

DESY 87-129  
October 1987



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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

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Abstract

An overview is given on the present status of electroweak radiative corrections in high energy reactions, in particular  $e^+e^-$  processes.

Talk given at the EPS Int. Conference on High Energy Physics 1987, Uppsala, Sweden

ELECTROWEAK RADIATIVE CORRECTIONS IN SU(2) x U(1)

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1. INTRODUCTION

In the electroweak Standard Model any observable quantity can in principle be calculated to an arbitrary order of perturbation theory. For an adequate analysis of the high precision experiments at LEP and SLC the inclusion of radiative corrections (RC) becomes a necessity. The physical motivation for studying RC are at least threefold:

- High precision experiments can test the validity of RC and thus test the Standard Model at the quantum level.
- Possible "new physics" will probably manifest as small deviations from the Standard Model predictions which have therefore to be known with high accuracy.
- From a practical point of view, RC can be large, in particular the bremsstrahlung corrections around the Z resonance. Their treatment constitutes the link between data taking and physics analysis.

2. RENORMALIZATION

Higher order calculations require the choice of a renormalization scheme which defines the free parameters and their relation to experimental quantities. In QED the preferred scheme is the on-shell scheme with the fine structure constant  $\alpha$  in Thomson scattering and the lepton masses as parameters. Its extension to SU(2) x U(1) leads to the electroweak on-shell scheme which makes use of the independent parameters

$$\alpha, M_W, M_Z, M_H, m_f \quad (1)$$

The simplest way to define the mixing angle is in terms of the weak boson masses:  $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ . This scheme has been widely used in practical applications (see e.g. refs. /1, 2/). The advantages are obvious: All parameters have an evident physical meaning and are (except  $M_H, m_t$ ) experimentally known; it has a natural separation into "QED corrections" (virtual and real bremsstrahlung) and infrared finite "weak corrections". This is of practical importance for the implementation into Monte Carlo programs.

3. THE  $M_W \leftrightarrow M_Z \leftrightarrow \sin^2\theta_W$  CORRELATION

Employing the on-shell scheme the muon lifetime, expressed by the muon

decay constant  $G_\mu$ , can be calculated in terms of the parameter set (1):

$$G_\mu = \pi\alpha/\sqrt{2} \left[ M_W^2(1-M_W^2/M_Z^2)(1-\Delta r) \right]^{-1}, \quad (2)$$

where  $\Delta r = \Delta r(\alpha, M_Z, M_W, M_H, m_t)$  summarizes the weak corrections of muon decay /3, 4/. (2) allows to replace  $M_W$  by the precisely measured  $G_\mu$ , or, equivalently, to express

$$\sin^2\theta_W = 1-M_W^2/M_Z^2 = \sin^2\theta_W(\alpha, G_\mu, M_Z, M_H, m_t) \quad (3)$$

The reward of eq. (2) is twofold:

- (i) It is interesting by itself since it allows a comparison of the  $M_W \leftrightarrow M_Z \leftrightarrow \sin^2\theta_W$  correlation with the measured quantities.
- (ii) It provides a value for  $M_W$  which can be used as input for numerical calculations of other observables of interest.

The uncertainty from the unknown Higgs mass can be expressed in terms of the total variation  $\Delta\sin^2\theta_W = 0.0048$  if  $M_H$  varies from 10 GeV to 1 TeV. It is smaller than the present experimental error  $(\Delta\sin^2\theta_W)_{\text{exp}} \gtrsim 0.005$  from  $\nu$  scattering.

The strong sensitivity of  $\Delta r$  on a large top quark mass  $m_t$  can be applied to obtain an upper limit for  $m_t$  from the  $M_W, M_Z$  values and their experimental errors (from /5/):  $m_t < 185$  GeV ( $1\sigma$ ). The hadronic uncertainty of  $\Delta r$  as given in /6/ is  $\delta(\Delta r) = 0.0007$ .

#### 4. RADIATIVE CORRECTIONS IN HIGH ENERGY PROCESSES

$e^+e^- \rightarrow \mu^+\mu^-$ :  $O(\alpha)$  corrections have been calculated by many authors /7 - 14/ (for a review see also /15/) and are available as Monte Carlo generators (Lynn, Kleiss, Stuart; Berends, Kleiss, Hollik). The most interesting measurable quantities on the Z peak are the forward-backward asymmetry

$$A_{\text{FB}} = \left[ \sigma(\theta < \pi/2) - \sigma(\theta > \pi/2) \right] / \left[ \sigma(\theta < \pi/2) + \sigma(\theta > \pi/2) \right] \quad (4)$$

and the polarization or left-right asymmetry

$$A_{\text{LR}} = \left[ \sigma(e_L^-) - \sigma(e_R^-) \right] / \left[ \sigma(e_L^-) + \sigma(e_R^-) \right]. \quad (5)$$

Both asymmetries can be utilized in two ways:

- (i) as functions  $A(M_Z, \sin^2\theta_W, M_H, m_t)$  in order to measure the value of  $\sin^2\theta_W$ . The sensitivity of  $A_{\text{LR}}$  to  $\sin^2\theta_W$  is significantly higher than of  $A_{\text{FB}}$  /13/ (see Figure 1).
- (ii) Replacing  $\sin^2\theta_W$  by means of (3) yields a prediction  $A(M_Z, G_\mu, m_H, m_t)$ . It can be used to delimit the unknown parameters  $M_H, m_t$  (see Figure 2).

The QED corrections for  $A_{\text{LR}}$  are almost negligible /10/. The expected experimental uncertainty of  $\Delta A_{\text{LR}} = 0.003$  and the weak corrections coming from the top and the Higgs make it possible to narrow the allowed mass ranges significantly.

$e^+e^- \rightarrow e^+e^-$ : The  $O(\alpha)$  full electroweak corrections /16/ have been calculated and are available as a Monte Carlo generator (Berends, Kleiss, Hollik). Other work done for Bhabha scattering is listed in /17/.

$e^+e^- \rightarrow F^+F^-$  with a heavy fermion F (top, heavy lepton):  $O(\alpha)$  electroweak corrections with soft photon bremsstrahlung have been calculated in /18/.

$e^+e^- \rightarrow W^+W^-$  is the process of prime importance at LEP 200. Earlier work of the  $O(\alpha)$  RC /19/ need to be checked. A new calculation is being performed in the on-shell parametrization (1) /20/.

$e^+e^- \rightarrow Z\gamma$  has attracted interest as a means of 'v counting' via detection of the  $\gamma$  and  $Z \rightarrow \nu\bar{\nu}$ .  $O(\alpha)$  virtual and soft photon corrections can be found in /21/; hard photons are treated via Monte Carlo in /22/.

Deep inelastic ep scattering: For HERA energies the  $O(\alpha)$  full electroweak RC have been calculated for the neutral current process  $ep \rightarrow eX(\gamma)$  /23/. Their implementation into a Monte Carlo generator is in progress.

Higher order QED around the Z resonance: Since the  $O(\alpha)$ QED corrections are large around the Z peak (typically - 40%) a careful study of the next order contributions becomes necessary for precision experiments like measurements of the mass and width of the Z boson. The main source for large negative corrections is the initial state bremsstrahlung where both soft and hard photons lead to a reduction of the peak cross section, roughly given by the factor

$$1 - 2\alpha/\pi \log(M_Z/\Gamma_Z) \log(M_Z^2/m_e^2) \simeq 0.6.$$

A partial inclusion of multiple photon emission consists of exponentiation of the infrared parts /8/ or of the leading log terms /24/. An exact treatment of the  $O(\alpha^2)$  initial state QED corrections to the integrated  $e^+e^- \rightarrow \mu^+\mu^-$  cross section has been performed in /25/. This allows to study the shift in the resonance peak which is crucial for the Z mass measurement: the shift of + 184 MeV from  $O(\alpha)$  is reduced by - 88 MeV due to the  $O(\alpha^2)$  contributions. The remaining uncertainty is estimated to be  $\simeq 15$  MeV /25/. A combination with the weak corrections contributing also to the Z shape is in progress /26/.

In general, the QED corrections to the  $e^+e^- \rightarrow \mu^+\mu^-$  cross section and  $A_{FB}$  constitute an obstacle in the physics analysis of high precision experiments since they are sensitive to the detector acceptance, to cuts (acollinearity,  $\gamma$  energy, ...) and to higher order contributions. The on-resonance left-right asymmetry  $A_{LR}$ , however, is practically free of the specific QED problems. Its high sensitivity to  $\sin^2\theta_W$  makes it a unique tool to test the internal structure of the Standard Model and to look for effects of "new physics", e.g. from more Higgs doublets /27/ or from the supersymmetric version of the Standard Model /28/.

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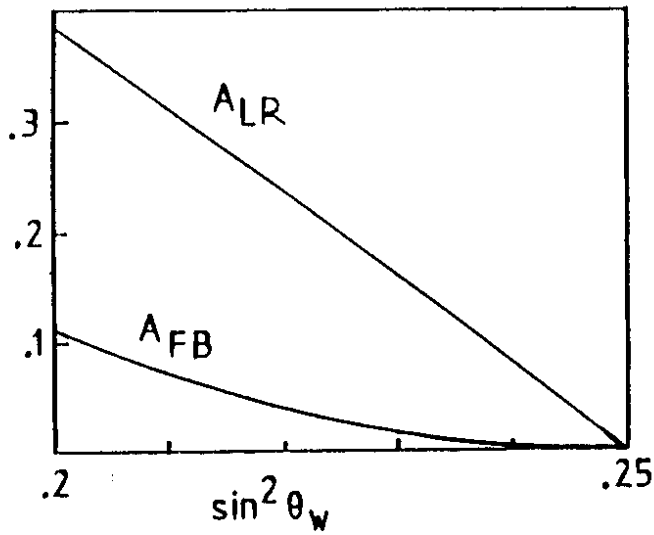


Fig. 1  $\sin^2\theta_W$ -dependence of the on-resonance forward-backward asymmetry  $A_{FB}$  and left-right asymmetry  $A_{LR}$ .  $M_Z = 93$  GeV.

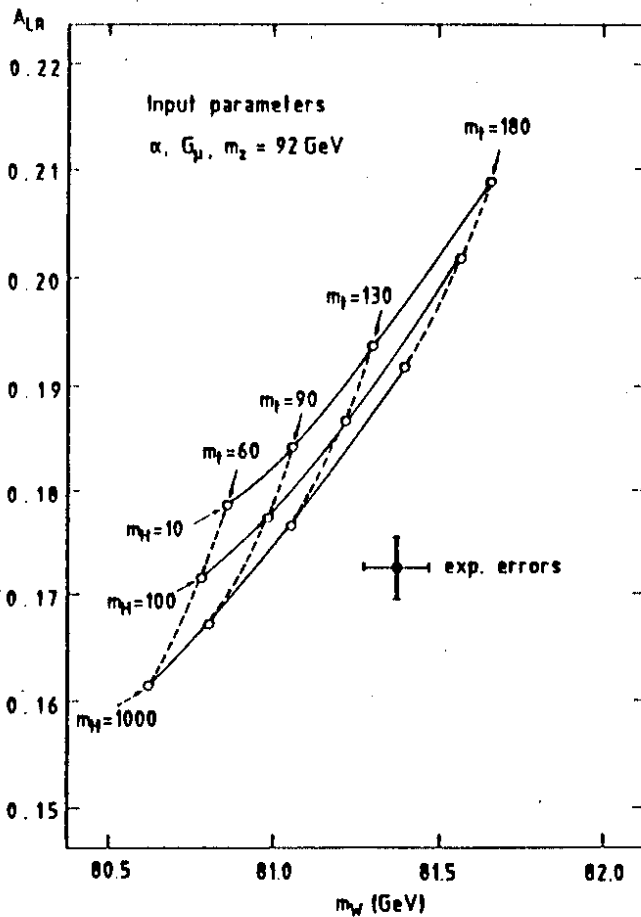


Fig. 2 Variation of the predictions for the left-right asymmetry  $A_{LR}$ , and for  $m_W$ , as a function of  $m_t$  and  $m_H$ , in comparison with the expected experimental precision of  $A_{LR}$  and  $m_W$ . (ref. /2/)