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HEAVY QUARKONIA

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# HEAVY QUARKONIA<sup>†</sup>

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## Abstract

We review a decade of studies devoted to the physics of heavy quark bound states  $c\bar{c}$  and  $b\bar{b}$ . After addressing the question of why, how and where to perform these investigations, we turn to an analysis of the spin (in-)dependent forces acting between heavy quarks. Processes involving large momentum transfer allow a determination of the coupling constant of the strong interaction. A critical evaluation is given on the possibility to accurately determine the strong coupling. For a description of hadronic transitions within heavy quarkonia non-perturbative methods have been developed. Except for one possible problem, these calculations will be shown to be in good agreement with data. Heavy quarkonia decays to light mesons provide the means to investigate known low mass states and to search for new forms of matter. Finally, an evaluation is attempted of the physics prospects which are achievable with significantly larger data sets than those available today.

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## Introduction

During the last decade heavy quarkonium states  $Q\bar{Q}$  have played a very prominent role in our understanding of the fundamental interactions. The study of such states is in many aspects very similar to the study of other two-particle systems like the hydrogen atom and the positronium. The main reason for the successful interplay between theory and experiment is that such systems are non-relativistic. Thus theoretical predictions of static properties and decay rates can easily be obtained with potential models. In addition, the large body of experimental data accumulated over the last years allows a detailed comparison of theoretical calculations with experiment.

The heaviest known quarkonium states are the  $\psi$  and the  $\Upsilon$  resonances, which test the  $Q\bar{Q}$  potential at distances between 0.1 and 1 fm. Analyzing the mass splittings between different  $Q\bar{Q}$  states allows a determination of not only the static potential between quarks, but also of contributions arising from the quarks' spin. Decays of heavy quarkonia provide direct means to measure the strong coupling of quarks by measurements of decay branching ratios. In addition, decays may also offer a source of new particles, such as the Higgs boson, the Axion, gluonic states and even supersymmetric particles.

The best place for the study of heavy quarkonia are electron-positron colliders. Heavy vector mesons like the  $J/\psi$  or the  $\Upsilon(1S)$  have the same quantum numbers as the photon,  $J^{PC} = 1^{--}$ , and are produced directly in  $e^+e^-$  interaction with a strength proportional to their leptonic partial width. As an example, the storage ring DORIS II at DESY produces about 10 000  $\Upsilon(1S)$  mesons per day. States with other than  $1^{--}$  quantum numbers cannot be produced directly, but must be studied in decays of the aforementioned vector mesons. Another production mechanism at  $e^+e^-$  storage rings proceeds via the two-photon formation  $\gamma\gamma \rightarrow Q\bar{Q}$ . Here states with positive charge conjugation and  $\text{spin} \neq 1$  can be produced. Unfortunately the two-photon luminosity decreases with the inverse mass of the produced state. Thus it is not surprising that the only heavy  $Q\bar{Q}$  state observed in two-photon processes is the  $\eta_c$ , with a total of about 30 events detected by four experiments.

Finally, heavy quarkonium states can also be produced in hadronic collisions. In fixed target experiments positive charge conjugation and  $\text{spin} \neq 1$  states are produced via gluon-gluon fusion processes. Due to the large background, only states decaying

into  $\mu^+\mu^-$  can be analyzed. Much cleaner is the direct production of heavy quarkonia in  $p\bar{p}$  annihilations with the additional advantage that all  $J^{PC}$  states can be formed. Here the  $\eta_c$ ,  $\chi_1^c$ ,  $\chi_2^c$  and possibly the  $h_c$  states in charmonium have been observed. In summary, it is obvious that  $e^+e^-$  storage rings are and will be our main tool for precision studies of heavy quarkonium states. Important additional and often complementary information, however, has and will come from two-photon induced reactions and from  $p\bar{p}$  annihilations.

This paper is organized as follows. In the next section we cover in detail the experimental techniques used to determine the mass spectra of  $Q\bar{Q}$  systems. This information will then be used to discuss our present understanding of spin-independent and spin-dependent forces between heavy quarks. Measurements of branching ratios, being inversely proportional to the decay width into three gluons, can be used to determine the strong coupling constant. We critically evaluate the possibilities and accuracies of such determinations. In the next section we discuss hadronic transitions within heavy quarkonium states, which play an important role in the study of non-perturbative methods. Following is a review of heavy quarkonia decays to light mesons. Such decays provide the means to investigate known low mass states and to search for new particles as the Axion and the Higgs. Finally we summarize and evaluate the physics prospects which are achievable with significantly larger data sets than those available today.

## Masses of Heavy Quarkonia

When positrons and electrons collide, they may scatter elastically or annihilate into a virtual photon of mass  $\sqrt{s} = W = 2E$ , where  $E$  is the beam energy. At total center-of-mass energies  $W \lesssim 10$  GeV the interaction proceeds primarily through the electromagnetic force and we may neglect the weak interaction. Neutral vector meson states  $V$  with the quantum numbers of the photon,  $J^{PC} = 1^{--}$ , are produced directly in  $e^+e^-$  interaction with a Breit-Wigner cross section:

$$\sigma_0(W) = \frac{3\pi}{W^2} \frac{\Gamma_{ee}\Gamma_{had}}{(W - M_V)^2 + \Gamma_{tot}^2/4}, \quad (1)$$

where  $\Gamma_{tot}$  is the total decay width,  $\Gamma_{ee}$  is the leptonic partial width and  $\Gamma_{had}$  is the hadronic width. The resonance is detected via its hadronic decays.

The emission of real and virtual photons in the process  $e^+e^- \rightarrow V$  modifies the lowest order cross section. These radiative corrections can be classified into initial

state hard- and soft-photon bremsstrahlung, initial state vertex corrections, and vacuum polarization of the intermediate virtual photon. They have originally been calculated [1] by Yennie *et al.* and by Bonneau and Martin. Several other theoretical analyses have appeared since [2,3,4,5] which in particular take into account the narrowness of heavy vector resonances. All four analyses can be stated as a convolution of the lowest order cross section  $\sigma_0(W)$  with a bremsstrahl-spectrum  $B(x, W)$

$$\bar{\sigma}(W) = \int dx \sigma_0(W(1-x)) B(x, W), \quad (2)$$

where  $x = k/E$  is the energy fraction carried away by all photons emitted. For the sake of simplicity we ignore hard bremsstrahlung, which would add a factor  $(1-x+x^2/2)$  to the integrand. Due to the narrow width of the  $\psi$  and  $\Upsilon$  resonances this part introduces a negligible contribution of at most 0.1%. The bremsstrahl spectra obtained by Greco [2], Jackson and Scharre [3], Tsai [4] and Kuraev and Fadin [5] are

$$\begin{aligned} B_{Greco} &= tx^{t-1} (1 + \delta_e + 2\Pi) \\ B_{Jackson} &= tx^{t-1} + (\delta_e + 2\Pi) \delta(x) \\ B_{Tsai} &= tx^{T-1} (1 + \delta_e) \\ B_{Kuraev} &= tx^{t-1} (1 + \delta_e). \end{aligned} \quad (3)$$

Ignored are higher order corrections as *e.g.* calculated in ref. [4,5]. In the above formulæ  $\delta_e = 3t/4 + (2\alpha/\pi) \times (\pi^2/6 - 1/4)$  arises from the vertex corrections and  $t = (2\alpha/\pi) \times (\ln(s/m_e^2) - 1)$  is the effective radiator thickness. The total vacuum polarization contribution  $\Pi = \sum_{i=e,\mu,\tau,q} \Pi_i$  arises from electron, muon, tau and all quark pairs loops. For example the electron loop contribution is given by  $\Pi_e = (\alpha/3\pi) \times (\ln(s/m_e^2) - 5/3)$ . At a center-of-mass energy  $W = 10$  GeV the numerical values are  $t = 8.7\%$ ,  $\delta_e = 7.2\%$ ,  $2\Pi_e = 2.8\%$  and  $2\Pi = 6.8\%$ .

It is apparent from the different bremsstrahl spectra in eq. 3 that the radiatively corrected cross section will differ for the different prescriptions and consequently the shape and magnitude of the cross section will differ. An inspection of the bremsstrahl spectra reveals the differences:

1. Jackson & Scharre and Greco include the vacuum polarization  $\Pi$  in their formulæ, whereas Kuraev & Fadin and Tsai do not. (Non-)inclusion of this term will change the normalization, but not the shape of the cross section. It is thus not important for a mass determination, but will be crucial for a measurement of  $\Gamma_{ee} B_{had}$  from the area under the resonance (see the section on *Total Widths*).

2. In addition Jackson and Scharre separate the vertex correction  $\delta_e$  from the exponentiated soft bremsstrahlung  $x^t$ , which reduces the cross section above the resonance mass. This approach was criticized by Kuraev and Fadin. Their criticism is corroborated by a recent calculation by Behrends *et al.* [6] of complete  $\mathcal{O}(\alpha^2)$  initial state radiative corrections.

Finally, the cross section  $\hat{\sigma}$ , eq. 2, has to be convoluted with the Gaussian energy distribution for the storage ring beams  $G(W) = (1/\sqrt{2\pi}\Delta) \times \exp(-(W - M)^2/2\Delta^2)$ . As the total width of the resonance  $\Gamma_{tot} \simeq 50$  keV is much smaller than the CMS-energy resolution  $\Delta \simeq 8$  MeV, we can approximate the cross section by  $\sigma_o(W) = A_o \delta(W - M)$  where  $A_o \equiv \int \sigma_o dW = (6\pi^2/M^2) \times \Gamma_{ee} B_{had}$  is the integral over the Breit-Wigner cross section of eq. 1. Accounting for the hadronic continuum contribution  $C$ , we obtain the observable cross section:

$$\sigma(W) = C/W^2 + A_o \int dx G(W(1-x)) B(x, M). \quad (4)$$

The integral in eq. 4 can be expressed [3] by the Gamma function and by Weber's parabolic cylinder function.

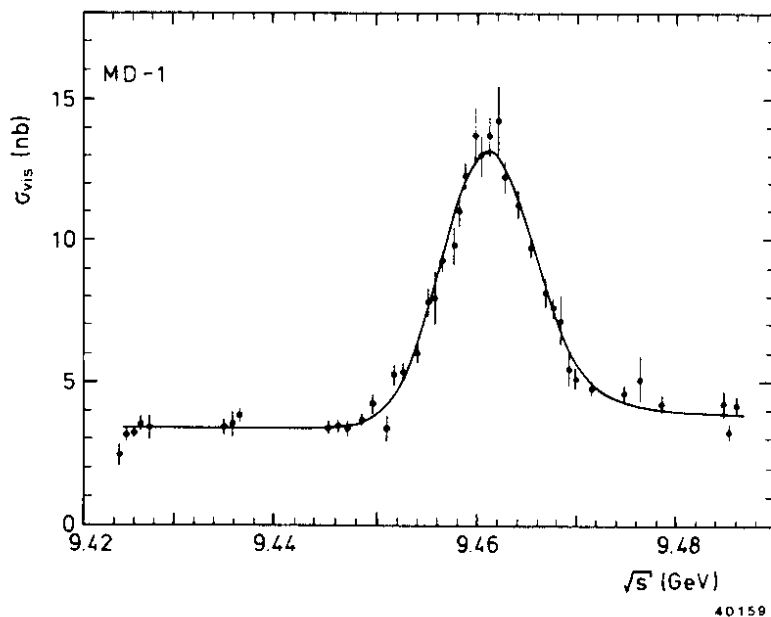


Figure 1: Observed hadronic cross section at the  $\Upsilon(1S)$  from Baru *et al.* [7], obtained with the MD-1 detector at Novosibirsk.

Figure 1 shows the visible hadronic cross section measured by Baru *et al.* [7] in the region of the  $\Upsilon(1S)$  resonance. The solid line in Fig. 1 shows the result of a fit with the

second order bremsstrahl spectrum calculated by Kuraev and Fadin. Together with the method of resonance depolarization of the transversely polarized beams they obtain [7] a very accurate mass of  $M(\Upsilon(1S)) = 9460.6 \pm 0.1$  MeV. Using the Jackson and Scharre formula leads to an increase in mass by 0.1 MeV. Thus an uncertainty of 0.1 MeV needs to be added to the experimental error for all previous mass determinations employing the Jackson and Scharre formalism. Table 1 shows the averaged masses [8] for the

Table 1: Experimental masses of heavy quarkonium  $c\bar{c}$  and  $b\bar{b}$  states below open flavor threshold. The preliminary  $\chi'_b$  masses are from CUSB [11,15]. Possible evidence for the  $h_c$  is from R704 [13] and for the  $h_b$  from CLEO [14]. All other masses are from the Review of Particle Properties [8].

Name	$Q\bar{Q}$ States		$M_{c\bar{c}}$ [MeV]	$M_{b\bar{b}}$ [MeV]
	$n^{2S+1}L_J$	$J^{PC}$		
$\eta_Q$	$1^1S_0$	$0^{-+}$	$2980.6 \pm 1.5$	
$J/\psi, \Upsilon$	$1^3S_1$	$1^{--}$	$3096.9 \pm 0.2$	$9460.0 \pm 0.2$
$\chi_0$	$1^3P_0$	$0^{++}$	$3414.9 \pm 1.1$	$9859.8 \pm 1.3$
$\chi_1$	$1^3P_1$	$1^{++}$	$3510.7 \pm 0.5$	$9891.9 \pm 0.7$
$\chi_2$	$1^3P_2$	$2^{++}$	$3556.3 \pm 0.4$	$9913.3 \pm 0.6$
$h_Q$	$1^1P_1$	$1^{+-}$	$3525.4 \pm 0.9$	$9894.8 \pm 1.5$
$\eta'_Q$	$2^1S_0$	$0^{-+}$	$3594.0 \pm 5.0$	
$\psi, \Upsilon$	$2^3S_1$	$1^{--}$	$3686.0 \pm 0.2$	$10023.4 \pm 0.3$
$\chi'_0$	$2^3P_0$	$0^{++}$		$10236.5 \pm 1.2$
$\chi'_1$	$2^3P_1$	$1^{++}$		$10255.3 \pm 0.6$
$\chi'_2$	$2^3P_2$	$2^{++}$		$10269.9 \pm 0.6$
$h'_Q$	$2^1P_1$	$1^{+-}$		
$\eta''_Q$	$3^1S_0$	$0^{-+}$		
$\psi, \Upsilon$	$3^3S_1$	$1^{--}$		$10355.5 \pm 0.5$

narrow vector states  $J/\psi$ ,  $\psi'$ , and  $\Upsilon(1S)$ ,  $\Upsilon(2S)$  and  $\Upsilon(3S)$ . All listed states are below the threshold for decay into mesons with open  $c$  or  $b$  flavor, respectively. Also shown in this table are the spectroscopic notations  $n^{2S+1}L_J$  and the resulting spin-parity-charge conjugation  $J^{PC}$ .

In addition we show in table 1 the masses of the  $\eta$  and  $\chi$  states. Most of this information stems from  $e^+e^-$  experiments. As these states are reached via photon transitions from the vector mesons, it is the detectors utilizing calorimeters with good photon energy resolution, which have added much knowledge on these transitions. As examples, we show in Fig. 2 the inclusive photon spectra obtained on the  $\psi'$ , the  $\Upsilon(2S)$  and the  $\Upsilon(3S)$  resonances. The first two spectra from the Crystal Ball collaboration [9,



10] are based on 1.8 million  $\psi'$  and 0.19 million  $\Upsilon(2S)$  events. With the newly installed BGO detector the CUSB group [11] has analyzed 0.47 million  $\Upsilon(3S)$  resonance decays. Additional information on the  $\eta_c$ , the  $\chi_1^c$  and  $\chi_2^c$  states was obtained by the R704 experiment [12] in  $p\bar{p}$  annihilations. Possible evidence for the last missing state below the  $\psi'$ , the spin-singlet state  $h_c$ , has been reported [13] by R704. The corresponding state in bottomonium, the  $h_b$ , may have been observed [14] by CLEO.

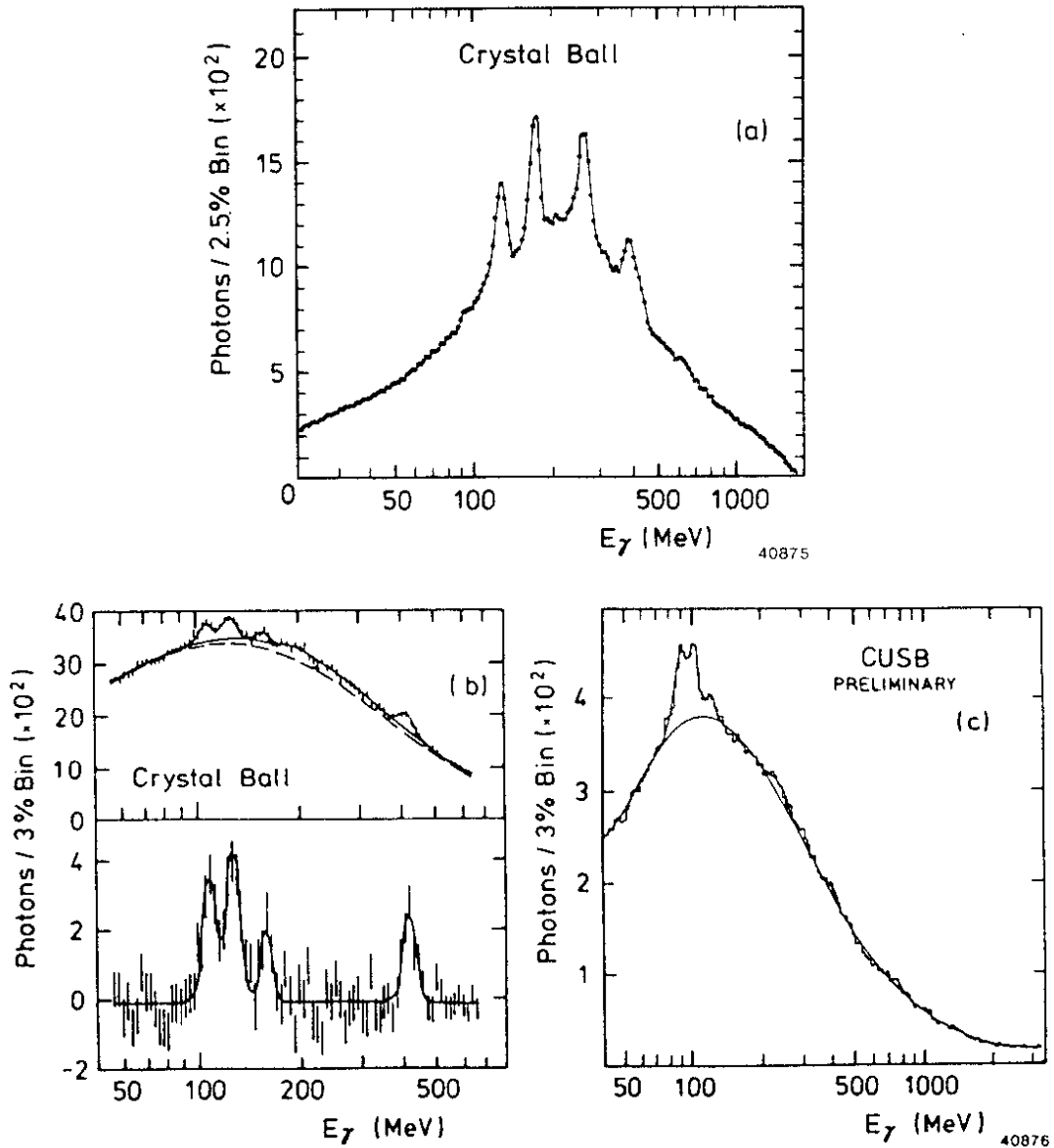


Figure 2: Inclusive photon spectrum obtained on a) the  $\psi'$ , b) the  $\Upsilon(2S)$  and c) the  $\Upsilon(3S)$  resonances. Figs. 2a and b) are from the Crystal Ball experiment [9,10]; Fig. 2c is preliminary data from the CUSB - BGO experiment [11].

## The Quark-Antiquark Potential

With Quantum Chromodynamics, the theory of the strong interactions, it should be possible to describe the interactions of quarks and gluons. As QCD perturbation theory is an inadequate tool to calculate the static properties and decay rates of mesons, models have to be used. At the present time, potential models seem to be the most useful method. However, evaluations of QCD on the lattice have added important information regarding the structure of the quark-antiquark potential. Many reviews have recently discussed the subject of potential models. A subjective selection of reviews dealing with heavy quark systems is given in Ref. [16].

As the strongly interacting constituents are heavy, relativistic effects are expected to be small and a sufficiently accurate approximation can be obtained by a non-relativistic treatment based on the Schrödinger equation  $[2m + p^2/m + V(r)]\psi(r) = E\psi(r)$ . The potential  $V(r)$  is the usual Coulombic plus linear confinement potential with modifications reflecting various relativistic effects [17,18,19,20]:

$$\begin{aligned}
 V(r) = & -\frac{4}{3} \frac{\alpha_s(r)}{r} + br + c \\
 & + \frac{1}{2m_Q^2} \left( \frac{4\alpha_s(r)}{r^3} - \frac{b}{r} \right) \vec{L} \cdot \vec{S} \\
 & + \frac{4}{3} \frac{\alpha_s(r)}{m_Q^2} \frac{1}{r^3} (3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2) \\
 & + \frac{4}{3} \frac{\alpha_s(r)}{m_Q^2} \frac{8\pi}{3} \vec{S}_1 \cdot \vec{S}_2 \delta(r)
 \end{aligned} \tag{5}$$

The Coulomb-like term arises from the one-gluon exchange between the quarks and dominates at short distances. The linear part is motivated by the string- (or chromo-electric tube-) picture of quark confinement and dominates at large distances. The constant term absorbs different choices in the quark masses. Such a Coulomb plus linear (plus constant) potential was first introduced by the Cornell group [21] and was found to very successfully fit the heavy  $\psi$  mass spectrum. Many other potential forms have been used. For example, Richardson [22] established the potential in momentum space with a single scale parameter  $\Lambda$ . All approaches lead to very similar potentials in the region of distances from about 0.1 to 1.0 fm, see Fig. 3a. However, the models differ substantially for inter-quark separations less than 0.1 fm.

Numerical studies of the interquark potential have been started using the lattice gauge theories of QCD. Calculations have been performed for the color group SU(3)

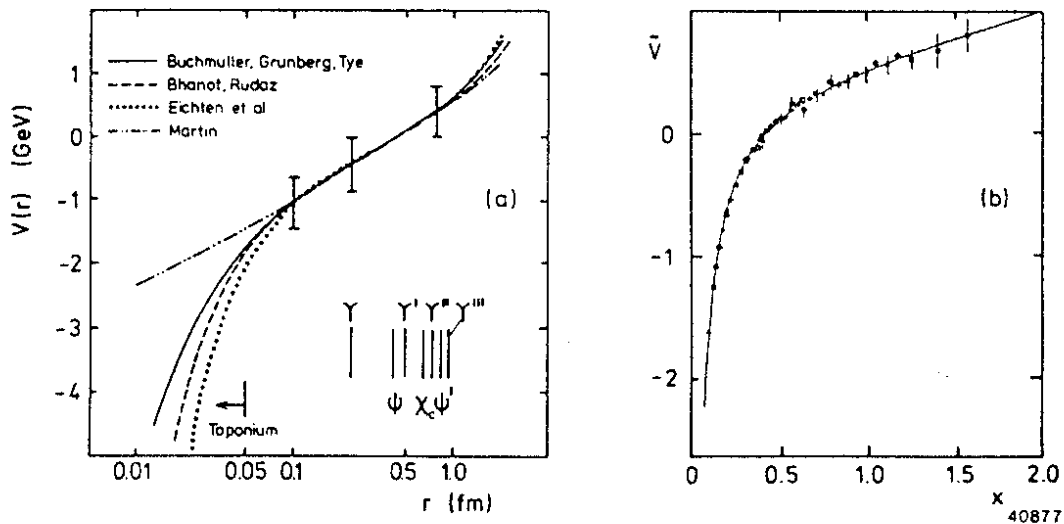


Figure 3: a) The radial dependence of some typical  $Q\bar{Q}$  potentials for heavy quarkonia (from Ref. [23]). The average radii of the observed  $c\bar{c}$  and  $b\bar{b}$  states are indicated. The potential models used are by Bhanot and Rudaz [24], Buchmüller *et al.* [23], Eichten *et al.* [21] and Martin [25]. b) Lattice calculation [26] of the  $Q\bar{Q}$  potential in quenched SU(3) (data points) with a fit to a Coulomb plus linear form.

in the quenched approximation, in which the production of quark-antiquark pairs is neglected. The results by Otto and Stack [26], shown in Fig. 3b, are in good agreement with the potential models discussed above. Laermann *et al.* [27] have recently included light quarks in a color SU(2) calculation. They find a softening of the potential at large distances, as expected from the break-up of the flux tube due to quark-pair creation.

The non-relativistic reduction of the Bethe-Salpeter equation with the one-gluon vector exchange interaction gives rise to the well known Breit-Fermi Hamiltonian, in direct analogy to the QED analysis of positronium. Thus the spin dependent terms, which are proportional to  $\alpha_s$ , are the same as in QED. However, spin-dependent terms arising from the confining potential depend on the choice of the transformation properties of this potential. The best agreement with data is obtained for a confining potential transforming like a scalar under Lorentz transformations. Confirming evidence of scalar confinement also comes from lattice gauge calculations [28]. The total spin-dependent potential, stated in eq. 5, consists of three terms inversely proportional to the square of the quark mass: they are the spin-orbit, the tensor and the spin-spin interaction potentials, respectively [17,18,19,20]. The scalar confinement potential contributes only to the spin-orbit term with a negative sign. Note that this sign would be reversed for a

vector confining potential.

For comparison with experimental mass splittings we take expectation values of the spin-dependent potentials. Introducing obvious abbreviations  $a, b, c$  we write the general spin-dependent energies

$$\langle V_{spin}(r) \rangle = a \langle \vec{L} \cdot \vec{S} \rangle + b \langle 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r}) - \vec{S}_1 \cdot \vec{S}_2 \rangle + c \langle \vec{S}_1 \cdot \vec{S}_2 \rangle. \quad (6)$$

The  $\chi$  states, which have orbital angular momentum  $L = 1$  and total spin  $S = 1$ , are split by the spin-orbit and tensor forces only. It is useful to define the ratio  $r = (M_2 - M_1)/(M_1 - M_0)$ , where  $M_J$  are the masses for the  $\chi$ -states with total spin  $J$ . In terms of  $a$  and  $b$  this ratio is given by  $r = (2a - 0.6b)/(a + 1.5b)$ . With the world-average  $\chi$  masses from table 1 we obtain  $a, b$  and  $r$  as stated in table 2. A pure Coulomb

Table 2: Expectation values of the spin-orbit ( $a$ ) and tensor ( $b$ ) potentials determined from the experimental  $\chi_c, \chi_b$  and  $\chi'_b$  masses. The ratio  $r$  is defined in the text.

$\chi$ -States	$a$ (MeV)	$b$ (MeV)	$r$
$\chi_c (c\bar{c})$	$34.9 \pm 0.3$	$40.4 \pm 0.8$	$0.48 \pm 0.01$
$\chi_b (b\bar{b})$	$14.1 \pm 0.4$	$12.0 \pm 0.9$	$0.67 \pm 0.05$
$\chi'_b (b\bar{b})$	$9.2 \pm 0.4$	$6.4 \pm 0.9$	$0.78 \pm 0.08$

potential like in QED yields the relation  $a = 1.5b$  and  $r = 0.8$ . The experimental values in table 2 indicate that both heavy  $Q\bar{Q}$  systems are close to this value. For the  $c\bar{c}$  system the relation  $a < b$  reveals the importance of the long-range component of the force. In particular, it is important to note that  $r < 0.8$  for all three  $\chi$ -states. A pure vector confining term would yield [17]  $r > 0.8$  whereas a scalar confining term yields  $r < 0.8$ . This is the experimental proof that the long-range confining potential transforms as a Lorentz scalar.

With the expectation values of the spin-dependent potentials in table 2 we can calculate how spin-orbit and tensor forces affect the unperturbed center-of-gravity of the  $\chi$ -states. The mass shifts due to these forces is sketched in figure 4 for the  $\chi_b$  and  $\chi'_b$  states. It is obvious that the mass splittings in both systems are of the same magnitude. For comparison, the potential model calculation by Gupta *et al.* [29] is included. The agreement with the experimentally determined masses is surprisingly good.

The spin-spin force is responsible for the mass splitting between vector and pseudoscalar states (*e.g.*  $J/\psi$ - $\eta_c$ ) and between the center-of-gravity of the  $\chi$ -states and the

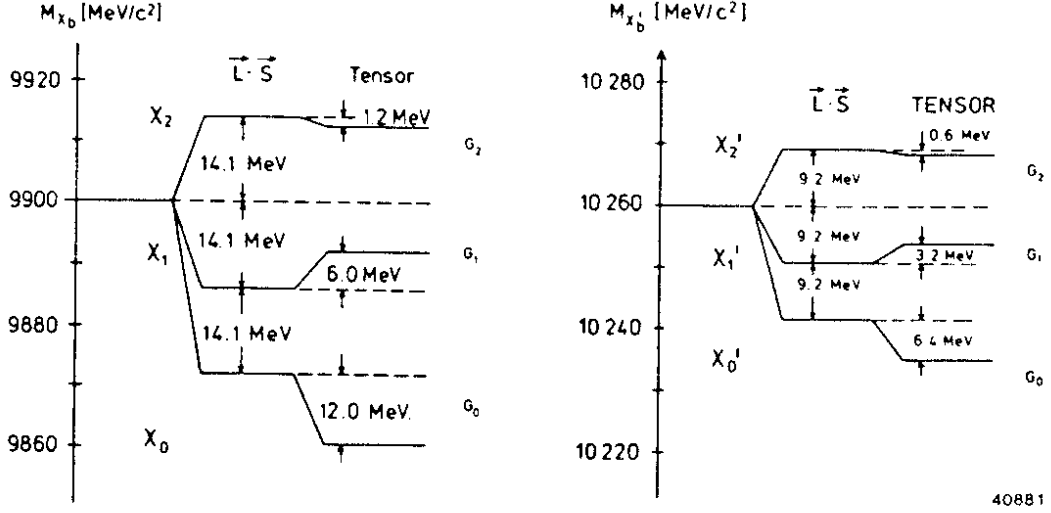


Figure 4: Mass splittings of the  $\chi_b$  and  $\chi'_b$  states due to spin-orbit and tensor forces.  $G_J$  denote the masses with spin  $J$  as predicted by Gupta *et al.* [29].

spin-singlet state  $h_Q$ . The spin-spin force has a contact  $\delta$ -function which does not contribute to  $P$ -wave states and higher angular momenta. Thus the center-of-gravity of the  $\chi$ -states and the spin-singlet state  $h_Q$  should be degenerate in the absence of a long range component. The R704 experiment has presented [13] possible evidence for the  $h_c$  with a mass of  $3525.4 \pm 0.9 \text{ MeV}$ , in perfect agreement with the  $\chi_c$  center-of-gravity. CLEO [14] has observed a weak  $2.5\sigma$  signal in the inclusive  $\pi^+\pi^-$  transition from the  $\Upsilon(3S)$  at a mass of  $9894.8 \pm 1.5 \text{ MeV}$ , which is  $5.4 \pm 1.6 \text{ MeV}$  below the center-of-gravity of the  $\chi_b$  states. The rough agreement of the theoretically predicted degeneracy confirms our notion that the spin-spin force is due to a Coulomb-like interaction. Indeed, it has been argued by Olsson and Suchyta [30], that the hyperfine splitting in the  $b\bar{b}$  system should be larger than in the  $c\bar{c}$  system. However, Igi and Ono [31] cannot accommodate the observed splittings in their potential model. It seems mandatory to get more information on the two spin-singlet states.

### Total Widths of Heavy Quarkonia

Information on the  $\chi$ -states has been obtained mostly from analyses of radiative transitions from radially excited vector states  $V'$ :  $V' \rightarrow \gamma\chi$ , see Fig. 2. Branching ratios for this process have been obtained for the  $\chi_c$ -states [8] by Mark I, SP27 and Crystal Ball and for the  $\chi_b$ -states [8] by ARGUS, CLEO, Crystal Ball and CUSB. Preliminary data

on the  $\chi'_b$ -states are available from CUSB [11]. Average values of the branching ratios  $B(V' \rightarrow \gamma\chi)$  are given in table 3.

Table 3: Experimental branching ratios for radiative transitions involving  $\chi$  states. Values involving the  $\chi_c$  and  $\chi_b$  are from [8], those for the  $\chi'_b$  are from CUSB [11,15] (prelim.). All branching ratios are in %.

Decay to/from	$\chi_0$	$\chi_1$	$\chi_2$
$B(\psi' \rightarrow \gamma\chi_c)$	$9.4 \pm 0.8$	$8.7 \pm 0.8$	$7.8 \pm 0.8$
$B(\chi_c \rightarrow \gamma J/\psi)$	$0.7 \pm 0.2$	$25.8 \pm 2.5$	$14.8 \pm 1.7$
$B(\Upsilon(2S) \rightarrow \gamma\chi_b)$	$4.3 \pm 1.0$	$6.7 \pm 0.9$	$6.6 \pm 0.9$
$B(\chi_b \rightarrow \gamma\Upsilon(1S))$	$< 6$	$35 \pm 8$	$22 \pm 4$
$B(\Upsilon(3S) \rightarrow \gamma\chi'_b)$	$4.8 \pm 1.4$	$12.0 \pm 2.6$	$12.8 \pm 2.9$
$B(\chi'_b \rightarrow \gamma\Upsilon(1S))$	$1.4 \pm 1.0$	$6.1 \pm 1.3$	$6.3 \pm 1.3$
$B(\chi'_b \rightarrow \gamma\Upsilon(2S))$	$6.9 \pm 3.8$	$24.7 \pm 6.9$	$18.9 \pm 5.3$

These branching ratios can be converted into partial widths with the total widths of the vector resonances from table 5. The resulting partial widths are listed in table 4 and compared with predictions from potential models. In order to facilitate this comparison, an average over several theoretical predictions [32] has been performed. The agreement with data is excellent. An early discrepancy between theory and experiment for the width  $\Gamma(\psi' \rightarrow \gamma\chi)$  was resolved after inclusion of resonance and continuum mixing [37] and of relativistic corrections [38].

Table 4: Comparison of experimental (Exp.) radiative widths with averaged theoretical (Th.) predictions [32]. All widths are in units of keV. A common systematic error from the uncertainty in the vector meson total widths (of about 20%) are not included in the errors. The errors on the theoretical prediction reflects their Gaussian spread.

Origin	Decay to/from	$\chi_0$	$\chi_1$	$\chi_2$
Exp.	$\Gamma(\psi' \rightarrow \gamma\chi_c)$	$24.1 \pm 2.0$	$22.3 \pm 2.0$	$20.0 \pm 2.0$
Th.	$\Gamma(\psi' \rightarrow \gamma\chi_c)$	$17 \pm 4$	$24 \pm 3$	$22 \pm 5$
Th.	$\Gamma(\chi_c \rightarrow \gamma J/\psi)$	$130 \pm 20$	$280 \pm 40$	$370 \pm 50$
Exp.	$\Gamma(\Upsilon(2S) \rightarrow \gamma\chi_b)$	$1.8 \pm 0.4$	$2.8 \pm 0.4$	$2.8 \pm 0.4$
Th.	$\Gamma(\Upsilon(2S) \rightarrow \gamma\chi_b)$	$1.2 \pm 0.2$	$2.1 \pm 0.3$	$2.2 \pm 0.3$
Th.	$\Gamma(\chi_b \rightarrow \gamma\Upsilon(1S))$	$27 \pm 3$	$33 \pm 3$	$39 \pm 4$
Exp.	$\Gamma(\Upsilon(3S) \rightarrow \gamma\chi'_b)$	$1.2 \pm 0.4$	$3.1 \pm 0.7$	$3.3 \pm 0.7$
Th.	$\Gamma(\Upsilon(3S) \rightarrow \gamma\chi'_b)$	$1.1 \pm 0.2$	$2.3 \pm 0.2$	$2.5 \pm 0.2$
Th.	$\Gamma(\chi'_b \rightarrow \gamma\Upsilon(2S))$	$12 \pm 2$	$14 \pm 2$	$16 \pm 2$

In addition, measurements of the branching ratio for the decay chains  $V' \rightarrow \gamma\chi$ ,  $\chi \rightarrow$

$\gamma V$ ,  $V \rightarrow \ell^+ \ell^-$  yield, after division by the leptonic branching ratio, the product branching ratio  $B(V' \rightarrow \gamma \chi, \chi \rightarrow \gamma V)$ . They were measured for the  $\chi_c$  [8] by Mark I, Mark II, DESY-Heidelberg, DASP and Crystal Ball, for the  $\chi_b$  [8] by the Crystal Ball and CUSB and for the  $\chi'_b$  preliminary data was presented by CUSB [15] at the Lepton and Photon Conference at Hamburg. Division of the product branching ratio by the primary transition strength  $B(V' \rightarrow \gamma \chi)$  yields the branching ratio  $B(\chi \rightarrow \gamma V)$ . Average values for these branching ratios are also stated in table 3.

The radiative branching ratios  $B_{E1} \equiv B(\chi \rightarrow \gamma V)$  can be converted into total widths of the  $\chi$ -states with  $\Gamma_{tot}(\chi) = \Gamma_{E1}/B_{E1}$  if we use some estimate for the radiative E1-width. Potential model predictions for these E1-widths are rather stable, especially since the transition dipole matrix elements involves wave functions with the same number of radial nodes. Again we average theoretical predictions [32] for the E1 transitions. These theoretical widths are also listed in table 4, with errors indicating the spread between different predictions. Combining them with the experimental branching ratios yield the  $\chi$ -widths listed in table 5.

Table 5: Total widths of heavy quarkonia states  $c\bar{c}$  and  $b\bar{b}$ . Widths of the  $\eta_c$  and the  $\psi, \Upsilon$  states have been measured directly, whereas those of the  $\chi$  states are inferred from measured radiative branching ratios. For the  $\chi_c$  states direct measurements also exist, with  $\Gamma(\chi_0^c) = 13 \pm 5$  MeV (from [39]) and  $\Gamma(\chi_2^c) = 2.6^{+1.4}_{-1.0}$  MeV (from [12]).

$Q\bar{Q}$ States		$\Gamma_{tot}(c\bar{c})$	$\Gamma_{tot}(b\bar{b})$
Name	$J^{PC}$	[keV]	[keV]
$\eta_Q$	$0^{-+}$	$(11.5 \pm 4.3) \cdot 10^3$	
$J/\psi, \Upsilon$	$1^{--}$	$74 \pm 8$	$51 \pm 3$
$\chi_0$	$0^{++}$	$(19 \pm 6) \cdot 10^3$	$> 510$
$\chi_1$	$1^{++}$	$(1.1 \pm 0.2) \cdot 10^3$	$92 \pm 23$
$\chi_2$	$2^{++}$	$(2.5 \pm 0.4) \cdot 10^3$	$177 \pm 37$
$\eta'_Q$	$0^{-+}$	$< 8 \cdot 10^3$	
$\psi, \Upsilon$	$1^{--}$	$256 \pm 48$	$42 \pm 9$
$\chi'_0$	$0^{++}$		$174 \pm 100$
$\chi'_1$	$1^{++}$		$57 \pm 18$
$\chi'_2$	$2^{++}$		$85 \pm 26$
$\eta''_Q$	$0^{-+}$		
$\psi, \Upsilon$	$1^{--}$		$26 \pm 6$

The total  $\chi$ -widths in the  $b\bar{b}$  system are about a factor of 10 smaller than those in charmonium. Theory predicts [40] the *hadronic* widths to be proportional to the

derivative of the wave function at the origin and to the mass:  $\Gamma(\chi \rightarrow \text{hadrons}) \propto \alpha_s^2 R'(0)^2/M^4$ . Two powers of mass yield a reduction in width by a factor of 10. This implies that  $R'(0)^2/M^2$  is about constant, in analogy to  $R(0)^2/M^2$  being about constant for vector state wave functions. An analysis of the leptonic vector meson widths has shown the latter constancy to hold to a good approximation. Within the  $b\bar{b}$  system the total  $\chi$ -widths decrease only by about a factor of two for the radial excitation. This decrease is again of the same size as the observed decrease in the leptonic widths for radial excitations of vector states. Both observations of decreasing widths for radial excitations are consistent with the potential being convex near the origin ( $d^2V/dr^2 < 0$ ).

Second order QCD calculations predict [41] for the ratio

$$\frac{\Gamma(\chi_0 \rightarrow \text{hadrons})}{\Gamma(\chi_2 \rightarrow \text{hadrons})} = \frac{15}{4} \left( 1 + \left\{ \begin{array}{c} 3.8 \\ 3.0 \end{array} \right\} \alpha_s \right) \approx \left( \begin{array}{c} 6.6 \\ 6.0 \end{array} \right) \text{ for } \left\{ \begin{array}{c} c\bar{c} \\ b\bar{b} \end{array} \right. \quad (7)$$

which is independent of the scale of the running coupling constant, but depends slightly ( $\mathcal{O}(10\%)$ ) on an estimate of the binding energy. Subtracting the theoretical E1-widths from the total widths stated in table 5 and forming ratios yields the following experimental numbers

$$\Gamma(\chi_0 \rightarrow \text{hadrons})/\Gamma(\chi_2 \rightarrow \text{hadrons}) = 8.9 \pm 3.2, > 2.6, 2.3_{-1.6}^{+4.1} \quad (8)$$

for  $(\chi_c, \chi_b, \chi_b')$ , respectively, in good agreement with the theoretically expected values.

In the section on *Masses of Heavy Quarkonia* it was explained that the integral over the resonance cross section yields an area of  $(6\pi^2/M^2) \times \Gamma_{ee} B_{had}$ . Depending on the theoretical bremsstrahl spectrum used in the fit, the area will differ. Consequently the meaning of  $\Gamma_{ee}$  will be different. Application of the prescription by Kuraev and Fadin [5] yields  $\Gamma_{ee}$  containing the vacuum polarization term. However, most previous measurements have used the formalism of Jackson and Scharre [3], which includes the *electronic* vacuum polarization loop. Application of this formalism results in something which is neither  $\Gamma_{ee}$  nor  $\Gamma_{ee}^{(0)}$ , the leptonic width in lowest order in QED. For a detailed discussion of this topic see ref. [42,43]. Both references also gives a re-evaluation of all previous  $\Gamma_{ee} B_{had}$  determinations. An increase between 5% and 14% results for the leptonic widths.

The Crystal Ball collaboration has recently analyzed [44] the resonance scan shown in Fig. 5. The visible hadronic cross section  $\sigma^{vis}$  is fit to the shape given in eq. 4 with the bremsstrahl spectrum calculated by Kuraev and Fadin. The solid line in Fig. 5 shows the



fit resulting in a preliminary leptonic width of  $\Gamma_{ee}(\Upsilon(1S)) = (1.33 \pm 0.03 \pm 0.06) \text{ keV}$ , where the first error is statistical, the second systematic. Using the Jackson-Scharre spectrum gives a 10% lower value.

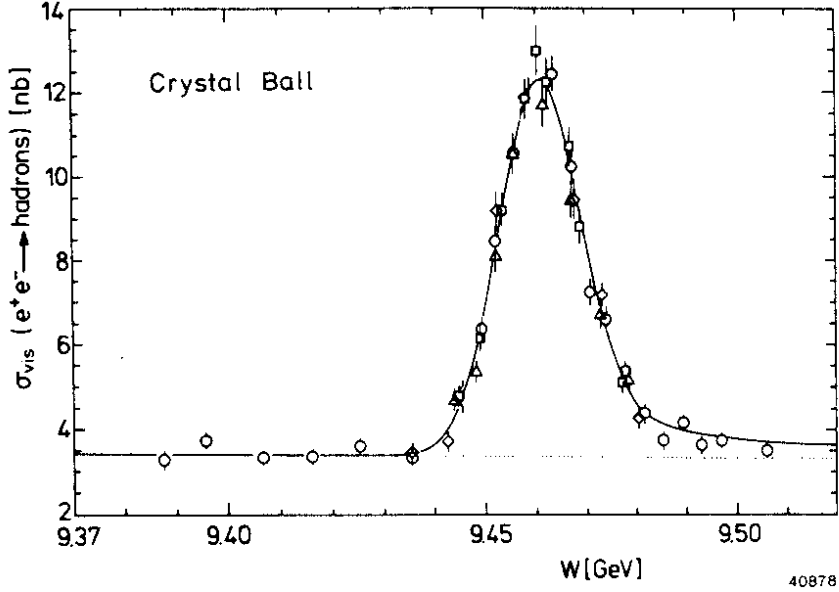


Figure 5: Visible hadronic cross section at the  $\Upsilon(1S)$  resonance, obtained with the Crystal Ball detector at DORIS II (preliminary result). The solid line is a fit to the data.

The average [42] of the Crystal Ball datum and the re-evaluated leptonic widths from other experiments yields the values in table 6. Also listed are the averages of the leptonic branching fraction,  $B_{\mu\mu}$ . It is important to note that both quantities contain the vacuum polarization term:  $\Gamma_{ee}$  contains it because we used the formalism of Kuraev and Fadin who do not include this term in their bremsstrahl spectrum.  $B_{\mu\mu}$  also contains this term as it is measured by including all  $\mu^+\mu^-$  decays with extra photons. By the Kinoshita-Lee-Nauenberg theorem [46], mass singularities cancel leaving as the only contribution the vacuum polarization term. As both,  $\Gamma_{ee}$  and  $B_{\mu\mu}$ , contain the vacuum polarization term, we may use the relation  $\Gamma_{tot} = \Gamma_{ee}/B_{\mu\mu}$  and obtain the total widths of the vector mesons stated in table 5. The proper treatment of the vacuum polarization term yields total widths that are on the average 20% larger than those stated in the 1986 Review of Particle Properties [8].

All vector meson total widths, with the exception of the  $\psi'$ , are on the order of 50 keV. The  $\psi'$  width of 256 keV is significantly larger than the  $\Upsilon(2S)$  width due to a large decay width for  $\psi' \rightarrow \pi\pi J/\psi$ . This transition is proportional to the fourth power

Table 6: Average leptonic widths and  $B_{\mu\mu}$  of heavy vector mesons (from ref. [42] with the inclusion of two new  $B_{\mu\mu}$  values from CUSB [45]). Note that  $\Gamma_{ee}$  and  $B_{\mu\mu}$  include the vacuum polarization term.

Vector Meson	$B_{\mu\mu}$ [%]	$\Gamma_{ee}$ [keV]
$J/\psi$	$6.9 \pm 0.6$	$5.1 \pm 0.3$
$\psi'$	$0.9 \pm 0.15$	$2.3 \pm 0.2$
$\Upsilon(1S)$	$2.59 \pm 0.13$	$1.33 \pm 0.04$
$\Upsilon(2S)$	$1.37 \pm 0.28$	$0.58 \pm 0.03$
$\Upsilon(3S)$	$1.64 \pm 0.38$	$0.43 \pm 0.03$

of the mean radius of the heavy quark system, thus it is about a factor 10 smaller in the bottomonium system. The theoretical prediction [40] for the *hadronic* vector meson width is proportional to the wave function at the origin and to the mass:  $\Gamma(V \rightarrow \text{hadrons}) \propto \alpha_s^3 R(0)^2/M^2$ . The decrease in width from charmonium to bottomonium is thus due to the running coupling constant  $\alpha_s$ , which gets smaller with larger constituent mass. Here again we have used the fact that the term  $R(0)^2/M^2$  is approximately constant.

### $\alpha_s$ Determination

The hadronic width of a vector meson is described in lowest order in QCD as the decay into three gluons [40]. Therefore the width depends on the strong coupling constant to the third power and should thus allow a very accurate determination  $\alpha_s$ . To eliminate the dependence on the *a priori* unknown wave function at the origin it is best to form ratios of widths. The following ratios are suitable from the point of view of experimental accessibility:  $\Gamma(V \rightarrow 3g)/\Gamma(V \rightarrow \mu^+\mu^-)$  and  $\Gamma(V \rightarrow \gamma gg)/\Gamma(V \rightarrow 3g)$ . As  $\alpha_s \sim 0.2$  is not a small quantity, it is important to know contributions from higher orders in QCD. Next-to-leading order calculations have been performed in the  $\overline{MS}$  scheme by Mackenzie and Lepage [47] for the decays to hadrons and by Barbieri *et al.* [41] for the leptonic decay. A summary of all the relevant formulæ, which we will not repeat here in detail, are collected *e.g.* by Buchmüller and Cooper [43].

The resulting ratios of widths are given by

$$\frac{\Gamma(V \rightarrow 3g)}{\Gamma(V \rightarrow \mu^+\mu^-)} = \frac{10(\pi^2 - 9)}{81\pi\alpha^2 e_Q^2} \alpha_s^3 \left[ 1 + \frac{\alpha_s}{\pi} \left\{ \frac{3\beta_0}{2} \ln \frac{\mu}{2m_Q} + 2.77\beta_0 - 14.0 \right\} \right] \quad (9)$$

$$\frac{\Gamma(V \rightarrow \gamma gg)}{\Gamma(V \rightarrow 3g)} = \frac{36\alpha e_Q^2}{5\alpha_s} \left[ 1 + \frac{\alpha_s}{\pi} \left\{ \beta_0 \ln \frac{\mu}{2m_Q} + 1.85\beta_0 - 11.8 \right\} \right],$$

where  $\beta_0 = 11 - 2n_f/3$  and  $n_f$  is the number of light flavors. The corrections to the hadronic  $3g$  and  $\gamma gg$  widths depend explicitly on the scale  $\mu$  where  $\alpha_s$  is to be evaluated. Different opinions exist on how to choose the scale. They amount to choosing a ‘natural scale’ like the vector meson mass  $M_V$ , or choosing the scale such that the next-to-leading order vanishes (Grunberg [48]). Note that the last scheme requires different scales for the two ratios of widths given above. A third alternative consists in choosing the scale such that there is no explicit dependence on the number of light flavors (Brodsky, Lepage, Mackenzie (BLM) [47]). This fixes the scale to  $\mu = 0.157M_V$  for both decay ratios. For bottomonium, the correction factors for the three scales are  $\Gamma(3g)/\Gamma(\mu\mu) \sim [1 + \alpha_s \{+2.9, 0, -4.5\}]$  and  $\Gamma(\gamma gg)/\Gamma(3g) \sim [1 + \alpha_s \{-1.8, 0, +0.7\}]$  for the two ratios of widths, respectively. The sequence of the corrections is {natural, Grunberg, BLM}. The Grunberg and BLM schemes yield the same correction factors for the  $c\bar{c}$  and  $b\bar{b}$  systems, whereas the factors for the natural scheme have been calculated for bottomonium. For charmonium, the corresponding values are approximately 20% larger. The correction is particularly large for  $B_{\mu\mu}$  in the BLM scheme. Most determinations of  $\alpha_s$  from  $B_{\mu\mu}$  have therefore used the Grunberg scheme.

Instead of choosing a particular scheme to evaluate  $\alpha_s$ , fig. 6a shows  $\alpha_s$  obtained from<sup>1</sup>  $B_{\mu\mu}$  as a function of the scale  $\mu$  (solid curve). The strong rise towards small  $\mu$  stems from the higher order corrections becoming large and negative. Note that the BLM scheme with a scale of  $\mu = 1.5 \text{ GeV}$  does not yield a sensible value for  $\alpha_s$ . The error on  $\alpha_s$  is typically 0.004 at  $\mu = 4 \text{ GeV}$  and 0.003 at  $\mu = 8 \text{ GeV}$ . With  $\alpha_s$  in second order,

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda_{\overline{MS}}^2}} \left( 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln \ln \frac{\mu^2}{\Lambda_{\overline{MS}}^2}}{\ln \frac{\mu^2}{\Lambda_{\overline{MS}}^2}} \right) \quad (10)$$

where  $\beta_1 = 102 - 38n_f/3$ , we evaluate the scale parameter of QCD for four light flavors,  $\Lambda_{\overline{MS}}^{(4)}$ , and obtain the solid curve shown in fig. 6b. Scales above  $\mu = 3 \text{ GeV}$  yield  $150 < \Lambda_{\overline{MS}}^{(4)} < 200 \text{ MeV}$  with a statistical error of at most 15 MeV. Given this extremely small error, the  $B_{\mu\mu}$  measurement would yield the most precise  $\Lambda_{\overline{MS}}^{(4)}$  determination if we knew more about the appropriate choice of scale.

The situation is better for an analysis of the ratio  $B_\gamma \equiv \Gamma(\Upsilon(1S) \rightarrow \gamma gg)/\Gamma(\Upsilon(1S) \rightarrow 3g)$ , where the different schemes yield much smaller corrections. These transitions have

<sup>1</sup>Note that  $B_{\mu\mu}$  should not contain the vacuum polarization term; it is therefore given by  $(1 - \Pi)^2 \times B_{\mu\mu}^{c2p}$ , where  $(1 - \Pi)^2 = 0.932$  at  $\Upsilon$  energies.

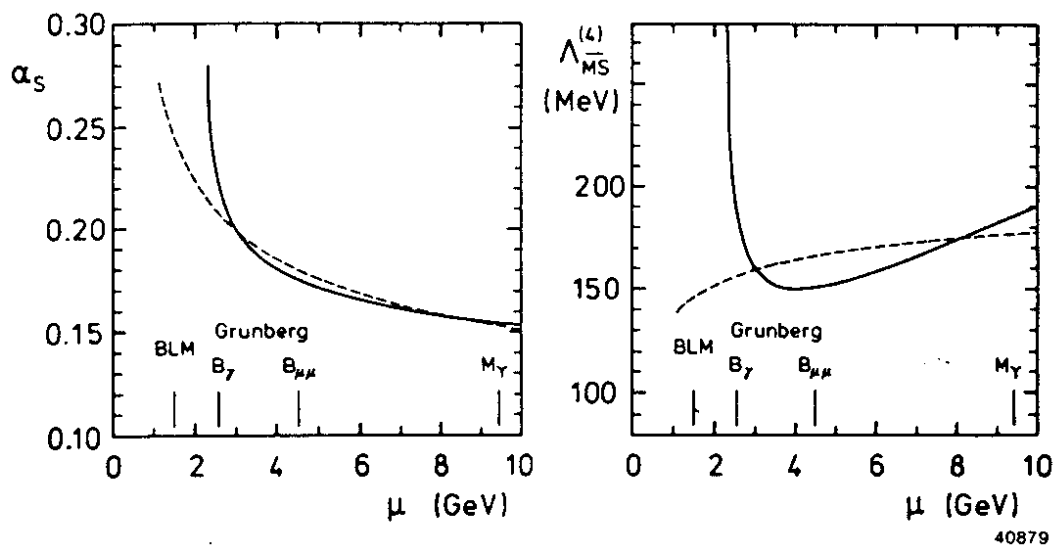


Figure 6: a)  $\alpha_s$  vs. the scale  $\mu$  at which it is evaluated. b) The same for  $\Lambda_{\overline{MS}}^{(4)}$ . The solid curves are from an evaluation of the  $\Upsilon(1S)$  leptonic branching ratio, the dashed curves are from  $B_\gamma$ , the decay of the  $\Upsilon(1S)$  into a photon plus hadrons. Also indicated are the specific scales discussed in the text.

been measured by CUSB [49] CLEO [50] and ARGUS [51]. They obtained branching ratios of  $(3.0 \pm 0.6)\%$ ,  $(2.5 \pm 0.2)\%$  and  $(3.0 \pm 0.2)\%$ , respectively. Unfortunately, the shape of the experimental spectra differ substantially, see fig. 7. CUSB obtained a spectrum consistent with the theoretical expectation from lowest order QCD [52], which is the same as that for ortho-positronium annihilation into three photons [53]. ARGUS' spectrum is in clear disagreement with the lowest order QCD spectrum. It agrees, however, with the much softer theoretical shape calculated by Field [54] with a parton shower Monte Carlo which includes the effect of the self-coupling of gluons. The spectrum from the CLEO collaboration, which is not shown here, favors the Field spectrum, but cannot rule out the harder QCD spectrum.

In conclusion, the experiments find rather different shapes of the photon spectrum. The branching ratios, however, agree rather favorably. It is thus of paramount importance to have another independent check on this spectrum. An indirect method of checking the spectrum consists in measuring exclusive radiative decays into light mesons. Assuming equal spectral shapes in decays of the  $J/\psi$  and the  $\Upsilon(1S)$  yields the prediction that branching ratios on the  $\Upsilon(1S)$  should be suppressed by a factor of 40 with respect to those on the  $J/\psi$  (for a discussion see ref. [55]).

Ignoring the problem of different spectral shapes we evaluate  $\alpha_s$  from the averaged

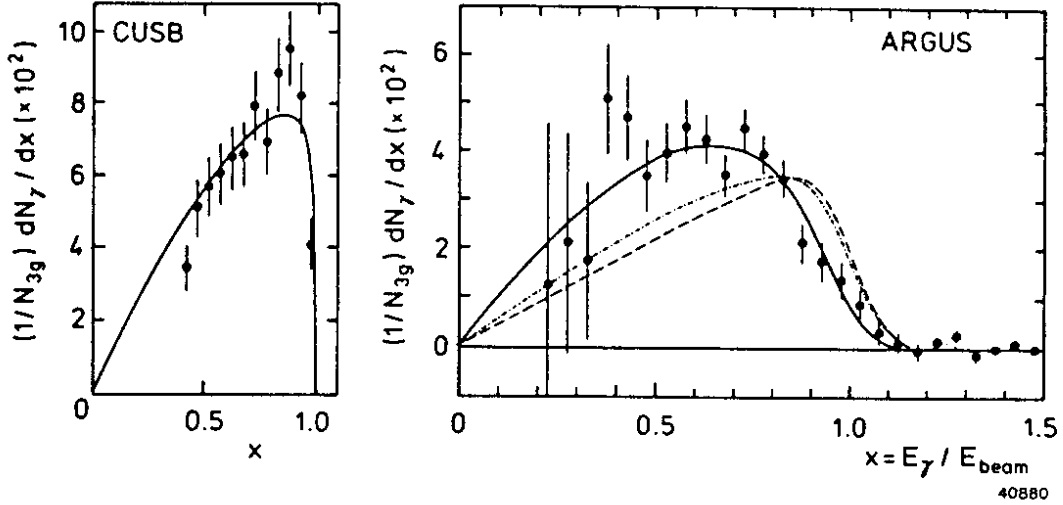


Figure 7: Direct photon spectrum from the  $\Upsilon(1S)$  resonance. a) from CUSB: the solid curve corresponds to the lowest order QCD prediction. b) from ARGUS: the dashed line is the lowest order QCD prediction, the solid line Field's prediction.

branching ratio  $B_\gamma = (2.8 \pm 0.2)\%$ . Fig. 6a shows the result as the dashed curve. As  $B_\gamma$  depends only on the first power of  $\alpha_s$ , the errors in  $\alpha_s$  are larger those obtained from  $B_{\mu\mu}$ . They are typically 0.01 over the full range of  $\mu$ . A calculation of  $\Lambda_{\overline{MS}}^{(4)}$  yields the dashed curve in fig. 6b. For  $\mu > 1$  GeV we obtain  $140 < \Lambda_{\overline{MS}}^{(4)} < 180$  MeV with an additional error of 40 MeV from the experimental uncertainty. This analysis shows that both,  $B_{\mu\mu}$  and  $B_\gamma$ , can in principle yield rather accurate  $\Lambda_{\overline{MS}}^{(4)}$  values. A determination with the former quantity is prone to theoretical uncertainties in the choice of the scale at which to evaluate the theoretical formula, whereas the latter quantity needs to be verified experimentally. In addition to these two experimental quantities, Kwong *et al.* [56] have used  $\eta_c$ ,  $J/\psi$  and  $\chi_2^c$  decays. Their analysis of  $\alpha_s$ , evaluated at the quark masses, yields values for  $\Lambda_{\overline{MS}}^{(4)}$  in the region 180 to 200 MeV, consistent with the above given determinations. For comparison, CELLO's fit [57] of all  $R$ -measurements from PETRA and PEP yields  $\alpha_s(34 \text{ GeV}) = 0.17 \pm 0.03$ .

As the heavy quarkonium potential itself depends on  $\alpha_s$ , it should be possible to obtain a value for  $\Lambda_{\overline{MS}}^{(4)}$  from an analysis of the inter-quark potential. Such analyses have been tried. There is a general consensus [20,58] that the QCD scale parameter must be larger than about 150 MeV. At present it seems difficult to obtain a firm upper bound on  $\Lambda_{\overline{MS}}^{(4)}$  from such analyses. Large values can be accommodated [59] by a corresponding change in the quark mass.

## Hadronic Transitions

Two-pion transitions are described in QCD as a two-step process. First the excited quarkonium state radiates (in lowest order) two gluons. Since the available energies are small and the relevant  $\alpha_s$  is large, perturbation methods are not applicable. However, Gottfried and Yan [60] have pointed out that a multipole expansion of the gluonic field converges rapidly since the dimensions of the radiating heavy quark system are small compared to the wavelength of the emitted gluons. In a second step the gluons fragment into light hadrons; here the properties of the  $\pi\pi$  system are determined by using partial conservation of axial-vector current and current algebra [60,61]. Absolute branching ratio predictions depend on the dynamics of the light hadron system. For transitions between vector meson states the transition strength in charmonium is used to predict those in bottomium. But for the transition to the  $h_b$  no analogous process exists in charmonium. Therefore Kuang & Yan [61] convert the gluons with probability 1 into  $\pi\pi$ , a method which also gives good agreement with the predictions for  $\pi\pi$  transitions between vector states.

The decays  $\Upsilon(2S) \rightarrow \pi\pi \Upsilon(1S)$  have been studied with high statistics by five groups at Cornell and DESY [62,63,64,65,66]. The averaged results for the branching ratios, collected in table 7, are in very good agreement with theoretical predictions [61]. The ratio of the branching ratios for the neutral pion decay mode to the charged mode indicates consistency with isospin conservation for this decay. The measured angular distributions were found to be consistent with those expected for a spin zero di-pion system emitted in an S-wave. Partial conservation of the axial-vector current together with the observed isotropic angular distributions predicts [60] the invariant  $\pi\pi$  mass spectrum to be peaked at high values, which indeed is being observed.

Previous studies [67,68] of the transition  $\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(1S)$  suggested that the  $\pi\pi$  invariant mass spectrum was approximately uniform, quite in contrast to the strong peaking observed for  $\Upsilon(2S) \rightarrow \pi\pi \Upsilon(1S)$ . With recently collected 165K  $\Upsilon(3S)$  events CLEO has again investigated [14]  $\pi\pi$  hadronic transitions. This was done for both, the exclusive decay mode, where the daughter  $\Upsilon$  resonance decays to either a pair of electrons or muons, and for inclusive hadronic decays of the daughter. Figure 8 shows the  $\pi^+\pi^-$  invariant mass distribution for the transitions to the  $\Upsilon(1S)$  and  $\Upsilon(2S)$ . The  $\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(1S)$  spectrum is rather flat, definitely not peaked at high masses and

Table 7: Averaged experimental results for two-pion transitions from the  $\psi'$  [8], the  $\Upsilon(2S)$  [62, 63,64,65,66] and the  $\Upsilon(3S)$  [67,68,14].

Transition	BR(%)
$\psi' \rightarrow \pi\pi J/\psi$	$50 \pm 4$
$\Upsilon(2S) \rightarrow \pi^+\pi^- \Upsilon(1S)$	$18.7 \pm 0.7$
$\Upsilon(2S) \rightarrow \pi^0\pi^0 \Upsilon(1S)$	$9.9 \pm 1.0$
$\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(1S)$	$3.9 \pm 0.3$
$\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(2S)$	$2.4 \pm 0.5$
$\Upsilon(3S) \rightarrow \pi^+\pi^- h_b$	$0.37 \pm 0.15$

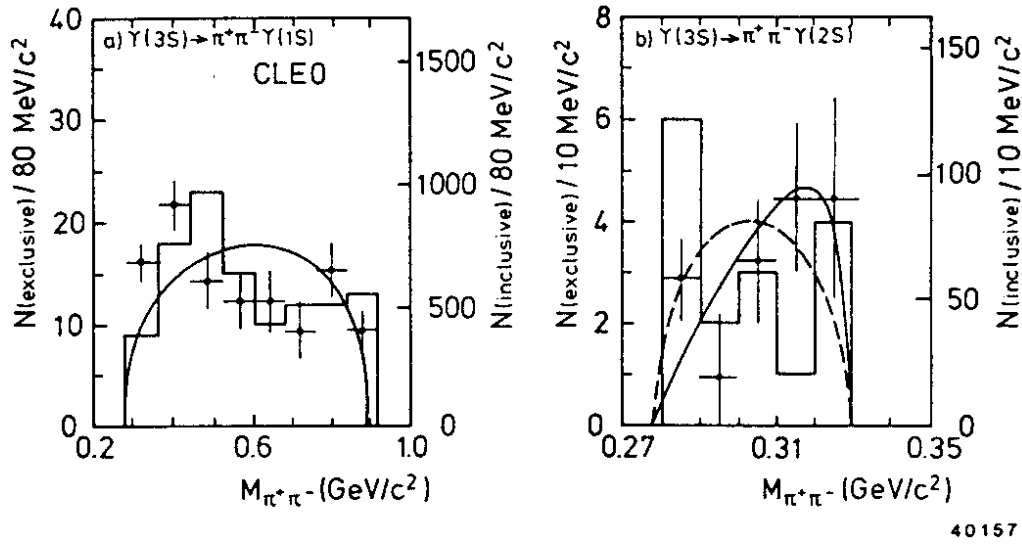


Figure 8: CLEO  $\pi^+\pi^-$  invariant mass distributions for a)  $\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(1S)$  and b)  $\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(2S)$ . The curves are explained in the text.

with a significant number of events immediately above threshold. A fit to the spectrum with *e.g.* the formula of Kuang & Yan [61] does not yield an acceptable description of the data (solid curve in fig. 8a).

One possible explanation for a rather uniform mass spectrum has been given by Voloshin [69]: a four-quark iso-vector resonance with mass close to the  $\Upsilon(3S)$  will cause a softening of the otherwise peaked mass distribution. The di-pion mass spectrum for the transition  $\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(2S)$  (fig. 8b) has insufficient statistics to distinguish between a peaked spectrum (solid curve, as found in  $\Upsilon(2S) \rightarrow \pi^+\pi^- \Upsilon(1S)$ ) or a flat spectrum (dashed curve, as found in  $\Upsilon(3S) \rightarrow \pi^+\pi^- \Upsilon(1S)$ ). Therefore no additional information can be obtained from this decay regarding the four-quark state. Hopefully this situation will be clarified with forthcoming results.

As was discussed in the section on the  $Q\bar{Q}$  potential, the  $h_Q$  state with  $J^{PC} = 1^{+-}$  is expected to be very close in mass to the center-of-gravity of the  $\chi$  states. The  $\Upsilon$  system offers a direct way to search for the  $h_b$  state: Kuang & Yan [61] have suggested the spin-flip  $\pi\pi$  transition  $\Upsilon(3S) \rightarrow \pi\pi h_b$ . CLEO [14] observes in the inclusive  $\pi\pi$  mass spectrum a  $2.5\sigma$  peak about 5.4 MeV below the center-of-gravity of the  $\chi_b$  masses. The branching fraction was determined to  $(0.37 \pm 0.15)\%$ .

Kuang & Yan [61] calculate a branching ratio of  $B(\Upsilon(3S) \rightarrow \pi^+\pi^- h_b) \simeq 0.4\%$ , in good agreement with the experimental datum. However, Voloshin [70] relates  $\langle \pi\pi | gg | 0 \rangle$  to matrix elements of the QCD energy momentum tensor. For transitions between vector meson states he obtains predictions similar to Kuang & Yan, but his calculated transition strength to the  $h_b$  is smaller by a factor of 150. More precise experimental information on the  $h_b$ , including for example the shape of the  $\pi\pi$  mass spectrum, may tell us which of the models is more applicable to soft pion transitions in  $\Upsilon$  decays [71].

## Radiative Decays

Radiative decays of heavy vector mesons offer a very clean way to study low mass mesons. This has been demonstrated by the many interesting physics results which emerged from analyses of radiative  $J/\psi$  decays. For a recent summary of results we refer to the references given in [72]. Of particular interest are here a search and identification of gluonic states and of hybrid mesons.  $\Upsilon(1S)$  radiative decays could add invaluable information regarding the existence of these important new forms of matter. Unfortunately, radiative decays from the  $\Upsilon(1S)$  are suppressed by about a factor of 40 with respect to those on the  $J/\psi$ . With the currently existing data sets, no radiative decay mode has been identified. The status is the same as was presented a year ago at the Conference on Physics in Collision [55].

However, new information on the decay  $\Upsilon(1S) \rightarrow \gamma axion$  has been presented at this conference [73]. The axion [74] is produced in this decay with a strength [75] proportional to a parameter  $1/x^2$ , which measures the ratio of vacuum expectation values for the two Higgs fields in the theory. In  $J/\psi$  decays the decay strength is proportional to  $x^2$ . Therefore the product of the  $\Upsilon(1S)$  and  $J/\psi$  radiative decay branching fractions is independent of this parameter. Specifically, the prediction is

$$B(J/\psi \rightarrow \gamma axion) \times B(\Upsilon(1S) \rightarrow \gamma axion) = (2.9 \pm 0.7) \times 10^{-9}, \quad (11)$$



where the error arises from the uncertainty in the  $c$  and  $b$  quark masses. With the Crystal Ball [76] limit  $B(J/\psi \rightarrow \gamma axion) < 1.4 \times 10^{-5}$  and a CUSB [77] limit of  $B(\Upsilon(3S) \rightarrow \gamma axion) < 1.2 \times 10^{-4}$  the axion seemed to be ruled out. Note that CUSB used  $\Upsilon(3S)$  decays; here the prediction on the product branching ratio is  $(2.3 \pm 0.8) \times 10^{-9}$ . However, recent calculations by Nason [78] of QCD radiative corrections to these processes indicate, that each branching ratio will be reduced by about 20%. This yields the prediction<sup>2</sup>

$$B(J/\psi \rightarrow \gamma axion) \times B(\Upsilon(1S) \rightarrow \gamma axion) = (1.8 \pm 0.4) \times 10^{-9} > 1.0 \times 10^{-9}, \quad (12)$$

where we have taken the theoretical error seriously and converted the prediction into a 90% CL lower limit.

The Crystal Ball experiment has analyzed [73] nearly 0.5 million  $\Upsilon(1S)$  decays for a signature of only one photon of beam energy in the detector. This restricts the search to the standard light Peccei-Quinn axion, which is long-lived and decays outside the detector volume into two photons. The null result of this search yields the limit  $B(\Upsilon(1S) \rightarrow \gamma axion) < 1.4 \times 10^{-5}$ . The product with the corresponding  $J/\psi$  branching fraction gives  $B(J/\psi \rightarrow \gamma axion) \times B(\Upsilon(1S) \rightarrow \gamma axion) < 0.2 \times 10^{-9}$ , a factor of 5 below the theoretical prediction eq. 12. The conclusion is that finally the standard axion is ruled out, leaving us with ‘invisible’ axions.

## Summary

The field of heavy quarkonium spectroscopy has matured very much in the last decade. The study of charmonium and bottomonium mass spectra and decay channels have provided us with a detailed understanding of the forces acting between quarks. In particular, the quark confining force contributing to the fine-structure splitting has been found to be consistent with being a Lorentz scalar. The short range force arising from one-gluon exchange behaves in every aspect identical to the electromagnetic force. Decays of heavy quarkonium states provide a very clean method to further study the strong interactions. With some advance in theoretical calculations and experimental accuracy, a very precise measurement of the strong coupling constant will be possible.

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<sup>2</sup>Recently, Aznauryan *et al.* [79] have calculated relativistic effects to the decay width. However, it is not clear whether they can be treated separately from QED radiative effects, which are of the same order. Therefore we ignore at the present time relativistic corrections, which at face value would amount to an additional reduction in width by 45% and 31% in  $J/\psi$  and  $\Upsilon(1S)$  decays, respectively.

Furthermore we need and hope to obtain in the near future confirmation on the spin-singlet states  $h_Q$ . This will provide a test of our understanding of the hyperfine force. The two-pion mass spectrum in the transition from the  $\Upsilon(3S)$  to the  $h_b$  will provide us with additional information, in analogy to the unexpected and not yet understood  $\pi\pi$  mass spectrum in  $\Upsilon(3S)$  transitions. Maybe the distortion of the observed mass spectrum is really the first clear-cut evidence of a four-quark resonance!

The study of  $J/\psi$  decays with nearly 10 million events has proved that narrow resonances are an ideal laboratory for detailed investigations of low mass spectroscopy. An analogous study of  $\Upsilon(1S)$  decays should help to resolve the many questions posed by the  $J/\psi$  data. In particular, radiative decays to low mass pseudo-scalar and tensor mesons will provide us with a better understanding of particles made of gluons only. Measurements of hadronic decays into *e.g.* vector plus pseudo-scalar will help to solve the puzzle of  $\rho\pi$  and  $K^*\bar{K}$  decays observed in  $J/\psi$  and  $\psi'$  decays. Precision studies of  $\Upsilon(2S)$  and  $\Upsilon(3S)$  decays will allow the discovery of the  $\eta_b$  and the  $D$  states. The latter states, with angular momentum  $L = 2$  between the quarks, will test the strong force at rather large distances, where it is dominated by the confining force. Finally,  $\chi_0^b$  and  $\chi_2^b$  decays offer the possibility to investigate gluon jets, which can be compared directly to two quark jets at the same center-of-mass energy. In summary, the  $\Upsilon(1S)$  states provide a rich laboratory for studying the strong force in both, the domain of asymptotic freedom and in the region of confinement. Very exciting physics results are to be expected.

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