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ABSTRACT

Leptonic decay constants of heavy pseudoscalar mesons are estimated in QCD by means of Hilbert transform power-moment sum rules at $Q^2=0$. The meson masses are also obtained in order to assess the reliability of these predictions. Our results are: $f_D/f_\pi = 1.7\pm0.2$, $f_F/f_\pi = 2.1\pm0.1$, and $f_B/f_\pi = 1.1-1.6$. As a byproduct a bound on f_{Bs}/f_B is obtained.

An accurate knowledge of the leptonic decay constants of heavy pseudoscalar mesons is quite important as they play the key role of absolute normalizations to a wide variety of heavy flavour weak transitions. In the absence of experimental data, considerable effort has been devoted to the theoretical estimate of f_D, f_F, f_B, etc., in particular in the framework of QCD sum rules [1] (for a review of other estimates see e.g. [2]). As is well known, this method relates through dispersion relations low energy parameters, such as particle masses and coupling constants, to the short distance Operator Product Expansion (OPE) of current correlators. This OPE is assumed to be valid in the presence of non-perturbative effects which are parametrized by a set of vacuum expectation values of quark and gluon fields. These vacuum condensates induce power corrections to asymptotic freedom [3]. Although this method seems well defined, there is some freedom in the choice of the optimal weight in the dispersion relation and in the model used for parametrizing the spectral function. Also, the method is sensitive to the number of loops considered in the calculation of the asymptotic freedom contribution, as well as to the number and actual values of the vacuum condensates included in the OPE. All this has resulted in a wide spectrum of predictions for f_D, f_B, etc, e.g. f_B = 60-200 MeV. Since on the one hand, better knowledge of these constants would allow for more accurate predictions in a number of semi-leptonic and non-leptonic transitions, and on the other hand there has been recent progress in the determination of vacuum condensates, we feel that a systematic reanalysis of this problem is needed. We attempt to do this here by choosing as a starting point the Hilbert transform power moment sum rules at $Q^2=0$. Aside from keeping the number of free parameters to a minimum, these sum rules provide a natural framework to study systems with a heavy quark, in the sense that at $Q^2=0$ non-perturbative effects are parametrized through the OPE as a series in inverse powers of the heavy quark mass m_Q, i.e. the only large mass scale. To this extent, $1/m_Q$ plays the unambiguous role of the short distance expansion parameter. The situation is not so transparent with Laplace transform sum rules as they involve an additional short distance parameter, i.e. the Laplace variable $\sigma \equiv 1/M^2$. Neverthe-

less, we shall also comment on this method at the end. In our analysis we incorporate the leading power corrections of dimension $d \leq 6$ and consider perturbative effects up to order $O(\alpha_s)$. Regarding the spectral function parametrization, in addition to the lowest pseudoscalar meson pole we include continuum contributions and compute the corrections they induce at the one and two-loop level. Finally, to assess the reliability of our results for the leptonic decay constants we also estimate the masses of the heavy pseudoscalar mesons.

We begin by defining the two-point function

$$\psi_5(q) = i \int d^4x \, e^{iqx} \langle 0 | T(\partial^\mu A_\mu(x) \, \partial^\nu A_\nu(0)) | 0 \rangle , \tag{1}$$

where

$$\partial^\mu A_\mu(x) = (m_q + m_Q) : \bar{q}(x) \, i\gamma_5 \, Q(x) : , \tag{2}$$

with $q(x)$ ($Q(x)$) being a light (heavy) quark field and $m_q(m_Q)$ its corresponding current mass. By selecting $q=u,d,s$ and $Q=c,b$ the axial-vector current divergences (2) will have the quantum numbers of D,F and B mesons. The function $\psi_5(q)$ satisfies the dispersion relation ($Q^2 \equiv -q^2 > 0$)

$$\psi_5(Q^2) = \frac{1}{\pi} \int_0^\infty ds \, \frac{\mathrm{Im}\,\psi_5(s)}{(s+Q^2)} + \text{subtractions}, \tag{3}$$

defined up to two subtractions, arising from external renormalization, which can be disposed of by taking at least two derivatives in (3). This procedure leads to the Hilbert transform or power moment sum rules, which at $Q^2=0$ become

$$\varphi^{(n)} \equiv \frac{(-)^{n+1}}{(n+1)!} \left(\frac{d}{dQ^2} \right)^{n+1} \psi_5(Q^2) \Big|_{Q^2=0} = \frac{1}{\pi} \int_0^\infty \frac{ds}{s^{n+2}} \, \mathrm{Im}\,\psi_5(s). \tag{4}$$

Up to the actual values of the vacuum condensates the moments

$\varphi^{(n)}$ can be computed in QCD and thus Eq.(4) relates the resonance parameters in the spectral function $\frac{1}{\pi}\mathrm{Im}\,\psi_5(s)$ to the fundamental perturbative and non-perturbative QCD parameters in $\varphi^{(n)}$. Although the quark and gluon condensates cannot yet be computed from first principles, they can be extracted from independent experimental data using QCD sum rules in other channels, e.g. the charmonium system, e^+e^- total cross sections, etc.

The perturbative QCD expression for the spectral function at the two-loop level, as first obtained correctly in [1.e], [4], is given by

$$\frac{1}{\pi}\mathrm{Im}\,\psi_5(q^2)\Big|_{A.F.} = \frac{3(m_q+m_Q)^2\,\bar{q}^4\,v}{8\pi^2 q^2}\Bigg\{1 + \frac{4\alpha_s}{3\pi}\Big\{\frac{3}{8}(7-v^2) + \sum_{i=1}^{2}\Big[(v+v^{-1})\big(\mathrm{Li}_2(\alpha_1\alpha_2) - \mathrm{Li}_2(-\alpha_i) - \ln\alpha_i\ln\beta_i\big) + A_i\ln\alpha_i + B_i\ln\beta_i\Big]\Big\}\Bigg\}, \tag{5}$$

where

$$\mathrm{Li}_2(x) = -\int_0^x dt\, t^{-1}\ln(1-t), \tag{6}$$

$$A_1 = \frac{3}{4}\left(\frac{3m_q+m_Q}{m_q+m_Q}\right) - \frac{19+2v^2+3v^4}{32\,v} - \frac{m_q(m_q-m_Q)}{\bar{q}^2 v(1+v)} \times\left(1+v+\frac{2v}{\alpha_1}\right) \tag{7}$$

$$B_1 = 2+2(m_q^2-m_Q^2)/\bar{q}^2 v\,;\quad \alpha_1 = \frac{m_q}{m_Q}\left(\frac{1-v}{1+v}\right); \tag{8}$$
$$\beta_1 = \sqrt{1+\alpha_1}\,(1+v)^2/4v,$$

and $\bar{q}^2 = q^2 - (m_q - m_Q)^2$, $v = (1 - 4m_q m_Q / \bar{q}^2)^{1/2}$. The expressions for A_2, B_2, α_2 and β_2 are obtained from (7) - (8) by the interchange $m_q \leftrightarrow m_Q$. In particular, in the limit $m_q \to 0$, well

justified for up- and down-quarks as in the case of D and $B_{u,d}$ mesons, as exact integration of (5) leads to [1.e]

$$\varphi^{(n)}_{A.F.} = \frac{3}{8\pi^2}\left(\frac{1}{m_Q^2}\right)^{n-1} B(n,3)\left(1+a_n^0\,\alpha_s\right), \tag{9}$$

where B(x,y) is the beta function and a_n^0 are the rational numbers

$$\frac{3\pi}{4}a_n^0 - \frac{\pi^2}{6} = 1 - \frac{2}{n+1} - \frac{6}{n+2} + \sum_{r=1}^{n+2}\Bigg\{\frac{1}{r^2} + \left[\frac{3}{2} - \frac{1}{n} - \frac{1}{(n+1)} - \frac{1}{(n+2)} + \frac{3}{(n+3)}\right]\times\frac{1}{r}\Bigg\}. \tag{10}$$

Parametrizing non-perturbative corrections to asymptotic freedom through the OPE and keeping the leading vacuum condensates of dimension $d \lesssim 6$, one has, always neglecting m_q,

$$\psi_5(q^2)\Big|_{N.P.} = \frac{m_Q^2}{m_Q^2-q^2}\,C_4\langle O_4\rangle + \frac{m_Q^3}{4}\,\frac{q^2}{(m_Q^2-q^2)^3}\,C_5\langle O_5\rangle + \frac{m_Q^2}{6}\left[\frac{2}{(m_Q^2-q^2)^2} - \frac{m_Q^2}{(m_Q^2-q^2)^3} - \frac{m_Q^4}{(m_Q^2-q^2)^4}\right]C_6\langle O_6\rangle, \tag{11}$$

where the $C_n\langle O_n\rangle$ are defined as

$$C_4\langle O_4\rangle = \left\langle \frac{\alpha_s}{12\pi}G^2 - m_Q\bar{q}q\right\rangle, \tag{12}$$

$$C_5\langle O_5\rangle = \left\langle g_s\,\bar{q}\,i\,\sigma\,\vec{G}\cdot\vec{\lambda}\,q\right\rangle, \tag{13}$$

$$C_6\langle O_6\rangle = \pi\alpha_s\left\langle(\bar{q}\gamma_\mu\vec{\lambda}q)\sum_q \bar{q}\gamma_\mu\vec{\lambda}q\right\rangle. \tag{14}$$

Computing the non-perturbative part of $\varphi^{(n)}$ through (11) and adding it to (9) one finds using (4) that for the first few moments, with increasing n the perturbative piece of $\varphi^{(n)}$ decreases in magnitude but at the expense of an increase in the non-perturbative contributions. As a compromise we then choose to consider only the first two moments; this will allow us to estimate f_p as well as M_P. From Eq.(11) one obtains

$$\varphi^{(1)}_{N.P.} = \frac{C_4 \langle O_4 \rangle}{m_Q^4} + \frac{3}{4} \frac{C_5 \langle O_5 \rangle}{m_Q^5} - \frac{5}{3} \frac{C_6 \langle O_6 \rangle}{m_Q^6}, \tag{15}$$

$$\varphi^{(2)}_{N.P.} = \frac{1}{m_Q^2} \left[\frac{C_4 \langle O_4 \rangle}{m_Q^4} + \frac{3}{2} \frac{C_5 \langle O_5 \rangle}{m_Q^5} - \frac{11}{3} \frac{C_6 \langle O_6 \rangle}{m_Q^6} \right]. \tag{16}$$

Adding (15) - (16) to (9) completes the theoretical calculation of the l.h.s. of (4) in QCD.

Turning to the hadronic spectral function appearing in the r.h.s. of (4) we choose the following parametrization

$$\frac{1}{\pi} \operatorname{Im} \psi_5(s)\Big|_{HAD.} = 2 f_P^2 M_P^4 \, \delta(s - M_P^2) + \theta(s - s_0) \frac{1}{\pi} \operatorname{Im} \psi_5(s)\Big|_{A.F.}, \tag{17}$$

where s_0 is the threshold for asymptotic freedom, the second term in (17) is given in (5), and

$$\langle 0 | A_\mu | P(p) \rangle = i \sqrt{2} f_P p_\mu, \tag{18}$$

defines the leptonic decay constant f_P; with this normalization e.g. $f_\pi = 93.2$ MeV. Given the rather large masses of the D,F and B mesons we would expect the spectral function to be relatively smooth for $s > M_P^2$, and thus the second term in (17), i.e. the perturbative QCD continuum, should be a reasonable parametrization of $\operatorname{Im}\psi_5(s)_{HAD}$ at such energies. In other words, we assume

that potential radial excitations of the heavy pseudoscalar mesons do not show up as prominent narrow peaks in the spectral function. In this case the first two moments read

$$\frac{2 f_P^2}{M_P^2} = \frac{1}{8\pi^2} \left[(1 - a_1) + \alpha_s (0.751 - b_1) \right] + \frac{C_4 \langle O_4 \rangle}{m_Q^4} + \frac{3}{4} \frac{C_5 \langle O_5 \rangle}{m_Q^5} - \frac{5}{3} \frac{C_6 \langle O_6 \rangle}{m_Q^6}, \tag{19}$$

$$\frac{2 f_P^2}{M_P^2} = \frac{M_P^2}{m_Q^2} \left\{ \frac{1}{32\pi^2} \left[(1 - a_2) + \alpha_s (1.706 - b_2) \right] + \frac{C_4 \langle O_4 \rangle}{m_Q^4} + \frac{3}{2} \frac{C_5 \langle O_5 \rangle}{m_Q^5} - \frac{11}{3} \frac{C_6 \langle O_6 \rangle}{m_Q^6} \right\}, \tag{20}$$

where a_1, a_2, b_1, b_2 are the continuum corrections which can be computed by integrating the second term in (17).

Concerning the values of the QCD parameters entering the sum rules (19) - (20) we use for the "on-shell" quark masses: $m_c(Q^2 = m_c^2) = 1.3$ GeV, $m_b(Q^2 = m_b^2) = 4.6$ GeV, $\alpha_s(m_c^2) = 0.296$, and $\alpha_s(m_b^2) = 0.214$. For the vacuum condensates we take $m_c\langle\bar{q}q\rangle = -0.014$ GeV^4 amd $m_b\langle\bar{q}q\rangle = -0.055$ GeV^4, which follow from the most recent determination of light quark masses and condensates [5]; $\langle\alpha_s G^2\rangle/12\pi = 0.03$ GeV^4 as extracted from e^+e^- data [6] and charmonium [7]. The quark-gluon condensate $\langle g_s \bar{q}\, i\sigma \vec{G}\cdot\vec{\lambda}\, q\rangle = 2M_0^2 \langle\bar{q}q\rangle$ seems to play a particularly important role in the QCD analysis of the baryon spectrum. The results obtained for the mass parameter M^2_0 are, however, somewhat controversial. We take $M^2_0 = (0.5 \pm 0.1)$ GeV^2 as a compromise between e.g. $M^2_0 \simeq (0.1\text{-}0.4)$GeV^2 [8] or $M^2_0 \simeq (0.6\text{-}1.0)$ GeV^2 [9], and $M^2_0 \simeq 0.3$ GeV^2 [10], the latter value being suggested by charmonium sum rules. We have found that the higher value $M^2_0 \simeq 1$ GeV^2 [11] is in disagreement with the observed D- and F- meson masses. Such a sensitivity to the quark-gluon condensate is actually an interesting aspect of Eqs. (19) - (20). Finally, for $C_6\langle O_6\rangle$ we use $C_6\langle O_6\rangle \simeq -9\pi\alpha_s\langle\bar{q}q\rangle^2$, which takes into account a factor of five increase in the factorization estimate according to recent results from an e^+e^- analysis [6] as well as previous claims from other sources [8], [11].

Starting with the D-meson and solving (19) - (20) for values of the asymptotic freedom threshold within the wide range $S_0 \simeq 2M_D^2 - 3M_D^2$ we obtain

$$M_D = 1.85 \pm 0.15 \text{ GeV}, \tag{21}$$

to be compared with $M_D|_{EXP} = 1.87$ GeV, and

$$f_D = 160 \pm 15 \text{ MeV} \tag{22}$$

Continuum corrections in this channel are important, i.e. without them $M_D \simeq 2\text{-}2.2$ GeV, but yet not so large as to spoil the predictive power of the sum rules. We wish to stress that S_0 is a free parameter in the sum-rule approach; the choice we made is an educated guess based on experience in other channels. The two important points are: (i) predictions should be reasonably stable under changes in S_0 within a (hopefully) wide region, and (ii) obtaining a value for f_D from the sum rules is not enough. Even with everything else fixed f_D is clearly a function of S_0. Hence, some other quantity whose experimental value is well known, e.g. M_D in our case, should be estimated simultaneously in order to assess the reliability of the prediction for f_D. It is rewarding that our results above meet these two requirements.

Considering next the F-meson, if we continue to neglect the light quark mass, m_s in this case, we would obviously find $M_F = M_D$, and $f_F = f_D$. Given the uncertainties in the values of the vacuum condensates and in S_0, which reflect themselves on the errors in (21) - (22), we will be unable to predict the small SU(3) breaking mass splitting $M_F - M_D$. Notice that the mass is

obtained from the ratio of two sum rules. However, the situation is better for f_F. For example, by retaining terms of order $O(\epsilon)$ and $O(\epsilon \ln\epsilon)$, with $\epsilon \equiv m_s/m_c$, Eq.(19) becomes

$$\frac{2f_F^2}{M_F^2} = \frac{1}{8\pi^2}\left\{1 + 3\epsilon + \alpha_s\left[0.751 + \frac{6}{\pi}\epsilon\left(1 + \frac{\pi^2}{9} + \ln\epsilon\right)\right]\right\}$$

$$+ \text{ continuum } + \text{ non-perturbative}. \tag{23}$$

A comparison with the corresponding relation for f_D leads to the approximate expression

$$\frac{f_F}{f_D} \simeq \frac{M_F}{M_D}\left(1 + \frac{3}{2}\epsilon\right) \simeq 1.2 . \tag{24}$$

We have solved the two sum rules in this channel by numerical integration of the imaginary part Eq.(5) retaining m_s but without approximations, i.e. without expanding in ϵ as in Eq.(23). Concerning the non-perturbative terms, at the present level of accuracy we have safely ignored the small SU(3) breaking in the quark condensates and used the same values of $C_n\langle O_n\rangle$ as before. In the broad duality region $S_0 \simeq 2M_F^2 - 3M_F^2$ we obtain

$$M_F = 1.9 \pm 0.1 \text{ GeV}, \tag{25}$$

as expected, and

$$f_F = 194 \pm 12 \text{ MeV}, \tag{26}$$

in nice agreement with the ϵ-expansion result Eq.(24).

Turning to the B-meson an inspection of the various contributions to (19) - (20) shows that except for $C_4\langle O_4\rangle$ the non-perturbative terms are unimportant. Continuum corrections in this channel are very large, a fact known from previous analyses [1.c], e.g. without them we would predict $M_B \simeq 8$ GeV as opposed to $M_B|_{EXP} = 5.27$ GeV. Including these corrections and allowing for the asymptotic freedom threshold to vary in the range $S_0 \simeq 1.1\, M_B^2 - 2\, M_B^2$ brings M_B down to $M_B \simeq (5.1 - 6.2)$GeV. Given the large value of the B-meson mass it would be reasonable to expect a precocious onset of asymptotic freedom in this channel, e.g.

$$S_0 \simeq (1.1 - 1.3)\, M_B^2, \tag{27}$$

which narrows down the resulting mass to

$$M_B = 5.2 \pm 0.2 \ \text{GeV}. \tag{28}$$

It should be emphasized, though, that (28) is not a genuine prediction. If we have not had beforehand information on M_B and had allowed s_0 to vary over a wider range the uncertainty would have been considerably larger. The above procedure is then to be understood more as a determination of S_0 than of M_B. However, since our purpose is to predict f_B, assessing the reliability of this prediction through the resulting value of M_B, we can use S_0 in the range (27) to obtain

$$f_B = 104 - 150 \ \text{MeV}. \tag{29}$$

Using the experimental value of M_B as input, the eigenvalue solution for S_0 is: $S_0 = 1.2\ M_B^2$, and $f_B = 127$ MeV. However, the result (29) provides a better feeling for the true uncertainties involved in this channel.

Concerning the B_s-meson, its decay constant is given by an expression analogous to Eq.(23) except that now $\epsilon \equiv m_s/m_b \simeq 0.04$. Given the smallness of ϵ and of the non-perturbative power corrections we expect the ratio f_{Bs}/f_B to be more accurately predicted than either f_{Bs} or f_B separately. Our result for this ratio is

$$\frac{f_{Bs}}{f_B} \gtrsim \frac{M_{Bs}}{M_B}, \tag{30}$$

which should be useful in connection with oscillations in the B^0-$\bar{B}^0$-meson system (for reviews see e.g. [2a], [13]).

The most severe source of uncertainty in the estimate of f_B, Eq.(29), is its power dependence on S_0. This is a characteristic feature of the Hilbert transform sum rules (4), which has a bigger impact on f_B than on f_D or f_F, Eqs.(22) and (26). To minimize this sensitivity to some extent, we have selected a duality region for S_0 by requiring that the ratio between

Eqs.(19) and (20) reproduce the experimental value of M_B within some error, e.g. 5%. In principle, Laplace transform QCD sum rules are expected to be far less sensitive to S_0 on account of their exponential weight. In the present application these sum rules are]1.f], [1.i], [1.j]

$$2 f_P^2 \frac{M_P^4}{m_Q^2} e^{-M_P^2/M^2} = \int_{m_Q^2}^{S_0} ds\, e^{-s/M^2} \frac{1}{\pi} \mathrm{Im}\, \psi_5(s)\Big|_{QCD}$$
$$+ e^{-m_Q^2/M^2} \left\{ C_4 \langle O_4 \rangle - \frac{1}{4} \frac{m_Q}{M^2} \left(1 - \frac{m_Q^2}{2M^2}\right) C_5 \langle O_5 \rangle \right. \tag{31}$$
$$\left. + \frac{1}{6M^2} \left(2 - \frac{m_Q^2}{2M^2} - \frac{m_Q^4}{6M^4}\right) C_6 \langle O_6 \rangle \right\},$$

where

$$\int_{m_Q^2}^{S_0} ds\, e^{-s/M^2} \frac{1}{\pi} \mathrm{Im}\, \psi_5(s)\Big|_{QCD} = \frac{3}{8\pi^2} \left\{ m_Q^4 \left[E_1\left(\frac{m_Q^2}{M^2}\right) \right.\right.$$
$$\left. - E_1\left(\frac{S_0}{M^2}\right)\right] + M^2(M^2 + m_Q^2)\, e^{-m_Q^2/M^2} - M^2(M^2 + S_0)\, e^{-S_0/M^2} \tag{32}$$
$$\left. - 2 m_Q^2 M^2 \left(e^{-m_Q^2/M^2} - e^{-S_0/M^2}\right) + O(\alpha_s) \right\}$$

with P = D, F, B, etc., and

$$E_1(z) = \int_z^\infty \frac{e^{-t}}{t}\, dt. \tag{33}$$

Earlier estimates based on these sum rules [1.f], [1.i], [1.j] have claimed somewhat smaller values of f_D and f_B than the ones obtained here through Hilbert transforms. However, the point we wish to raise here is that although Eq.(31) does exhibit a better (softer) behaviour on S_0, it has conceptual as well as numerical disadvantages. First, the power corrections in Eq.(31) appear dominated by $C_4\langle O_4 \rangle$ on account of the smallness of $C_5\langle O_5 \rangle$ and

$C_6\langle O_6\rangle$ rather than because of the usual Laplace suppression; notice that the terms multiplying $C_5\langle O_5\rangle$ and $C_6\langle O_6\rangle$ are roughly comparable. This means that inside the "sum rule window" ($M^2 \simeq 0.8 - 2.1\ GeV^2$ for P=D and $M^2 \simeq 3.3 - 4.2\ GeV^2$ for P=B) the Laplace variable M^2 has lost the unambiguous character of short distance expansion parameter it usually has in the familiar applications to light quark systems. A second cause of discomfort is the pronounced sensitivity of the perturbative contribution Eq.(32) to the values of m_Q and M^2. In fact, the dependence on m_Q is in this case exponential. For instance, in the case of f_B a 4% increase in m_b from m_b = 4.6 GeV to m_b = 4.8 GeV produces a change in the perturbative contribution of a factor of 2-3 for $M^2 \simeq 3\text{-}5\ GeV^2$. On the other hand, for fixed S_0 and m_Q, Eq.(32) changes by a factor of 8-16 for P=D and by a factor of 40 for P=B, inside the "sum rule window" in M^2. The corresponding changes in the non-perturbative piece are roughly a factor of 3 and 20 (in the same direction), respectively. In the case of f_D these extreme variations are offset by corresponding large variations in the exponential on the l.h.s. of Eq.(31), so that in the end the result for f_D appears somewhat stable. All things considered we find $f_D \simeq 120\text{-}150$ MeV inside the "window" $M^2 \simeq 1.5\text{-}3\ GeV^2$, and for $S_0 \simeq (1.5\text{-}3)M_D^2$, which is consistent with Eq.(22). However, in view of the above remarks we would not attach much significance to this result. For f_B the huge variations of the perturbative and non-perturbative contributions are not entirely offset by the variation of the exponential in the l.h.s. of Eq.(31), so that uncertainties are beyond the 100% level. We wish to point out that such a dangerous situation is not encountered in the usual applications of Laplace sum rules to light quark systems. Also, these problems do not affect the Hilbert transform power moments at Q^2=0, which in our view, are more reliable to treat charm and beauty mesons as they lead to stable predictions.

In conclusion, collecting our results and normalizing to f_π = 93.2 MeV we have found

$$\frac{f_D}{f_\pi} = 1.7 \pm 0.2\,, \quad \frac{f_F}{f_\pi} = 2.1 \pm 0.1\,, \quad \frac{f_B}{f_\pi} = 1.1 - 1.6\,, \quad \frac{f_{B_s}}{f_B} \gtrsim \frac{M_{B_s}}{M_B}. \qquad (34)$$

The quality of these predictions is gauged by the results obtained for the meson masses in the same framework, i.e. Eqs. (21), (25), and (28). As usual, however, the ultimate test will have to come from experiment. In this connection we wish to mention a promising proposal [14] to measure these decay constants form various exclusive B-decays.

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