_{y-\frac{y_{24}}{1-\frac{y_{13}}{y_{134}}}} du \right.$$
$$\left. + \int_{y(1-\frac{y_{13}}{y_{134}})}^{y} dy_{24} \int_0^1 du \right\} y_{24}^{-\varepsilon}\, u^{-\varepsilon}(1-u)^{-\varepsilon} \tag{12}$$

where $u = y_{34}/(y_{134}-y_{13})$. We work in $n = 4-2\varepsilon$ space time dimensions for the purpose or regularisation. We have kept the ε-dependence in the four particle phase space (12), because P_1B_{34} is singular in the limit $\varepsilon \to 0$. The singularity comes from the region ($u \to 0 \Leftrightarrow y_{34} \to 0$). Therefore it is isolated in the second part of (12). It is a non-leading singularity ($\sim \varepsilon^{-1}$) and the logarithm associated with it is also non-leading ($\sim \ln y$). Therefore we are left with the first part of (12). Putting $\varepsilon = 0$ we get

$$P_1 B_{34} \approx \frac{1}{2} \int_0^y dy_{13} \int_y^1 dy_{134} \frac{1-y_{134}}{y_{134}-y_{13}} \int_0^{y(1-\frac{y_{13}}{y_{134}})} dy_{24} \int_{y-\frac{y_{24}}{1-\frac{y_{13}}{y_{134}}}}^1 \frac{du}{u} \frac{1-u}{y_{24}+(y_{134}-y_{13})u} \frac{1}{y_{13}+(y_{134}-y_{13})u} \tag{13}$$

in the leading and next-to-leading logarithmic approximation. It is now a question of some analysis to prove that in the leading and next-to-leading approximation

$$P_1 B_{34} = \frac{1}{12} \ln^4 y \tag{14}$$

as claimed. Let us remark that we have checked this result by calculating $P_{13}B_{13} + P_{34}B_{34}$ and $P_o b_o$ independently. Both expressions carry leading singularities ($\sim \varepsilon^{-4}$) which drop out only in the difference. These leading singularities generate the leading logarithms in the following sense:

$$P_{13}B_{13} + P_{34}B_{34} = \Big[\frac{5}{2} y^{-2\varepsilon} - \frac{25}{12} y^{-3\varepsilon} + \frac{7}{12} y^{-4\varepsilon} - \frac{1}{2} y^{-5\varepsilon} + \frac{1}{3} y^{-6\varepsilon} - \frac{1}{12} y^{-7\varepsilon} \Big] \varepsilon^{-4} + O(\varepsilon^{-3}) \tag{15}$$

$$P_o b_o = \Big[\frac{5}{2} y^{-2\varepsilon} - 2 y^{-3\varepsilon} + \frac{1}{4} y^{-4\varepsilon} \Big] \varepsilon^{-4} + O(\varepsilon^{-3}) \tag{16}$$

Expanding (15) and (16) into powers of ε we obtain the correct leading logarithmic contributions. There are no other sources of leading logarithms. One may note that high powers of $y^{-\varepsilon}$ as in (15) are driven by two sources. One is a high number of y's as integration boundaries and the other is a high number of partial fractioned factors in the integrand /7/. In order to prove that there is no next-to-leading difference between $P_{13}B_{13} + P_{34}B_{34}$ and $P_o b_o$ we also quote here the $O(\varepsilon^{-3})$-corrections to equations (15) and (16). They are equal and given by

$$(15,16) \longrightarrow (15,16) + \big[5 y^{-2\varepsilon} - 2 y^{-3\varepsilon} \big] \varepsilon^{-3} + O(\varepsilon^{-2}) \tag{17}$$

For the convenience of the reader who wants to verify our calculation, we give specific leading and nonleading contributions separately

$$(y_{134}<y)\; b_0 = \frac{3}{4\varepsilon^4} + \frac{3}{2\varepsilon^3} + \frac{4-3\zeta_2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(10-6\zeta_2-\frac{15}{2}\zeta_3\right) + 24-16\zeta_2$$

$$-15\zeta_3 - \frac{9}{4}\zeta_4 + \left[-\frac{3}{2\varepsilon^3} - \frac{3}{\varepsilon^2} + \frac{6\zeta_2-8}{\varepsilon} - 20 + 12\zeta_2 + 15\zeta_3\right]\ln y$$

$$+\left[\frac{3}{2\varepsilon^2} + \frac{3}{\varepsilon} + 8 - 6\zeta_2\right]\ln^2 y - \left(\frac{1}{\varepsilon}+2\right)\ln^3 y + \frac{1}{2}\ln^4 y \qquad (18)$$

$$(y_{134}<y,\, y_{234}<y)\, b_0 = \frac{3}{4\varepsilon^4} - \frac{2\zeta_2}{\varepsilon^2} - \frac{11}{2\varepsilon}\zeta_3 - \frac{57}{8}\zeta_4 + \left[-\frac{3}{\varepsilon^3} + \frac{8\zeta_2}{\varepsilon} + 22\zeta_3\right]\ln y$$

$$+\left[\frac{6}{\varepsilon^2} - 16\zeta_2\right]\ln^2 y - \frac{8}{\varepsilon}\ln^3 y + 8\ln^4 y$$

$$(19)$$

$$(y_{134}<y)\,\mathcal{B}_{34} = \frac{13}{36\varepsilon^4} + \frac{7}{12\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{4}{3} - \frac{49}{36}\zeta_2 + \frac{\gamma}{2}\right) + \frac{1}{\varepsilon}\left(\frac{25}{12} - \frac{7}{3}\zeta_2 - \frac{10}{3}\zeta_3\right.$$

$$\left.+\frac{5}{2}\gamma - \frac{1}{4}\gamma^2\right) + \left[-\frac{7}{12\varepsilon^3} - \frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{25}{12}\zeta_2 - \frac{5}{2} - \gamma\right) - 4 + \frac{7}{2}\zeta_2\right.$$

$$\left.+3\zeta_3 - 5\gamma + \frac{1}{2}\gamma^2\right]\ln y$$

$$+\left[\frac{3}{8\varepsilon^2} + \frac{3}{4\varepsilon} + \frac{9}{4} - \frac{9}{8}\zeta_2 + \gamma\right]\ln^2 y$$

$$-\left[\frac{1}{24\varepsilon} - \frac{1}{4}\right]\ln^3 y - \frac{13}{96}\ln^4 y \qquad (20)$$

$$(y_{134}<y,\, y_{234}<y)\,\mathcal{B}_{34} = \frac{13}{36\varepsilon^4} - \frac{23\zeta_2}{18\varepsilon^2} - \frac{5\zeta_3}{2\varepsilon} + \frac{23}{12}\zeta_4$$

$$+\left[-\frac{7}{6\varepsilon^3} + \frac{25\zeta_2}{6\varepsilon} + 7\zeta_3\right]\ln y + \left[\frac{7}{4\varepsilon^2} - \frac{23}{4}\zeta_2\right]\ln^2 y$$

$$-\frac{19}{12\varepsilon}\ln^3 y + \frac{47}{48}\ln^4 y \qquad (21)$$

$$(y_{13}<y,\, y_{14}<y,\, y_{134}>y)\,\mathcal{B}_{13} = \frac{2}{3\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{8}{3} - \frac{2}{3}\zeta_2\right) + \frac{1}{3\varepsilon}\left(23-8\zeta_2-5\zeta_3\right)$$

$$+\left[\frac{2}{3\varepsilon^3} + \frac{1}{\varepsilon}\left(\frac{\zeta_2}{3}-3\right) - 11 + 4\zeta_2 + 3\zeta_3\right]\ln y$$

$$+\left[-\frac{3}{2\varepsilon^2} - \frac{2}{\varepsilon} + \zeta_2 - \frac{1}{2}\right]\ln^2 y + \left[\frac{5}{3\varepsilon} + \frac{10}{3}\right]\ln^3 y$$

$$-\frac{29}{24}\ln^4 y \qquad (22)$$

$$(y_{134}<y)\,\mathcal{B}_{13} = \frac{7}{18\varepsilon^4} + \frac{7}{6\varepsilon^3} + \frac{1}{\varepsilon^2}\left(\frac{11}{3} - \frac{17}{18}\zeta_2 - \gamma\right) + \frac{1}{\varepsilon}\left(\frac{73}{6} - \frac{14}{3}\zeta_2 - \frac{11}{3}\zeta_3 - 5\gamma + \frac{\gamma^2}{2}\right)$$

$$+\left[-\frac{2}{3\varepsilon^3} - \frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-6+\frac{11}{3}\zeta_2+2\gamma\right) - 21 + 9\zeta_2 + 10\zeta_3 + 10\gamma - \gamma^2\right]\ln y$$

$$+\left[\frac{1}{2\varepsilon^2} + \frac{3}{2\varepsilon} + 4 - \frac{7}{2}\zeta_2 - 2\gamma\right]\ln^2 y - \left[\frac{1}{6\varepsilon} + \frac{1}{2}\right]\ln^3 y - \frac{1}{24}\ln^4 y \qquad (23)$$

$$(y_{13}<y,\ y_{24}<y,\ y_{134}>y,\ y_{234}>y)\, b_0 = \frac{1}{\varepsilon^2} + \frac{4-2\zeta_3}{\varepsilon} + 12 - 2\zeta_2 - 2\zeta_3 - \frac{5}{4}\zeta_4$$

$$+ \left[\frac{2}{\varepsilon^2} + \frac{2}{\varepsilon} - 4\zeta_2 + 6\zeta_3\right] \ln y + \left[\frac{1}{\varepsilon^2} - \frac{1}{2\varepsilon} - 8 - 2\zeta_2\right] \ln^2 y$$

$$+ \left[\frac{19}{3} - \frac{3}{\varepsilon}\right] \ln^3 y + \frac{11}{24} \ln^4 y \qquad (24)$$

In these expressions γ is the Euler number and ζ_n are the usual ζ functions.

So we see that the difference in the leading logarithm behaviour of the N_c-term comes from different definitions of the two-jet region in the two approaches. In the so-called singular approach the two-jet region was defined in terms of P_o given in (5), saying that all configurations, where either three partons or two pairs of partons have invariant squared masses less than y are considered two jets. If we apply the two-jet constraint to the original four-particle configuration this is the correct kinematic definition for two jets. It is in complete analogy to the two-jet region used for the other pole terms, as for example the y_{13}-pole term in the C_F and N_c-contributions /3/. But, as already mentioned, the region P_o is not the complement of the three-plus four-jet region used for calculating three- and four-jet cross sections, so that with P_o the total cross section could not be matched. This is possible only if we use the kinematic region P_{34} in the y_{34}-pole term. As already mentioned above, P_{34} can be characterized as the procedure that the two-jet criteria is applied to the three-jet configuration, described by the variables $y_{I\ III} = y_{134}$ and $y_{II\ III} = y_{234}$ and not to the original four-particle configuration. Since both procedures are legitimate we

have no reason to prefer either one. Of course, only the approach with P_{34} matches the three- and four-jet calculation. In case we would prefer P_o instead of P_{34}, we must transfer the terms included in the three- and four-jet calculation, i.e. the term (14) and the additional one-leading terms, to the two-jet cross section. The fact, that different two-jet constraints, as our P_o and P_{34}, lead to different results for cross section, has been found also recently by studying the recombination dependence of the $O(\alpha_s^2)$ three-jet cross section /8/.

4. Conclusions

We have found an additional leading logarithm in the two-jet cross section contrary to naive expectations. We have traced back its origin to the definition of the three-jet cross section as used in all earlier calculations /6, 9/.

The importance of the additional leading logarithm became clear to us when we tried to reproduce the total cross section at very small values of y (y = 0.001). However, it plays a role even at physical values of y ($0.02 \leq y \leq 0.05$), where the leading and next-to-leading contribution of the C_F-term almost compensate each other. This is to say, the nonabelian (N_c-) part of the theory is very much influenced by this additional term and depends very much on how the two-jet cross section is defined.

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