

DEUTSCHES ELEKTRONEN-SYNCHROTRON **DESY**

DESY 87-117
September 1987



PHOTON-PHOTON INCLUSIVE SCATTERING AND PERTURBATIVE QCD

by

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ISSN 0418-9833

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PHOTON-PHOTON INCLUSIVE
SCATTERING AND PERTURBATIVE QCD

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Abstract

Perturbative QCD expectations and problems associated with the study of the photon structure function data are reviewed. An assessment is given for the viability and sensitivity of photon-photon scattering as a decisive tool for the determination of the QCD scale. Particular attention is given to the theoretical problems of singularity cancellations at $x=0$ and threshold-associated difficulties at $x=1$ and their implications on the actual data analysis. It is concluded that the experimental results, while not providing a decisive verification of QCD at small distances, do add to other independent experiments which are all consistent with the theory and suggest a reasonably well defined QCD scale parameter. The importance of the small Q^2 limit to photon-photon analysis is discussed and the data is examined in an attempt to identify and isolate the contributions of the hadronic and point-like sectors of the target photon.

I) Introduction

Quantum Chromodynamics (QCD) is accepted as the theory of strong interactions. The theory, in its perturbative form, can be tested experimentally provided we confine our investigation to short distance phenomena. Indeed, the theory derives its experimental support from a diversified class of experiments: deep inelastic lepton-nucleon scattering, high energy e^+e^- and $p\bar{p}$ annihilation, jets, large p_T scattering, the Drell-Yan process, quarkonium spectroscopy, single photon physics and more. None of these experiments provides a unique or decisive test of QCD. They are all, nevertheless, consistent with the theory and are compatible with a QCD scale parameter $\Lambda = 150-200$ MeV.

Over the last ten years, following Witten's paper¹⁻², it has become a matter of common wisdom to anticipate that the experimental study of the photon structure function would provide a clean test of QCD and a unique determination of its scale. This anticipation led to a very extensive experimental program aimed at a detailed study of photon-photon inclusive reactions³⁻⁷. In the following I shall discuss the critical problems associated with this research. With most of the present experiments approaching their termination, it is appropriate to assess both the achievements and misconceptions of this very ambitious research effort.

The plan of this review is as follows: a summary of the formalism describing $\gamma\text{-}\gamma$ inclusive reactions is given in Sec. II. Sec. III is devoted to the quark-parton model and Sec. IV to QCD. Theoretical problems associated with $x=0$ and $x=1$ are discussed in Sec. V. Sec. VI deals with the separation of the target photon into two components and its implications to the low Q^2 analysis. Our conclusions are summarized in Sec. VII.

II) General Formalism

Photon-photon interactions are studied from the reaction (Fig. 1a):

$$e^+e^- \rightarrow e^+e^- + \gamma \quad (1)$$

in which we shall consider the Q^2 photon as a probe and the P^2 photon as a target. We have thus an $e\text{-}\gamma$ deep inelastic scattering experiment on a photon target. For simplicity we shall confine most of our remarks to single tag experiments where the target photon is quasi real $P^2=0$. The cross section for the reaction

$$e(p_1) + \gamma(q_2) \rightarrow e(p_2) + X(p_3) \quad (2)$$

is given by⁸:

$$\frac{d\sigma}{dx dy} = \frac{4\pi\alpha^2(P_1 \cdot q_2)}{Q^4} [1 + (1-\gamma)^2] \cdot \left\{ 2x F_T^\gamma + \epsilon(y) F_L^\gamma + \epsilon(y) \epsilon(z) F_X^\gamma \right\} \quad (3)$$

where:

$$X = \frac{Q^2}{2(q_1 \cdot q_2)}, \quad \gamma = \frac{q_1 \cdot q_2}{P_1 \cdot q_2}, \quad z = \frac{q_1 \cdot q_2}{q_1 \cdot P_2} \quad (4)$$

$\epsilon(y)$ and $\epsilon(z)$ are the polarizations of Q^2 and P^2 .

To simplify the formalism we introduce the photon structure functions:

$$\begin{aligned} F_T^\gamma(x, Q^2) &= F_T^\gamma(x, Q^2) \\ F_2^\gamma(x, Q^2) &= 2x F_T^\gamma(x, Q^2) + F_L^\gamma(x, Q^2) \\ F_3^\gamma(x, Q^2) &= F_X^\gamma(x, Q^2) \end{aligned} \quad (5)$$

Once we neglect the photon longitudinal components, we reproduce the Callan-Gross relation $F_2^\gamma = 2xF_1^\gamma$. The photon structure function provides us, thus, with a simple relation between the photon-photon total cross section and the quark-antiquark distributions within the target photon, as we have:

$$F_2^\gamma(x, Q^2) = 2x F_F^\gamma(x, Q^2) = x \sum_{F\bar{F}} e_q^2 [\bar{q}(x, Q^2) + \bar{q}(x, Q^2)] \quad (6)$$

and

$$F_2^{\gamma}(X, Q^2) = \frac{Q^2}{4\pi^2\alpha} \sigma_{\gamma\gamma}(Q^2, W) \quad (7)$$

where:

$$W^2 = P_3^2 = Q^2 \frac{1-x}{x}$$

The study of $F_2^{\gamma}(X, Q^2)$ offers some unique observations¹⁻² which are derived from the fact that the photon can couple directly to a quark-antiquark pair (Fig. 1b). This coupling has to be corrected for QCD-gluon emission and absorption (Fig. 1c). We therefore anticipate that:

- 1) Whereas the standard hadronic structure function peaks at small x and rapidly falls off with increasing x , the photon structure function is expected to peak at high x and then rapidly decrease to zero at $x=1$, as required kinematically.
 - 2) $F_2^{\gamma}(X, Q^2)$ has a positive scale breaking dependence $\lg \frac{Q}{\Lambda^2}$. This overall increase with Q^2 provides a definite advantage over hadronic structure functions which have a positive scale breaking at very small x and negative scale breaking at higher x . In particular, the study of the photon structure function is free from higher twist effect ambiguities.
 - 3) $F_2^{\gamma}(X, Q^2)$ is supposed to be completely determined in QCD. Technically one calculates the F_2^{γ} moments in perturbative QCD and then extracts F_2^{γ} from the inverse Mellin transform. The scope of QCD analysis of the hadronic structure functions is much more limited. Perturbative QCD enables us to calculate the evolution, but not the normalization, of $F_2^{\text{HAD}}(X, Q^2)$.
 - 4) $F_2^{\gamma}(X, Q^2)$ depends on $\frac{1}{\alpha_S}$ as compared with $(1+\frac{S}{4\pi})$ in e^+e^- annihilation. It makes the normalization of the photon structure function conveniently sensitive to QCD parameters.
- These optimistic expectations are confronted with two fundamental difficulties that must be resolved so as to make the QCD analysis of the data viable.
- 1) The dramatic features of F_2^{γ} , i.e. the high x peaking and positive scale breaking are not exclusive signatures of QCD, but rather general signatures of the parton structure of the photon. In the study of hadrons the parton model predicts scaling and it is the scale breaking which provides

a clean signature of QCD. In our case, F_2^{γ} scaling is broken on the parton level. In order to establish the onset of QCD, we need to secure that QCD predictions are indeed exclusive. We note that $F_L^{\gamma} = 0$ in the parton model and differs from zero in QCD. However, F_L^{γ} is a difficult experimental quantity to study.

2) The photon is special in the theory of partons and their interactions due to its two component structure. The photon can interact with quarks either directly through its point-like coupling, or collectively through its coupling to vector mesons (VDM). Our theoretical expectations were derived for the photon's point-like component. However, the actual data contains the two components added incoherently. As we shall see the two components have an intricate and complex relationship, the understanding of which is essential for our ability to isolate the point-like component.

A better understanding of the above two outstanding problems is the main theme of this review.

III) The Quark-Parton Model

The photon's two component structure may be viewed from a phenomenological point of view by studying the space-time properties of the probing photon and its coupling to the target photon through a $q\bar{q}$ pair. For small P_T , the lifetime of the $q\bar{q}$ state is long and we have a large overlap with the low-lying vector mesons ρ, ω, ϕ etc. Hence the target photon hadronic component. For high P_T the lifetime of the $q\bar{q}$ state becomes so short that we have a diminishing overlap with the hadron states and hence the point-like component. This situation is beautifully demonstrated in Fig. 2 which shows the cross section for a charged final state hadron, in the reaction $e^+e^- \rightarrow e^+e^-h^+X$, as a function of P_T . We can vividly observe the transition from hadron-like to point-like as P_T is increased.

The quark-parton model-QPM, is a simple QED estimate of the point-like sector. The cross section for the reaction $\gamma\gamma \rightarrow q\bar{q}$ is derived from the box diagrams (Fig. 1b). For a single tag arrangement ($P^2=0$) we get³:

$$G_{QPM}^{\nu\nu}(\alpha, W) = \frac{8\pi\alpha^2}{(W^2 + \alpha^2)^3} 3 \sum_{F_i} \left\{ L \left[\frac{1}{2}(W^2 + \alpha^2)^2 \right. \right. \\ \left. \left. + 2m_i^2 W^2 - 4m_i^4 - \alpha^2 W^2 \right] - \frac{2\Delta t}{W^2 + \alpha^2} \left[\frac{1}{4}(W^2 + \alpha^2)^2 + m_i^2 W^2 - \alpha^2 W^2 \right] \right\} \quad (8)$$

where

$$L = \frac{2}{W^2 + \alpha^2} \frac{g \left[(W + \sqrt{W^2 - 4m_i^2}) / 2m_i \right]}{\sqrt{W^2 - 4m_i^2}}$$

$$\Delta t = \frac{W^2 + \alpha^2}{W} \sqrt{W^2 - 4m_i^2}$$

This cross section corresponds to a structure function:

$$F_2^{\nu\nu}(x, \alpha^2) = \frac{3\alpha}{\pi} \sum_{F_i} e_{q_i}^4 \left[x^2 - (1-x)^2 \right] g \frac{W^2}{m_i^2} + \delta x^2 (1-x) - x \quad (9)$$

The constituent quark masses to be used are the standard values of $m_u = m_d = 300$ MeV, $m_s = 500$ MeV and $m_c = 1500$ MeV. We neglect, in this discussion, the contributions from heavier quarks.

As we shall see, QCD calculations do not relate to the non-leading part of (9). QCD corrections change only the fine details of the leading x dependence, but not the two striking features of high x peaking and positive scale breaking.

Our results are very encouraging, on the one hand, as we have obtained very clean signatures for the parton character of the target photon. On the other hand, we realize that QCD verification is bound to be rather difficult. In the continuation we shall have to examine under what conditions are the QCD characteristics of $F_2^{\nu\nu}$ different enough from QPM so as to make a QCD study reliable.

IV) QCD

In order to facilitate QCD calculations we shall utilize the Lipatov-Altarelli-Parisi evolution equations¹⁰. Witten's calculation¹¹, as well as many of the calculations that followed¹², have used the Operator Product Expansion (OPE) method, which yields the same results. However, following the photon's evolution equations has the advantage that they can be readily compared with the better known hadron's equations.

Let us define the moments of the quarks and gluon distributions:

$$q(n, t) = \int dx q(x, t) X^{n-1} \quad (10)$$

$$G(n, t) = \int dx G(x, t) X^{n-1}$$

To leading order in α_s we get^{9,12}:

$$\frac{\partial q(n, t)}{\partial t} = e_q^2 d(n)_{Born} + \frac{1}{2\pi b t} \left[A_{qq}(n) q(n, t) + \right. \\ \left. + A_{GG}(n) G(n, t) \right] \quad (11)$$

$$\frac{\partial G(n, t)}{\partial t} = \frac{1}{2\pi b t} \left[\sum_{q\bar{q}} A_{Gq}(n) q(n, t) + A_{GG}(n) G(n, t) \right]$$

where $2\pi b = \frac{1}{6}(33-2N_f)$ and we define the born term

$$q(x, \alpha^2)_{Born} = 3 e_q^2 \frac{\alpha}{2\pi} [x^2 + (1-x)^2] g \frac{\alpha^2}{\lambda^2} \equiv \\ \equiv e_q^2 d(x)_{Born} g \frac{\alpha^2}{\lambda^2} \quad (12)$$

$$d(n)_{Born} = \int dx d(x)_{Born} X^{n-1}$$

The QCD anomalous dimension matrix elements are

$$A_{qq}(n) = \frac{4}{3} \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2 \sum_{j=1}^n \frac{1}{j} \right]$$

$$A_{Gq}(n) = \frac{4}{3} \frac{2+n+n^2}{n(n^2-1)}$$

$$A_{qG}(n) = \frac{1}{2} \frac{2+n+n^2}{n(n+1)(n+2)}$$

$$A_{GG}(n) = 3 \left[-\frac{1}{6} + \frac{2}{n(n-1)} + \frac{(n+1)(n+2)}{(n+1)(n+2)} - 2 \sum_{j=1}^n \frac{1}{j} - \frac{n}{9} \right]$$

The evolution equations (11) are solved by choosing a normalization point t_0 and are conveniently presented by the non-singlet and singlet moments:

$$\Delta(n, s) = q_{2/3}(n, t) - q_{1/3}(n, t) \quad (14)$$

$$\Sigma(n, s) = \sum_{F_i} [q(n, t) + \bar{q}(n, t)]$$

$$s = g \frac{t}{t_0}$$

The important distinction of the evolution equation for $q(n,t)$ is that it is inhomogeneous. This is different from the hadron's evolution equation which is homogeneous. Indeed, the solution to (11) is the sum of the homogeneous equation (without the box-Born term) and a particular solution of the inhomogeneous equation which would be just the box diagram corrected for gluon radiation. We are, thus, led to a natural identification of the inhomogeneous solution as the photon's point-like component. The homogeneous equation presents the hadron-like component and suffers from our inability to calculate its normalization.

In the following we shall concentrate on the F_2^Y non-singlet moments that are easier to handle:

$$M_n^Y(Q^2) = \int dx F_2^Y(x, Q^2) x^{n-2}$$

and utilize the more convenient OPE notation¹³. The solutions for the NS moments are given by:

$$M_n^x(Q^2) = A_n(\mu^2) \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{d_n} + \frac{a_n}{\alpha_s(\mu^2)} \frac{1}{d_n+1} \left\{ 1 - \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{d_n+1} \right\} + \frac{b_n}{d_n} \left\{ 1 - \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{d_n} \right\} \quad (16)$$

where μ^2 is an arbitrary renormalization point ($\alpha_s(\mu^2) \ll 1$), A_n corresponds to the unknown hadronic component of the photon and d_n are derived from the anomalous dimension matrix and are tabulated in refs. 11 and 14. a_n and b_n are numbers we know how to calculate^{11, 15}.

For fixed n and in the high Q^2 limit (16) becomes:

$$\lim_{Q^2 \rightarrow \infty} M_n^x(Q^2) = \frac{1}{\alpha_s(Q^2)} \frac{a_n}{d_n+1} + \frac{b_n}{d_n} \quad (17)$$

n fixed

i.e. we have obtained a result which does not depend on the unknown hadronic component and which depends on one parameter

α_s which is just the object of our investigations. Equivalent result is obtained when we utilize the evolution equations^{14, 16}.

V) Problems Associated with QCD Analysis

Witten's first calculation¹ was done in the leading log (LL) approximation and is shown in Fig. 3 as compared with the QPM results of the box calculation. The QCD results are different enough from QPM so as to suggest a clean QCD test and a measure of α_s or Λ .

Unfortunately, higher order calculations turned out to be not as encouraging. We encounter two classes of problems: The higher order calculations suggest that the difference between QCD and QPM for realistic Q^2 values is smaller than anticipated. We have to re-consider the exclusivity of the QCD analysis. The other class of problems is more fundamental. Higher order QCD calculations result¹¹⁻¹⁶ in negative $F_2^Y(x, Q^2)$ in the vicinity of $x=0$ and $x=1$. These are caused by zeroes of d_n and d_{n+1} . Moreover these singularities are getting worth with each order of the calculation¹⁴. Actually, a small spike at $x=0$ is seen already in Witten's LL result (see Fig. 3). If we denote $y = \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}$ and $d = d_n$ or d_{n+1} , we can readily see that our problems result from

$$\lim_{d \rightarrow 0} \frac{1}{d} (1-y^d) = -\lg y \quad (18)$$

typical of Eq. (16). We shall discuss in some detail mostly the $x=0$ problems. As for $x=1$, the situation is similar, only that we note that $x=1$ is not attained kinematically so the problems are not so severe.

A clue as to the understanding and solution of these problems was given¹⁷ in the case of double tag arrangement where $p^2 > 0$. In this case we can calculate perturbatively also the A_n elements (for $p^2 \gg \Lambda^2$). This leads to a cancellation of the singularities and finite positive results for $F_2^Y(x, Q^2, p^2)$. However, we lose the dependence on Λ as $\lg \frac{Q^2}{\Lambda^2}$ is replaced by $\lg \frac{Q^2}{p^2}$. Nevertheless, this has inspired several regularization schemes^{14-16, 18} for the real photon calculation in which a similar cancellation takes place.

The suggested solutions were developed according to schemes which are not always compatible with each other. Regularization schemes based on OPE have the singularity cancellation between the point-like and the hadronic components. This calls for a completely new understanding of the hadron sector which is usually estimated by VDM and is finite. Schemes based on the evolution equations claim that the singularity cancellation takes place separately in the point-like and hadron sectors and that each is finite. The difference between those points of view is fundamental and calls for added clarifications.

However, in actual analysis, all schemes cut on the soft processes so as to enhance the point-like QCD sector and add a phenomenological hadronic sector. They obtain, therefore, results that are quite similar. The question is what is the sensitivity of these calculations. The problem of QCD sensitivity is summarized in Figs. 4-6. Fig. 4 shows higher order QCD fits to F_2 with $\Lambda_{\overline{MS}} = 150-200$ MeV. The success of these QCD fits is seen in Fig. 5 which shows F_2/α integrated over $0.3 < x < 0.8$. However, a phenomenological fit to the same data gives $A = 158 \pm 88$ MeV demonstrating the insensitivity of the QCD result. We also show in Fig. 6 a beautiful overall fit²⁰ based on QPM + hadron model only. It is thus clear that at present experimental Q^2 values, QCD provides good but not exclusive reproduction of the data. Moreover, the sensitivity of a QCD calculation to A is low, making a clean scale determination impossible.

VI) The Photon's Hadronic Component

As we have noted all regularization schemes are utilizing essentially the same strategy for their data analysis. The first step is to cut on the soft (hadronic component rich) sector. This can be done by either cutting¹⁵ on $x > 0.3$ or¹⁶ on $P_T > 1$ GeV. The overlap between those cuts is high. In either case we are left with data which is predominantly, but not entirely, point-like, and we have to provide some input for the hadronic component. The early naive believe that once Q^2 is reasonably high, one can securely assume the data to be point-like is clearly not realistic! The problem is that the necessity to include the hadron component introduces a model

dependence to a calculation that was supposed to be model independent.

A coupled problem relates to the understanding of the low Q^2 ($Q^2=0$ included). It is clear that this data has a predominant hadronic component, but is this the only component? In an OPE regularization scheme we expect both components to co-exist at any Q^2 to provide the needed singularity cancellation (In this context the Antoniadis-Grunberg fit¹⁵ with a diminishing hadron sector is problematic!). In a scheme like the one suggested by Field et al.¹⁶, one expects the low Q^2 to be almost free of the point-like component and to exhibit precautions approximate scaling. The experimental situation is not clear.

Let us briefly review the various phenomenological descriptions of the hadron sector and their implications for low Q^2 . The most common estimate is based⁸ on VDM. The recipe calls for an estimate of the pion structure function from $\pi^+ p \rightarrow \mu^+ \bar{\nu} x$, assume that the structure functions of the pion and the vector mesons are identical, and get:

$$F_2^{\nu}(x, Q^2)_{VDM} \approx \sum_V \left(\frac{e}{g_V}\right)^2 F_2^{\pi}(x) \approx 0.2 \alpha(1-x) \quad (19)$$

This is the estimate used in most experimental studies. It provides for a finite and scaled hadronic structure function peaked at $x=0$.

An alternative approach is to use the GVD model²¹. GVDM does not have an explicit x dependence but it can be acquired once we parametrize the input $Q^2=0$ cross section as $\sigma_{\gamma\gamma} = A + \frac{B}{W}$. We get then:

$$\sigma_{\gamma\gamma}^{GVDM} = \left(A + \frac{B}{W}\right) F_{GVDM}(Q^2) \quad (20)$$

which translates to a non scaling structure function which peaks at $x=1$. Both models, as different as they are, produce acceptable fits to the data once combined with QPM or QCD.

Recently an attempt has been made²⁰ to re-examine all PEIRA data³⁻⁶ with a model that combines a parameter free QPM and a hadronic parametrization which contains a specific threshold factor and is compatible with high Q^2 scaling:

$$\sigma_{\gamma\gamma}(\theta^2, W)_{HAD} = (1-X)^2 \left(\frac{A}{Q^2+b} + \frac{B}{W\sqrt{Q^2+b}} \right) \quad (21)$$

The analysis covers the complete available range of $0 < Q^2 < 100 \text{ GeV}^2$ and $1 < W < 25 \text{ GeV}$ and produces a remarkably good fit of $\frac{X^2}{df} = 1$. The results are shown in Fig. 7 for a cross section fit and the resulting structure function was given in Fig. 6.

Regardless of the details, the conclusion of such an investigation and similar ones is that we have to take into account both components for any realistic combination of (Q^2, W) or (X, Q^2) . This introduces an unavoidable model dependence to any future QCD analysis which may be minimized by appropriate cuts, but not entirely eliminated.

VII) Conclusions

Our main conclusion is that a QCD study of inclusive $\gamma\gamma$ reactions is possible and most interesting. However, the early hope that this kind of experiment would provide a clean and decisive test of perturbative QCD turned out to be premature. Whereas the theoretical problems of the actual QCD calculation can be solved, we realize that the actual analysis is not entirely model dependent free because of the need for the hadronic component. Moreover, for Q^2 which can be attained at present or near future e^+e^- experiments, QCD offers a good but not exclusive reproduction of the data.

Our experimental support for perturbative QCD goes back if so, to our amazing ability to fit so many diversified short distance experiments with a consistent scale value.

Acknowledgements

I wish to thank the DESY directorate and the DESY Theory Group for their kind hospitality and the Minerva Foundation for financial support. I have benefitted from discussions with G. Alexander, J.H. Field and E. Gotsman.

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Figure Captions

Fig. 1: a) The two photon diagram $e^+e^- \rightarrow e^+e^- + x$

b) QPM box diagrams

c) QCD diagrams: (i) valence part, (ii) sea part

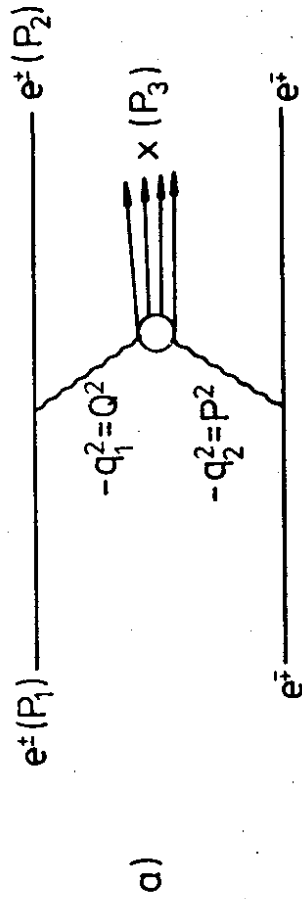


Fig. 2: Cross sections for single charged hadron

$$e^+e^- \rightarrow e^+e^- h^\pm x$$

Fig. 3: The photon structure function

A) The LL part of QPM

B) Full QPM

C) Witten QCD LL result

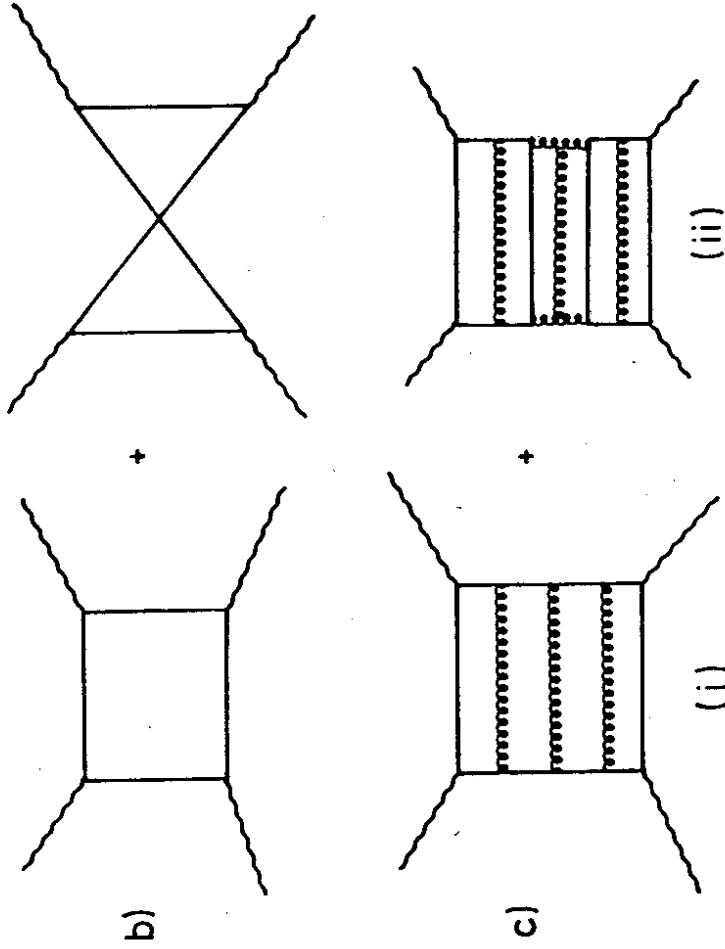


Fig. 4: Higher order QCD fits to F_2^Y

Fig. 5: Fits to $\langle F_2 \rangle / \alpha$ with $0.3 < x < 0.8$.
QCD fit is with $\Lambda_{\overline{MS}} = 183$ MeV,
phenomenological fit is with $\Lambda = 158$ MeV.

Fig. 6: $F_2^Y(x, Q^2)$ fits with QPM + hadronic model²⁰

Fig. 7: $\sigma^{\gamma\gamma}(Q^2, \omega)$ data and fit²⁰

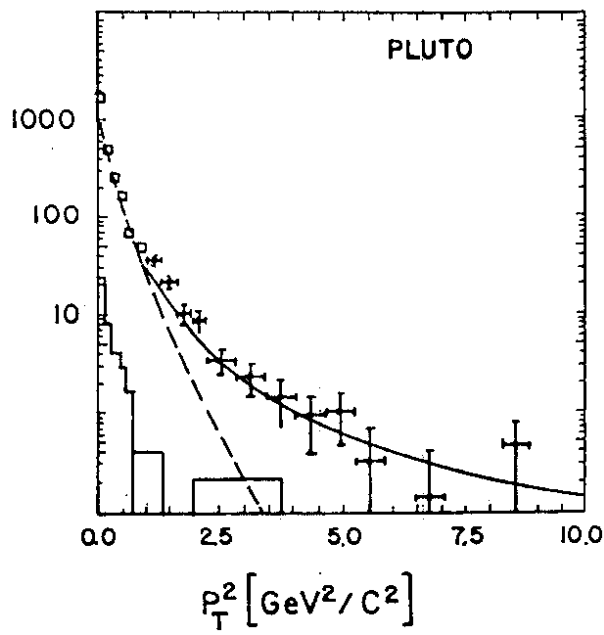
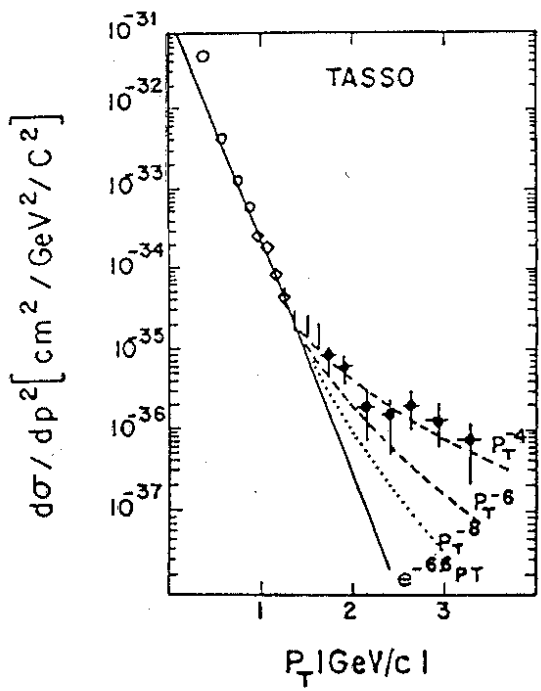


Fig.2

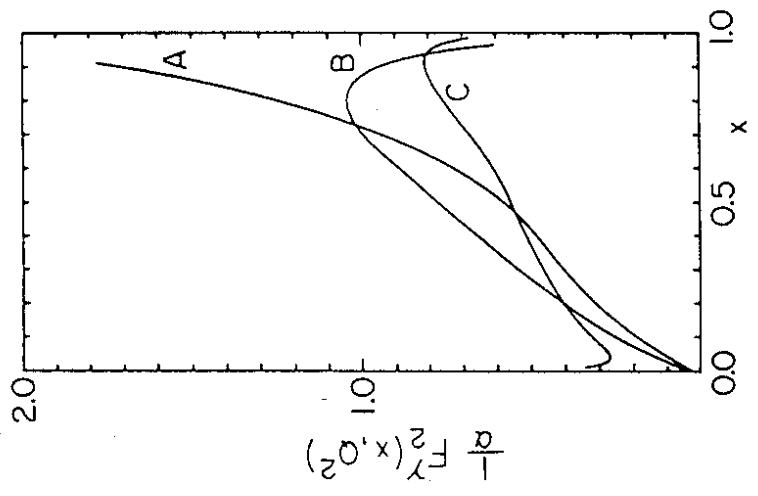


Fig.3

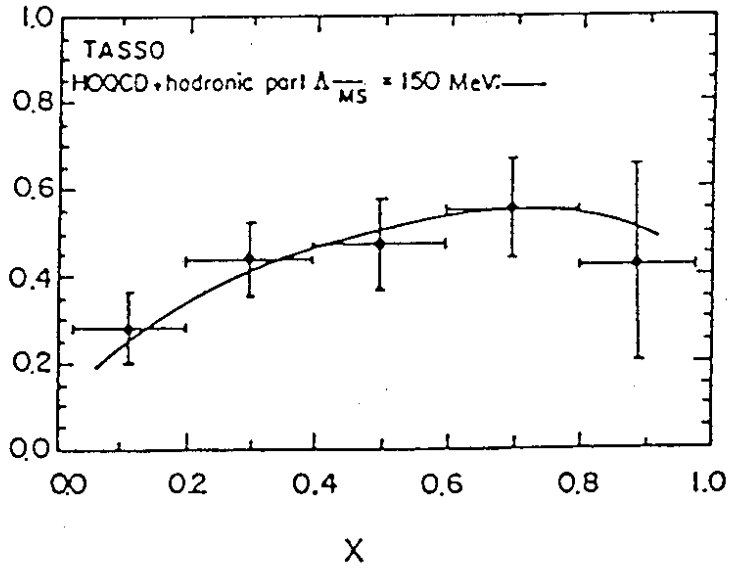
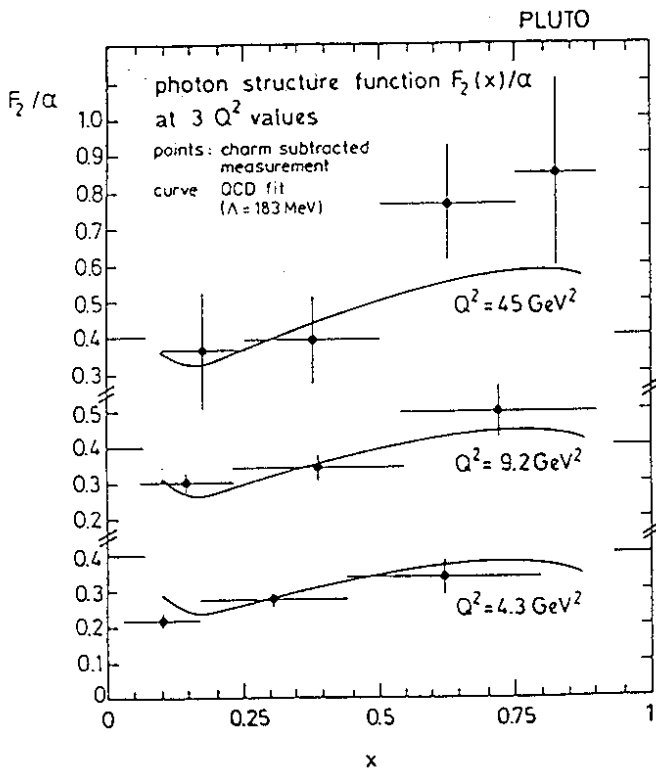


Fig.4

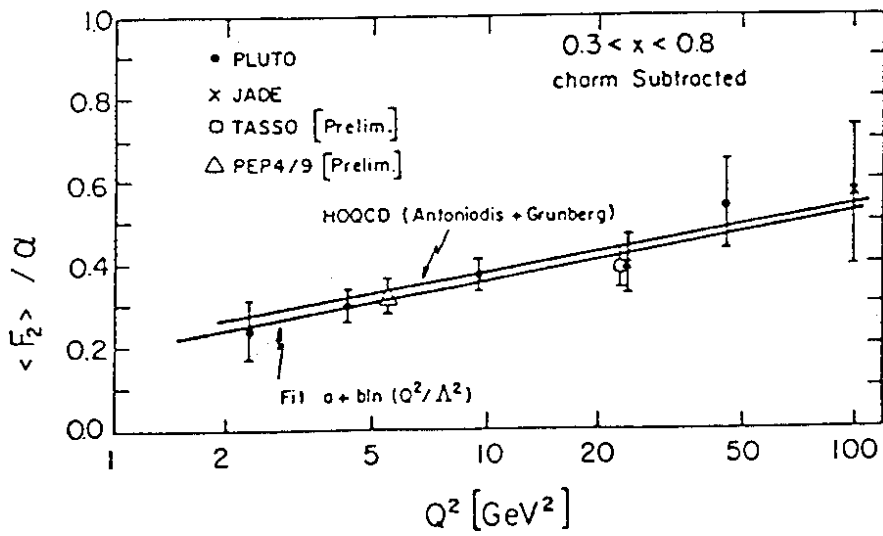


Fig.5

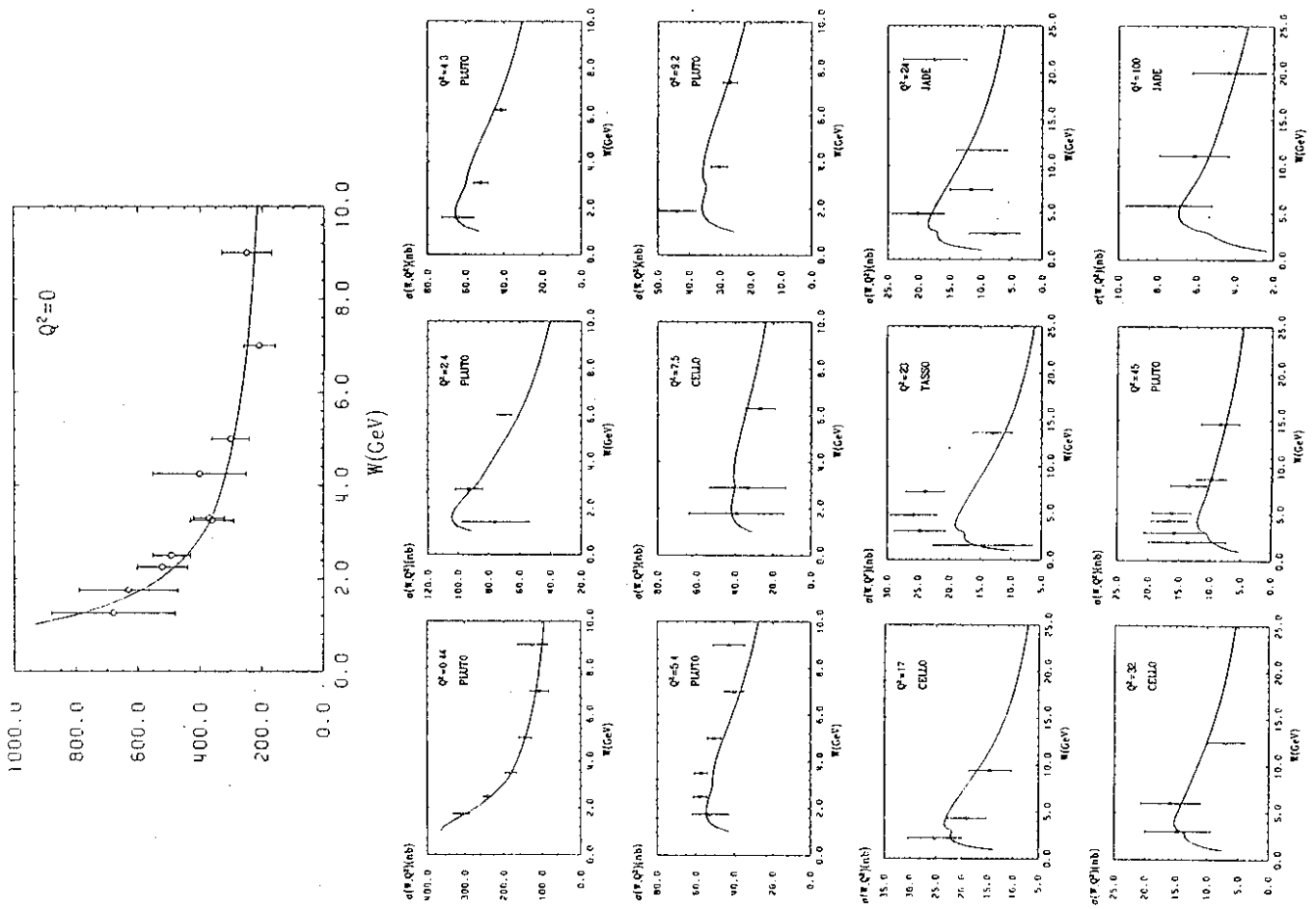


Fig. 7

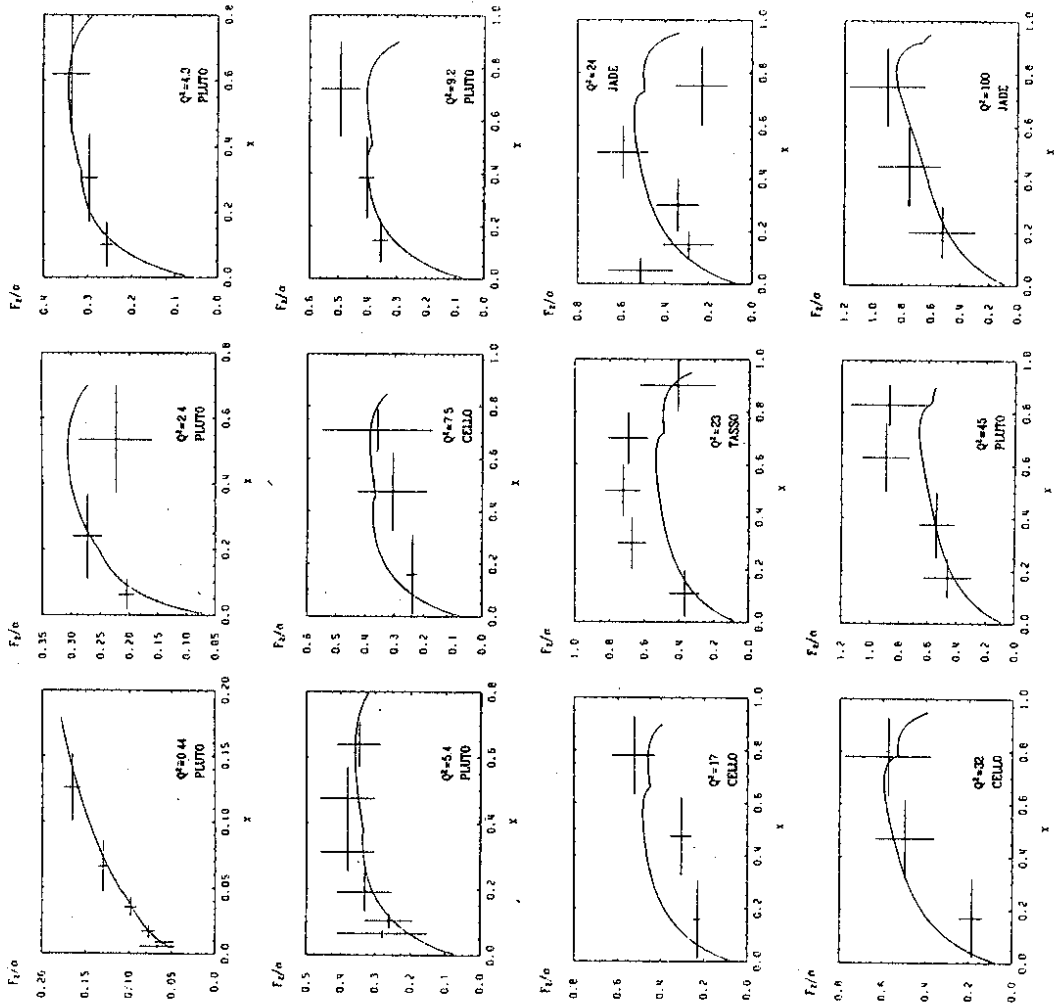


Fig. 6