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B°-$\bar{B}^\circ$ MIXINGS - A REAPPRAISAL

by

A. Ali

Deutsches Elektronen-Synchrotron DESY, Hamburg

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B^o-$\bar{B}^o$ Mixings - A Reappraisal *

A. Ali

Deutsches Elektronen-Synchrotron DESY, Hamburg

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B^o-$\bar{B}^o$ Mixings - A Reappraisal

A. Ali
Deutsches Elektronen-Synchrotron DESY,
Hamburg, Federal Republic of Germany

Abstract

We review the implications of recent experimental results on B^o-$\bar{B}^o$ mixings for the parameters of the standard model. Some popular extensions of the standard model are also discussed. We emphasize crucial experimental tests which are necessary to check the consistency of the standard model in this sector.

Introduction

By popular accounts [1], the field of Astrophysics is right now enjoying its "Sternstunde"! The discovery of the Supernova 1987a [2] and the correlated measurements of the expected neutrino flux at the KAMIOKANDE and IMB detectors [3] have also sent a wave of excitement in the high energy physics community. The ν-measurements at these non-accelerator experiments have provided us invaluable information about quantum flavour dynamics, QFD, involving leptons. Though it is still too early to assess the final impact of the SN 1987a measurements on our understanding of neutrino masses, mixings and other static properties, yet preliminary studies show that their sensitivity is at least comparable to the present neutrino accelerator experiments [4].

While the SN 1987a data were on their way, mankind had learnt to do experiments with accelerators; in particular the UA1 [5] and ARGUS [6] collaborations were able to measure B^0-$\bar{B}^0$ mixings! These accelerator experiments have also provided us invaluable new information on some aspects of QFD involving quarks. In particular, they have given first measurements of the quantities related to the very tiny mass differences $m(B_H) - m(B_L) \sim 0(10^{-6}\ \mathrm{eV})$ related to the neutral bottom meson complexes $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$, namely

$$x_d \equiv (\Delta M/\Gamma)_d$$
$$x_s \equiv (\Delta M/\Gamma)_s \qquad (1.1)$$

where

$$\Gamma_i = 1/2\,(\Gamma_L + \Gamma_H)_i \qquad i = d, s$$
$$(\Delta M)_i = (M_H - M_L)_i$$

and the subscripts stand for heavier and lighter of the two mass-

eigenstates, with definite decay widths Γ_H and Γ_L, respectively.

One cannot resist the temptation of pointing out the similarity of the particle physics interest in the ν-SN 1987a experiments and the present measurements of the ARGUS and UA1 experiments, namely that both attempt at measuring tiny quantities - mass differences and mixing angles, one involving neutral leptons and the other involving neutral hadrons. It would be an exciting theoretical challenge to relate the mixing angles in the two sectors if and when such information is available from the lepton sector.

In the standard model, the information on x_d and x_s can be cast in terms of relations involving the top quark mass m_t and the Cabibbo-Kobayashi-Maskawa matrix elements [7], V_{tb}, V_{ts} and V_{td}. Since these matrix elements had not been previously measured, the results by ARGUS and the UA1 collaboration constitute, in principle, the first measurements of these quantities. On the other hand previous experiments and unitarity of the CKM matrix did provide non-trivial constraints on the parameters of the standard model [8] and the overriding interest right now is to check the consistency of the standard model against the new ARGUS and UA1 results. This has been the subject of very many recent theoretical papers [9]-[16]. Since exotic virtual states can also contribute to the mass difference x_i, the possibility of new interactions and/or new particles having a hand at work has also received a renewed interest in this context [17]-[19].

Of course, unlike the bonanza of the SN 1987a neutrinos, the measurements by the ARGUS and UA1 collaboration have not entirely come out of the blue! These effects were more or less predicted long ago based on theoretical calculations in the CKM model [20], [21]. The prediction [21] that the mixing in the $B_s^0 - \bar{B}_s^0$ sector is expected to be large has survived the present measurements as well as the earlier experimental discoveries of long bottom meson lifetime [22] and present bounds on the CKM angle V_{bu} [23]. On the other hand one should hasten to add that the ARGUS result on $B_d^o - \bar{B}_d^o$ mixing, namely $\chi_d = 0.73 \pm 0.17$ has come as a mild surprise to most of the updated (anno 1986) versions of the earlier predictions [24]. Reflecting back, there were three essential inputs that had provided the "small χ_d-scenario", namely

(i) As a result of an earlier UA1 measurement [25], it was generally believed that the top quark mass $m_t \simeq 40$ GeV
(ii) $B_d^o - \bar{B}_d^o$ transition rates are Cabibbo suppressed [21],[26]

$$\chi_d \sim \left(|V_{td}|/|V_{bc}|\right)^2 \simeq \sin^2\theta_c$$

(iii) $f_{B_d} \leqslant f_\pi = 130$ MeV

This resulted into the estimates [28]

$$\chi_d = 0.05\left(m_t/40\,\mathrm{GeV}\right)\left(B_d f_{B_d}^2/0.012\,\mathrm{GeV}^2\right) \times \left(V_{td}/\sin^3\theta_c\right)^3 \tag{1.2}$$

Thus, allowing a factor 2 uncertainty on χ_d for a given value of m_t, a value of $\chi_d \simeq 0.1$ could have equally been accommodated for $m_t \simeq 40$ GeV. On the other hand, the ARGUS result implies $\chi_d > 0.44$ at 90 % C.L., which is at least a factor 5 too large for the specified values of the parameters in (1.2). Retrospectively, a factor of ~ 10 between the estimates (1.2) and the ARGUS result can be gotten in a variety of ways by scaling up any of the three quantities shown in the brackets in (1.2). Armed with the ancient rules of multiplication and division, it is not difficult to show that any of the following modifications will resolve the apparent discrepancy:

a) $m_t \gtrsim 100$ GeV with (fB_d) and (V_{td}) in the same ball-park as shown in (1.2)

b) $(Bf_{B_d}) \simeq O(300$ MeV) with m_t and V_{td} close to their assumed values

c) $V_{td} = V_{td}^{max.} \simeq 0.02$, with an additional factor of 4 gotten from somewhat larger values for m_t and/or f_{Bd}

Which of these solutions, if any, is preferred by nature is hard to predict with the available information, particularly since it concerns future! With the hurried interest in pushing up the top quark mass limit in some experimental quarters, all one could risk to say at this stage is that a larger value of m_t in the standard model can more comfortably accommodate the ARGUS result on χ_d. I personally don't like the scenario (b) above, since such large values of Bf_B are not supported by recent theoretical calculations based on the QCD sum rule approach [27] or potential models [26],[29]. Clearly, this is at best a plausibility argument. Perhaps, the theoretical predictions [24],[28] on χ_d were all made in downbeat moods! A somewhat more "inflationary scenario" nominally pushing each of the three parameters in (1.2) by ~ 2 will make the standard model come up to the ARGUS result on χ_d, yet not be in conflict with present phenomenology. This looks like a boring solution but that does not make it unworthy, or does it?

Anyway, there is a fourth scenario possible which is very exciting

d) The standard model contribution does not exhaust χ_d and χ_s.

Among various scenarios that can give additional positive contributions to χ_d and χ_s there are two which deserve careful consideration, namely the existence of a fourth family of quarks and leptons[19] and/or the existence of supersymmetric interactions with relatively light gluinos and squarks [17],[18]. The former scenario is plausible only if the matrix elements $V_{t'b}$ and $V_{t'd}$ are relatively large so that they don't offset the gain due to $m_{t'}^2$. Present phenomenological constraints allow such a possibility [30]. Incidentally, this is probably the only scenario which could enhance $B_d^o - \bar{B}_d^o$ mixing rate such that $r_d \gg r_s$. As we are going to discuss later most other extensions of the standard model preserve the relation obtaining in the standard model

$$\frac{x_s}{x_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 (1+\delta) \quad , \ \delta > 0 \tag{1.3}$$

which necessarily gives $r_d < r_s$. The present data are certainly compatible with (1.3) though the possibility $r_d \gg r_s$ cannot be ruled out yet experimentally.

The supersymmetric scenario is probably the most exciting. As is by now well known, the realistic SUSY models allow flavour changing neutral currents, FCNC, transitions due to the existence of effective $-(\hat{g}\,\tilde{d}_i\,d_j)$, $i \neq j$ interactions. This can give rise to $|\Delta B| = 2$, $\Delta Q = 0$ transitions which, however, are appreciable in the present context only if $\tilde{m}_g$ and $\tilde{m}_q$ are very close to their present experimental bounds, namely $\tilde{m}_g$, $\tilde{m}_q \simeq 0(50\ \text{GeV})$. In that case, one also expects FCNC transitions of the type $b \to s\gamma$ and $b \to sg$. The latter can attain as large a value as 0(10 %) [32], thereby explaining also the so-called charm deficit in non-leptonic b decays [33]. On the other hand, the charm deficit may itself have become defunct by now [34]. In that case, there won't be an appreciable contribution to x_d from the SUSY interactions. The data are not crying for either! Anyway, it is an exciting possibility to think about.

The plan of this write-up is as follows. In Section 2, I will review the current experimental situation on $B^o - \bar{B}^o$ mixing. Section 3 contains a reappraisal of the present constraints on the CKM matrix including the UA1 and ARGUS results. In Section 4 we briefly discuss the Left-Right symmetric models, SUSY and 4th-family scenario. We conclude in Section 5 by reiterating some tests in

$B^o - \bar{B}^o$ mixings to understand the origin of the mass mixings.

2. Experimental Status of B^o-$\bar{B}^o$ Mixings

The quantities of phenomenological interest for the discussion of the present e^+e^- and $p\bar{p}$ experiments are time-integrated ratios

$$\begin{aligned} x_q &= (\Delta M/\Gamma)_q \\ y_q &= (\Delta\Gamma/2\Gamma)_q \qquad q = d, s \end{aligned} \tag{2.1}$$

where $(\Delta\Gamma)_q = (\Gamma_H - \Gamma_L)_q$. All current experiments use the prompt leptons from the semileptonic decays of the bottom quark $b \to \ell^- \nu_e X$. The appropriate measure of $B^o - \bar{B}^o$ mixing is then the relative fraction of the wrong-sign lepton defined as

$$\chi_q \equiv \frac{\Gamma(B_q \to \ell^- X)}{\Gamma(B_q \to \ell^\pm X)} \tag{2.2}$$

or the related quantity [35]

$$r_q \equiv \frac{\Gamma(B_q \to \ell^- X)}{\Gamma(B_q \to \ell^+ X)} \tag{2.3}$$

with the obvious relation $\chi_q = r_q/1+r_q$. In terms of the time-integrated quantities x_q and y_q defined earlier, one has the relations

$$r_q = \frac{x_q^2 + y_q^2}{2 + x_q^2 - y_q^2} \tag{2.4}$$

$$\chi_q = \frac{x_q^2 + y_q^2}{2(1 + x_q^2)} \tag{2.5}$$

For the sake of orientation, the corresponding quantities x and y for the K^0-$\bar{K}^0$ complex have been experimentally determined to have the values (36)

$$\begin{aligned} x(K_L - K_S) &\simeq -0.945 \\ y(K_L - K_S) &\simeq 1.00 \end{aligned} \tag{2.6}$$

giving $\chi(K_L - K_S) = 0.498$, very close to the theoretical maximum limit $\chi = 0.5$. In the case of bottom hadrons there is general consensus that for both the B_d - $\bar{B}_d^0$ and B_s^0 - $\bar{B}_s^0$ complexes, the relation

$$y_q \ll x_q \qquad q = d,s \tag{2.7}$$

holds (37). Thus, experimental measurements of r_q are essentially measurements of the mass difference dependent quantities x_q.

Let us now translate the results of the various experimental collaborations in terms of the quantities r_d and r_s (or χ_d and χ_s). Concentrating first on the results from the $\Upsilon(4s)$ decays, we recall that the $B^0\bar{B}^0$ pair is produced in a P-wave, so that r_d is given by the ratio (38)

$$r_d = \frac{N(B_d^0 B_d^0) + N(\bar{B}_d^0 \bar{B}_d^0)}{N(B_d^0 \bar{B}_d^0)} \tag{2.8}$$

What actually have been measured in the various experiments are the same-sign $N(l^+l^+)$ and $N(l^-l^-)$ and opposite-sign $N(l^+l^-)$ di-leptons (ll = ee, eμ, μμ). Taking into account the fact that in the decays of $\Upsilon(4s)$ one observes not only $B_d^0\bar{B}_d^0$ but also $B_u^+B_u^-$,

$$\Upsilon \begin{array}{l} \rightarrow B_u^+ B_u^- \rightarrow \ell^+\ell^- \\ \rightarrow B_d^0 \bar{B}_d^0 \rightarrow \ell^\pm\ell^\pm,\ \ell^+\ell^- \end{array} \tag{2.9}$$

the quantity r_d can be related to the experimental numbers $N(l^\pm l^\pm)$ and $N(l^+l^-)$ using the obvious relation

$$r_d = \frac{[N(\ell^+\ell^+) + N(\ell^-\ell^-)](1+\lambda_\Upsilon)}{N(\ell^+\ell^-) - [N(\ell^+\ell^+) + N(\ell^-\ell^-)]\lambda_\Upsilon} \tag{2.10}$$

where

$$\lambda_\Upsilon = (f^+/f^0)(BR_u/BR_d)^2 \tag{2.11}$$

and

$$\begin{aligned} f^+ &\equiv BR(\Upsilon \rightarrow B_u^+ B_u^-) \\ f^0 &\equiv BR(\Upsilon \rightarrow B_d^0 \bar{B}_d^0) \\ BR_u &\equiv BR(B_u^+ \rightarrow \ell^+ X) \\ BR_d &\equiv BR(B_d^0 \rightarrow \ell^\pm X) \end{aligned} \tag{2.12}$$

Assuming $\lambda_\Upsilon = 1.2$, the ARGUS Collaboration has determined r_d to be

$$r_d = 0.21 \pm 0.08 \tag{2.13}$$

or equivalently $\chi_d = 0.17 \pm 0.05$. For equal semileptonic branching ratios, this implies $f^0 \simeq 45$ % and $f^+ \simeq 55$ %. The dependence of r_d on λ_Υ is shown in fig. (1), from which it is clear that a larger value of λ tends to increase r_d. As an amusing thought if one had a good measurement of r_d available, this could have been used to determine f^0 and f^+!

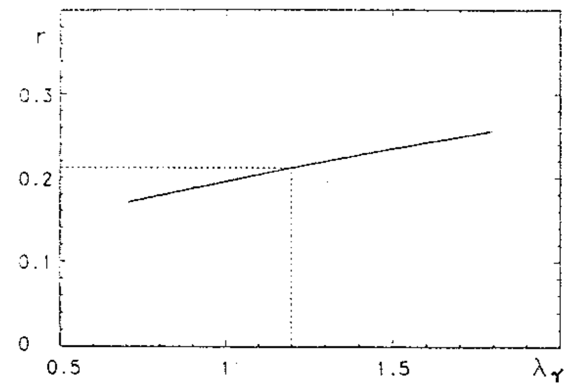


Fig. 1. The mixing parameter r_d as the function of the parameter λ_Υ defined in eq. (2.11). The dotted line indicates the choosen value by the ARGUS Collaboration (from ref. 6)

We also recall that the dominant background to $N(l^+l^+)$ and $N(l^+l^-)$ arises from both the processes $\Upsilon \to B_d^0\bar{B}_d^0$ and $\Upsilon \to B_u^+B_u^-$ in which one of the two leptons is due to the secondary $b \to c \to (s,d)l^+\nu_1$ decays. This background can at best be estimated and the ARGUS Collaboration has quoted an error of $\pm$ 25 % on such estimates. With these assumptions the ARGUS measurement (2.13) can be translated in terms of χ_d, giving [6]

$$\chi_d = 0.73 \begin{array}{l} +0.17 \\ -0.19 \end{array} \qquad (2.14)$$

Using the relation $\chi_d = (2r_d/1-r_d)^{1/2}$, the measurement (2.14) gives a lower limit on χ_d [6]

$$\chi_d > 0.44 \text{ (90 \% C.L.)} \qquad (2.15)$$

Let us contrast the ARGUS measurements with the ones reported by the CLEO Collaboration at CESR [39]. The upper limit on the quantity r_d from the CLEO measurements is shown in fig. (2) as a function of the ratio of the semileptonic branching ratios, for

assumed values of f^0 and f^+. Using $BR_u = BR_d$ and $f^+/f^0 = 1.5$ (corresponding to $\lambda_\Upsilon = 1.5$) CLEO obtained an upper bound [39]

$$r_d < 0.24 \quad \text{(90 \% C.L.)} \qquad (2.16)$$

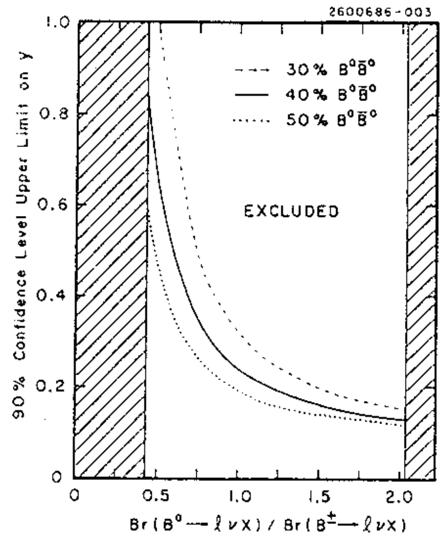


Fig. 2. Upper limits from the CLEO Collaboration on the mixing parameter y $(=r_d)$ for several assumed B_d^0 to $B_u^\pm$ production ratios, as a function of the ratio of the semilept. branching ratios.The shaded regions are excluded by the branching ratio measurements (from ref. 39)

It is clear from both the $r_d - \lambda_\Upsilon$ dependence of fig. 1 as well as the f^+/f^0 dependence of fig. 2 that a value $\lambda_\Upsilon = 1.5$ used by the CLEO Collaboration would imply a higher value of r_d from the measurements of the ARGUS Collaboration, almost coinciding their central value for r_d with the 90 % CLEO upper bound (2.16). On the contrary, decreasing $\lambda_\Upsilon = 1.5$ to $\lambda_\Upsilon = 1.2$, the value assumed by the ARGUS Collaboration, the CLEO bound (2.16) becomes even more stringent, namely $r_d < 0.22$ as can be seen in fig. 2. Neglecting this mild dependence of the measurements of r_d on λ_Υ, the joint results of ARGUS and CLEO imply the following bounds for the quantity χ_d,

$$0.44 < \chi_d < 0.78 \qquad (90\ \%\ \text{C.L.}) \qquad (2.17)$$

Given the various theoretical and experimental uncertainties, this is an impressive result for first generation experiments on $B_d^0 - \bar{B}_d^0$ mixing. Also, it lies roughly an order of magnitude above the estimate (1.2). Stretching the parameters $B f_{B_d}$ and V_{td} to their plausible extremes, this would imply a lower bound on m_t, namely $m_t > 50$ GeV, as already noted by the ARGUS Collaboration [6].

Let us now turn to the discussions of the continuum experiments in e^+e^- and $\bar{p}p$ collisions. First, we recall that, unlike the ARGUS and CLEO results, the experiments in the continuum are sensitive to both χ_d and χ_s. However, in inclusive measurements one can only measure a weighted average of χ_d and χ_s. For $B^0 - \bar{B}^0$ mixing this measure is defined as

$$\chi \equiv \frac{\Gamma(B \to \ell^- X)}{\Gamma(B \to \ell^{\pm} X)} \qquad (2.18)$$

with the obvious relation

$$\chi = \frac{1}{\langle BR \rangle}\left[\chi_d P_d\, BR_d + \chi_s P_s\, BR_s\right]$$
$$\simeq \chi_d P_d + \chi_s P_s \qquad (2.19)$$

where P_d and P_s are, respectively, the branching ratios $b \to B_d^0 X$ and $b \to B_s^0 X$ and we have again assumed the equality of the various bottom hadron semileptonic branching ratios $(BR)_d = (BR)_s = \langle BR \rangle$. The parameters P_d and P_s are the continuum analogues of the parameter λ_Υ for the decays $\Upsilon \to B_d^0 \bar{B}_d^0$, $B_u^+ B_u^-$.

Very much like λ_Υ, at present they can only be guessed. A reasonable range is exemplified by

$$0.375 \leqslant P_d \leqslant 0.40$$
$$0.10 \leqslant P_s \leqslant 0.20 \qquad (2.20)$$

with probably $P_d \approx 0.375$ and $P_s = 0.15$ being more suggestive, by the dominance of $C \to (D, D^*)X$ and the world average $P(s\bar{s})/P(d\bar{d}) = 0.33 \pm 0.03$, [40] expressing the SU(3) breaking effects in the production of $q\bar{q}$ pairs in the usual process of quark fragmentation. The UA1 Collaboration has measured the ratio $\chi = \chi_d P_d + \chi_s P_s$, and obtained $\chi = 0.121 \pm 0.047$. This gives the lower bound [5]

$$\chi > 0.065 \qquad (90\ \%\ \text{C.L.}) \qquad (2.21)$$

On the other hand, there also exist upper bounds on the quantity χ from continuum e^+e^- experiments. Using very much the same method as employed later by UA1, the MARK II Collaboration had set an upper limit (41)

$$\chi < 0.12 \qquad (90\%\ \text{C.L.}) \qquad (2.22)$$

The corresponding limit on χ from the JADE method, namely the $B-\bar{B}$ mixing induced reduction in the forward-backward charge asymmetry of the bottom quark is [42]

$$\chi < 0.13 \qquad (90\ \%\ \text{C.L.}) \qquad (2.23)$$

The situation concerning B^0-$\bar{B}^0$ mixings in the continuum is interestingly very similar to the one at the Υ(4s), namely the central value of χ in the UA1 measurement almost coincides with the 90 % C.L. upper limit by the MARK II Collaboration! However, to be fair one should remark that it is conceivable that the branching ratios P_d and P_s are appreciably different in e^+e^- annihilation at $\sqrt{s} = 29 - 35$ GeV and $p\bar{p}$ collisions at $\sqrt{s} = 546 - 630$ GeV. Again, in my opinion it is rather unlikely, though a slight energy dependence is certainly admissable. The least that can be said about the continuum limits (2.21)-(2.23) is that they are compatible with each other. It is also obvious that there is no conflict between the $\Upsilon(4s)$ result on r_d, (2.17), the expectations for P_d and P_s (2.20) and the UA1 result $\chi = 0.121 \pm 0.047$. This can be seen in fig. 3 where we have plotted the constraints on the parameters χ_d and χ_s (or r_d and r_s), following from the 90 % C.L. upper limits on χ (from MARK II), r_d (from CLEO) and the corresponding lower limits on χ (from UA1) and r_d (from ARGUS). The curves assume our best guess for P_d and P_s, $P_d = 0.375$ and $P_s = 0.15$ for both the MARK-II and UA1 data. A number of observations are in order.

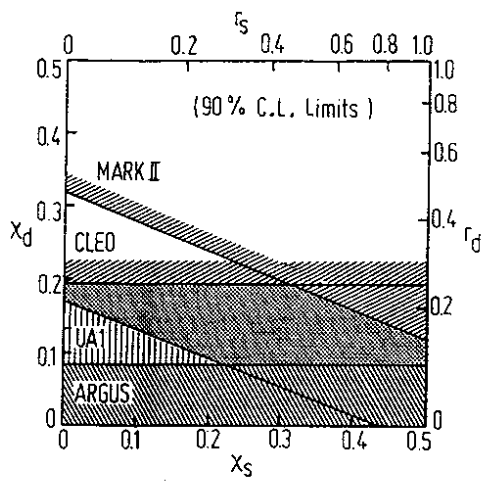


Fig. 3. The allowed (shaded) and excluded (// lines) region in the $\chi_d - \chi_s$ plane resulting from the 90 % upper limits from the MARK II (ref. 41) CLEO (ref. 39) and the corresponding lower limits from the UA1 (ref. 5) and ARGUS (ref. 6) measurements.

The present data still admit a region in the $r_d - r_s$ plane for which $r_s \simeq 0$ and $r_d \simeq 0.2$, which is quite amusing since as we are going to elaborate further in the next section, the standard

3-family model makes a firm prediction, namely $r_s/r_d > 1$. At the same time the region favoured by the standard model, namely $r_s = 0.4 - 0.5$ is still allowed by the present experimental constraints. It is imperative to narrow down experimentally the presently allowed hexagone region in fig. 3 in the $r_d - r_s$ plane. With the present experimental situation, it is likely that $B_d^0 - \bar{B}_d^0$ mixing contributes a substantial fraction (1/4-1/2) to the parameter χ measured by the UA1 collaboration.

3. Implications of B^0-$\bar{B}^0$ Mixings in the Standard Model

3.1 An update on the CKM Matrix

As is by now folklore, the standard model with three generations of quarks and leptons describes the generation mixing among quarks by a 3x3 unitary matrix, first introduced by Kobayashi and Maskawa [7]. Written symbolically as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{3.1}$$

a particularly useful parametrization is due to Wolfenstein [43]

$$V_{CKM\text{-}W} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{3.2}$$

where $\lambda = \sin\theta_c = 0.221 \pm 0.002$. Given λ, the experimental bound on the ratio

$$\bar{R} \equiv \frac{\Gamma(b \to u\, l\, \nu_l)}{\Gamma(b \to c\, l\, \nu_l)} \tag{3.3}$$

can be used to bind the quantity $\rho^2+\eta^2$. The relation between $\bar{R}$ and $|V_{bu}/V_{bc}|^2$ involves evaluation of the phase space and QCD corrections. Defining

$$\left|\frac{V_{bu}}{V_{bc}}\right|^2 \equiv \bar{R}\ \phi\left(m_b, m_c, m_u, \Lambda_{QCD}\right) \tag{3.4}$$

present experimental bounds [43] on m_b, m_c from semileptonic lepton spectra give [44]

$$\phi = 0.45 - 0.54 \tag{3.5}$$

where $O(\alpha_s)$ QCD corrections have been taken into account. The largest value for V_{bu} obtains for the largest allowed value of ϕ given $\bar{R}$. For $\bar{R} < 0.08$ we obtain

$$\left|\frac{V_{bu}}{V_{bc}}\right| = \lambda\left(\rho^2+\eta^2\right)^{1/2} < 0.21 \tag{3.6}$$

giving

$$\alpha \equiv \left(\rho^2+\eta^2\right)^{1/2} < 0.95 \tag{3.7}$$

A careful determination based on the average bottom meson lifetime $\tau_b = (1.11 \pm 0.16) \times 10^{12}$ sec and the average semileptonic branching ratio $\langle BR \rangle = (11.7 \pm 0.6)$ % gives [44]

$$|V_{bc}| = 0.051 \begin{matrix} +0.008 \\ -0.009 \end{matrix} \tag{3.8}$$

which in turn leads to the determination of the parameter A in the matrix (3.2),

$$A = 1.05 \pm 0.17 \tag{3.9}$$

In this parametrization the two matrix elements $|V_{bc}|$ and $|V_{ts}|$ are equal. Knowing V_{ts}, it is possible to put an upper bound on V_{td} using the relation

$$\begin{aligned} \gamma &= \lambda\sqrt{(1-\rho)^2+\eta^2} \\ &= \lambda\sqrt{1+\alpha^2-2\rho} \end{aligned} \tag{3.10}$$

where

$$\gamma \equiv |V_{td}/V_{ts}|$$

With the present bound (3.7) on α, the maximum of γ occurs for $\alpha = -\rho = 0.95$. This gives numerically

$$\gamma < 1.95\,\lambda < 0.43 \tag{3.11}$$

This for $|V_{ts}| = |V_{bc}|$ estimated in (3.8) would then give

$$|V_{td}| < 0.024 \tag{3.12}$$

This exercise, however, ignores the information from CP violation in the kaon sector. Taking this into account puts a lower bound on η which is correlated with m_t. This would marginally reduce the upper bound on V_{td}/V_{ts} (in 3.11). These bounds have been worked out in refs. (13)-(17) but we shall ignore them for the time being in the discussion of $|V_{ij}|$. Thus, the combined effect of all present measurements on the CKM matrix elements can be expressed as follows:

$$|V| = \begin{pmatrix} 0.972 - 0.974 & 0.219 - 0.223 & 0 - 0.013 \\ 0.219 - 0.223 & 0.972 - 0.974 & 0.042 - 0.059 \\ 0 - 0.024 & 0.042 - 0.059 & 0.998 - 0.999 \end{pmatrix} \tag{3.13}$$

The upper bounds on $|V_{bu}|$ and $|V_{td}|$ obtained here are larger than some other recent determinations, the reasons for which can be traced to the use of an unjustified smaller value of $\bar{R}$ and/or neglect of QCD corrections in the estimates of Γ_{SL}. It is very similar though to the estimates presented recently in refs. (19). Again, we emphasize that the explanation of ϵ_K in the CKM model is only possible if all the angles are non-zero. This would imply a non-zero lower bound on both $|V_{bu}|$ and $|V_{td}|$ and we shall discuss them later.

3.2 Standard Model Estimates of x_d and x_s

Having updated the CKM matrix elements, let us now discuss the quantities x_d and x_s relevant for B^0-$\bar{B}^0$ mixings. In the standard model, one normally uses the box diagrams to evaluate $(\Delta M)_i$, i = d,s. This gives

$$x_d = \tau_{B_d} \frac{G_F^2 m_t^2}{6\pi^2} m_{B_d} \eta_a (\mathcal{B}_d f_{B_d}^2) F(y_t) |V_{td}^* V_{tb}|^2$$

$$x_s = \tau_{B_s} \frac{G_F^2 m_t^2}{6\pi^2} m_{B_s} \eta (\mathcal{B}_s f_{B_s}^2) F(y_t) |V_{ts}^* V_{tb}|^2 \tag{3.14}$$

where η_a is a QCD correction factor estimated to be $\eta_a \simeq 0.85$. The function $F(y_t)$ is defined as ($y_t = m_t^2/m_w^2$)

$$F(y_t) \equiv \frac{1}{4} + \frac{9}{4(1-y_t)} - \frac{3}{2(1-y_t)^2} - \frac{3}{2}\frac{y_t^2 \ln y_t}{(1-y_t)^3} \tag{3.15}$$

$F(y_t)$ is a slowly monotonously decreasing function with

increasing y_t and assumes the value 1, 0.86, 0.75, 0.54 for m_t = 0, 50 GeV m_w, and 180 GeV, respectively. The coupling constants $(\mathcal{B}_d f_{B_d}^2)$ and $(\mathcal{B}_s f_{B_s}^2)$ are defined using the relationships

$$\langle B_d^0 | J_\mu^+ J^\mu | \bar{B}_d^0 \rangle = \frac{4}{3} \mathcal{B}_d f_{B_d}^2$$

$$\langle B_s^0 | J_\mu^+ J^\mu | \bar{B}_s^0 \rangle = \frac{4}{3} \mathcal{B}_s f_{B_s}^2 \tag{3.16}$$

Knowing m_t, the pseudoscalar coupling constants $\mathcal{B}_d f_{B_d}$ and the B_d lifetime τ_B, a measurement of x_d would be essentially a measurement of the CKM factor $|V_{td}^* V_{tb}|$. Similarly, a measurement of x_s is essentially a measurement of the CKM factor $|V_{ts}^* V_{tb}|$. Since in the standard model, $|V_{tb}| \simeq 1$, measurements of x_d and x_s amount to actually measuring V_{td} and V_{ts}, respectively. The CKM model makes a firm prediction for the ratio x_s/x_d even without the knowledge of m_t and the absolute values of the coupling constants, namely that this ratio is bounded from below:

$$\frac{x_s}{x_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 (1+\delta) = \gamma^{-2}(1+\delta) \tag{3.17}$$

where δ is an SU(3)-breaking factor and in all likelihood it is positive definite. Having just determined in (3.11) that in the CKM model $|V_{td}/V_{ts}| < 0.43$, this would predict

$$\frac{x_s}{x_d} > \gamma^{-2} > 5.4 \tag{3.18}$$

Thus, the ARGUS lower bound on x_d, namely $x_d > 0.44$ (90 % C.L.) would imply $x_s > 2.4$ predicting $\Upsilon_s > 0.72$. In fig. 4 we show again the allowed region in the χ_d-χ_s plane from the present experiments. The allowed region expected in the standard model following from the inequality

$$\chi_s > \chi_d / [\gamma^4 (1 - 2\chi_d) + 2\chi_d] \quad (3.19)$$

is also shown. It is remarkable that the present experiments and the inequality (3.18) based on the CKM model have more or less pinned down the quantities χ_d and χ_s to lie in the range

$$0.08 < \chi_d < 0.12$$
$$0.42 < \chi_s < 0.50 \quad (3.20)$$

We note that the upper bound on χ_d in all likelihood is more stringent than the CLEO upper bound $\chi_d < 0.19$.

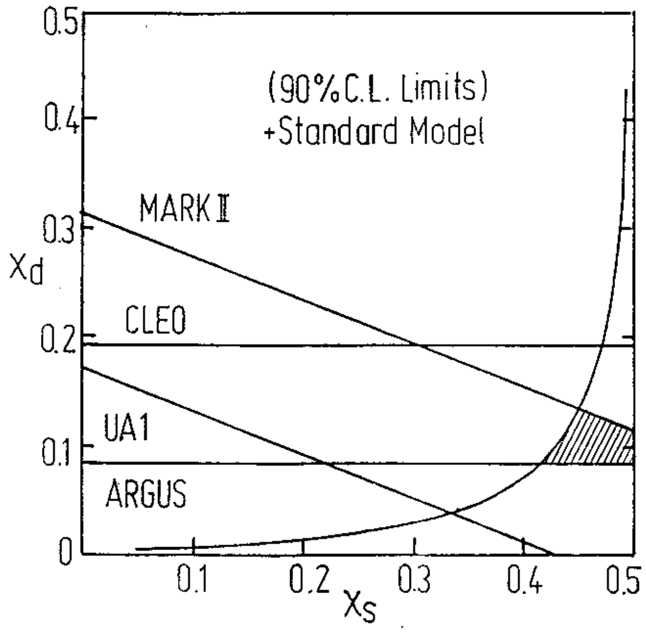


Fig. 4. The experimentally allowed region in the χ_d-χ_s plane from fig. 3 and the CKM model constraint (3.18). The shaded area is the region allowed by the present experiments and the CKM model.

This a firm prediction and in my opinion constitutes one of the most important tests of the CKM model in this sector, hence the urgency in reducing the presently allowed region in the χ_d-χ_s plane in precision experiments.

Let us now leave the firm grounds and enter into the more slippery domain of predicting χ_d and χ_s individually. We have already discussed that in the CKM model, present phenomenology determines

$$|V_{td}^* V_{tb}| < 0.024$$
$$|V_{ts}^* V_{tb}| \simeq V_{bc} = 0.051 {}^{+0.008}_{-0.009}$$

No constraints on the top quark mass exist except the firm PETRA limit, $m_t > 23$ GeV, and the constraint from $\Delta\rho$ giving [45] $m_t > 180$ GeV. We briefly discuss the present estimates of the coupling constant factors $B_d f_{B_d}^2$ and $B_s f_{B_s}^2$ entering in (3.14).

There are essentially three kinds of estimates available in literature.

(i) Potential models [29],[46]

In non-relativistic potential models, $f_P^{Pot.}$ ($P = Q\bar{q}$) is related to the wave-function at the origin $\psi(o)$, by the expression

$$f_P^{Pot.} = \sqrt{\frac{12}{m_P}}\, |\psi(0)| \quad (3.21)$$

with the normalization $\int d^3r\, |\psi(r)|^2 = 1$, $\psi(r)$ being the radial wave function in the $Q\bar{q}$ c.m. system. In such models, $\psi(0)$ is governed by the reduced mass. To estimate $\psi(0)$ for the $\Upsilon = b\bar{b}$ system we can use the experimental decay width $\Gamma(\Upsilon \to e^+e^-)$. This gives [46] (in units of $f_\pi = 130$ MeV)

$$f_{B_d}^{Pot.} \simeq 160 \text{ MeV}, \quad f_{B_s}^{Pot.} \simeq 200 \text{ MeV} \quad (3.22)$$

A particularly relevant question is if $f_P^{Pot.}$ evaluated from (3.21) and f_P needed for x_d and x_s are the same quantities. It has recently been argued that the coupling constants $f_P^{Pot.}$ must be rescaled. The point is that in the non-relativistic models, $(f_P^2)^{Pot.}$ is proportional to the probability of annihilation of quarks with the characteristic off-shellness of order R^{-1}, where R is a typical hadronic radius. On the other hand the current entering the definition of the physical annihilation constants f_P must be normalized at $\mu \simeq m_Q$. This brings about a rescaling factor (47) between f_P and $f_P^{Pot.}$,

$$f_P = f_P^{Pot.} \alpha_s(m_Q)^{-2/b} \tag{3.23}$$

where $b = 1/3(33 - 2n_f)$. This would then imply a reduction of f_P from $f_P^{Pot.}$ by about 30-40 %, resulting in the values

$$f_{B_d} = 115 \pm 15 \text{ MeV}$$

$$f_{B_s} = 140 \pm 20 \text{ MeV} \tag{3.24}$$

In fact, one could use this approach to relate f_{B_d} and f_D obtaining (48)

$$\frac{f_{Bd}}{f_D} = \left(\frac{m_D}{m_B}\right)^{1/2} \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{2/b} = 0.7 \pm 0.1 \tag{3.25}$$

Recalling that the MARK III Collaboration has set an upper limit on f_D (49)

$$f_D < 340 \text{ MeV}$$

Eq. (3.25) would suggest an upper bound $f_B < 240$ MeV, which is mildly interesting.

(ii) Hyperfine mass splitting (50)

Here one can relate f_B to the hyperfine mass splitting $m_{B*}-m_B$ by the relation

$$m_{B^*} - m_B = 32\pi\alpha_s|\Psi(0)|^2/9m_b m_u \tag{3.26}$$

This gives

$$f_{B_d}^2 \simeq (120 \text{ MeV})^2/\alpha_s \tag{3.27}$$

The big uncertainty in this approach is the value of α_s but it is interesting that the ball-park of f_{B_d} is not too far from the estimates (3.25) from the renormalized potential models.

(iii) QCD sum rule approach

The situation until the summer of 1986 in this field has been summarized by Shifman (47). There is a certain amount of dispersion in the two theoretical estimates in this framework giving

$$\begin{aligned} f_{Bd} &= (115 \pm 15) \text{ MeV} \quad \text{ref. (51)} \\ &= (190 \pm 30) \text{ MeV} \quad \text{ref. (52)} \end{aligned} \tag{3.28}$$

Recently, a new calculation has been attempted in this approach by using the Hilbert transform power-moment sum rule at $Q^2 = 0$ (53), which attempts at a simultaneous fit of the pseudoscalar meson masses and coupling constants. The authors of ref. (53) have posted the following values,

$$f_D/f_\pi = 1.7 \pm 0.2$$

$$f_F/f_\pi = 2.1 \pm 0.1$$

$$f_{Bd}/f_\pi = 1.1 - 1.6 \qquad (3.29)$$

which gives $f_{B_d} = 145 - 220$ MeV. These values are closer to the estimates in ref. (52) and somewhat larger as compared to the ones in ref. (51). Clearly more experimental and theoretical information is called for. There also exists at least one calculation of the hadronic bag constant B_K for the kaons in the QCD sum rule approach [54], which gives $B_K = 0.84 \pm 0.08$ and differs sharply from the current algebra and SU(3) estimates [55] of $B_K = 0.4$. Hopefully, for the bottom hadrons $B_d = B_s \simeq 1$. In view of the present uncertainty, a reasonable estimate for the quantity $B_d f_{B_d}^2$ seems to be

$$B_d f_{Bd}^2 = (150 \pm 50 \text{ MeV})^2 \qquad (3.30)$$

Let us now use the mean value for $B_d f_B^2 = (150 \text{ MeV})^2$ and set $|V_{td}| = |V_{td}|^{max.} = 0.024$ to get $m_t(\text{min.})$ from the ARGUS lower bound $\chi_d > 0.44$. We get

$$m_t^{min} = 45 \text{ GeV} \qquad (3.31)$$

This is not too far from the realistic bound obtained in ref. (9) but is much lower than the ones reported in refs. (10) and (16). It still allows detection of top quarks at LEP-I! We point out that the present phenomenology allows the combination $\tau_B V_{td}^2$ to be as large as 6.7×10^{-16} sec. We remark that $m_t^{min} = 45$ GeV can quite comfortably accommodate the present value of ϵ_K, particularly because the present limit on α is quite large, namely $\alpha = \sqrt{\rho^2+\eta^2} < 0.95$. We are presently going to discuss at length the constriants on the CKM model parameters from the joint analysis of r_d and ϵ_K. As already pointed out as early as 1978, the mixing in the B_s-$\bar{B}_s$ sector is expected to be significant in the standard model. For $m_t^{min} = 45$ GeV, the limit just obtained from the ARGUS result, we get

$$\chi_s(m_t = 45 \text{ GeV}) \simeq 2.4 \, (B_s f_{Bs}^2)/(0.15 \text{ GeV})^2 \qquad (3.32)$$

For higher values of m_t, χ_s rises very fast; for example, $\chi_s = 7.1$, 24 for $m_t = m_W$ and 180 GeV, respectively. This translates into $\chi_s = 0.425$, 0.490, 0.499 for $m_t = 45$, m_W and 180 GeV, respectively. It is interesting to note that if m_t is large, for example $m_t \gg m_W$, one looses sensitivity on χ_s. In that case, though one would be able to measure χ_s from the continuum (e^+e^- and $p\bar{p}$) experiments, but it would be difficult to precisely determine χ_s! It would then necessitate time dependent measurements like for example suggested for the Tevatron II/SSC experiments [56]. Hopefully, by that time both χ_d and χ_s would have been measured and we would know the top quark mass to have a more precise theoretical estimate for χ_s.

3.3 Constraints on the CKM matrix elements from r_d

As we have already discussed, the measurement of χ_d can be used to determine the CKM matrix element $|V_{td}|$ given m_t. Not knowing m_t, we plot in fig. (5) the matrix element $|V_{td}|_{min}$ as a function of m_t, using the experimentally allowed value of χ_d, (3.20) and $B_d f_{B_d}^2 = 0.15 \pm 0.05$ GeV^2. From the upper bound $m_t \ll 180$ GeV, one can determine a lower bound on the matrix element $|V_{td}|$

$$|V_{td}| > 0.008 \qquad (3.33)$$

while for $m_t = 50$ GeV, one expects $V_{td} \simeq 0.020$.

It is worth pointing out that even with $V_{td} = V_{td}^{max} = 0.024$, the inclusive branching ratio $BR(t \to dx)$ is expected to be less than 5×10^{-4}. Thus, in the standard CKM model, the prospects of directly measuring the V_{td} matrix elements at future high energy machines like LEP, LHC, SSC etc. are not very bright. It is remarkable that low energy experiments on B decays like ARGUS have provided a non-trivial lower bound on V_{td}! Mass-shift measurements have an incredibly long lever arm!!

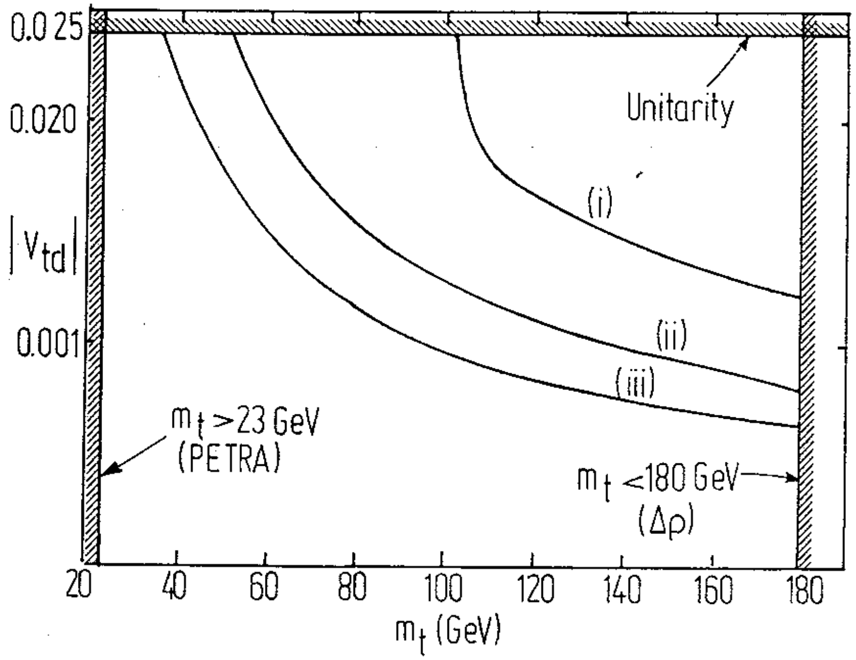


Fig. 5. Upper bound on the matrix element $|V_{td}|$ from unitarity and $\bar{R} < 0.08$, and the lower bounds from the ARGUS result $\chi_d > 0.44$ (90 % C.L.) for three values of the coupling constants (i) $B_d f_B^{\,2} = (0.10\ \mathrm{GeV})^2$, (ii) $B_d f_B^{\,2} = (0.15\ \mathrm{GeV})^2$ and (iii) $B_d f_B^{\,2} = (0.2\ \mathrm{GeV})^2$

The measurement of Υ_d imposes very interesting constraints on the parameters of the standard model. To see this clearly, let us recast eq. (3.14) for χ_d recalling that in the Wolfenstein parametrization

$$|V_{td}| = A\lambda^3\sqrt{1+\alpha^2-2\rho}$$
$$\alpha = \sqrt{\rho^2+\eta^2} \qquad (3.36)$$

Then

$$x_d = C_d(1+\alpha^2-2\rho)\, y_t F(y_t) \qquad (3.37)$$

where

$$C_d \equiv \frac{A^2\lambda^6}{6\pi^2} G_F^2 m_{B_d} m_W^2 \eta_Q \tau_{B_d} (B_d f_{B_d})^2$$
$$= 0.35^{+0.48}_{-0.25} \qquad (3.38)$$

for $Bf_{Bd} = 0.15 \pm 0.05$ GeV. Our aim is to ultimately get a limit on the V_{bu} matrix element which is directly proportional to α. Solving (3.37) for α gives the desired form

$$\alpha^2 = (2\rho-1) + \frac{x_d}{C_d}\left[y_t F(y_t)\right]^{-1} \qquad (3.39)$$

Using the 90 % C.L. bound on χ_d from ARGUS and CLEO, namely

$$0.44 < \chi_d < 0.78 \qquad (3.40)$$

and assuming a value for m_t, this can be solved for α and ρ. We also recall that α is bounded from $\bar{R} < 0.08$ to be $\alpha < 0.95$, and the condition $\alpha > |\rho|$ must also be satisfied.

The analysis of Υ_d can be combined with the one of $|\epsilon_K|$[13],(14),(17). In the Wolfenstein parametrization, the parameter $|\epsilon_K|$ is given by the following expression:

$$|\epsilon_K| = C_K B_K A^2\lambda^6\eta\, y_c$$
$$\times\left[-\eta_1 + \eta_2 (y_t/y_c) F(y_t) A^2\lambda^4(1-\rho) + \eta_3 G(y_t)\right] \qquad (3.41)$$

where $\eta_c = m_c^2/m_W^2$ and η_1, η_2 and η_3 are QCD correction factors, calculated to be $\eta_1 = 0.7$, $\eta_2 = 0.6$, $\eta_3 = 0.4$, the constant

$c_K = G_f^2 f_K^2 m_K m_W^2 / 6\pi^2\sqrt{2}\,\Delta m_K = 4 \times 10^4$ and the function $G(y_t)$ is given by

$$G(y_t) = \ln\left(y_t/y_c\right) - 3/4\; y_t/(1-y_t)\left(1+\left(y_t/(1-y_t)\right)\ln y_t\right)$$

Eq. (3.41) can be solved for η given ρ, m_t and the bag constant B_K. Numerically for $\epsilon_K = 2.3 \times 10^{-3}$ one gets

$$B_K\eta = 1.33\left(\frac{1.5\,\mathrm{GeV}}{m_c}\right)^2\left(1.05/A\right)^2 \times\left[-\eta_1 + 2.4\times 10^{-3}\left(A/1.05\right)^2\eta_2\left(y_t/y_c\right)F(y_t)(1-\rho) + \eta_3\,G(y_t)\right]^{-1} \qquad (3.42)$$

Since, without loss of generality η can be choosen to satisfy $\eta \geqslant 0$, one can use the relation $\eta = \sqrt{\alpha^2-\rho^2}$ with $-\alpha < \rho < \alpha$ and get another relationship between α and ρ. The overlap for the solutions of Eqs. (3.39) and (3.42) gives the allowed region in the $(\alpha-\rho)$ plane for given values of m_t, B_K and Bf_{B_d}.

We are going to show the correlations in the $(\alpha-\rho)$ plane presently, following from the 90 % C.L. measurement of x_d (3.40) and ϵ_K. However, it is obvious that present phenomenology allows a realistic determination of the quantity $\tau_b\, V_{bc}^{\;2}$ to only within $\pm$ 35 %, giving

$$\tau_b\, V_{bc}^{\;2} = (4.55 \pm 1.65) \times 10^9\ \mathrm{GeV}^{-1} \qquad (3.43)$$

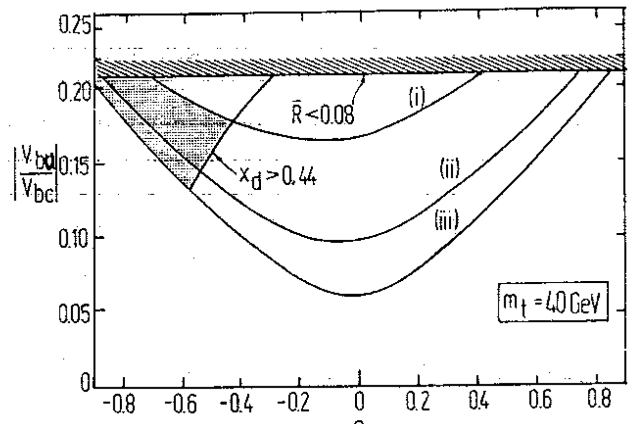
and the theoretical uncertainty on $Bf_{B_d}^2$ is a factor 4 ranging from 0.01 GeV^2 to 0.04 GeV^2. Thus, even given m_t, one has a rather large uncertainty in the x_d analysis alone. The analysis for ϵ_K suffers from the poor knowledge of the bag-parameter B_K, though as we have already noted it is more likely to be closer to 1 than to 0.4. The uncertainty on m_c is on top of that. Thus, it is hard to make very precise determination, except if $m_t \simeq$ 40-50 GeV in that case the CKM parameters are rather tightly constrained.

In figs. 6(a) - 6(d) we show the correlations between $|V_{bu}/V_{bc}|$ and ρ allowed by the experimental constraints on x_d, (3.40), $|\epsilon_K| = 2.3 \times 10^{-3}$, $|V_{bu}/V_{bc}| < 0.21$ (from $\bar{R} < 0.08$), the parameter c_d having the extreme value $c_d = 0.83$ and the product Am_c^2/B_K in eq. (3.42) having a value in the range 3.63 $\mathrm{GeV}^2 \geqslant Am_c^2/B_K \geqslant 1.75$ GeV^2 for representative values m_t = 40 GeV, 60 GeV, 80 GeV and 100 GeV, respectively. It is obvious that the correlations for this extreme case are going to be very weak. Despite this certain general remarks are in order.

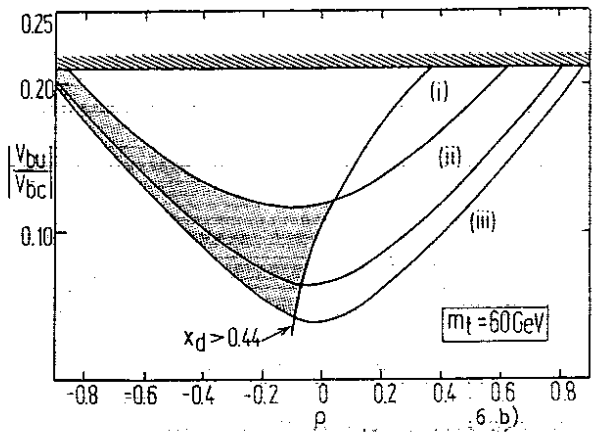
1) m_t = 40 GeV is still allowed by the present 90 % C.L. measurements of x_d, though only for the extreme values of the parameter c = 0.83. In this case $V_{bu}/V_{bc} > 0.135$, leading to $V_{bu} > 7 \times 10^{-3}$. (See fig. 6a). We don't think this is a likely situation, though it is allowed by the present theoretical uncertainty.

2) The bound on $|V_{bu}/V_{bc}|$ deteriorates rapidly with increasing m_t, with x_d measurements still providing a better bound than ϵ_K for $m_t \leqslant 60$ GeV with $B_K = 1$. (See fig. 6b). Of course, for smaller values of c_d, r_d provides better bounds for larger values of m_t.

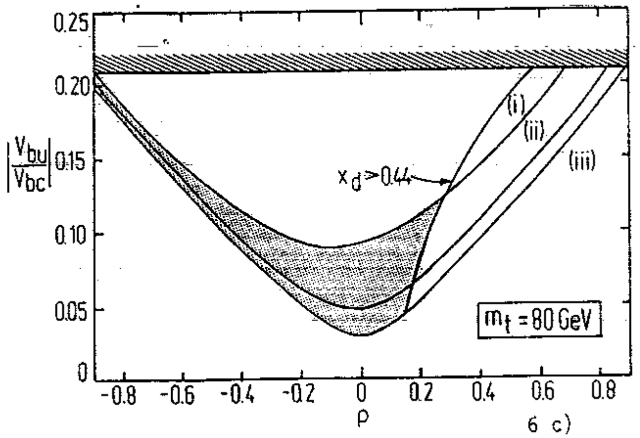
3) For $m_t > 70$ GeV, and $c_d = 0.83$ the constraints from x_d become very thin and $|V_{bu}/V_{bc}|^{min}$ is the one given by ϵ_K. For $m_t > 100$ GeV, one obtains $V_{bu} \simeq 0(10^{-3})$. (See figs. 6c and 6d)
For the mean values $c_d = 0.35$, $A = 1.05$ and $m_c = 1.5$ GeV, the r_d-bounds on $|V_{bu}/V_{bc}|$ are more constraining than the ones from ϵ_K up to m_t = 100 GeV for B_K in the range $0.4 \leqslant B_K \leqslant 1.0$.
This can be seen in figs. 7a-7c, which corresponds to the central values for the r_d-parameters and m_t = 55 GeV, 75 GeV and 95 GeV, respectively. The values of $|V_{bu}/V_{bc}|^{min}$ for this choice of parameters - which we consider to be more realistic than the one shown in figs. 6 - are plotted as a function of m_t in fig. 8. This suggests that $|V_{bu}/V_{bc}| > 0.05$ for $m_t \leqslant 100$ GeV with probably $|V_{bu}/V_{bc}| > 0.10$ for $m_t \leqslant 75$ GeV. Thus, it seems to us that the present limit on the matrix element V_{bu} is probably not too far above its logical value, hoping that $m_t \leqslant m_W$.



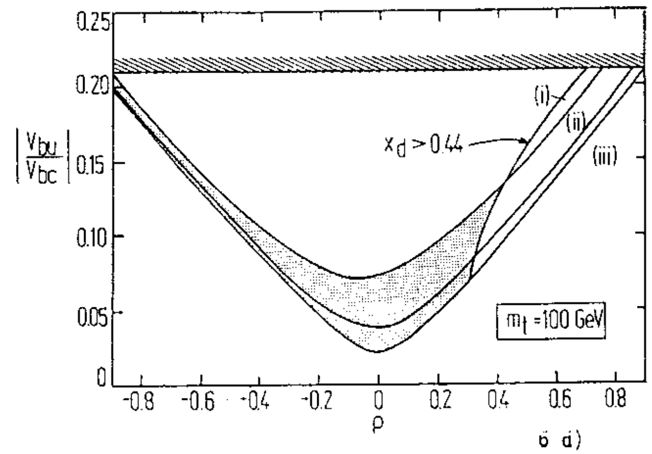
6 a)



6 b)



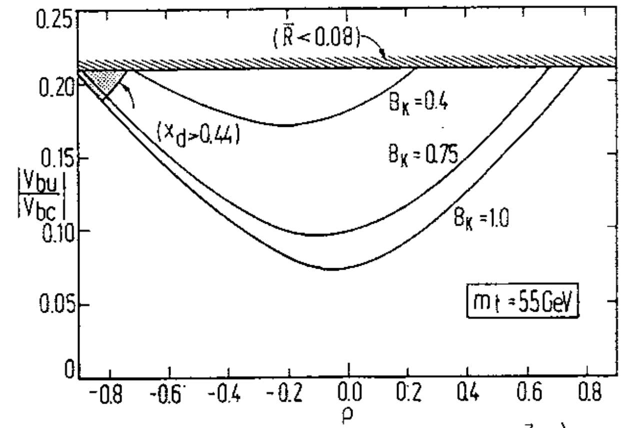
6 c)



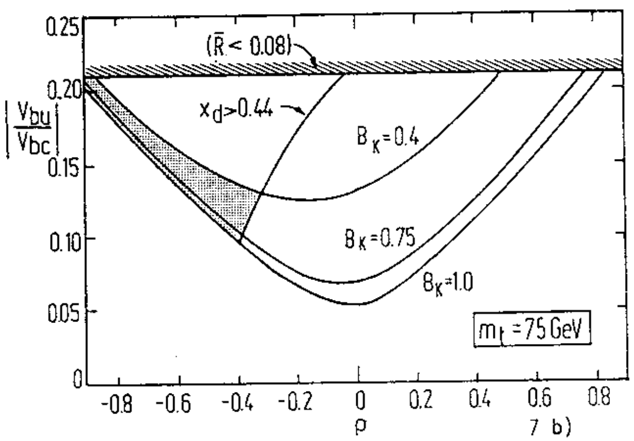
6 d)

Fig. 6. Constraints on the CKM parameters ρ and $|V_{bu}/V_{bc}| = \lambda\alpha$ from the joint analysis of $|\epsilon_K|$ and x_d. The three curves from ϵ_K analysis correspond to the values (i) $Am_c^2/B_K = 1.75$ GeV^2, (ii) 2.57 GeV^2, (iii) 3.63 GeV^2 The curve shown for x_d corresponds to the least restrictive value of the parameter $c_d = 0.83$.

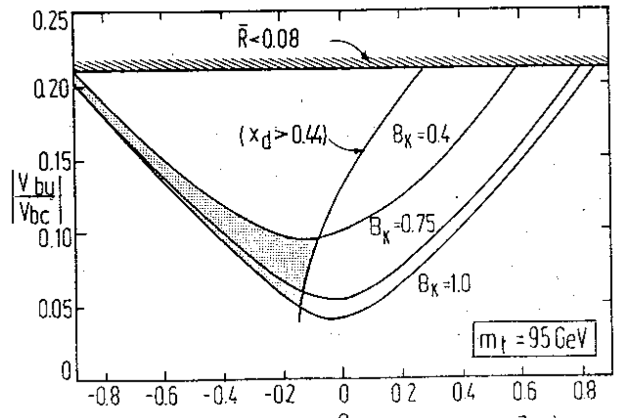
a) $m_t = 40$ GeV, b) $m_t = 60$ GeV, c) $m_t = 80$ GeV
d) $m_t = 100$ GeV



7 a)



7 b)



7 c)

Fig. 7. Constraints on the CKM parameter ρ and $|V_{bu}/V_{bc}|$ from the joint analysis of χ_d and $|\epsilon_K|$ for the central values of the parameters $c_d = 0.35$, $A = 1.05$, $m_c = 1.5$ GeV and the bag parameter $B_K = 1.0$, 0.75 and 0.4. a) $m_t = 55$ GeV, b) $m_t = 75$ GeV, c) $m_t = 95$ GeV.

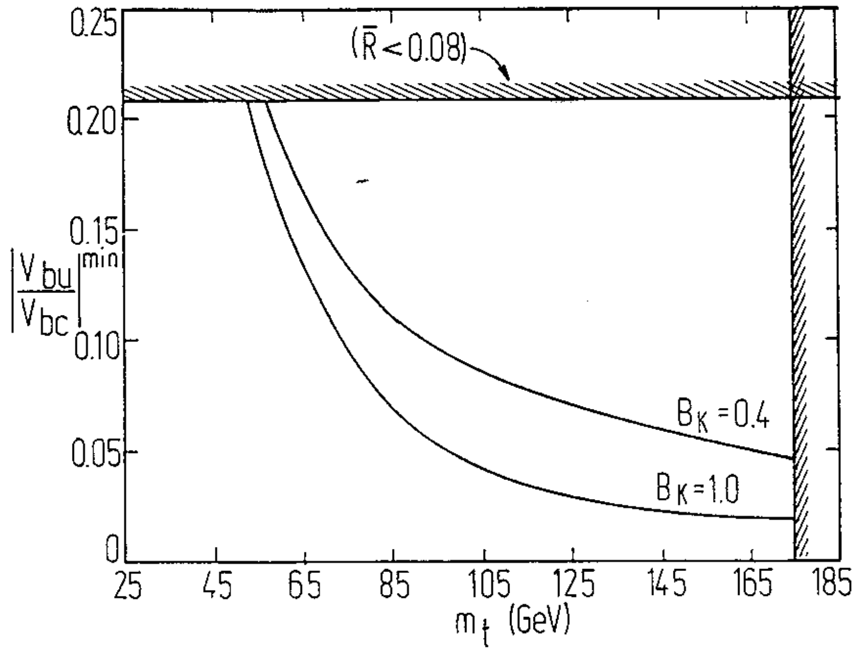


Fig. 8. The ratio of the CKM matrix elements $|V_{bu}/V_{bc}|^{min}$ obtained from the analysis of $|\epsilon_K|$ and $\chi_d > 0.44$ plotted as a function of the top quark mass for $A = 1.05$, $B_d f_B^2 = (0.15\ \text{GeV})^2$, $m_c = 1.5$ GeV and two representative values of the bag constant $B_K = 0.4$ and 1.0. The upper bound on m_t from $\Delta\rho$ and on V_{bu}/V_{bc} from $\bar{R} < 0.08$ are also indicated

Let us now briefly discuss some experimental consequences of the present experimental bounds (3.20) on χ_d and χ_s. It is possible to make a firm prediction for the quantity χ in the continuum experiments. We remark that for the probability $P_s = 0.20$, there is already a conflict between the ARGUS and MARK II results and the CKM prediction that $\chi_s \geq 0.42$. Thus, very probably $P_s < 0.20$ and this problem is easily evaded. For the expected choice $P_d = 0.375, P_s = 0.15$, the prediction is

$$\chi\,(\text{cont.}) \geq 0.10$$

While for the pessimistic choice $P_d = 0.4$, $P_s = 0.10$, one expects

$$\chi\,(\text{cont.}) \geq 0.075$$

Based on this argument, at least one of the following developments must take place

1) Experiments in the continuum at PEP/PETRA, LEP/SLC, $Sp\bar{p}S$ / Tevatron etc. must experimentally measure χ at or above 7.5 % (preferably $\sim$ 10 %).

2) The 90 % C.L. lower bound by ARGUS must ultimately settle to a lower number.

3) The CKM contribution does not saturate χ_d and χ_s.

In this context, the UA1 lower limit $\chi > 0.06$ (90 % C.L.) augers well for the CKM model, though one would like to see more positive measurements from e^+e^- experiments to confirm the standard model case.

4. B^0-$\bar{B}^0$ Mixings - other Alternatives

The discussion of the last section can be summarized in one sentence: The standard model can accommodate the present results on B^0-$\bar{B}^0$ mixing if $m_t \geq 50$ GeV. It is nevertheless conceivable

that a conflict for the standard model is already programmed. It is worthwhile to sample a few plausible extensions of the standard model vis à vis B^0-$\bar{B}^0$ mixings. We briefly review some of these alternatives here.

4.1 Minimal Left-Right Symmetric Models

The minimal LR symmetric models have been discussed extensively in literature [57]. For the present discussion it is sufficient to recall that the charged current interactions in such models can be parametrized as [58]

$$\mathcal{L}_{cc} = \frac{g_R}{2\sqrt{2}} \Big\{ \bar{u}_i \big[\cos\xi (K_L)_{ij} \gamma_\mu P_L - e^{-i\lambda} \sin\xi (K_R)_{ij} \gamma_\mu P_R \big] d_j W_L^\mu + \bar{u}_i \big[e^{i\lambda} \sin\xi (K_L)_{ij} \gamma_\mu P_L + \cos\xi (K_R)_{ij} \gamma_\mu P_R \big] d_j W_R^{+\mu} + h.c. \Big\} \tag{4.1}$$

$$P_{\binom{L}{R}} = (1 \mp \gamma_5)$$

where $W_{L,R}$ are the two mass eigenstates, $K_L (K_R)$ = Left (Right) CKM matrix, and ξ, λ are L-R mixing parameters. In terms of the Higgs expectation values v and ω

$$\xi = \frac{2|v\omega|}{|v|^2 + |\omega|^2} \frac{M_L^2}{M_R^2} \leq O(10^{-2})$$

$$e^{i\lambda} = -\frac{v\omega^*}{|v\omega|} \tag{4.2}$$

where the Higgs fields transform as (1/2, 1/2, 0) under $SU(2)_L \times SU(2)_R \times U(1)$ and

$$\langle \Phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & \omega \end{pmatrix} \tag{4.3}$$

Neglecting ξ, the charged current has the simple form

$$\mathcal{L}_{cc} = \frac{g_R}{2\sqrt{2}} \bar{u}_i \left[W_L^{\mu} (K_L)_{ij} \gamma_\mu P_L + W_R^{\mu} (K_R)_{ij} \gamma_\mu P_R \right] d_j + \text{h.c.} \tag{4.4}$$

Expressing the additional contributions to x_d and x_s by the quantities $(\Delta x_d)^{LR}$ and $(\Delta x_s)^{LR}$, one can formally write

$$(x_{d,s})^{LR} \equiv (x_{d,s})^{LL} \left(1 + (\Delta x_{d,s})^{LR} \right) \tag{4.5}$$

where $(x_{d,s})^{LL}$ correspond to the standard $SU(2)_L \times U(1)$ estimates (3.14) and the additional contributions $(\Delta x_{d,s})^{LR}$ have the form (with the tacit assumption $K_L = K_R$) (58)

$$(\Delta x_{d,s})^{LR} = 3/2 \; m_{W_L}^2 / m_{W_R}^2 \left(m_{B_{d,s}}^2 / (m_b + m_{d,s})^2 + 1/6 \right) \Big[\eta_{t_1}^{LR} \left((4 - y_t)/(1 - y_t) + \frac{4 - 2y_t + y_t^2 \ln y_t}{(1 - y_t)^2} \right) + \eta_{t_2}^{LR} \ln \, m_{W_L}^2 / m_{W_R}^2 \Big] \left(\eta_t F(y_t) \right)^{-1} \tag{4.6}$$

where $\eta_{t_1}^{LR}$ and $\eta_{t_2}^{LR}$ are QCD correction factors estimated to be $\eta_{t_1}^{LR} = \eta_{t_2}^{LR} = 1.8$. We note the factorization property of $(x_{d,s})^{LR}$. Thus, also in this scenario

$$(x_s/x_d)^{LR} \simeq (x_s/x_d)^{LL} \tag{4.7}$$

However, the L-R contribution $(\Delta x_{d,s})^{LR}$ cannot be very significant due to the similar bounds on the K_L-K_S mass difference. In ref. (59), it has been shown that such bounds lead to $m_R > 2.5$ TeV and $m_{H^\pm} > 10$ TeV. This gives (58)

$$(\Delta x_{d,s})^{LR} = 0.06 \left(\frac{2.5\,\text{TeV}}{m_R} \right)^2 + 0.25 \left(\frac{10\,\text{TeV}}{m_{H^\pm}} \right)^2 \leqslant 0.3 \tag{4.8}$$

Quite amusingly, it has been pointed out in ref. (17) that $(\Delta x_{d,s})^{LR} < 0$ for all physically allowed values 23 GeV $< m_t <$ 180 GeV, leading to a decrease of $(x_{d,s})$ as compared to the standard model. Probably minimal L-R symmetric models (with the assumption $K_L = K_R$) are not very relevant if one is looking for enhancements of $x_{d,s}$.

4.2 Supersymmetric Models

Supersymmetric models have a peculiar source of flavour-changing neutral current (FCNC) transitions through the non-diagonal gluino-squark-quark coupling — $(\tilde{g}\,\tilde{d}_i\, d_j)$. The source of such interactions is the renormalization of the left-handed down squark masses due to the presence of the charged Higgsino - d type-squark - u type quark $\tilde{H}^\pm \tilde{d} U$ interactions. The 6x6 mass matrix of the scalar partners of the d-type quarks is then generally of the form (60)

$$\begin{pmatrix} (\tilde{m}_D^2)_{LL} & (\tilde{m}_D^2)_{LR} \\ (\tilde{m}_D^2)_{RL} & (\tilde{m}_D^2)_{RR} \end{pmatrix} = \begin{pmatrix} \mu_L^2 + m_D^\dagger m_D + c\, m_U^\dagger m_U & \mu m_D \\ \mu m_D & \mu_R^2 + m_D^2 \end{pmatrix} \tag{4.9}$$

where D(U) = d-type (u-type) quarks and the parameters μ_L, μ_R and μ are of the order of global SUSY breaking scale. The parameter c can in principle be calculated in terms of μ_L given the top mass m_t and is found to lie in the range (18),(61)

$$-0.7 < c < -0.4$$

with c increasing with m and decreasing with m_t. No FCNC effects are caused due to the right-handed squark mass matrix since $(\tilde{m}_D^2)_{RR} \sim \mu_R^2 + m_D^2$, and in view of what has been said about the L-R contribution to χ_d, χ_s we can ignore the LR mixing $(\tilde{m}_D^2)_{LR} \sim \mu m_D$ Thus, one need consider only the consequences of the left-handed mass matrix for the squarks. Written in the basis where the U and D quarks are diagonal it reads as

$$\tilde{m}^2_{DLL} = \mu^2_L + m_D^2 + cV^+ m_U^2 V \qquad (4.10)$$

where $V = V_{CKM}$ is the 3x3 matrix of the standard model. Now, clearly only the possibility $\mu_L \leqslant 0(m_W)$ can influence low energy phenomenology like the values of χ_d, χ_s. There are two additional parameters μ_L and $\tilde{m}_g$ (apart from the parameters that enter in the standard model analysis) in the SUSY scenario. The combined effect of the SUSY interactions can be expressed in the form

$$(\chi_{d,s})^{SUSY+SM} = (\chi_{d,s})^{SM} \left(1 + \Delta\chi_{d,s}^{SUSY}\right) \qquad (4.11)$$

where the SUSY contribution is found to be (17),(18),(61)

$$\Delta\chi_{d,s}^{SUSY} = 3(\alpha_s^2/\alpha_W^2)(m_W^2/m_{\tilde{g}}^2) \times F^{SUSY}(\tilde{y}_b, \tilde{y}_b)/(y_t F(y_t)) \qquad (4.12)$$

where

$$F^{SUSY}(\tilde{y}_b, \tilde{y}_b) = f(\tilde{y}_b, \tilde{y}_b) - 2f(\tilde{y}_b, \tilde{y}_d) + f(\tilde{y}_d, \tilde{y}_d)$$
$$\tilde{y}_i = \tilde{m}_i^2/m_W^2, \quad i = \tilde{d}, \tilde{s} \qquad (4.13)$$

and

$$f(x,y) = 22/27\, f_2(x,y) - 16/27\, f_1(x,y)$$
$$f_1(x,y) = 1/(y-x)\{x/(x-1)^2 \ln x - 1/(x-1) - x \to y\}$$
$$f_2(x,y) = 1/(x-y)\{(x/(x-1))^2 \ln x - 1/(x-1) - x \to y\} \qquad (4.14)$$

The SUSY contribution $\Delta\chi_{d,s}^{SUSY}$ has been numerically calculated in refs. (17) and (18) with amusingly rather different conclusions. We show in fig. (9) the values of $\Delta_{x,d}^{SUSY}$ from ref. (18) taking $\tilde{m}_g = \tilde{m}_b$ (the lightest down squark) for two representative values of m_t, m_t = 60 and 100 GeV. For $\tilde{m}_g = \tilde{m}_b \simeq 60$ GeV, limits close to the ones from the UA1 experiments,(31) $\Delta_{x,d}^{SUSY} << 1$ for the parameter c in the range (4.10). If c is close to 1, the effects get enhanced as pointed out in ref. (17). Thus, either the gluino mass is just around the corner of the UA1 limit, or else SUSY interactions will not significantly enhance $\chi_{d,s}$.

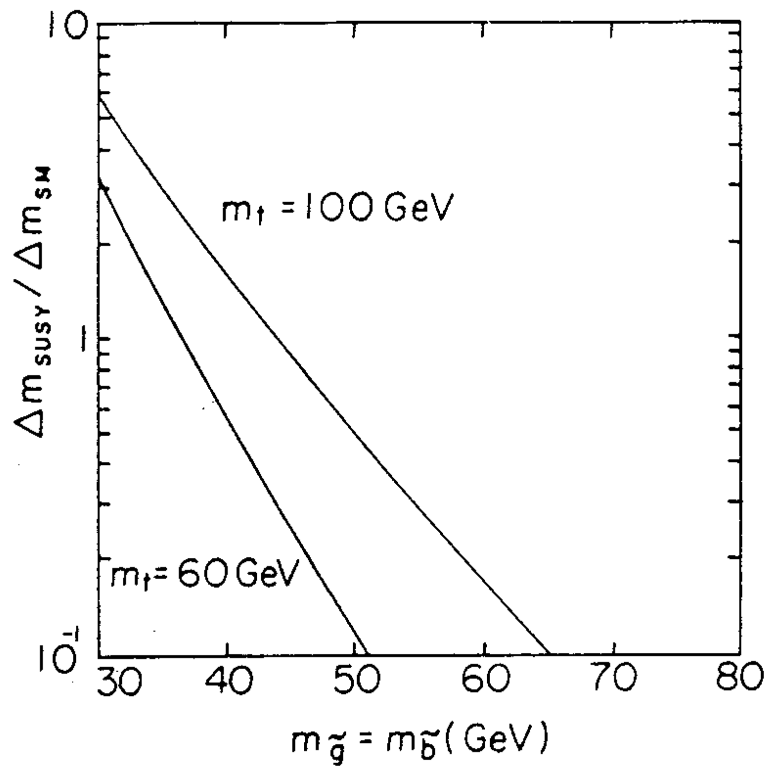


Fig. 9. The ratio $\Delta\chi_d^{SUSY} = \Delta M_d^{SUSY}/\Delta M_d^{SM}$ for the B_d^o-$\bar{B}_d^o$ system as a function of the gluino and the lightest squark mass (taken here to be equal) for m_t = 60 GeV and 100 GeV (from ref. 18)

We remind that as a consequence of eq. (4.10), the relationship

$$(x_s/x_d)^{SUSY+SM} \simeq (x_s/x_d)^{SM} \tag{4.15}$$

still holds, and the standard model expectations for r_s given r_d as shown in fig. (4) also hold for the SUSY scenario.

It is worth pointing out that if the SUSY induced FCNC effects are detectable in $\chi_{d,s}$, then very probably so are the FCNC transitions giving rise to $b \to s\gamma$ amd $b \to sg$. In ref. (18), it has been argued that branching ratios $b \to s\gamma \simeq 0(10^{-3})$ and $b \to sg \simeq 0(10^{-1})$ are conceivable for smaller values of $\tilde{m}_g$. This prospect would have become enormously fascinating if the charm deficit in hadronic bottom decays had not been explained away by the new measurement of the $D \to K\pi$ branching ratio by the MARK III collaboration.[34] Sometimes a small step in the right direction prevents a giant leap in the wrong!

4.3 Fourth Family of Quarks and Leptons

At present there does not exist a single piece of experimental information which even vaguely points in the direction of a fourth family of quarks and leptons. There are already some astrophysical constraints that suggest that the existence of a fourth neutrino is almost on the verge of exclusion [62]. In the near future this question is expected to be settled by the measurements of the Z^o decay width at SLC and LEP, which is sensitive to the number of neutrinos, with typically $\Gamma(Z^o \to \nu\bar{\nu})$ = 170 MeV for each neutrino species. Since these experiments have not yet been done, there is no firm experimental evidence against a fourth family either.

Such models have received renewed interests in the context of B_d^o-$\bar{B}_d^o$ mixing [19],[64] though they were put forward quite some time ago as possible extensions of the standard CKM model [63].

The parameters of the 4x4 matrix, which now contains 6 angles and 3 phases cannot all be determined. However, one could impose constraints following from the unitarity of the 4x4 matrix and present experimental information to narrow down the allowed range of parameters. Such an exercise was first done in ref. (30) and recently updated in ref. (19). Calling the fourth doublet of quarks (b',t'), symbolically one can write the 4x4 flavour mixing matrix,

$$V_4 = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix} \qquad (4.16)$$

The unitarity constraints can be written as [19]

$$V_4 = \begin{pmatrix} 0.972\text{-}0.976 & 0.218\text{-}0.222 & 0.0\text{-}0.12 & 0.0\text{-}0.08 \\ 0.18\text{-}0.23 & 0.84\text{-}0.98 & 0.04\text{-}0.06 & 0.0\text{-}0.50 \\ 0.0\text{-}0.13 & 0.0\text{-}0.47 & 0.0\text{-}0.99 & 0.0\text{-}0.99 \\ 0.0\text{-}0.14 & 0.0\text{-}0.49 & 0.0\text{-}0.99 & 0.0\text{-}1.0 \end{pmatrix} \qquad (4.17)$$

The additional matrix element that are relevant for B^0-$\bar{B}^0$ mixings are $V_{t'd}$, $V_{t's}$ and $V_{t'b}$ and the present phenomenology still allows these couplings to be sizeable. Since both b' and t' are heavy they don't contribute to $\Gamma(b)$ or $\Delta\Gamma_{d,s}$ but they do contribute to $\Delta m_{d,s}$. Taking into account the contribution of the quarks in the box diagram, one can express

$$\chi^{4F}_{d,s} = \chi^{SM}_{d,s}\left[1 + \Delta\chi^{4F}_{d,s}\right] \qquad (4.18)$$

where

$$\Delta\chi^{4F}_{d} = \left|V^*_{t'b} V_{t'd}\right|^2 / \left|V^*_{tb} V_{td}\right|^2 m_{t'}^2/m_t^2 + 2\left(\left|V^*_{t'b} V_{t'd}\right| / \left|V^*_{tb} V_{td}\right|\right) m_{t'}^2/(m_{t'}^2 - m_t^2) \ln \frac{m_{t'}^2}{m_t^2}$$

$$\Delta\chi^{4F}_{s} = \Delta\chi^{4F}_{d} \left\{ \begin{matrix} V_{t'd} \to V_{t's} \\ V_{td} \to V_{ts} \end{matrix} \right\} \qquad (4.19)$$

Based on our present knowledge of V_4,(4.17), it is conceivable that $\Delta\chi^{4F}_d \geq 1$ but $\Delta\chi^{4F}_s \ll 1$. In that case one would be able to explain the ARGUS result on χ_d even with a modest value of m_t and in addition there would be no natural explanation why the relationship (3.17) should hold. Thus, $\chi_d \geq \chi_s$ is still allowed in this scenario. As we have discussed earlier, probably this is the only plausible extension of the standard model where the value of χ_s is not bounded from below knowing χ_d. The present experimental region in the χ_d-χ_s plane to the left of the CKM curve in fig. (4) would be an allowed region. Thus, it is imperative to improve the present precision on χ_d and χ_s.

5. Conclusion and Outlook

The discovery of B^0-$\bar{B}^0$ mixings by the UA1 collaboration and the unexpectedly high B^0_d-$\bar{B}^0_d$ rate measured by the ARGUS collaboration are probably the most significant result in accelerator experiments since the discovery of the $W^{\pm}$ and Z^0 bosons. This is already evidenced by the large theoretical interest [(8)-(19)] that these experiments have stimulated in the recent past. In particular, they constrain the CKM model parameters in the "hidden sector", namely the top quark. Given m_t, measurements of χ_d and χ_s amount to measurement of the matrix elements $|V_{td}|$ and $|V_{ts}|$ respectively. Knowing the present bounds on these angles from unitarity, one can predict a lower bound on m_t, which turns out to be 0(50 GeV). The measurement of m_t has now assumed an additional phenomenological aspect. Even without the precise knowledge of m_t, the consistency of the standard model can be checked by the measurements of the relative rate r_s/r_d. In the CKM model present phenomenological constraints give $\chi_s/\chi_d > 5.4$. Violation of this bound is expected only in the four -family models. This is a good example of probing heavy new scales by low energy precision experiments.

We have critically reviewed the present experimental situation and found that at 90 % C.L. limits, there is no conflict between the various data. Yet, the central value of the ARGUS result $\Upsilon_d = 0.21 \pm 0.08$ is on the verge of admissability given the MARK II bound $\chi < 0.12$ and the standard model constraints. This would become acute if future $e^+e^-/p\bar{p}$ experiments do not find χ at $\sim$ 10 % level. By the same argument, ongoing e^+e^- experiments at PEP/PETRA better soon produce positive evidence of B^0-$\bar{B}^0$ mixings!

Disentangling Υ_d and Υ_s in continuum experiments will require the measurements of flavour correlations. In this respect the rate $b \rightarrow l^+K^-K^-X$ is a good measure of Υ_s (65). Amusingly, correlations of the type $p\bar{p} \rightarrow \ell^+\ell^+\Lambda^0 X$ and $p\bar{p} \rightarrow \ell^-\ell^-\bar{\Lambda}^0 X$ were reported by UA1 in the first reports on the processes $p\bar{p} \rightarrow \mu^\pm\mu^\pm X$ (66) and were interpreted as evidence of flavour correlations from B_s^0-$\bar{B}_s^0$ mixing (26). UA1 collaboration and Tevatron experiments are well advised to disentangle Υ_d and Υ_s from the inclusive measurements using such flavour correlations. Some suggestions for B_s^0 tagging with high energy bottom hadron beams have also been discussed in ref. (67). The best way to measure Υ_s is, of course, by collecting large data samples above the $B_s\bar{B}_s$ threshold above the $\Upsilon(4s)$ region. In this context, the resonances $\Upsilon(5s)$ and $\Upsilon(6s)$ are of particular importance. Better theoretical calculations are also necessary to determine the branching ratios $\Upsilon(5s, 6s, \cdots) \rightarrow B_s\bar{B}_s$. Seen in this perspective bottom factories optimized in luminosity near $\sqrt{s}$ = 10-12 GeV would be particularly useful machines.

The large value of χ_d, if confirmed by subsequent experiments, augers well for a number of measurements in future high statistics bottom hadron experiments. One such example is the prospects of measuring CP violation in exclusive non-leptonic bottom decays (69). For example, the time integrated asymmetry defined as (68),(69)

$$c_f \equiv \left(\Gamma(B_d^0 \rightarrow f) - \Gamma(\bar{B}_d^0 \rightarrow \bar{f}) \right) / \left(\Gamma(f) + \Gamma(\bar{f}) \right) \tag{5.1}$$

has been shown to be given by (70)

$$c_f = \frac{2 x_d \, Im(\Delta A)}{2 + x_d^2 + x_d^2 (\Delta V)} \tag{5.2}$$

where

$$\Delta A \equiv \frac{\bar{A}(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)}$$
$$\Delta V \equiv V_{tb} V_{td}^* / V_{tb}^* V_{td} \tag{5.3}$$

The present measurements $0.44 < x_d < 0.56$ allow the asymmetry c_f to reach 0(20-30) % and make CP violation accessible in future bottom hadron facilities, like the one being discussed at UCLA. Some of the favourable two-body channels in the non-leptonic sectors are:

$$B_d^0, \bar{B}_d^0 \rightarrow \pi^+\pi^-, K^+\pi^-, D^+\pi^-, J/\psi K_s, \phi K_s, D^+D^-, \pi^0 K_s, \eta K_s, D^0 K_s \tag{5.4}$$

Recent updates based on three standard deviation signatures have been presented in refs. (69). Defining

$$N_{b\bar{b}} = 9 / c_f^2 \, BR(f+\bar{f}) \, \varepsilon_f \tag{5.5}$$

with ε_f being the detection efficiency of the final state f, it seems that $N_{b\bar{b}} \simeq 0(10^7)$ would make most of the channels in (5.4) experimentally interesting as far as CP violation is concerned.

The expectations that $\chi_s > 0.4$ in the CKM model could render the determination of x_s a very difficult proposition in time integrated measurements. If $\chi_s > 0.45$, then the sensitivity of χ_s on x_s becomes very weak since the saturation limit $\chi_s = 0.5$ is reached very slowly. This circumstance would necessitate measurements of time evolution of the bottom hadron beams starting for example as a pure B_s^o or $\bar{B}_s^o$, in analogy to the K_L-K_S regeneration experiments. These experiments are feasible at Tevatron II, LHC/SSC and the fixed target version of the UNK machine, since the Lorentz factor γ would easily make the bottom hadron track length $L = \tau\gamma\beta c \simeq 0(1\text{-}10\ \mathrm{cm})$ at these energies. Such proposals have been discussed in refs. (56).

The measurements of x_d have an indirect impact on all quantities that depend on m_t, examples of which are rare kaon and beon decays. Thus the transitions $K \rightarrow \pi\nu\bar{\nu}$ and $B \rightarrow Ke^+e^-$, $B \rightarrow K\nu\bar{\nu}$ etc. get enhanced. This is being discussed elsewhere (70).

Finally, the most important aspect of the B^o-$\bar{B}^o$ mixings is that it constrains the parameters of the CKM model. Such constraints become especially interesting when combined with the corresponding ϵ_K analysis. One such constraint is that $m_t \gtrsim 50$ GeV. Another exciting constraint is that for $m_t < 70$ GeV, very probably $V_{bu}/V_{bc} > 0.1$. We wait eagerly for new experimental input in this sector!

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References

1) See, for example, Time Magazine, March 23 (1987) 40

2) I. Shelton, International Astronomical Union (IAU) circular No. 4316 (1987)

3) K. Hirata et al. (Kamiokande II Collaboration), Phys. Rev. Lett. 58 (1987) 1490;
R.M. Bionta et al. (IMB III Collaboration), Phys. Rev. Lett. 58 (1987) 1494

4) See, for example, J. Ellis, Summary talk at the International Conference on "Neutrino Masses and Neutrino Astrophysics", Ashland, Wisconsin, U. S. A. (1987)

5) C. Albajar et al. (UA1 Collaboration), Phys. Lett. 186B (1987) 427

6) H. Albrecht et al. (ARGUS Collaboration), DESY Report, DESY 87-029 (1987)

7) N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531;
M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49 (1973) 652

8) A. Ali, B. van Eijk and I. ten Have, DESY Report, DESY 86-108 (1986)

9) J. Ellis, J.S. Hagelin and S. Rudaz, CERN Report CERN-TH 4679/87 (1987)

10) J. Maalampi and M. Roos, Univ. of Helsinki Report HU-TFT-87-12 (1987)

11) V. Barger et al., Univ. of Wisconsin-Madison Report, MAD/PH/341 (1987)

12) L.-L. Chau and W.-Y. Keung, Univ. of Calif.-Davis Report,

UCD-87-02 (1987)

13) H. Harari and Y. Nir, SLAC Report, SLAC-PUB-4341 (1987)

14) V.A. Khoze and N.G. Uraltsev, Leningrad Nucl. Phys. Inst. Report 1290 (1987)

15) I. Bigi and A. Sanda, SLAC Report SLAC-PUB-4299 (1987)

16) L. Angelini et al., Univ. of Bari Report, Bari-GT/2-87 (1987)

17) G. Altarelli and P. Franzini, CERN Report, CERN-TH 4745 (1987)

18) S. Bertolini, F. Borzumati and A. Masiero, Carnergie-Mellon Univ. Report CMU-HEP 87-07 (1987)

19) E. Paschos, Univ. of Dortmund Report DO-TH 87/8 (1987)

20) J. Ellis et al., Nucl. Phys. B131 (1977) 285

21) A. Ali and Z.Z. Aydin, Nucl. Phys. B148 (1979) 165

22) D.M. Ritson, in Proceedings of the XXIIIrd International Conference on High Energy Physics, Berkeley, U.S.A., 1986

23) M. Gilchriese, in Proceedings of the XXIIIrd International Conference on High Energy Physics, Berkeley, U.S.A., 1986

24) See, for example, G. Altarelli, in Proceedings of the XXIIIrd Int. Conference on High Energy Physics, Berkeley, U.S.A., 1986 and references therein to earlier work.

25) G. Arnison et al. (UA1 Collaboration), Phys. Lett. 147B (1984) 493

26) A. Ali and C. Jarlskog, Phys. Lett. 114B (1984) 266

27) M.A. Shifman and M.V. Voloshin, ITEP Reports 86-54, 86-83 (1986); T.M. Aliev and V.L. Eletsky, Sov. J. Nucl. Phys. 38 (1983) 396;
C. Dominguez and N. Paver, DESY Report, DESY 87-60 (1987)

28) A. Ali, DESY Report, DESY 86-108 (1986) and ref. (8)

29) S. Godfrey, Phys. Rev. D33 (1986) 1391

30) K. Kleinknecht, in Proceedings of the International Symposium on Production and Decay of Heavy Hadrons, Heidelberg, Germany, 1986

31) R. Batley (UA1 Collaboration), in Proceedings of the 6th Topical Workshop on p-$\bar{p}$ Collider Physics, Aachen, Germany, 1986;
A. Honma (UA1 Collaboration), in Proceedings of the XXIIIrd International Conference on High Energy Physics, Berkeley, U.S.A., 1986

32) S. Bertolini, F. Borzumati and A. Masiero, Carnegie-Mellon Univ. Reports 86-19, 87-03, 87-07;
N.G. Deshpande et al., Univ. of Oregon report OTIS-346 (1986)

33) For a recent review see E. Thorndike, in Proceedings of the 1987 APS/DPF Meeting, Salt Lake City, U.S.A., 1987

34) D. Hitlin, Invited talk at the 2nd Topical Seminar on "Heavy Flavours", S. Miniato, Italy (1987)

35) L.B. Okun, V.I. Zacharov and B.M. Pontecorvo, Lett. Nuovo Cim. 13 (1975) 218;
A. Pais and S.B. Treiman, Phys. Rev. D12 (1975) 244

36) Particle Data Group, Phys. Lett. 170B (1986) 1

37) J.S. Hagelin, Nucl. Phys. B193 (1981) 123;

A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B245 (1984) 369

38) L.B. Okun et al. in ref. (35);
I. Bigi and A. Sanda, Phys. Lett. 171B (1986) 320

39) A. Bean et al. (CLEO Collaboration), Phys. Rev. Lett. 58 (1987) 183

40) See, for example, D.H. Saxon, Rutherford Appleton Lab. Report RAL-86-057 (1986)

41) T. Schaad et al. (MARK II Collaboration), Phys. Lett. 160B (1985) 188

42) W. Bartel et al. (JADE Collaboration), Phys. Lett. 171 (1986) 320

43) L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945

44) A. Ali, DESY 86/41 (1986) and in "Physics at LEP", CERN 86/02, vol. 2 (Ed. J. Ellis and R. Peccei), 1986

45) W. Marciano, in Proceedings of the 1987 APS/DPF Meeting, Salt Lake City, U.S.A., 1987

46) H. Krasemann, Phys. Lett. 96B (1980) 397;
Ali and Jarlskog (ref. 26)

47) M. Shifman, in Proceedings of the International Symposium on Production and Decay of Heavy Hadrons, Heidelberg, Germany, 1986

48) Shifman et al. in ref. (27)

49) D. Hitlin, in Proceedings of the Internatinal Symposium on "The Fourth Family of Quarks and Leptons", Santa Monica, Calif., U.S.A. (1987)

50) M. Suzuki, Phys. Lett. 142B (1984) 207 and 162B (1985) 392

51) Aliev and Eletsky in ref. 27)

52) L.J. Reinders et al., Phys. Rep. C127 (1985) 1

53) Dominguez and Paver in ref. 27)

54) L.J. Reinders and S. Yazaki, Nucl. Phys. B288 (1987) 789;
K. Chetyrkin et al., Phys. Lett. 174B (1986) 104

55) J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Lett. 135B (1984) 481

56) M. Arenton et al., Fermilab-Proposal P771 (1986). See also, B. Cox and D.E. Wagoner, Fermilab Report, Fermilab-Conf. 86/167 (1986)

57) J.C. Pati and A. Salam, Phys. Rev. D10 (1974) 275;
R.N. Mohapatra and J.C. Pati, Phys. Rev. D11 (1975) 566, 2558;
R.N. Mohapatra and G. Senjanovic, Phys. Rev. D12 (1975) 1502, Phys. Rev. Lett. 44 (1980) 912; Phys. Rev. D23 (1981) 165

58) G. Ecker and W. Grimus, Z. Phys. C30 (1986) 293; R. Decker and U. Türke, Zeit. f. Phys. C26 (1984) 117

59) G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. 48 (1982) 848;
G. Ecker, W. Grimus and H. Neufeld, Phys. Lett. 127B (1983) 365;
R.N. Mohapatra, G. Senjanovic and M.D. Tran, Phys. Rev. D28 (1983) 546;
F.J. Gilman and M.H. Reno, Phys. Rev. D29 (1984) 937

60) See for example H.P. Nilles, Phys. Rep. 110 (1984) 1;
H. E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 17 and

references to earlier work therein

61) J.M. Gerard et al., Phys. Lett. 140B (1984) 349;
J.M. Gerard et al., Phys. Lett. 141B (1984) 79, Nucl. Phys. B253 (1985) 93;
M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. B255 (1985) 413;
M.B. Gavela et al., Phys. Lett. 154B (1985) 147

62) J. Yang et al., Ap. J. 281 (1984) 493. However, J. Ellis et al, Phys. Lett. 167B (1986) 457 get an upper bound $N_\nu \lesssim 5.5$ by relaxing some of the parameters in their analysis. For a critique see G. Steigman et al., UMN-TH-562/86 (1986)

63) S.K. Bose and E.A. Paschos, Nucl. Phys. 169B (1980) 384

64) A. Anselm et al., Phys. Lett. 156B (1985) 102;
M. Shin, in Proceedings of "The Fourth Family of Quarks and Leptons", Santa Monica, Calif., U.S.A. (1987);
Bigi and Sanda (ref. 15)

65) A. Ali and F. Barreiro, Zeit. f. Phys. C30 (1986) 635

66) C. Rubbia, in Proceedings of the "4th Topical Workshop on Proton-Antiproton Collider Physics", Bern, Switzerland (1984);
K. Eggert, in Proceedings of the International Symposium on "Cosmic Rays and Particle Physics", Tokyo, Japan (1984)

67) D. Cline, Univ. of Wisconsin-Madison Report, WISCV-EX-86-270 (1986)

68) See, for example, I. Bigi and A. Sanda, Nucl. Phys. 281 (1987) 41 and references therein to earlier work

69) D. Du and Z. Zhao, Institute for Advanced Study, Princeton Preprint IASSNS-HEP-87/23 (1987). For earlier work see
D. Du, I. Dunietz and D-d. Wu, Phys. Rev. D34, 3414 (1986)

70) G.Hou and A.Soni, Pittsburg Report PITT-87-05(1987) and these proceedings