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Standard Model of the Electroweak Interactions†

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Abstract

The standard model of electroweak interactions is briefly surveyed, and the implications of recent precise measurements of neutral current processes and the W and Z masses are described. The weak angle is found to be $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2 = 0.230 \pm 0.0048$, where the error includes full statistical, systematic, and theoretical uncertainties. Allowing $\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W)$ to vary one obtains $\rho = 0.998 \pm 0.0086$. Consequences for tests of the standard model at the level of radiative corrections, non-standard Higgs representations, m_t and heavy fourth family fermions, grand unification, and possible additional Z bosons are discussed.

The Standard Model

The standard electroweak model is based on the gauge group[1] $SU_2 \times U_1$, with gauge bosons W_μ^i, $i = 1, 2, 3$, and B_μ for the SU_2 and U_1 factors, respectively, and the corresponding gauge coupling constants g and g'. The left-handed fermion fields $\begin{pmatrix} \nu_i \\ e_i^- \end{pmatrix}$ and $\begin{pmatrix} u_i \\ d_i' \end{pmatrix}$ of the i^{th} fermion family transform as doublets under SU_2, where $d_i' \equiv \sum_j V_{ij} d_j$, and V is the quark mixing matrix. The right-handed fields are SU_2 singlets. The U_1 charge is $Y \equiv Q - T_3$, where Q and T_3 are the electric charge and 3^{rd} component of SU_2, respectively. (Some authors define Y as $2(Q - T_3)$). In the minimal model there are three fermion fields and a single complex Higgs doublet $\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.

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After spontaneous symmetry breaking the part of the Lagrangian involving fermions is

$$\begin{aligned} L_F &= \sum_i \bar{\psi}_i (i \not\partial - m_i - \frac{m_i H}{\nu}) \psi_i - \frac{g}{2\sqrt{2}} (J_W^\mu W_\mu^- + J_W^{\mu\dagger} W_\mu^+) \\ &- e J_{EM}^\mu A_\mu - \frac{g}{2\cos\theta_W} J_Z^\mu Z_\mu, \end{aligned} \tag{1}$$

where $\theta_W \equiv \tan^{-1}(g'/g)$ is the weak angle, $e = g \sin\theta_W$ is the positron electric charge, and $A \equiv \cos\theta_W B + \sin\theta_W W^3$ is the (massless) photon field. $W^{\pm} \equiv (W^1 \mp i W^2)/\sqrt{2}$ and $Z \equiv -\sin\theta_W B + \cos\theta_W W^3$ are the massive charged and neutral weak boson fields, respectively.

In (1), ψ_i is the ith fermion field with mass m_i. For the quarks these are the current masses, which for the light quarks are estimated[2] to be $m_u \simeq 5.6 \pm 1.1\ MeV$, $m_d \simeq 9.9 \pm 1.1\ MeV$, $m_s \simeq 199 \pm 33\ MeV$, and $m_c \simeq 1.35 \pm 0.05\ GeV$, (these are running masses evaluated at 1 GeV). For the heavier quarks $m_b \simeq 5\ GeV$, and $m_t > O(50)\ GeV$.

The Higgs sector

L_F in (1) is written assuming the minimal Higgs sector. H is the physical Higgs scalar that remains after removing the "eaten" fields: $\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$, where $\nu = 2M_W/g = (\sqrt{2}G_F)^{-1/2} \simeq 246\ GeV$ is the vacuum expectation value (VEV) of $\sqrt{2}\varphi^o$. The Yukawa coupling of H to ψ_i, which is flavor diagonal in the minimal model, is $-m_i/\nu = -g m_i / 2M_W$. The H mass is not predicted by the model. A variety of experimental constraints involving nuclear transitions and rare K, B, and Υ decays exclude most of the mass range below $M_H \sim 3.9\ GeV$, but windows from $\simeq 18$ to $50\ MeV$ and from $\simeq 211\ MeV$ to several hundred MeV are still possible. The theoretical requirement that the $SU_2 \times U_1$ breaking vacuum be the global minimum of the Higgs potential implies

$$\begin{aligned} M_H &> \frac{1}{4\pi} \frac{(6M_W^4 + 3M_Z^4)^{1/2}}{\nu} \left[1 - \frac{4m_t^4}{2M_W^4 + M_Z^4} \right]^{1/2} \\ &\simeq 7.0\ GeV \left[1 - \left(\frac{m_t}{79\ GeV} \right)^4 \right]^{1/2}. \end{aligned} \tag{2}$$

(for more than 3 fermion families m_t^4 in (2) is replaced by $\sum_i m_i^4$), while cosmological arguments lead to a limit that is larger by $\sqrt{2}$. Various arguments based on the assumption that the Higgs self-interaction is not strong suggest that M_H is less than several hundred GeV to 1 TeV.

In non-minimal models there are extra neutral and charged Higgs particles. The Yukawa couplings can no longer be precisely predicted but are still typically of order m_i/ν, and the neutral Higgs particles generally can mediate flavor-changing neutral currents unless they are forbidden to do so by extra symmetries. The $K_L - K_S$ mass difference requires $h/M < O(10^{-7}\ GeV^{-1})$, where h is the Yukawa coupling of a neutral Higgs of mass M to $\bar{d}s$. Typically h is around 10^{-3} (unless it is zero due to an extra symmetry), implying $M > 10\ TeV$. The theoretical and cosmological lowers limits no longer hold in non-minimal models, but the non-rigorous upper bounds continue to apply to the lightest neutral Higgs.

The charged current and QED

The second term in L_F represents the charged current weak interaction, with (for massless neutrinos)

$$J_W^{\mu\dagger} = (\bar{u}\ \bar{c}\ \bar{t})\ \gamma^\mu(1+\gamma^5)V \begin{pmatrix} d \\ s \\ b \end{pmatrix} + (\bar{\nu}_e\ \bar{\nu}_\mu\ \bar{\nu}_\tau)\ \gamma^\mu(1+\gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix}, \tag{3}$$

where V is the Kobayashi-Maskawa-Cabibbo mixing matrix. For momenta small compared to M_W one has the effective four-fermion interaction $-L_{eff}^{CC} = \frac{G_F}{\sqrt{2}} J_W^{\mu\dagger} J_{W\mu}$, with the Fermi constant given (at tree level, *i.e.* lowest order in perturbation theory) by $G_F/\sqrt{2} = g^2/8M_W^2 = 1/2\nu^2$. The standard model predictions for the weak charged current have been extensively tested[4] in μ and τ decay; semi-leptonic β, hyperon, and meson decays; νe and ν-hadron (*e.g.* quasi-elastic and deep inelastic) scattering; and, less quantitatively, in $|\Delta S| = 1$ non-leptonic decays and $\Delta S = 0$ parity-violating interference effects. CP violation is incorporated in the standard model by a single observable phase in V (or by $(F-1)(F-2)/2$ phases for F fermion families). All existing results are in agreement with standard model predictions.

The third term in L_F describes electromagnetic interactions (QED). The current is

$$J_{EM}^\mu = \sum_i q_i\ \bar{\psi}_i\gamma^\mu\psi_i = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \bar{e}\gamma^\mu e + \cdots \tag{4}$$

where q_i is the charge of ψ_i in units of e.

The neutral current

The last term in L_F is the weak neutral current interaction. The massive Z boson couples to the current

$$\begin{aligned} J_Z^\mu &= \sum_i t_{3L}(i)\ \bar{\psi}_i\gamma^\mu(1+\gamma^5)\psi_i + t_{3R}(i)\ \bar{\psi}_i\gamma^\mu(1-\gamma^5)\psi_i - 2\sin^2\theta_W J_{EM}^\mu \\ &= \sum_i \bar{\psi}_i\gamma^\mu(V^i + A^i\gamma^5)\psi_i, \end{aligned} \tag{5}$$

where $t_{3L}(i)$ is the T_3 eigenvalue of the left-handed component of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and e_i). Similarly, $t_{3R}(i)$ is the T_3 eigenvalue of the right-handed component of ψ_i. It is zero in the standard model but could be non-zero in generalizations with exotic fermions in right-handed doublets. The vector and axial couplings are $V^i \equiv t_{3L}(i) + t_{3R}(i) - 2\sin^2\theta_W q_i$ and $A^i \equiv t_{3L}(i) - t_{3R}(i)$. (There are many different conventions concerning factors of two in J^μ, V, and A, as well as in the sign of γ^5). At low momenta the neutral current interaction is described by the effective interaction

$$- L_{eff}^{NC} = \rho \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}, \tag{6}$$

where $\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W)$ is unity in the minimal model and in generalizations with extra Higgs doublets, but can differ from one in models which have Higgs triplets, *etc.*, with nonzero VEV's.

The neutral current interaction has been observed and quantitatively tested in a wide variety of weak processes, including deep inelastic $\overset{(-)}{\nu}_\mu$ N scattering from isoscalar and proton targets, elastic $\overset{(-)}{\nu}_\mu\, p$ scattering, coherent $\nu N \to \nu\pi^0 N$ scattering, elastic $\overset{(-)}{\nu}_i$ e $(i = e, \mu)$ scattering, and $e^+e^- \to$ hadrons. In addition, weak- electromagnetic interference has been studied in polarized eD and μC scattering, atomic parity violation, and forward-backward asymmetries in $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$, $c\bar{c}$, and $b\bar{b}$. All processes are in excellent agreement with the standard model predictions. Quantitative results[5] are detailed below.

Renormalization and radiative corrections

The minimal model (with $\rho = 1$) has three parameters (not counting M_H and the fermion masses and mixings). These could be taken to be g, g', and ν, but it is convenient to replace these with parameters more closely related to experiment. A particularly useful set is: (a) the fine structure constant $\alpha = 1/137.036$ *, determined from the Josephson effect. (b) The Fermi constant, $G_F = 1.16637 \times 10^{-5} GeV^{-2}$, determined from the muon lifetime formula (including lepton mass and $O(\alpha)$ radiative corrections):

$$\tau_\mu^{-1} = G_F^2\, P \left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left(1 + \frac{2\alpha}{3\pi}\ \ln\ \frac{m_\mu}{m_e}\right)\right], \tag{7}$$

where

$$P = \left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{3}{5}\ \frac{m_\mu^2}{M_W^2}\right] \frac{m_\mu^5}{192\pi^3}. \tag{8}$$

*α is dependent upon the energy scale of the process in which it is measured. This value is appropriate for low energy. At energies of order M_W the value 1/128 is applicable.

(c) $\sin^2\theta_W$, determined from neutral current processes and the W and Z masses. The value of $\sin^2\theta_W$ depends on the renormalization prescription. A very useful scheme[6] is to take the tree level formula $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ as the definition of the renormalized $\sin^2\theta_W$ to all orders in perturbation theory[†]. An alternative is to use the modified minimal subtraction ($\overline{MS}$) quantity $\sin^2\hat{\theta}_W(\mu)$, where μ is conveniently chosen to be M_W for electroweak processes. The two definitions are related by $\sin^2\hat{\theta}_W(M_W) = C(m_t, M_H)\sin^2\theta_W$, where $C = 0.9907$ for $m_t = 45\ GeV$, $M_H = 100\ GeV$. Yet another possibility is to take M_Z rather than $\sin^2\theta_W$ as the third fundamental parameter. This will be useful when very precise values of M_Z are determined at SLC and LEP.

Experiments are now at a level of precision that complete $O(\alpha)$ radiative corrections must be applied[‡]. These corrections are conveniently divided into two classes:

1. QED diagrams involving the emission of real photons or the exchange of virtual photons in loops, but not including vacuum polarization diagrams. These graphs yield finite and gauge invariant contributions to observable processes. However, they are dependent on energies, experimental cuts, *etc.*, and must be calculated individually for each experiment.

2. Electroweak corrections, including $\gamma\gamma$, γZ, ZZ, and WW vacuum polarization diagrams, as well as vertex corrections, box graphs, *etc.*, involving virtual W's and Z's. Many of these corrections are absorbed into the renormalized Fermi constant defined in (7). Others modify the tree level expressions for neutral current amplitudes in several ways:

   (a) Contributions proportional to $\langle f|J_Z^\mu|i\rangle$ (*e.g.* for $\nu i \to \nu f$) can be absorbed into a parameter ρ^{NC} that multiplies the tree level amplitude.

   (b) Similarly, terms proportional to $\langle f|J_{EM}^\mu|i\rangle$ can be absorbed into a coefficient κ of $\sin^2\theta_W$. Both ρ^{NC} and κ depend on the reaction and the relevant momentum Q^2. They (as well as Δr defined below) depend rather sensitively on m_t and any fourth family fermion masses if they are sufficiently large (*e.g.* $> 100\ GeV$). The dependence on M_H is small but not entirely negligible at the present level of exerimental precision.

   (c) Other contributions induce new effective operators in L_{eff}^{NC} with coefficients λ. These are generally very small and can usually be neglected.

In addition, vacuum polarization diagrams modify the tree level expressions

[†]This definition is used for the results given below.

[‡]Radiative corrections also play a crucial role in the charged current, such as in the verification of three-generation unitarity of the KMC matrix[6].

for M_W and M_Z. In the $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ scheme one has

$$\begin{aligned} M_W &= \frac{A_o}{\sin\theta_W(1-\Delta r)^{\frac{1}{2}}} \\ M_Z &= \frac{M_W}{\cos\theta_W} \end{aligned} \tag{9}$$

where $A_o = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.281\ GeV$. The radiative correction parameter Δr is predicted to be 0.0713 ± 0.0013 for $m_t = 45\ GeV$ and $M_H = 100\ GeV$, while $\Delta r \to 0$ for $m_t \sim 245\ GeV$. If M_Z is regarded as fundamental then

$$\begin{pmatrix} \sin^2\theta_W \\ \cos^2\theta_W \end{pmatrix} = \frac{1}{2}\left[1 \mp \left(1 - \frac{4A_o^2}{M_Z^2(1-\Delta r)}\right)^{1/2}\right] \tag{10}$$

are derived parameters, and $M_W = M_Z \cos\theta_W$.

Cross section and asymmetry formulas

It is convenient to write the terms in $-L_{eff}^{NC}$ relevant to ν-hadron, νe, and e-hadron processes in a form that is valid in an arbitrary gauge theory (assuming massless left-handed neutrinos). One has

$$-L^{\nu H} = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^\mu(1+\gamma^5)\nu \left\{\sum_i [\epsilon_L(i)\,\bar{q}_i\gamma_\mu(1+\gamma^5)q_i + \epsilon_R(i)\,\bar{q}_i\gamma_\mu(1-\gamma^5)q_i]\right\}, \tag{11}$$

$$-L^{\nu e} = \frac{G_F}{\sqrt{2}}\,\bar{\nu}_\mu\gamma^\mu(1+\gamma^5)\nu_\mu\ \bar{e}\gamma_\mu(g_V^e + g_A^e\gamma^5)e \tag{12}$$

(for $\overset{(-)}{\nu}_e e$ the charged current contribution must be included), and

$$-L^{eH} = \frac{G_F}{\sqrt{2}}\sum_i [C_{1i}\,\bar{e}\gamma_\mu\gamma^5 e\ \bar{q}_i\gamma^\mu q_i + C_{2i}\,\bar{e}\gamma_\mu e\ \bar{q}_i\gamma^\mu\gamma^5 q_i] \tag{13}$$

The standard model expressions for $\epsilon_{L,R}(i)$, $g_{V,A}^e$ and C_{ij} are given in Table 1.

At present the most precise determinations of $\sin^2\theta_W$ are from deep inelastic neutrino scattering from (approximately) isoscalar targets. The ratio $R_\nu \equiv \sigma_{\nu N}^{NC}/\sigma_{\nu N}^{CC}$ of neutral to charged current cross sections has been measured to 1% accuracy by the CDHS[7] and CHARM[8] collaborations, so it is important to obtain theoretical expressions for R_ν and $R_{\bar{\nu}} \equiv \sigma_{\bar{\nu}N}^{NC}/\sigma_{\bar{\nu}N}^{CC}$ (as functions of $\sin^2\theta_W$) to comparable accuracy. Fortunately, most of the uncertainties concerning the strong interactions (as well as neutrino spectra) cancel in the ratio. For neutral current parameters in the vicinity of the standard model $\simeq 90\%$ of R_ν can be predicted from isospin alone[9]. The remaining 10% (from such effects as quark mixing and

Table 1: Standard model expressions for the neutral current parameters for ν-hadron, νe, and e-hadron processes. If radiative corrections are ignored, $\rho = \kappa = 1$, $\lambda = 0$. At $O(\alpha)$, $\rho_{\nu N}^{NC} = 1.00074$, $\kappa_{\nu N} = 0.9902$, $\lambda_{u_L} = -0.0031$, $\lambda_{d_L} = -0.0026$, and $\lambda_{u_R} = \frac{1}{2}\lambda_{d_R} = 3.5 \times 10^{-5}$ for $m_t = 45\ GeV$, $M_H = 100\ GeV$, $\sin^2\theta_W = 0.23$, and $\langle Q^2 \rangle = 20\ GeV^2$. For νe scattering $\kappa_{\nu e} = 0.9897$ and $\rho_{\nu e} = 1.0054$ (at $\langle Q^2 \rangle = 0.$). For atomic parity violation, $\rho'_{eq} = 0.9793$ and $\kappa'_{eq} = 0.9948$. For the SLAC polarized electron experiment $\rho'_{eq} = 0.970$, $\kappa'_{eq} = 0.993$, $\rho_{eq} = 0.993$ and $\kappa_{eq} = 1.03$ after incorporating additional QED corrections.

Quantity	Standard Model Expression
$\epsilon_L(u)$	$\rho_{\nu N}^{NC}[\frac{1}{2} - \frac{2}{3}\kappa_{\nu N}\sin^2\theta_W + \lambda_{u_L}]$
$\epsilon_L(d)$	$\rho_{\nu N}^{NC}[-\frac{1}{2} + \frac{1}{3}\kappa_{\nu N}\sin^2\theta_W + \lambda_{d_L}]$
$\epsilon_R(u)$	$\rho_{\nu N}^{NC}[-\frac{2}{3}\kappa_{\nu N}\sin^2\theta_W + \lambda_{u_R}]$
$\epsilon_R(d)$	$\rho_{\nu N}^{NC}[\frac{1}{3}\kappa_{\nu N}\sin^2\theta_W + \lambda_{d_R}]$
g_V^e	$\rho_{\nu e}[-\frac{1}{2} + 2\kappa_{\nu e}\sin^2\theta_W]$
g_A^e	$\rho_{\nu e}[-\frac{1}{2}]$
C_{1u}	$\rho'_{eq}[-\frac{1}{2} + \frac{4}{3}\kappa'_{eq}\sin^2\theta_W]$
C_{1d}	$\rho'_{eq}[\frac{1}{2} - \frac{2}{3}\kappa'_{eq}\sin^2\theta_W]$
C_{2u}	$\rho_{eq}[-\frac{1}{2} + 2\kappa_{eq}\sin^2\theta_W]$
C_{2d}	$-C_{2u}$

the s sea) is strongly constrained by independent measurements involving deep inelastic e, μ, and charged-current ν scattering, including dimuon production, and can be estimated to the necessary (10%) accuracy.

A simple zeroth order approximation (ignoring quark mixing, the s and c sea, and certain tiny higher twist effects) is

$$\begin{aligned} R_\nu &= g_L^2 + g_R^2 r \\ R_{\bar\nu} &= g_L^2 + \frac{g_R^2}{r}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} g_L^2 &\equiv \epsilon_L(u)^2 + \epsilon_L(d)^2 \simeq \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \\ g_R^2 &\equiv \epsilon_R(u)^2 + \epsilon_R(d)^2 \simeq \frac{5}{9}\sin^4\theta_W, \end{aligned} \tag{15}$$

and $r \equiv \sigma^{CC}_{\bar\nu N}/\sigma^{CC}_{\nu N}$ is the ratio of $\bar\nu$ and ν charged current cross sections, which can be measured directly. (In the simple parton model, ignoring hadron energy cuts, $r \simeq (\frac{1}{3} + \epsilon)/(1 + \frac{1}{3}\epsilon)$, where $\epsilon \sim 0.125$ is the ratio of the fraction of the nucleon's s momentum carried by antiquarks to that carried by quarks. *i.e.* $\epsilon \equiv (\bar U + \bar D)/(U + D)$, where $U \equiv \int_o^1 xu(x)dx$ is the first moment of the u quark distribution.) In practice, (14) must be corrected for quark mixing, the s and c seas, c quark threshold effects (which mainly affect σ^{CC} - these turn out to be the largest theoretical uncertainty), non-isoscalar target effects, $W - Z$ propagator differences, and radiative corrections (which lower the extracted value of $\sin^2\theta_W$ by ~ 0.009.). Details of the neutrino spectra, experimental cuts, x and Q^2 dependence of structure functions, and longitudinal structure functions enter only at the level of these corrections and therefore lead to very small uncertainties. Altogether, the theoretical uncertainty is $\Delta\sin^2\theta_W \sim \pm 0.005$, which would be very hard to improve in the future. The formulas for deep inelastic scattering from non-isoscalar targets and for $\nu p \to \nu p$ elastic scattering can be traced back from ref. 5.

The laboratory cross section for $\overset{(-)}{\nu}_\mu\, e \to \overset{(-)}{\nu}_\mu\, e$ elastic scattering is

$$\frac{d\sigma_{\nu_\mu,\bar\nu_\mu}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi}\left[(g_V^e \pm g_A^e)^2 + (g_V^e \mp g_A^e)^2(1-y)^2 - (g_V^{e2} - g_A^{e2})\frac{y m_e}{E_\nu}\right], \tag{16}$$

where the upper (lower) sign refers to $\nu_\mu(\bar\nu_\mu)$, $y \equiv T_e/E_\nu$ (which runs from 0 to $(1 + \frac{m_e}{2E_\nu})^{-1}$) is the ratio of the kinetic energy of the recoil electron to the incident $\overset{(-)}{\nu}$ energy, and $G_F^2 m_e/2\pi = 4.31 \times 10^{-42}\ cm^2/GeV$. For $E_\nu \gg m_e$ this yields a total cross section

$$\begin{aligned} \sigma &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(g_V^e \pm g_A^e)^2 + \frac{1}{3}(g_V^e \mp g_A^e)^2\right] \\ &\sim \frac{G_F^2 m_e E_\nu}{2\pi}\begin{cases} 1 - 4\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W, & \nu_\mu e \\ \frac{1}{3} - \frac{4}{3}\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W, & \bar\nu_\mu e \end{cases} \end{aligned} \tag{17}$$

The most accurate leptonic measurements[10,11] of $\sin^2\theta_W$ are from the ratio $R \equiv \sigma_{\nu_\mu e}/\sigma_{\bar\nu_\mu e}$, in which many of the systematic uncertainties cancel. Radiative corrections, which are small compared to the precision of present experiments, increase the extracted $\sin^2\theta_W$ by $\simeq 0.002$. The cross sections for $\overset{(-)}{\nu}_e\, e$ may be obtained from eqn (16) by replacing $g^e_{V,A}$ by $g^{e\prime}_{V,A} \equiv g^e_{V,A} + 1$, where the 1 is due to the charged current contribution.

The SLAC polarized-electron experiment[12] measured the parity-violating asymmetry

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \tag{18}$$

where $\sigma_{R,L}$ is the cross section for the deep inelastic scattering of a right (left)-handed electron: $e_{R,L}N \to eX$. In the quark parton model

$$\frac{A}{Q^2} = a_1 + a_2\frac{1-(1-y)^2}{1+(1-y)^2}, \tag{19}$$

where $Q^2 > 0$ is the momentum transfer and y is the fractional energy transfer from the electron to the hadrons. For the deuteron or other isoscalar target one has, neglecting the s quark and antiquarks,

$$\begin{aligned} a_1 &= \frac{3G_F}{5\sqrt{2}\pi\alpha}(C_{1u} - \frac{1}{2}C_{1d}) \simeq \frac{3G_F}{5\sqrt{2}\pi\alpha}(-\frac{3}{4} + \frac{5}{3}\sin^2\theta_W) \\ a_2 &= \frac{3G_F}{5\sqrt{2}\pi\alpha}(C_{2u} - \frac{1}{2}C_{2d}) \simeq \frac{9G_F}{5\sqrt{2}\pi\alpha}(\sin^2\theta_W - \frac{1}{4}), \end{aligned} \tag{20}$$

where $3G_F/5\sqrt{2}\pi\alpha = 2.16\times10^{-4}\ GeV^{-2}$. Radiative corrections lower the extracted value of $\sin^2\theta_W$ by ~ 0.005.

Experiments measuring atomic parity violation[13] are now very precise, and the uncertainties associated with atomic wave functions are relatively small (especially for cesium). For heavy atoms one determines the "weak charge"

$$\begin{aligned} Q_W &= -2[C_{1u}(2Z+N) + C_{1d}(Z+2N)] \\ &\simeq Z(1-4\sin^2\theta_W) - N. \end{aligned} \tag{21}$$

Radiative corrections increase the extracted $\sin^2\theta_W$ by ~ 0.008.

The forward-backward asymmetry for $e^+e^- \to l\bar{l}$, $l = \mu$ or τ, is defined as

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \tag{22}$$

where σ_F (σ_B) is the cross section for l^- to travel forward (backward) with respect to the e^- direction. A_{FB} and R, the total cross section relative to pure QED, are given in a model with a single Z by

$$\begin{aligned} R &= F_1 \\ A_{FB} &= 3F_2/4F_1 \end{aligned} \tag{23}$$

where

$$F_1 = 1 - 2\chi_0 V^e V^l \cos\delta_R + \chi_0^2 (V^{e2} + A^{e2})(V^{l2} + A^{l2})$$
$$F_2 = -2\chi_0 A^e A^l \cos\delta_R + 4\chi_0^2 A^e A^l V^e V^l, \tag{24}$$

where

$$\tan\delta_R = \frac{M_Z \Gamma_Z}{M_Z^2 - s}$$
$$\chi_0 = \frac{\rho G_F}{2\sqrt{2}\pi\alpha} \frac{s M_Z^2}{[(m_Z^2 - s)^2 + m_Z^2 \Gamma_Z^2]^{\frac{1}{2}}} \tag{25}$$

and $\sqrt{s}$ is the CM energy. In the standard model $V^e = V^l = (-\frac{1}{2} + 2\sin^2\theta_W)$ and $A^e = A^l = -\frac{1}{2}$. Equation (24) is valid at tree level. If the data are radiatively corrected for QED effects (as described above) then the remaining electroweak corrections can be incorporated[14] (in an approximation adequate for existing PEP and PETRA data) by replacing χ_0 by $\chi(s) \equiv \chi_0(s)\alpha/\hat{\alpha}(s)$, where $\hat{\alpha}(s)$ is the running QED coupling. Numerically, $\alpha/\hat{\alpha}(s) \sim 1 - \Delta r$ if Δr is evaluated for $m_t < 100\ GeV$. (Numerically similar results apply to A_{FB} in a popular alternative scheme in which the data is corrected only for QED corrections to the one photon annihilation diagram, including vacuum polarization.) Formulas for $e^+e^- \to$ hadrons may be found in ref. 15.

At present energies the A_{FB} are dominated by the linear (in χ) term in F_2, the value of which is almost independent of $\sin^2\theta_W$ for reasonable values. The major $\sin^2\theta_W$ constraint is from the weak-electromagnetic interference (χ) term in F_1, which would be important for $\sin^2\theta_W$ much different from $\frac{1}{4}$. In fact, for $\sin^2\theta_W$ near 0.23 the pure weak (χ^2) term in F_1 is numerically larger than the linear term.

At SLC and LEP it should be possible to measure A_{FB} for $e^+e^- \to l^+l^-$ at the Z pole to high precision. Similarly, the left-right asymmetry

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}, \tag{26}$$

where σ_L (σ_R) is the cross section for a left (right)-handed incident electron will be measured very precisely at SLC and possibly at LEP. It may also be possible to measure the final polarization asymmetry for the case $l = \tau$:

$$A_{pol} \equiv \frac{\sigma(\tau_L) - \sigma(\tau_R)}{\sigma(\tau_L) + \sigma(\tau_R)}, \tag{27}$$

where $\sigma(\tau_L)$ ($\sigma(\tau_R)$) is the cross section to produce a left (right)-handed τ^-. At tree level and neglecting terms of order $(\Gamma_Z/M_Z)^2$, one has

$$A_{FB} \simeq 3\eta_l \frac{\eta_e + \frac{1}{2}P_e}{1 + 2P_e\eta_e}$$
$$A_{LR} \simeq 2\eta_e \tag{28}$$
$$A_{pol} \simeq 2\eta_l,$$

where P_e is the initial e^- polarization and

$$\eta_i \equiv \frac{V^i A^i}{V^{i2} + A^{i2}}, \; i = e, \; \mu, \; \tau. \tag{29}$$

The high precision measurements will require careful application of both QED and electroweak radiative corrections[14] to (28).

Neutral current results

$\sin^2 \theta_W$ and, equivalently, M_Z have been determined from the W and Z masses and in a variety of neutral current processes spanning a very wide Q^2 range. The results[5], shown in Table 2 and Figure 1, are in impressive agreement with each other, indicating the quantitative success of the standard model. The best fit to all data yields $\sin^2 \theta_W = 0.230 \pm 0.0048$ and $M_Z = 92.0 \pm 0.7 \; GeV$, where the errors (as well as those given below for other neutral current parameters) include full statistical, systematic, and theoretical uncertainties. The corresponding value of $\sin^2 \hat{\theta}_W(M_W)$ (for fixed $m_t = 45 \; GeV$, $M_H = 100 \; GeV$) is 0.228 ± 0.0044. This is larger by $\simeq 2.5 \; \sigma$ than the prediction $0.214^{+0.003}_{-0.004}$ of minimal SU_5 (for $\Lambda^{(4)}_{\overline{MS}} = 150^{+150}_{-75} \; MeV$) and other "great desert" models. It is closer to (but still somewhat below) the prediction of the simplest supersymmetric GUTs. (Typically $0.237^{+0.003}_{-0.004}$ for $M_{SUSY} \sim M_W$, decreasing by ~ 0.003 for $M_{SUSY} \sim 10 \; TeV$). Similar conclusions hold for all values of m_t and M_H, as can be seen in Figure 2.

The radiative corrections are sensitive[18] to the isospin breaking associated with a large m_t. The $\sin^2 \theta_W$ values determined from the various reactions are consistent with each other for $m_t \ll 200 \; GeV$, but disagree for very large m_t, as can be seen in Figure 1. A simultaneous fit to $\sin^2 \theta_W$ and m_t requires $m_t < 180 \; GeV$ at 90% *c.l.* for $M_H \leq 100 \; GeV$, with a slightly weaker limit for larger M_H. The allowed region in $\sin^2 \theta_W$ and m_t is shown in Figure 3. Similar limits hold for the mass splittings between fourth generation quarks or leptons. These results assume that there is no compensating new physics such as non-doublet Higgs representations (discussed below). In the future precise determinations of the Z mass at SLC and LEP, combined with existing neutral current data (and eventually with e^+e^- asymmetries), will tightly constrain m_t or imply new physics[5].

The measured values of M_W and M_Z are given in Table 3. They are in excellent agreement with the predictions of the standard model when full radiative corrections (to both the W and Z mass formulas and to deep inelastic scattering) are included, but disagree significantly when the corrections are excluded. From the data one obtains $\Delta r = 0.077 \pm 0.037$, in excellent agreement with the value 0.0713 ± 0.0013 predicted for $m_t = 45 \; GeV$ and $M_H = 100 \; GeV$. The allowed region for $\sin^2 \theta_W$ and Δr is shown in Figure 4.

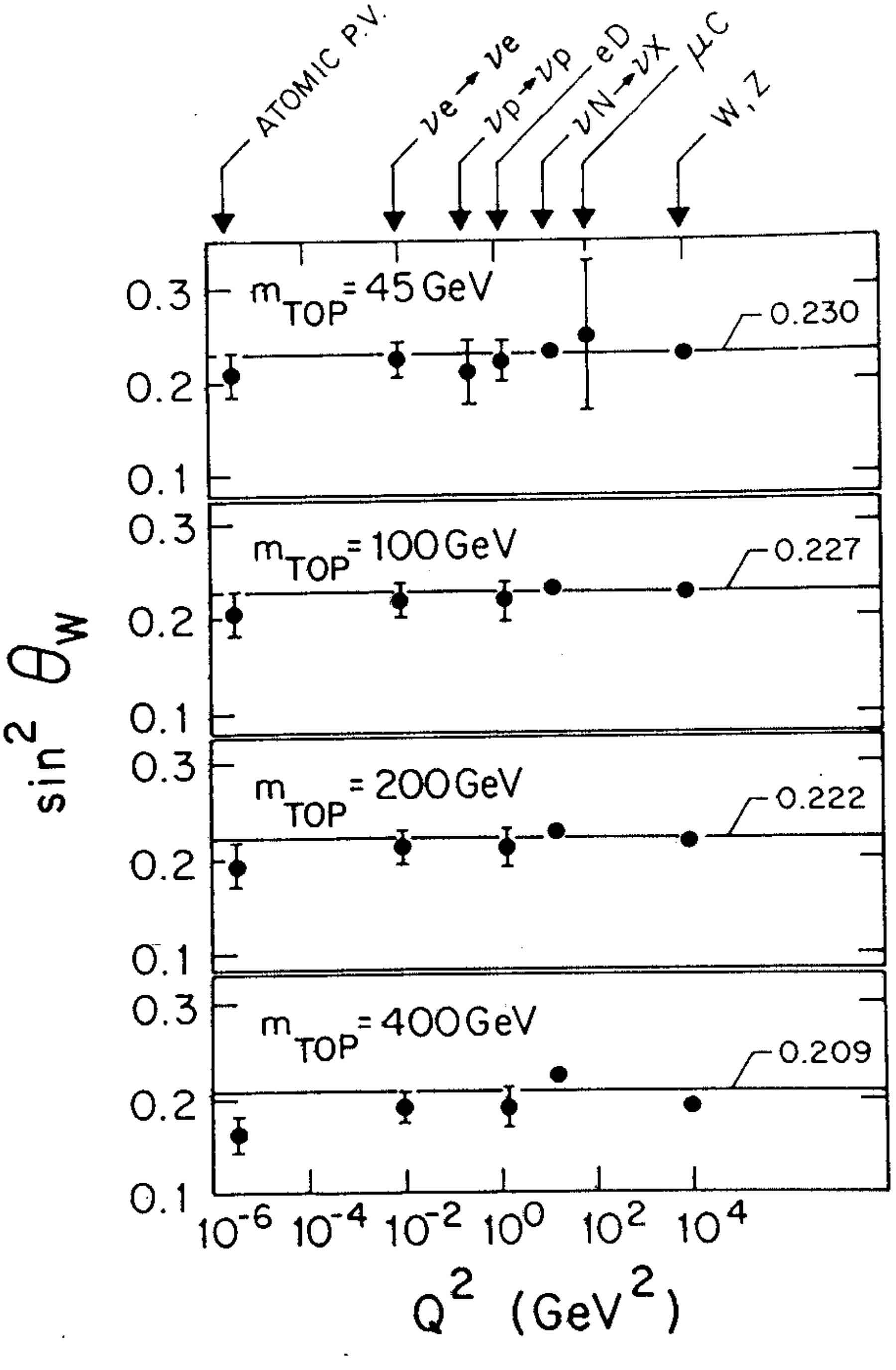

Figure 1: (a) $\sin^2\theta_W$ for various reactions as a function of the typical Q^2, determined for $m_t = 45\ GeV$. The best fit line $\sin^2\theta_W = 0.230$ is also shown. (b-d) $\sin^2\theta_W$ values determined for $m_t = 100, 200$, and $400\ GeV$.

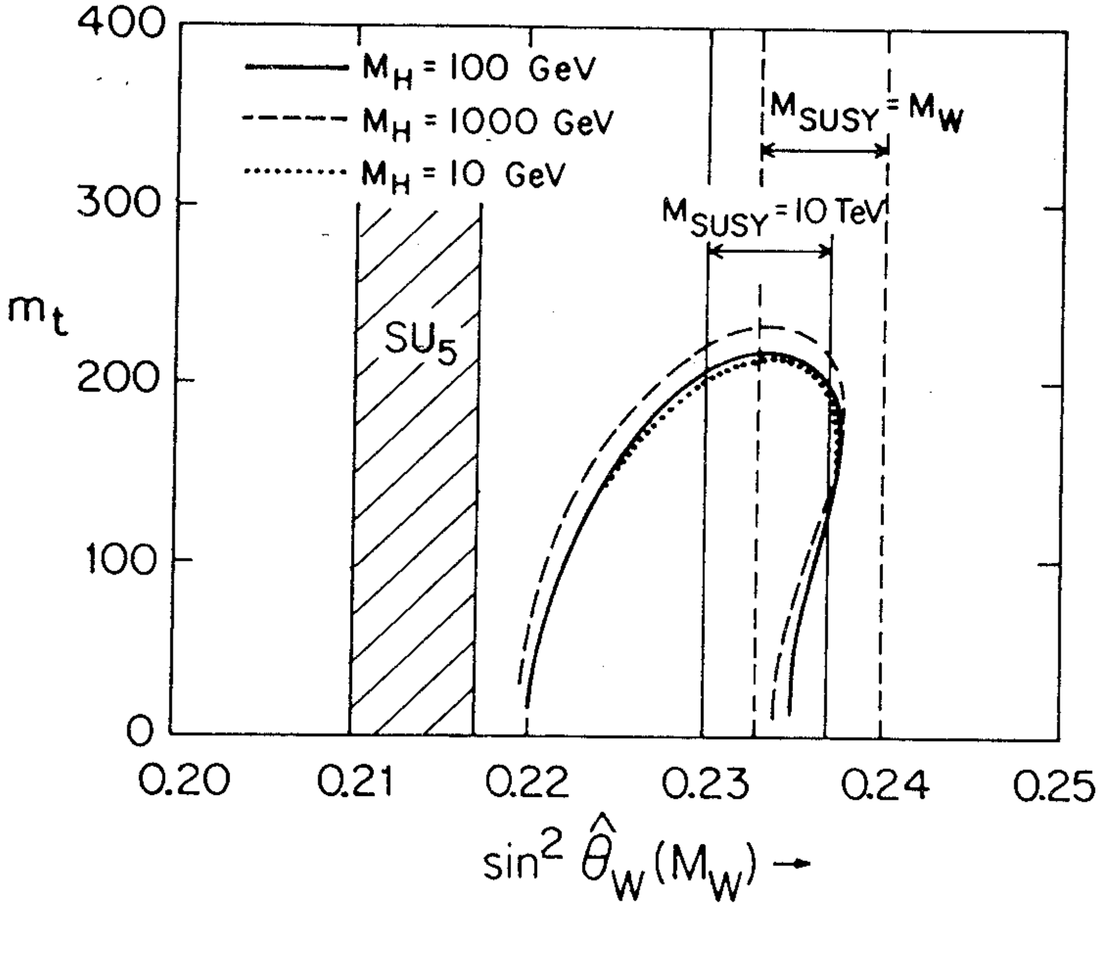

Figure 2: Allowed regions (90% c.l.) in $\sin^2 \hat{\theta}_W(M_W)$ and m_t for fixed values of M_H. Also shown are the predictions of ordinary and supersymmetric GUTs, assuming no new thresholds between M_W or M_{SUSY} and the unification scale.

Table 2: Determination of $\sin^2\theta_W$ and M_Z (in GeV) from various reactions. The central values of all fits assume $m_t = 45\ GeV$ and $M_H = 100\ GeV$ in the radiative corrections. Where two errors are shown the first is experimental and the second (in square brackets) is theoretical, computed assuming 3 fermion families, $m_t < 100\ GeV$, and $M_H < 1\ TeV$. In the other cases the theoretical and experimental uncertainties are combined. When m_t is allowed to be totally arbitrary the fits to all data yield $\sin^2\theta_W = 0.229 \pm 0.007$ and $M_Z = 91.8 \pm 0.9\ GeV$. The existing e^+e^- data do not yield a useful determination of $\sin^2\theta_W$: at PEP and PETRA energies the asymmetries are nearly an absolute prediction of the model, and all values of $\sin^2\theta_W$ from 0.1 to 0.4 give a good description of the data. (The e^+e^- asymmetries are nearly independent of m_t as well.)

Reaction	$\sin^2\theta_W$	M_Z
Deep inelastic (isoscalar)	$0.233 \pm .003 \pm [.005]$	$91.6 \pm 0.4 \pm [0.8]$
$\overset{(-)}{\nu}_\mu\, p \to \overset{(-)}{\nu}_\mu\, p$	$0.210 \pm .033$	95.0 ± 5.2
$\overset{(-)}{\nu}_\mu\, e \to \overset{(-)}{\nu}_\mu\, e$	$0.223 \pm .018 \pm [.002]$	93.0 ± 2.7
W, Z	$0.228 \pm .007 \pm [.002]$	92.3 ± 1.1
Atomic parity violation	$0.209 \pm .018 \pm [.014]$	95.1 ± 3.9
SLAC eD	$0.221 \pm .015 \pm [.013]$	93.3 ± 2.7
μC	$0.25 \pm .08$	89.6 ± 9.7
All data	0.230 ± 0.0048	92.0 ± 0.7

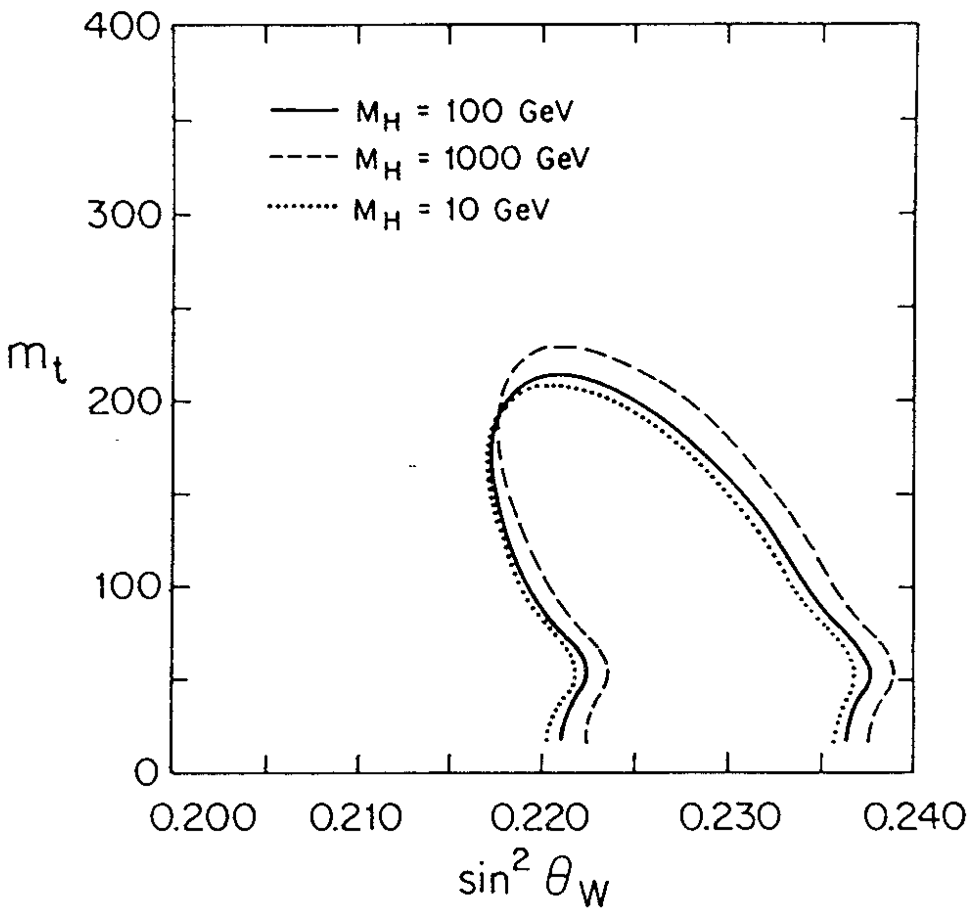

Figure 3: Allowed regions (90% c.l.) in $\sin^2\theta_W - m_t$ for Higgs masses of 10, 100, and 1000 GeV.

Table 3: The W and Z masses (in GeV). The first uncertainties are mainly statistical and the second are energy calibration uncertainties that are 100% correlated between M_W and M_Z for each group. The last two rows are predictions of the standard model, using $\sin^2\theta_W$ determined from deep inelastic scattering, with and without radiative corrections, respectively.

Group	M_W	M_Z
UA2 (ref 16)	$80.2 \pm 0.8 \pm 1.3$	$91.5 \pm 1.2 \pm 1.7$
UA1 (ref 17)	$83.5^{+1.1}_{-1.0} \pm 2.7$	$93.0 \pm 1.4 \pm 3.0$
UA1 + UA2 combined	80.9 ± 1.4	91.9 ± 1.8
Prediction from deep inelastic (with radiative corrections; $\sin^2\theta_W = 0.233 \pm .006$)	80.2 ± 1.1	91.6 ± 0.9
Prediction from deep inelastic (without radiative corrections; $\sin^2\theta^o = 0.242 \pm .006, \delta_W = 0$)	75.9 ± 1.0	87.1 ± 0.7

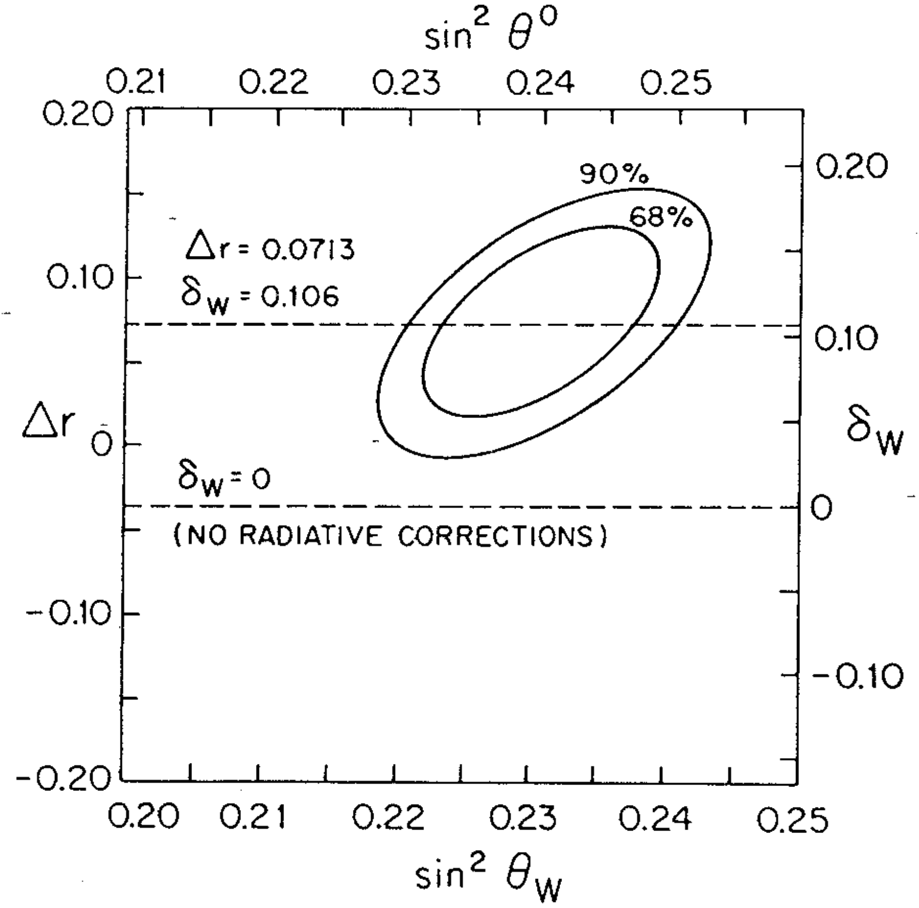

Figure 4: The allowed region in the $\sin^2\theta_W - \Delta r$ (or $\sin^2\theta^o - \delta_w$) plane determined from deep inelastic (isoscalar) data and the W and Z masses.

A related parameter[5] δ_W is defined by

$$M_W = \frac{A_o}{\sin\theta^o(1-\delta_W)^{\frac{1}{2}}}, \tag{30}$$

where $\sin^2\theta^o$ is the value ($.242 \pm .006$) obtained for the weak angle from deep inelastic scattering if all radiative corrections (to both σ^{NC} and σ^{CC}) are ignored. δ_W, which incorporates all of the radiative corrections relating deep inelastic scattering, the W and Z masses, and muon decay, is found to be 0.112 ± 0.037. This agrees with the prediction 0.106 ± 0.004 (for the same m_t and M_H as above) and establishes the existence of radiative corrections at the 3σ level.

W and Z decays

The partial decay width for gauge boson V to decay into massless fermions $f_1\bar{f}_2$ is

$$\Gamma(V \to f_1\bar{f}_2) = C\frac{M_V}{12\pi}\left(|\alpha|^2 + |\beta|^2\right), \tag{31}$$

where α and β are defined by the coupling

$$-L = V^\mu \bar{f}_1\gamma_\mu(\alpha + \beta\gamma^5)f_2. \tag{32}$$

For leptons $C = 1$, while for quarks $C = 3(1 + \frac{\alpha_s(M_V)}{\pi})$, where the 3 is due to color and the factor in parentheses is a QCD correction[19]. Corrections to (31) for massive fermions are given in Reference 19.

The appropriate α and β for $V = W^\pm$ or Z can be read off from equations (1), (3), and (5). One obtains

$$\begin{aligned}
\Gamma(W^+ \to e^+\nu_e) &= \frac{G_F M_W^3}{6\sqrt{2}\pi} \simeq 230\ MeV \\
\Gamma(W^+ \to u_i\bar{d}_i) &= \frac{C G_F M_W^3}{6\sqrt{2}\pi}|V_{ij}|^2 \simeq 717|V_{ij}|^2\ MeV \\
\Gamma(Z \to \psi_i\bar{\psi}_i) &= \frac{\rho C G_F M_Z^3}{6\sqrt{2}\pi}\left[V^{i2} + A^{i2}\right] \\
&\simeq \begin{cases} 170\ MeV\ (\nu\bar{\nu}),\ 85.4\ MeV\ (e^+e^-), \\ 305\ MeV\ (u\bar{u}),\ 394\ MeV\ (d\bar{d}), \end{cases}
\end{aligned} \tag{33}$$

where the numerical values assume $\sin^2\theta_W = 0.230$, $M_W = 80.7\ GeV$, and $M_Z = 91.9\ GeV$. (The factor of ρ in Γ_Z is relevant for generalizations of the standard model with $\rho \neq 1$.) Expressing the widths in terms of $G_F M_{W,Z}^3$ incorporates the bulk of the electroweak radiative corrections[19]. The remaining corrections introduce negligibly small corrections $1+\epsilon$, where ϵ is channel dependent; typically $|\epsilon| < (2-3) \times 10^{-3}$.

For 3 fermion families the total widths are (for the same $\sin^2\theta_W$, M_W, M_Z) $\Gamma_Z \sim (2.85-2.55)\ GeV$ and $\Gamma_W \sim (2.84-2.12)\ GeV$, where the range corresponds to m_t varying from zero to very large values, and the other fermion masses have been neglected. For $m_t = 45\ GeV$, for example, one has $\Gamma_Z \sim 2.58\ GeV$, $\Gamma_W \sim 2.52\ GeV$, and branching ratios

$$\begin{array}{llllll} B(W^+ \to e^+\nu_e) & = & 0.091 & B(W^+ \to u\bar{d}) & = & 0.29|V_{ud}|^2 \\ B(Z \to \nu_e\bar{\nu}_e) & = & 0.067 & B(Z \to e^+e^-) & = & 0.034 \\ B(Z \to u\bar{u}) & = & 0.12 & B(Z \to d\bar{d}) & = & 0.15, \end{array} \tag{34}$$

with analogous formulas for the other families. Additional light neutrino flavors are expected to increase Γ_Z by 170 MeV each and should be observable in precision measurements at SLC and LEP.

Deviations from the Standard Model

The W and Z masses and neutral current data can be used to search for and set limits on deviations from the standard model. For example,

$$\rho \equiv M_W^2/(M_Z^2\cos^2\theta_W) \tag{35}$$

can differ from unity if there are Higgs multiplets with weak isospin $> \frac{1}{2}$ with significant vacuum expectation values. One has the tree level result

$$\rho = \frac{\sum_i (t_i^2 - t_{3i}^2 + t_i) < \phi_i >^2}{\sum_i 2t_{3i}^2 < \phi_i >^2} \tag{36}$$

where $< \phi_i >$ is the VEV of a Higgs field ϕ_i with weak isospin and z-component t_i and t_{3i}, respectively. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters. It is convenient to take these as α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\sin^2\theta_W$ and ρ can be considered dependent parameters defined by

$$\sin^2\theta_W \equiv A_o^2/M_W^2(1-\Delta r) \tag{37}$$

and by equation (35), respectively.

If the new physics which yields $\rho \neq 1$ is a small perturbation which does not significantly affect the radiative corrections, one can use the standard model expressions for Δr and the other radiative correction parameters in Table 1. Then ρ can be regarded as a phenomenological parameter which multiplies L_{eff}^{NC} in (6). (Also, the expression for M_Z in (9) is divided by $\sqrt{\rho}$; the M_W formula is unchanged.) The allowed regions in the $\rho - \sin^2\theta_W$ plane are shown in Figure 5, and the values of $\sin^2\theta_W$ and ρ from various reactions are given in Table 4. A global

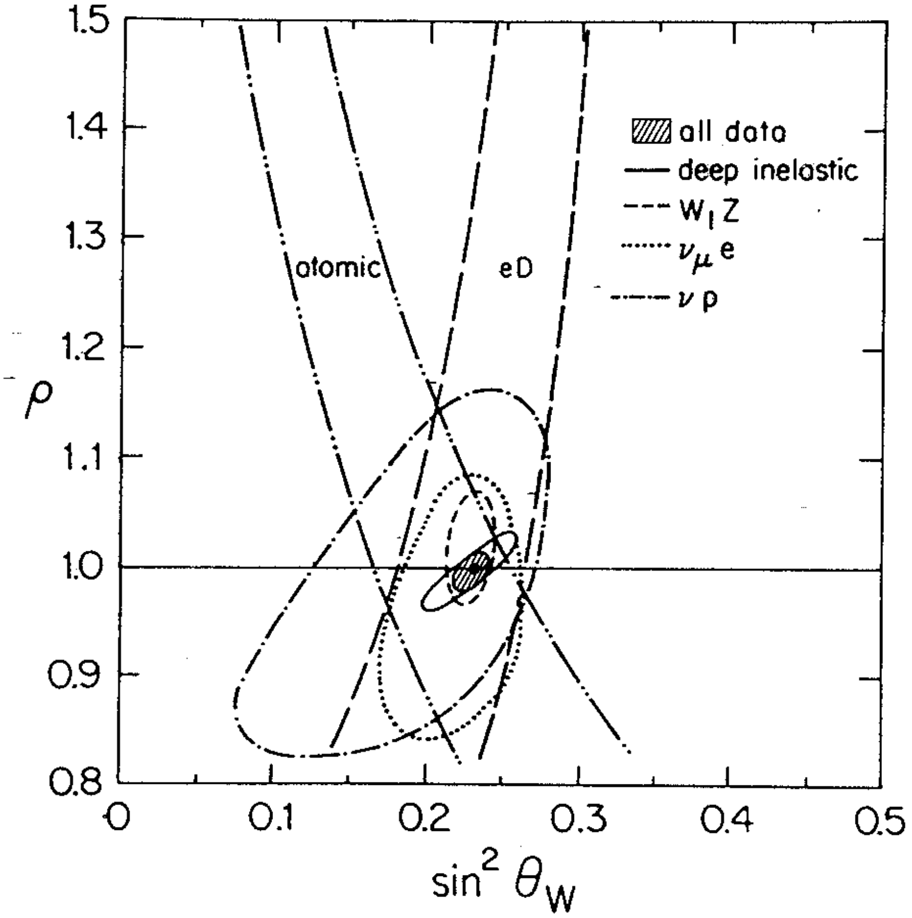

Figure 5: The allowed regions in $\sin^2\theta_W - \rho$ at 90% *c.l.* for various reactions.

Table 4: Determination of ρ and $\sin^2\theta_W$ from various reactions. Where two errors are shown the first is experimental and the second (in square brackets) is theoretical.

Reaction	$\sin^2\theta_W$	ρ	Correlation
deep inelastic (isoscalar)	$0.232 \pm 0.014 \pm [.008]$	$0.999 \pm .013 \pm [.008]$	.90
$\overset{(-)}{\nu}_\mu\, p \to \overset{(-)}{\nu}_\mu\, p$	$0.205 \pm .041$	$0.98 \pm .06 \pm [.05]$	—
$\overset{(-)}{\nu}_\mu\, e \to \overset{(-)}{\nu}_\mu\, e$	$0.221 \pm .021 \pm [.003]$	$0.976 \pm .056 \pm [.002]$	.12
$W,\ Z$	$0.228 \pm .008 \pm [.003]$	$1.015 \pm .026 \pm [.004]$	.19
all data	$0.229 \pm .0064$	$0.998 \pm .0086$	.63

Table 5: Values of $t_{3R}(i)$ (predicted to vanish in the standard model) obtained from neutral current data[5,20] assuming $\rho = 1$. Almost identical results are obtained if ρ is allowed to vary. Also shown are the 90% *c.l.* upper limits on the mixing $\sin^2 \alpha_{iH}$ between ψ_{Ri} and an exotic heavy fermion H_R, computed for the important special case that $t_{3R}(H)$ is the same as the T_3 eigenvalue $t_{3L}(i)$ of ψ_{Li}. (This would hold if H_R is a mirror fermion or occurs in a 27-plet of E_6). The value of $t_{3R}(b)$ incorporates $B^0 - \bar{B}^0$ mixing.

i	$t_{3R}(i)$	$\sin^2 \alpha_{iH}$
u	0.003 ± 0.010	0.035
d	0.007 ± 0.012	0.032
e^-	-0.001 ± 0.022	0.074
μ^-	0.035 ± 0.038	0.090
τ^-	-0.039 ± 0.054	0.23
c	-0.15 ± 0.15	0.34
b	0.04 ± 0.15	0.45

fit to all data yields[5]

$$\begin{aligned} \sin^2 \theta_W &= 0.229 \pm 0.0064 \\ \rho &= 0.998 \pm 0.0086, \end{aligned} \tag{38}$$

remarkably close to unity (justifying the neglect of $\rho - 1$ in the radiative corrections). This implies 90% *c.l.* upper limits of 0.047 and 0.081 for the VEVs (relative to those of Higgs doublets) for Higgs triplets with $t_{3i} = 0$ or ± 1, respectively. The theoretical component of the errors in (38) and Table 4 are computed assuming $m_t < 100$ *GeV* and $M_H < 1$ *TeV*. For m_t completely arbitrary there is a set of fine-tuned solutions in which the effects of large m_t and non-doublet Higgs fields (which can yield $\rho < 1$) approximately cancel. One then has the very weak limit $m_t < 530$ *GeV* at 90% *c.l.*, with the largest m_t corresponding to $\rho \sim 0.92$.

The neutral current data also place stringent constraints on the $SU_2 \times U_1$ assignments of the fermions. The standard model assumes $t_{3R}(i) = 0$ (defined in eqn. (5)). However, mixing between the right-handed component ψ_{Ri} of ψ_i and a heavy exotic right-handed fermion H_R with T_3 eigenvalue $t_{3R}(H)$ would induce $t_{3R}(i) = t_{3R}(H) \sin^2 \alpha_{iH}$, where α_{iH} is the $\psi_{Ri} - H_R$ mixing angle. Significant limits can be determined for all of the ordinary fermions except the s quark. They are listed in Table 5 along with the corresponding 90% *c.l.* limits on $\sin^2 \alpha_{iH}$ for an important special case. These limits assume that the $t_{3L}(i)$ (defined in (5)) are canonical. This is strongly suggested by numerous charged current constraints; *i.e.* mixings of ψ_{Li} with exotic left-handed fermions would affect charged-current universality.

Closely related to the $t_{3R}(i)$ constraints are the values of the axial vector couplings $A^i \equiv t_{3L}(i) - t_{3R}(i)$ of ψ_i to Z. For $i = \mu,\ \tau,\ c$, and b these are determined by the forward-backward asymmetries in $e^+e^- \to \psi_i\bar{\psi}_i$. Assuming canonical A^e (the vector couplings are also assumed canonical, but the correlations are extremely weak) one obtains $A^\mu = -0.54 \pm 0.03$, $A^\tau = -0.46 \pm 0.05$, $A^c = 0.65 \pm 0.15$, and $A^b = -0.54 \pm 0.15$, to be compared with the values $-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$, and $-\frac{1}{2}$, respectively, predicted by the standard model and by $e-\mu-\tau$, $u-c$, and $d-s-b$ universality. The value of A^b rules out almost all topless models, which generally predict 0 or $+\frac{1}{2}$. (Many topless models are also excluded by the long b lifetime and by limits on flavor-changing neutral current b decays[21].)

Many extensions the standard model predict the existence of additional Z bosons. For the simplest case (one extra Z) the physical (mass eigenstate) bosons are

$$\begin{aligned} Z_1 &= Z_1^o \cos\theta + Z_2^o \sin\theta \\ Z_2 &= -Z_1^o \sin\theta + Z_2^o \cos\theta \end{aligned} \tag{39}$$

where, under reasonable assumptions, the lighter boson Z_1 is the particle observed by UA1 and UA2, Z_1^o is the $SU_2 \times U_1$ boson which couples to $g_1 J_Z^\mu/2$, where $g_1 \equiv g/\cos\theta_W$, and J_Z^μ is defined in (5); Z_2^o couples to a new current $g_2 J_2^o$, and θ is a mixing angle. The extra Z manifests itself *(a)* because the Z_1 mass is reduced by mixing, *(b)* because the Z_1 couplings are modified by mixing, and *(c)* by Z_2 exchange.

As an important example of an extra Z, we consider $Z(\beta) = \cos\beta Z_\chi + \sin\beta Z_\psi$, with $(g_2/g_1)^2 = 5\lambda_\beta \sin^2\theta_W/3$. This is the extra boson in E_6 models which break to $G \times U_{1\beta}$, where G contains the standard model. Z_χ and Z_ψ are the gauge bosons which occur for $SO_{10} \to SU_5 \times U_{1\chi}$ and $E_6 \to SO_{10} \times U_{1\psi}$, respectively. The special case $Z_\eta = \sqrt{\frac{3}{8}} Z_\chi - \sqrt{\frac{5}{8}} Z_\psi = -Z(\beta = \pi - \tan^{-1}\sqrt{\frac{5}{3}})$ occurs in many superstring models.

We will follow the formalism in Ref. 22. We assume $\lambda_\beta = 1$, which occurs if the underlying group breaks directly to $SU_3 \times SU_2 \times U_1 \times U_1'$ (limits on M_{Z_2} and θ scale roughly as $\sqrt{\lambda}$ and $1/\sqrt{\lambda}$, respectively). Limits on M_{Z_2} and θ are presented for two cases:

a) The constrained Higgs case. This case, which is the analog of $\rho = 1$ in $SU_2 \times U_1$, occurs if all $SU_2 \times U_1$ breaking is due to Higgs doublets (this is expected in superstring models). The free parameters are $\sin^2\theta_W$, M_{Z_2}, and θ. M_{Z_1} is related by

$$\tan^2\theta = \frac{M_o^2 - M_{Z_1}^2}{M_{Z_2}^2 - M_o^2}, \tag{40}$$

where $M_o = M_W/\cos\theta_W$ would be the Z_1^o mass in the absence of mixing. *(b)* The unconstrained Higgs case. This is the analog of $\rho \neq 1$ and occurs if there are Higgs triplets, *etc.*, with significant VEV's. In this case $\sin^2\theta_W, M_{Z_1}, M_{Z_2}$, and θ are all arbitrary.

The 90% *c.l.* lower limits on the Z_2 masses for the constrained and unconstrained cases, respectively, are 273 and 249 GeV (Z_χ), 154 and 151 $GeV(Z_\psi)$, 111 and 112 $GeV(Z_\eta)$, and 325 and 343 GeV (Z_{LR}, the extra boson which occurs in $SU_{2L}\times SU_{2R}\times U_1$ models.) The 90% *c.l.* lower limits on M_{Z_2} and θ range are shown for $Z(\beta)$ as a function of $\cos\beta$ in Figure 6. $|\theta|$ must be smaller than $\simeq 0.05$ except for a small region near the Z_η. The relative weakness of the constraints on Z_2 is due to the fact that most theoretically favored extra Z's tend to couple fairly weakly to the ordinary quarks and leptons (and more strongly to exotic fermions such as heavy Majorana neutrinos[23]).

At the present time the indirect limits on heavy Z's (with $\lambda\sim 1$) from the neutral current§ are somewhat more stringent[22,25] than limits from direct searches $\bar{p}p\to Z_2+X,\ Z_2\to l^+l^-$ at the $S\bar{p}pS$ except for a small region in β near the Z_η. Nevertheless, the limits (typically $120-300$ GeV) are still relatively weak. In constrast, there is a non-rigorous but plausible lower limit[26] from the K_L-K_S mass difference of several TeV on the mass of the new charged bosons in many $SU_{2L}\times SU_{2R}\times U_1'$ models. This situation will presumably change in the near future: for example, the FNAL $\bar{p}p$ collider should be sensitive to bosons up to around 400 GeV and the SSC would be sensitive up to several TeV; indirect searches at LEP, SLC, and HERA should also be very useful[27], especially for constraining the mixing angle θ.

Most of the parameters relevant to ν-hadron, νe, e-hadron, and e^+e^- processes are now determined uniquely and precisely from the data in "model independent" fits (*i.e.* fits which allow for an arbitrary electroweak gauge theory). The values for the parameters defined in Equations (11) - (13) are given in Table 6 along with the predictions of the standard model for $\sin^2\theta_W=0.230$. The agreement is excellent. The e^+e^- results are difficult to present in a model independent way because Z-propagator effects are non-negligible at PETRA and PEP energies. However, assuming $e-\mu-\tau$ universality the lepton asymmetries imply $A^e=-0.511\pm0.013$, in good agreement with the standard model prediction $-\frac{1}{2}$. The vector coupling is not well determined by existing e^+e^- data: values of V^e from -0.3 to 0.3 are allowed.

All existing data are consistent with the standard $SU_2\times U_1$ model, which is now clearly established as correct to first approximation. Electroweak radiative corrections have been established at the 3σ level, and there are significant limits on many possible deviations. Future high precision measurements such as M_Z, e^+e^-

§In some models comparable limits have been obtained[24] from electroweak radiative corrections to charged current data.

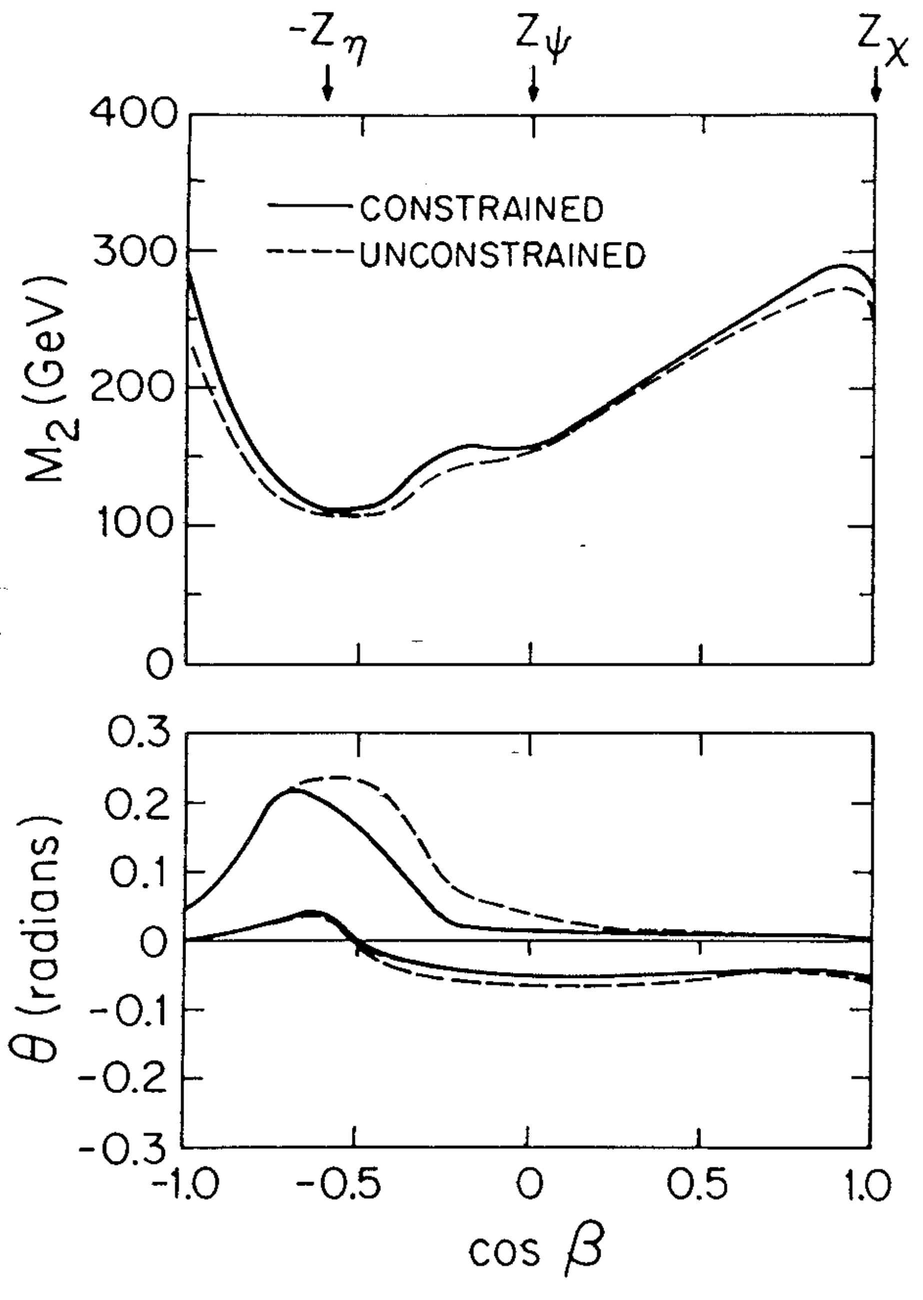

Figure 6: Lower limits on M_{Z_2} and allowed θ range (both at 90% *c.l.*) for an E_6 boson $Z(\beta) = \cos\beta\ Z_\chi + \sin\beta\ Z_\psi$ for constrained and unconstrained Higgs. The special cases Z_χ, Z_ψ, and $-Z_\eta$ are indicated.

Table 6: Values of the model independent neutral current parameters, compared with the standard model prediction for $\sin^2\theta_W = 0.230$. Correlations are not given for the neutrino-hadron couplings because of the non-Gaussian χ^2 distributions. However, the neutrino-hadron constraints are accurately represented by the ranges listed for the variables $g_i^2 \equiv \epsilon_i(u)^2 + \epsilon_i(d)^2$ and $\theta_i \equiv \tan^{-1}(\epsilon_i(u)/\epsilon_i(d))$, $i = L$ or R, which are very weakly correlated. There is a second $g_{V,A}^e$ solution, given approximately by $g_V^e \leftrightarrow g_A^e$, which is eliminated by e^+e^- data under the assumption that the neutral current is dominated by the exchange of a single Z.

Quantity	Experimental Value	Standard Model Prediction	Correlation	
$\epsilon_L(u)$	$0.339 \pm .017$	0.345		
$\epsilon_L(d)$	$-0.429 \pm .014$	-0.427		
$\epsilon_R(u)$	$-0.172 \pm .014$	-0.152		
$\epsilon_R(d)$	$-0.011^{+.081}_{-.057}$	0.076		
g_L^2	0.2996 ± 0.0044	0.301		
g_R^2	0.0298 ± 0.0038	0.029		
θ_L	2.47 ± 0.04	2.46		
θ_R	$4.65^{+0.48}_{-0.32}$	5.18		
g_A^e	$-0.498 \pm .027$	-0.503		-0.08
g_V^e	$-0.044 \pm .036$	-0.045		
C_{1u}	-0.249 ± 0.071	-0.191	-0.98	-0.88
C_{1d}	0.381 ± 0.064	0.340		0.88
$C_{2u} - \frac{1}{2}C_{2d}$	0.19 ± 0.37	-0.039		

polarization and forward-backward asymmetries at SLC and LEP, and planned and proposed νe elastic scattering experiments are expected to extend the sensitivity of these tests and searches dramatically.

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