DEUTSCHES ELEKTRONEN – SYNCHROTRON DESY

DESY 87-154
November 1987

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by

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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 87-154
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ISSN 0418-9833

THE MASS OF THE HIGGS PARTICLE[1)]

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ABSTRACT

I review bounds on the mass of the Higgs particle and discuss the naturalness of the Fermi scale within the context of the standard model. For a Higgs particle mass below 140 GeV the standard model is consistent up to energy scales around the Planck mass. The small ratio between the Fermi scale and the Planck mass M_p is as natural as the ratio m_e/M_p for QED.

1) Talk presented at the Trieste conference "Search for Scalar Particles: Experimental and Theoretical Aspects", July 1987

1. INTRODUCTION

A gauge theory for electroweak interactions with massive W and Z bosons is not renormalizable unless W and Z acquire their mass from spontaneous symmetry breaking. This necessitates introduction of a scalar field[1] (operator) whose vacuum expectation value (vev) is different from zero, implying that the symmetry of the ground state is lower than the symmetry of the action. In the standard model[2] this scalar belongs to the so called weak doublet. The excitations above the ground state comprise massless and massive spin one bosons and one neutral massive scalar, the Higgs scalar. If we could move the mass of the Higgs scalar to infinity while keeping its vev fixed, the scalar mode would decouple from the effective low energy theory and we would recover the unrenormalizable model with massive W and Z without a scalar excitation. We therefore know that there must be some upper bound on the mass of the Higgs particle at which the renormalizability of the standard model breaks down. To get a first qualitative intuition on the magnitude of this bound we may look at tree level scattering $W^+W^- \rightarrow W^+W^-$. For low enough Higgs particle mass M_H the graphs with exchange of a Higgs particle (see fig 1) partially cancel the contributions from other graphs and the amplitude is consistent with unitarity. For very high M_H, however, the Higgs exchange graphs can be neglected in a region where the dynamical variables are in the range of a few TeV or below. Tree level unitarity breaks down in this energy region. Lee, Quigg and Thacker[3] have estimated the tree level unitarity bound $M_H<1\text{TeV}$, whereas validity of perturbation theory needs $M_H<600\text{GeV}$.

This leads us to a first important conclusion on possible alternatives for physics in the TeV region:

A) Either the description of physics by the standard model breaks down at an energy scale Λ around 1 TeV. New physics appears at these energies and the standard model is only an approximation for momenta smaller than Λ. The scale Λ may be viewed as a physical cutoff for the standard model.

B) Or the scale of new physics is higher (Λ >5TeV). In this case it must be possible to associate to the symmetry breaking scalar operator a (low energy) scalar field and the Higgs particle exists as an observable scalar excitation. Its mass M_H must be below 1 TeV. We will formulate later the bound on M_H in dependence on Λ more precisely.

At this place we should mention that in principle

there could be a possible intermediate case between alternatives A) and B), namely a strongly interacting Higgs scalar with mass in the TeV region. We will see, however, that all evidence suggests that such a high scalar mass requires new physics with Λ in the same energy region. We therefore can include this case into A).

In the standard model, spontaneous symmetry breaking is described by the minimum of an effective potential for the weak doublet φ. In the tree approximation it has the form

$$V_0(\varphi) = -\mu_\varphi^2 \varphi^+\varphi + \frac{1}{2}\lambda(\varphi^+\varphi)^2 \quad (1)$$

The minimum occurs for

$$|\langle\varphi\rangle|^2 = \varphi_0^2 = \mu_\varphi^2/\lambda \quad (2)$$

and the mass of the physical neutral Higgs scalar and the W boson obtains as

$$M_H^2 = 2\lambda\varphi_0^2 \quad (3)$$

$$M_W^2 = \frac{1}{2} g_2^2 \varphi_0^2 \quad (4)$$

The Fermi scale φ_0 can be directly measured from muon decay etc.

$$\varphi_0 = 174\ \text{GeV} \quad (5)$$

It seems actually more natural to parametrize the effective potential in the spontaneously broken phase by directly observable renormalized parameters φ_0 and λ (or M_H)

$$V_0 = \frac{1}{2}\lambda\,(\varphi^+\varphi - \varphi_0^2)^2 \qquad (6)$$

In the tree approximation λ is directly related to the ratio of physical masses M_H/M_W:

$$\lambda = \frac{M_H^2}{4M_W^2}\, g_2^2 \,. \qquad (7)$$

The two main parts of this talk give a (personal) summary of what is known theoretically about the two parameters λ and φ_0. I will first discuss bounds on M_H (or rather on M_H/M_W) and subsequently turn to the Fermi scale φ_0 and the associated "naturalness" questions. Throughout this talk I remain in the context where low energy physics is described by the standard model with only one low energy scalar doublet. Most of the arguments can be generalized to extensions like supersymmetric theories or the case of several doublets. I will argue, however, that such extensions are not necessary.

A couple of years ago Cabibbo, Maiani, Parisi and Petronzio[4] have investigated the conditions that a perturbative treatment of the standard model remains valid up to a high scale $\Lambda \approx 10^{15}$ GeV.

Their bounds on M_H and the top quark mass m_t are shown in fig.2. I will argue that (at least part of) these bounds not only indicate a breakdown of perturbation theory but rather are consistency conditions of the full theory if the standard model remains valid up to a given physical cutoff. On the other hand, these are the <u>only</u> consistency conditions. For any value of M_H and m_t well within the allowed region the standard model can be extended far above the Planck mass M_p. Around M_p gravity effects become important. One expects that this scale corresponds to a physical cutoff at which the standard model should be replaced by a unified theory including gravity. Nevertheless, a theory of electroweak and strong interactions consistent up to $\Lambda \approx M_p$ has the same profound status as QED.

The central tool for a qualitative and quantitative understanding of bounds on M_H is the one loop renormalization group equation (RGE) for λ. (This is similar to QCD where overall features like asymptotic freedom and the appearance of Λ_{QCD} as a scale related to confinement are already manifest in the one loop RGE for the strong gauge coupling.) Neglecting contributions from Yukawa couplings of light quarks and leptons one obtains[5] ($t=\ln\mu$)

$$\frac{d\lambda}{dt} = \beta_\lambda = \frac{1}{16\pi^2}\left\{12\lambda^2 - 12h_t^4 + 12h_t^2\lambda\right.$$

$$+\frac{9}{4} g_2^4 + \frac{9}{10} g_2^2 g_1^2 + \frac{27}{100} g_1^4 \tag{8}$$
$$- 9 g_2^2 \lambda - \frac{9}{5} g_1^2 \lambda \Big\}$$

The one loop RGE for the Yukawa coupling h_t of the top quark ($m_t = h_t \varphi_0$) is

$$\frac{dh_t}{dt} = \beta_t = \frac{1}{16\pi^2}\left\{\frac{9}{2} h_t^3 - 8 g_3^2 h_t - \frac{9}{4} g_2^2 h_t - \frac{17}{20} g_1^2 h_t\right\} \tag{9}$$

Here g_3, g_2 and g_1 are the gauge couplings of SU(3), SU(2) and U(1). We will see that except for a possible small region aroung the physical cutoff equations (8) and (9) can be supposed to be reliable approximations. This is firmly established for the case of small gauge couplings (as observed) and small h_t, whereas for large h_t nonperturbative work remains to be done in order to substantiate further this conclusion.

2.) TRIVIALITY OF φ^4-THEORY

Let me first consider the case $g_i^2 \ll \lambda$, $h_t^2 \ll \lambda$. This is related to the bound on M_H in the region indicated in fig.3. We can neglect all other degrees of freedom and therefore deal with a pure (four component) φ^4 theory. The one loop RGE becomes

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2} \lambda^2 \tag{10}$$

and has the simple solution

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{4\pi^2} \ln \frac{\Lambda}{\mu} \tag{11}$$

One immediately notices that for any arbitrary positive coupling $\lambda(\Lambda)$ at the scale Λ the renormalized coupling at some physical scale μ goes logarithmically to zero as Λ/μ goes to infinity. This is the heart of the so called triviality[6] of the φ^4 theory. (It is the analogue to asymptotic freedom in QCD. However, since the sign of the β function is opposite, the infrared and ultraviolet limits are interchanged.) We may turn this argument around and draw trajectories (fig.4) for a given value of the renormalized coupling $\lambda(\varphi_0)$ at the Fermi scale. For nonvanishing positive $\lambda(\varphi_0)$ the theory is consistent only for energies below a critical scale μ_c. This gives an upper bound on the physical cutoff $\Lambda \leqslant \mu_c$. One concludes that the φ^4 theory with $\lambda(\varphi_0) > 0$ cannot be extended to infinitely short distances!

However, the scale dependence of λ is only logarithmic. For $M_H \leqslant 130$ GeV the theory remains consistent far beyond M_p. In the one loop approximation the situation is then analogue to the Landau pole in QED. (The RGE for the electromagnetic coupling has a similar structure as (10)). If one believes that a more unified theory should replace the standard model at a scale Λ around or below M_p, one obtains an upper bound on the Higgs particle mass in dependence of this physical cutoff through the inequality

$$\lambda(\varphi_0) < \frac{4\pi^2}{3\ln(\Lambda/\varphi_0)} \qquad (12)$$

Parametrizing $\Lambda/\varphi_0 = 10^x$ leads to the simple formula

$$M_H \lesssim \bar{M}_H = x^{-\frac{1}{2}} \cdot 590\ GeV \qquad (13)$$

I give a few values in table 1. One finds for Λ below 1 TeV that Λ and $\bar{M}_H$ are in the same order of magnitude and perturbation theory becomes unreliable. Also, the bound has to be corrected for the contributions of g_i and h_t to the RGE, which lead to the more precise estimate in fig2.

So far I have used the one loop RGE to demonstrate "triviality" and derive the bound on M_H. Triviality is connected to the divergence of $\lambda(\mu)$ for $\mu \to \mu_c$ (fig.4). In the region near μ_c the coupling λ grows huge and perturbation theory breaks down. Thus triviality can only be established by a nonperturbative treatment. Actually, the one loop RGE suggests that even starting with a very strong $\lambda(\Lambda)$ the Higgs coupling falls very rapidly and perturbation theory applies already at a scale $\mu_p \approx (1/2)\Lambda$. It would be sufficient to establish this feature nonperturbatively, namely that for arbitrary $\lambda(\Lambda)$ perturbation theory becomes valid at a scale

$\mu_p \approx (1/c)\Lambda$ with c not much bigger than one. If this holds, the bound on M_H derived within perturbation theory is valid up to an uncertainty in the ratio μ_p/Λ. Details of the nonperturbative treatment would only be needed to determine c. (For μ_p near Λ the value of c depends on the regularization used to define the φ^4-theory).

Nonperturbative studies of the lattice φ^4-theory over the last couple of years show convincing evidence that this theory is indeed "trivial". The methods used include strong coupling expansions[7], Monte Carlo simulations[8], exact inequalities[9] and rigorous block spin renormalization group equations[10]. Recently, Lüscher and Weisz[11] have performed a three loop perturbation analysis combined with a strong coupling expansion for the (one component) lattice φ^4 theory. They found that the expansions for weak and strong λ overlap and that perturbation theory indeed becomes a valid approximation for $\mu_p \approx (1/2)\Lambda$. Their results for the evolution of $\lambda(\mu)$ for $\lambda(\Lambda) \to \infty$ are shown in fig.5. They are confirmed by a high statistics numerical simulation[12]. In fig.6 I reproduce their result for the bound on M_H/φ_0 in dependence on Λ/M_H. There is little doubt that similar results apply to the four component φ^4-theory.

We still have to ask if for the standard model the approximation of negligible gauge couplings and Yukawa couplings remains valid in the nonperturbative region where λ becomes strong. Fortunately, there is a Schwinger-Dyson type expansion in the small couplings g_i and h_t which remains valid even for strong λ[13]. One finds that the β function for g_i is always $\sim g^3_i$ and β_t has contributions $\sim h^3_t$, $g^2_i h_t$. For small enough g_i and h_t the corresponding β functions are therefore small and these couplings evolve only slowly. If the approximation g^2_i, $h^2_t \ll \lambda$ is valid at the scale φ_0 it remains valid over the whole range of interest $\varphi_0 < \mu < \Lambda$. In particular, the nonperturbative results for the pure φ^4 theory apply to the standard model.

This allows to draw conclusions for the standard model in the case where the heaviest quark mass is not very large (m_t < 120 GeV):

1) For large $\lambda(\varphi_0)$ (say M_H > 140 GeV) the standard model becomes inconsistent at some scale Λ as a result of triviality of φ^4 theory. At or below this scale a consistent description of nature needs the introduction of new physics. This necessity arises independently of gravity. For M_H > 170 GeV the scale Λ must be substantially lower than the Planck mass M_p.

2) A strong coupling $\lambda(\Lambda)$ decreases very rapidly below the cutoff. Even for $\lambda(\Lambda)\to\infty$ one finds $\lambda(\mu)$ within the range of validity of perturbation theory for $\mu_p \lesssim (1/2)\Lambda$. This implies that no

genuine strong coupling φ^4-theory exists! A strongly coupled Higgs (M_H > 600 GeV) is only possible if there is also additional new physics at Λ around 1 TeV. In this case high energy experiments may be more successful by looking directly for signals of this new physics rather than specializing to the Higgs mass. The Higgs scalar would no longer be singled out particularly among other excitations.

3) The one loop RGE is justified for a qualitative analysis. It reproduces important features of nonperturbative physics even outside the region of its applicability. The quantitative results of pertubative RGE apply (up to a small uncertainty how "near" the cutoff Λ it remains valid). This has an analogue in QCD where perturbation theory remains valid down to energy scales not far from Λ_{QCD}.

4) For a given physical cutoff Λ there is indeed an upper bound on $\lambda(\varphi_0)$ and therefore on M_H.

5) For small $\lambda(\varphi_0)$ (M_H < 140 GeV) perturbation theory remains valid far above M_p. The issue of "triviality" of the standard model depends now also on the behaviour of g_i and h_t.

3.) INFRARED FIXPOINT AND INTERVAL FOR STRONG YUKAWA COUPLING

Let me now turn to the case of stronger Yukawa couplings (m_t > 120 Gev) connected to the bound on M_H in the region depicted in Fig.7. We can neglect the gauge couplings in a first approximation and the one loop RGE simplifies

$$\frac{d\lambda}{dt} = \frac{3}{4\pi^2}\left\{\lambda^2 + h_t^2\lambda - h_t^4\right\} \qquad (14)$$

$$\frac{dh_t}{dt} = \frac{9}{32\pi^2} h_t^3 \qquad (15)$$

The RGE (15) has a solution similar to (11). This suggests again that an arbitrary strong Yukawa coupling $h_t(\Lambda)$ becomes weak very fast and that perturbation theory applies for the coupled system of scalar and fermions at a scale not far below the cutoff, $\mu_p = (1/c)\Lambda$. I will assume that this is indeed the case, although I should emphasize that a nonperturbative proof of this statement remains to be done. With this assumption the coupled system of scalars and fermions is "trivial": both $\lambda(\mu)$ and $h_t(\mu)$ vanish for $\Lambda/\mu \to \infty$.2)

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2) If the coupled system is not trivial, there should exist a new phase with strong h_t and λ. This could have important consequences for an explanation of the gauge hierarchy[13].

The RGE (14) has an interesting structure: for $h_t^2 >> \lambda$ the coupling $\lambda(\mu)$ increases due to the term $\sim - h_t^4$ as μ becomes smaller, whereas for $h_t^2 << \lambda$ it decreases. This suggests that there is some intermediate value for λ/h_t^2 where both effects become comparable. Indeed, for strong enough h_t the RGE for the ratio λ/h_t^2,

$$\frac{d}{dt}\left(\frac{\lambda}{h_t^2}\right) = \frac{h_t^2}{16\pi^2}\left\{12\left(\frac{\lambda}{h_t^2}\right)^2 + 3\,\frac{\lambda}{h_t^2} - 12\right\} \qquad (16)$$

is governed by an infrared fixpoint.[13] The ratio λ/h_t^2 remains constant if the right hand side of (16) vanishes. This happens for

$$\frac{\lambda}{h_t^2} = \left(\frac{65}{64}\right)^{1/2} - \frac{1}{8} = x_0 \qquad (17)$$

and corresponds to a mass ratio

$$\frac{M_H}{m_t} \approx 1.3 \qquad (18)$$

Corrections due to nonvanishing gauge couplings lower the fixpoint ratio M_H/m_t somewhat. The fixpoint corresponds ot the edge in the upper right corner of the allowed region in fig.7. The situation is similar[13] if a heavy top quark is replaced by a heavy quark pair of a possible fourth generation.

If the coupled system of scalar and quarks has no ultraviolet fixpoint (as for a "trivial" theory) the infrared fixpoint value (17), (18) would be an exact prediction as $\Lambda/\varphi_0 \to \infty$. With a physical cutoff Λ at or below M_p, however, the ratio Λ/φ_0 is finite. The prediction of values for the coupling constants being given by the infrared fixpoint is replaced by a finite <u>infrared interval</u> I_0 of allowed couplings at the scale φ_0. (The size of I_0 shrinks to zero as Λ/φ_0 goes to infinity.) In our context this can be understood by noting that the β function (16), which determines the approach to the fixpoint, is proportional to the Yukawa coupling h^2_t. For small values of h_t the function is small even for large or small ratios λ/h^2_t. The ratio λ/h^2_t changes therefore very little between Λ and φ_0 even if it is not close to the fixpoint value (17). The existence of an infrared fixpoint becomes irrelevant for the evolution over a finite ratio Λ/φ_0. In contrast, for large h_t the β function (16) takes large positive or negative values unless λ/h^2_t is near the fixpoint (17). In this case the fixpoint is approached very rapidly and (18) becomes a prediction.

More generally, the analysis of IR fixpoints should be replaced by an analysis of IR intervals in the case of "trivial" theories with finite cutoff Λ. The IR intervals are situated around IR fixpoints. Any interval I_Λ of allowed couplings at the cutoff Λ will be mapped by the renormalization group into an interval I_0 for the renormalized couplings at $\varphi_0 < \Lambda$. If I_Λ is within the range of

attraction of an IR fixpoint within I_0, then I_0 is always contained in I_Λ and smaller than I_Λ. For small couplings the β functions are small so that I_0 and I_Λ approximately coincide. For "trivial" theories an infinite I_Λ will be mapped onto a finite I_0. This implies bounds on the allowed couplings in dependence on Λ/φ_0. The bounds on M_H and m_t in fig.3 and 7 correspond to the IR interval for the combined system of λ and h_t.

For an analysis of I_0 for the ratio λ/h^2_t we introduce new variables x and s

$$x = \frac{\lambda}{h_t^2} - x_0 \qquad (19)$$

$$\frac{ds}{dt} = h_t^2(t) \qquad (20)$$

with h(t) the Yukawa coupling running according to (15). With the new evolution parameter s the RGE for the deviation x of λ/h^2_t from the fixpoint x_0 reads

$$\frac{dx}{ds} = \frac{3}{4\pi^2}\, x \left(x + 2x_0 + \frac{1}{4} \right) \qquad (21)$$

For $x \gg 2x_0 + 1/4$ we find the same behaviour as the one loop RGE (10) for λ in the pure φ^4 theory. Arbitrarily large values of x at Λ get therefore renormalized to values of the order x_0 if $s(\Lambda) - s(\mu)$ is of order one. The only difference from (11) is that $\ln\Lambda/\mu$ is now replaced by

$$s(\Lambda) - s(\mu) = \frac{32\pi^2}{9} \ln \frac{h_t(\Lambda)}{h_t(\mu)} . \qquad (22)$$

On the other hand, values $|x| \ll 2x_0 + 1/4$ follow a RGE with anomalous dimension

$$\frac{dx}{ds} = A_x x \quad , \quad A_x = \frac{3}{4\pi^2} \left(2x_0 + \frac{1}{4} \right) \qquad (23)$$

which leads to a typical power law behaviour

$$\frac{x(\mu)}{x(\Lambda)} = \left(\frac{h_t(\mu)}{h_t(\Lambda)} \right)^{\frac{16x_0+2}{3}} \qquad (24)$$

Combining the results for small and large $|x|$ one sees immediately that the interval I_Λ : $-x_0 < x < \infty$ shrinks to a finite interval I_0 for the allowed ratios $\lambda(\varphi_0)/h^2_t(\varphi_0)$. Very large λ/h^2_t ($x \to \infty$) or small λ/h^2_t ($x \to -x_0$) become inconsistent. The size of I_0 depends on the ratio $h_t(\Lambda)/h_t(\varphi_0)$. For $h_t(\Lambda) \to \infty$ the interval I_0 shrinks to the fixpoint. As mentioned before, this is the edge in the upper right hand corner of fig.7. For finite values $h_t(\Lambda)$ the allowed interval I_0 is finite and it increases as $h_t(\Lambda)$ decreases. This explains the opening of the allowed region in fig.7 as m_t becomes smaller than its maximum allowed value $\bar{m}_t$ ($\approx$330 GeV for $\Lambda = 10^{15}$ GeV). For strong Yukawa couplings the Higgs coupling λ has not a very independent existence anymore. It has to move in a sort of slavery, dominated by the evolution of h_t. I expect this feature to generalize to a nonperturbative treatment of the system.

For the region 80 GeV $< m_t <$ 120 GeV the Yukawa coupling is small and the evolution of λ does not "feel" the existence of an IR fixpoint in λ/h^2_t. Nevertheless, the term $-(3/4\pi^2) h_t^4$ in eq.(8) dominates β_λ for small λ. It excludes values of λ near zero since the IR interval does not extend to $\lambda = 0$. This explains the lower bound in fig.2 for 80 GeV $< m_t <$ 120 GeV. (The lower bound on M_H for $m_t >$ 80 GeV assumes $\lambda(\Lambda) \geq 0$. It is not completely proven so far that this requirement is necessary.)

Let me summarize this section by stressing two points:

1) In theories with a finite physical cutoff Λ the allowed low energy values of couplings are determined by infrared intervals. These intervals shrink to associated fixpoints for $\Lambda \to \infty$.

2) The one loop RGE define a hierarchy of couplings. At the first level are the gauge couplings g. Their one loop β functions do not depend on any other couplings of the standard model. Second one has Yukawa couplings h with one loop functions depending on g and h, but independent of scalar selfinteractions. There is an IR fixpoint in the ratio h^2/g^2. For strong enough h and g the low energy ratio h^2/g^2 is restricted by the corresponding infrared interval. Due to the existence of the chiral UV fixpoint for h=0 small Yukawa couplings get renormalized slowly with a typical power law behaviour. The IR interval for the ratio h^2/g^2 is always bounded by zero.

Finally, the β function of scalar self-couplings λ depends on g,h and λ. For nonvanishing g and h no fixpoint is associated with $\lambda=0$. For strong enough h (or g) the infrared behaviour of λ is determined by the evolution of g and h whereas λ moves in a sort of slavery.

4.) Coleman-Weinberg symmetry breaking

I finally come to the lower bound on the Higgs particle mass for $m_t < 80$ GeV (compare fig.2). In this region one can use the approximation $\lambda \ll g^2_i$ to solve the RGE:

$$\frac{d\lambda}{dt} = \beta_\lambda = \frac{1}{16\pi^2}\left\{\frac{9}{4}g_2^4 + \frac{9}{10}g_2^2 g_1^2 + \frac{27}{100}g_1^4 - 12h_t^4\right\} \quad (25)$$

Approximating β_λ by a constant gives the typical logarithmic behaviour for $\lambda(\varphi)$

$$\lambda(\varphi) = \lambda(\Lambda) - \beta_\lambda \ln\frac{\Lambda}{\varphi} = \frac{1}{2}\beta_\lambda\left(\ln\frac{\varphi^2}{\mu_0^2} + c_0\right) \quad (26)$$

From this one obtains[14] the one loop effective potential[3)]

$$V = -\mu_\varphi^2(\varphi)\varphi^\dagger\varphi + \frac{1}{2}\lambda(\varphi)(\varphi^\dagger\varphi)^2$$

+ correction terms

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3) The correction terms in (27) and the constant c_0 in (26) depend on the precise definition of $\lambda(\varphi)$.

$$= -\mu_\varphi^2\varphi^\dagger\varphi + \frac{1}{4}\beta_\lambda(\varphi^\dagger\varphi)^2\left(\ln\frac{\varphi^\dagger\varphi}{\mu_0^2} - \frac{1}{2}\right) \quad (27)$$

The parameter $\lambda(\varphi_0)$ is eliminated in favour of the parameter μ_0 according to (26). This is called dimensional transmutation.

Consider first the case $\mu_\varphi^2 = 0$. The minimum of V occurs for

$$\varphi_0^2 = \mu_0^2 \quad (28)$$

The curvature around the minimum leads to the Higgs particle mass

$$(M_H^{CW})^2 = \beta_\lambda\varphi_0^2 = \frac{3}{16\pi^2\varphi_0^2}\left(2M_W^4 + M_Z^4 - 4m_t^4\right) \quad (29)$$

For small top mass one finds $M_H^{CW} \approx 10$ GeV. for increasing m_t the value of M_H^{CW} decreases until the point

$$m_t \approx 80\,\text{GeV}, \quad M_H^{CW} < 1\,\text{GeV} \quad (30)$$

(Around this point effects from the running of g_i and h_t and two loop corrections should be included for a determination of M_H^{CW}.)

At the point (30) the one loop β-function for λ vanishes. This point plays a special role in the context of dilatation symmetry. In the standard model without gravity the only mass parameters in the classical action appear in the Higgs potential: the scalar mass term μ_φ^2 and a possible classical cosmological constant $\varkappa_0 = V(\varphi = 0)$. There are two ways how to implement classical dilatation symmetry in the standard model:

A) One introduces a Goldstone boson σ and replaces 15]16]

$$\mu_\varphi^2 \longrightarrow \mu_\varphi^2 \exp\frac{2\sigma}{M}$$

$$\varkappa_0 \longrightarrow \varkappa_0 \exp\frac{4\sigma}{M} \qquad (31)$$

In this case one obtains no information on μ_φ^2 and $\varkappa_0$.

B) All classical mass parameters vanish

$$\mu_\varphi^2 = 0 \quad , \varkappa_0 = 0 \qquad (32)$$

Due to dilatation symmetry the values (32) correspond to a fixpoint in the RGE and are in this sense "natural" (see later). If perturbation theory is valid, weak symmetry breaking must be of the Coleman-Weinberg type and leads to (29). Furthermore, the effective cosmological constant $\varkappa$ is determined by the potential (27) with $\mu_\varphi^2(\varphi) = 0$

$$\varkappa = V(\varphi_0) + \text{QCD contributions}$$

$$V(\varphi_0) = -\beta_\lambda \varphi_0^4 \qquad (33)$$

One knows from observation that $\varkappa$ must be tiny. Since the QCD effects should not exceed $0(\Lambda_{QCD}^4)$

one concludes that β_λ must be smaller than about 10^{-12}. The alternative (B) therefore predicts[17] in perturbation theory $m_t \approx 80$ GeV, $M_H < 1$ GeV.[4)]

The lower bound[18] in fig.2 for $m_t < 80$ GeV needs a discussion of V (27) for $\mu_\varphi^2 \neq 0$. I show the qualitative dependence of V on μ_φ^2 in fig.8. For increasing μ_φ^2 the potential approaches the form of the classical potential. For a positive scalar mass term ($\mu_\varphi^2 < 0$), however, a second minimum develops at $\varphi = 0$. For $\mu_\varphi^2 = \mu_\varphi^{c2}$ the two minima correspond to the same value of V. For $\mu_\varphi^2 < \mu_\varphi^{c2}$ the lowest minimum is the symmetric one at $\varphi = 0$ and one expects a ground state without weak symmetry breaking. (For small enough $\mu_\varphi^2 < \bar{\mu}_\varphi^2$ there is only one minimum at $\varphi = 0$.) This behaviour is characteristic for a <u>first order phase transition</u> at $\mu_\varphi^2 = \mu_\varphi^{c2}$. At the transition point one finds $M_H = \frac{1}{\sqrt{2}} M^{CW}_H$ and derives the lower limit on M_H from (29).

At the end of this section one word of caution is in order. The validity of perturbation theory has not been proven for the spontaneously broken phase

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4) Possible tests for the idea of classical scale invariance involve a new intermediate range force[16] or modifications of standard cosmology[17]. Both effects depend on the dilatation anomaly in the quantum theory.

in the standard model. This comment may seem surprising since the values of all dimensionless couplings at the Fermi scale are small for the discussion of this section. Obviously, two loop effects for the RGE are supressed compared to the one loop contribution. Nevertheless, validity of perturbation theory not only requires small enough couplings but also the validity of a saddle point approximation in the functional integral. There are still open topics like the question if the perturbative results for the Coleman Weinberg first order phase transition hold quantitatively. (I believe that the qualitative features are true for the elctroweak gauge theory). Another question concerns a possible nontrivial interplay between strong and weak interactions due to nonperturbative chiral symmetry breaking in QCD. Finally, I should mention that the standard model with small enough λ and h_t possibly has a nontrivial continuum limit for $\Lambda \to \infty$. Since this is mostly of theoretical interest I comment on this topic in the appendix.

5.) Naturalness of the Fermi scale

Why is

$$\frac{\varphi_0}{M_p} \approx 10^{-17} \qquad ? \qquad (34)$$

This small dimensionless quantity is crucial for the weakness of gravitational interactions. This question is the heart of the "gauge hierarchy

problem"[19]. So far the ratio (34) is not understood. Why is

$$\frac{\varphi_0}{\Lambda_{QCD}} \approx 10^{3} \qquad ? \qquad (35)$$

This ratio (together with dimensionless gauge and Yukawa couplings) determines the scales in atomic physics. Our world would look completely different if (35) is modified. The fact that φ_0/Λ_{QCD} is many orders of magnitude smaller than M_p/Λ_{QCD} may be called the "connection problem" between electroweak and strong interactions. What has φ_0 to do with Λ_{QCD}?

It is not excluded that (34) is a consequence of (35) and the small ratio Λ_{QCD}/M_p (which can be understood from the logarithmic evolution of the strong coupling constant). I will not pursue this possibility in this lecture and only discuss (34) neglecting nonperturbative QCD effects. Is the small ratio φ_0/M_p natural? Three main objections have been given against the naturalness of (34) in the context of the standard model embedded in some unified theory with a large characteristic mass scale M near M_p:

i) The scalar mass term μ_φ^2 is quadratically divergent in perturbation theory. Its natural value should therefore be given by the physical cutoff Λ^2.[20]

ii) In order to obtain $|\mu_\varphi^2| \ll M^2$ the couplings of the theory have to be adjusted with very high accuracy in every order in perturbation theory.[19] This argument has first been given in the context of grand unification.

iii) The value $\mu_\varphi^2 = 0$ does not correspond to an enhanced symmetry of the theory. Small values of μ_φ^2 are therefore not natural[21] (in contrast to the small Yukawa coupling of the electron, for example).

New theories have been advocated to cure the alleged "unnaturalness" of (34) in the standard model: low energy supersymmetry, technicolour, compositeness...Is new physcis really needed? Does the observed scale φ_0 give any information about the scale Λ where physics beyond the standard model becomes necessary? My personal answer to this question is negative. I will argue[22] that the small ratio φ_0/M_p is as natural (or unnatural) as the ratio m_e/M_p in quantum electrodynamics coupled to gravity. (To underline the importance of this statement I recall that nobody would advocate a necessary extension of QED at a scale somewhat above m_e for reasons of unnaturalness of the small ratio m_e/M_p).

i) Quadratic divergences

In perturbation theory the scalar mass term has contributions (compare fig.9) proportional to the cutoff momentum squared:

$$\mu_\varphi^2 = \bar{\mu}^2 + c\Lambda^2 \qquad (36)$$

The quadratically divergent term $c\Lambda^2$, however, is purely an artefact of the regularization procedure and contains no physics! First of all, the value of c depends on the regularization scheme chosen. In particular, there exist regularizations without quadratic divergences[23] (c=0) in all orders in perturbation theory. (Supersymmetry is not needed for this.) In the language of statistical mechanics the quadratic divergence $c\Lambda^2$ is not a universal quantity. The situation is completely analogous for massive fermions. Depending on the regularization the fermion mass has linearly divergent contributions or not.

To illustrate this, let me consider QED (with electrons) regularized on a lattice with lattice distance $a \sim \Lambda^{-1}$. The "Hopping parameter" (corresponding to the "bare mass" $\bar{m}_e$) must be fine tuned in order to obtain $m_e \ll 1/a$. The electron mass has a linear divergence

$$m_e = \bar{m}_e + c\Lambda \tag{37}$$

Again the value of c depends on details of the regularization, for example if the lattice is cubic or triangular etc. This does not mean that a value m_e = 511 keV is unnatural - the electron mass simply sets the scale of QED in this model. Presence of other interactions with a much higher characteristic scale M (like gravity) opens the question why m_e/M is very small, but this is not related to the issue of the linear divergence. It is just the same for electroweak interactions. If we would not know about M_p (or a unification scale M_X) the scale of the theory is set by φ_0. Any value of φ_0 is equally consistent[5] and "natural", irrespective if the regularization induces a quadratic divergence or not. It is sometimes argued that fundamental unification will decide on the "correct" or "physical" regularization. This is true, but leads nowhere unless we know what is the "correct" definition of the bare mass and the corresponding "correct" value of c. The QED

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5) This is not true for the complete standard model with QCD. Due to chiral symmetry breaking the scale of weak interactions is bounded from below by Λ_{QCD}.

example illustrates that the pure existence of regularization schemes with $c \neq 0$ is certainly not sufficient to conclude the necessity of new physics. The argument for new physics at the TeV scale to cure the quadratic divergence of the Higgs mass is not better than an argument for an extension of QED at a scale of a few MeV to cure the linear divergence of the electron mass.

ii) Fine tuning

Grand unfied theories have a scale φ_X (not far from M_p) characteristic for spontaneous breaking of the unification group. A naive perturbative expansion for the doublet mass term μ_φ^2 gives

$$\mu_\varphi^2 = \underbrace{\nu^2 + c_1 g^2 \varphi_X^2}_{\text{tree}} + \underbrace{c_2 g^2 \nu^2 + c_3 g^4 \varphi_X^2}_{\text{one loop}} + \dots \tag{38}$$

(with g the gauge coupling, ν^2 a mass parameter and c_i containing ratios of dimensionless couplings and logarithms). Such a perturbation series has bad convergence properties if the first terms are of the order $g^{2N}\varphi_X^2$, and therefore many orders of magnitude bigger than $|\mu_\varphi^2|$. As a consequence, the parameter ν^2 or the tree mass $\mu_0^2 = \nu^2 + c_1 g^2 \varphi_X^2$ needs to be fine tuned differently in every order in perturbation theory. I will argue that this "fine tuning problem" is only a consequence of a "bad choice" of the perturbative expansion series for μ_φ^2. Using perturbative renormalization group methods the problem disappears.

The RGE for the scalar mass term is given by an anomalous dimension A

$$\frac{d\mu_\varphi^2}{dt} = A\mu_\varphi^2 \tag{39}$$

For the one loop approximation in the standard model one has

$$A = \frac{1}{16\pi^2}\left(6\lambda + 6h_t^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2\right) \tag{40}$$

One defines the physical short distance mass

$$\mu_0^2 = \mu_\varphi^2(M_p) \tag{41}$$

(In contrast to the bare mass this is a physical (universal) quantity. The value of μ_φ^2 at an arbitrary scale M may be called the tree approximation.) The scalar mass term at the Fermi scale is then given by

$$\mu_\varphi^2(\varphi_0) = \mu_0^2 \exp\left(-\int_0^{\ln M_\varphi/\varphi_0} A(t)\,dt\right) \tag{42}$$

For the approximation of constant A this has the solution

$$\mu_\varphi^2(\varphi_0) = \mu_0^2\left(\frac{\varphi_0}{M_p}\right)^A \tag{43}$$

(Otherwise a mean value of A has to be taken according to (42)). Several comments are in order:

- For small values $|\mu_0^2|$ the scalar mass term is also small. For A positiv it is even smaller than $|\mu_0^2|$.

- A perturbative expansion corresponds to an expansion of A in powers of g^2. This gives a good series (the first terms converge for g^2 sufficiently small) for the ratio $\mu_\varphi^2(\varphi_0)/\mu_0^2$. No fine tuning of μ_0^2 is needed order by order in perturbation theory.

- As expected from the previous discussion, no trace of quadratic divergences appears in the RGE for physical quantities.

- For comparison, let me give an example for an RGE where a real fine tuning problem would exist. I have drawn in fig.10 the renormalization group trajectories for the RGE $d\mu_\varphi^2/dt = A\mu_\varphi^2 + B\varphi_x^2$ for vanishing and nonvanishing B. For $B \neq 0$ the scale M where $\mu_\varphi^2/(M)=0$ would vary in different orders in perturbation theory. To obtain the coincidence $M \approx \varphi_0$ would require fine tuning of parameters order by order in perturbation theory.

- It can be shown[22] that the RGE for the doublet mass term has always the form (39)(B=0), even if the standard model is embedded in a unified theory. This is a consequence of the essential second order character of the weak phase transition. (The small Coleman Weinberg first order effects do not change this conclusion.)

iii) Naturalness and Symmetry

The small electron mass is often considerd as natural since for $m_e=0$ the theory has an enhanced symmetry: chiral symmetry. In contrast, the value $\mu_\varphi^2=0$ is not singled out by an additional symmetry.[6] I will argue that despite this apparent difference the situation is essentially analogous to the electron mass in QED. As a consequence of chiral symmetry the RGE for m_e has always a fixpoint for $m_e=0$.

$$\frac{dm_e}{dt} = A_e m_e \tag{44}$$

This implies (for $|A_e|<<1$) that an arbitrarily small electron mass does not change its order of magnitude by the renormalization group evolution. In this sense it is stable. The fixpoint for vanishing mass, however, does not necessarily require an additional symmetry. Indeed, the RGE for μ_φ^2 has a fixpoint at $\mu_\varphi^2=0$. Rather than to an

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6) It corresponds to classical dilatation symmetry, but this symmetry has anomalies.

exact symmetry it is due to the second order character of an associated phase transition. (This has a relation to classical dilatation symmetry.) One may argue that t'Hooft's naturalness criterion should be generalized for fixpoints of arbitrary origin.

Even more, in the spontaneously broken phase the vev φ_0 is a more natural parameter than μ_φ^2. (For example, higher dimensional theories would directly predict φ_0 and not the infinitely many scalar mass terms.) For $\varphi_0=0$ the ground state (not the theory) has an enhanced symmetry: SU(2) x U(1) remains unbroken.[7] Generalizing t'Hooft's criterion also for symmetries of the ground state one could call a small value of φ_0 "natural". In this context the small mass term μ_φ^2 is a consequence of the small value φ_0. (It is calculable as a function of λ and φ_0). The stable behaviour of the RGE for μ_φ^2 can be viewed as a consequence of the stability of φ_0. The analogy between φ_0 and m_e becomes even closer if one notes that the small Yukawa coupling of the electron in the effective low energy theory may well be a consequence of a small scale φ_g of spontaneous generation symmetry breaking[24]

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7) More precisely one should speak about different realizations of a gauge symmetry.

(compared to the unification scale). The electron mass would vanish for $\varphi_0 \to 0$ and $m_e=0$ would correspond to an enhanced symmetry of the ground state of the unified theory. I conclude that a small ratio φ_0/M_p within the standard model is as natural as a small ratio m_e/M_p in QED.

6.) Conclusions

The small ratio φ_0/M_p still waits for an explanation. Nevertheless, there is no technical problem, no consistency problem and no fine tuning problem associated with this ratio. In my opinion, new physics at the TeV scale should therefore not be motivated by "improving the gauge hierarchy problem" with respect to quadratic divergences or fine tuning. (An explanation of the ratio φ_0/M_p would of course be a very valid motivation for a more complete theory.) The observed value of the Fermi scale φ_0 gives no information on the physcial cutoff Λ where new physics is expected to become important.

Within the standard model there are still several theoretical gaps to be filled by a nonperturbative treatment: The triviality of the coupled system of scalar and fermions should be established, including a quantitative estimate at which scale $\mu_p = (1/c)\Lambda$ perturbation theory becomes valid for arbitrarily strong Yukawa couplings at the cutoff Λ. The Coleman-Weinberg symmetry breaking should be

confirmed nonperturbatively for the electroweak gauge theory. The interplay between QCD and electroweak interactions for weak symmetry breaking needs to be investigated. These are only a few "practical" tasks immediately relevant for predictions of the standard model. Beyond this, there are other questions of a more theoretical nature.

The most important task remains for our experimental colleagues: to put experimental bounds on the theoretically allowed values of M_H and m_t and finally find the Higgs scalar and the top quark. This may give information on the physical cutoff Λ, depending on the range of M_H:

$10\ \text{GeV} < M_H < 140\ \text{GeV}$

The standard model is consistent up to energies $\gtrsim M_p$. It is possible that new physics sets in only near the Planck scale.

$140\ \text{GeV} < M_H < 600\ \text{GeV}$

Information about new physics at a scale below M_p becomes possible. The upper bound on the physical cutoff Λ, where the standard model needs to be extended, depends on the value of the top quark mass.

$M_H > 600\ \text{GeV}$

New physics is expected around or below 1 TeV. The standard model description breaks down at this scale. There is no reason why the Higgs particle should be singled out particularly amongst other excitations of the new theory for an experimental detection of the new physics.

$M_H < 10$ GeV

Perturbation theory predicts constraints on the top mass in dependence on M_H. A Higgs mass in this region would give interesting additional information on dilatation symmetry.

Acknowledgement

I would like to thank M. Lüscher for interesting discussions on his results on triviality of φ^4 theory.

Appendix

In this appendix I comment that it is not unlikely that the standard model has a nontrivial continuum limit if the renormalized Yukawa and scalar couplings at the Fermi scale are not too large[25].

This is equivalent to the statement that all couplings $g^2_i(\Lambda)$, $h^2(\Lambda)$, $\lambda(\Lambda)$ remain finite and positiv (or zero) as Λ goes to infinity. If the number of generations N_G is smaller than five the gauge couplings g_2 and g_3 are asymptotically free in perturbation theory

$$\lim_{\Lambda\to\infty} g_2(\Lambda)\to 0 \;, \quad \lim_{\Lambda\to\infty} g_3(\Lambda)\to 0 \qquad (A.1)$$

In contrast, $g_1(\Lambda)$ increases for increasing Λ. In the short distance (UV) region we only need to consider the abelian gauge theory of the U(1) factor of the standard model.

The one loop RGE for the ratio h_t/g_1 is

$$\frac{d}{d\tilde{t}}\left(\frac{h_t}{g_1}\right) = \frac{9}{2}\left(\frac{h_t}{g_1}\right)^3 - \left(\frac{17}{20}+b_1\right)\frac{h_t}{g_1} = \beta_{hg} \qquad (A.2)$$

$$\frac{d\tilde{t}}{dt} = \frac{g_1^2(t)}{16\pi^2} \qquad (A.3)$$

$$\frac{dg_1}{dt} = \frac{b_1}{16\pi^2}\, g_1^3 \;, \quad b_1 = \frac{4}{3}N_G + \frac{1}{10} \qquad (A.4)$$

I show β_{hg} in fig.11. There is an UV fixpoint for $h_t/g_1 = 0$ (corresponding to chiral symmetry) and an IR fixpoint for nonzero $(h_t/g_1)_c$. If the Yukawa coupling is small enough so that h_t/g_1 is in the range of attraction of the UV fixpoint ($h_t/g_1 < (h_t/g_1)_c$) we can neglect the Yukawa coupling compared to g_1 in the UV region. The situation for the other Yukawa couplings is similar.

It remains the coupled system of λ and g_1 with the following one loop RGE for the ratio:

$$\frac{d}{dt}\left(\frac{\lambda}{g_1^2}\right) = 12\left(\frac{\lambda}{g_1^2}\right)^2 - \left(\frac{9}{5}+2b_1\right)\left(\frac{\lambda}{g_1^2}\right) + \frac{27}{100} = \beta_{\lambda g} \quad (A.5)$$

From fig.12 one learns that there is both an IR and an UV fixpoint for λ/g^2_1. Region I corresponds to section 2 where the cutoff cannot be moved to infinity. For regions II and III, however, the ratio λ/g^2_1 is in the range of attraction of an UV fixpoint. (Region III includes the Coleman Weinberg symmetry breaking of section 4.) If g_1 remains finite for $\Lambda \to \infty$, also λ will remain finite[25].

The perturbative analysis of the abelian U(1) theory (A.4) would suggest that it is trivial. However, nonperturbative results for the abelian lattice gauge theory with fermions may indicate[26] that there is a second order phase transition with a corresponding UV fixpoint for the gauge coupling. If this is confirmed, the β function for g_1 would ressemble fig.13. I conclude that there is a nontrivial continuum limit for the standard model if the following conditions are met:[8)]

8) These conditions are sufficient, but may not be necessary. Another (more remote) possibility is a new phase in the coupled fermion-scalar system for strong Yukawa couplings.

i) There is a nonperturbative UV fixpoint for g_1 in the abelian gauge theory with chiral fermions and scalars.

ii) The UV fixpoint in the ratio λ/g^2_1 persists nonperturbatively and λ is in the range of attraction of this fixpoint.

iii) The Yukawa couplings are small enough so that h/g_1 is in the range of attraction of the chiral UV fixpoint.

iv) Asymptotic freedom for g_2 and g_3 are not disturbed by g_1 and λ of the order of their UV fixpoint values.

I should emphasize again that the possibility of a nontrivial continuum limit is merely of theoretical interest. In the real world one expects a physical cutoff $\Lambda \lesssim M_p$ where the standard model should be extended to include gravity. There is therefore no need for the couplings to be within the range of attraction of a possible nontrivial UV fixpoint. No additional bounds on M_H should be derived from such a requirement.

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Figure captions

Fig.1: Tree level W^+W^- scattering.

Fig.2: Allowed region[4] for M_H and m_t if perturbation theory is valid up to $\Lambda = 10^{15}$ GeV.

Fig.3: Region of large λ, small h_t.

Fig.4: Renormalization group trajectories of $\lambda(\mu)$ for three values of $\lambda(\varphi_0)$

Fig.5: Two loop and three loop perturbative expansion and strong coupling expansion for $\lambda(M_H)$ in the one component φ^4 theory[11]. Also shown are two points of a Monte Carlo simulation[12] with statistical errors.

Fig.6: Upper bound on M_H as a function of Λ[11] (with estimated errors).

Fig.7: Region of large Yukawa coupling.

Fig.8: Qualitative behaviour of the Coleman Weinberg effective potential for different values of μ_φ^2.

Fig.9: Contribution to quadratic divergence of the scalar mass term from a fermion loop.

Fig.10: Renormalization group trajectories for μ_φ^2 for B=0 and B$\neq$0.

Fig.11: β-function for the ratio h_t/g_1.

Fig.12: β-function for the ratio λ/g_1^2.

Fig.13: Speculated form of the β function for the abelian gauge coupling g_1.

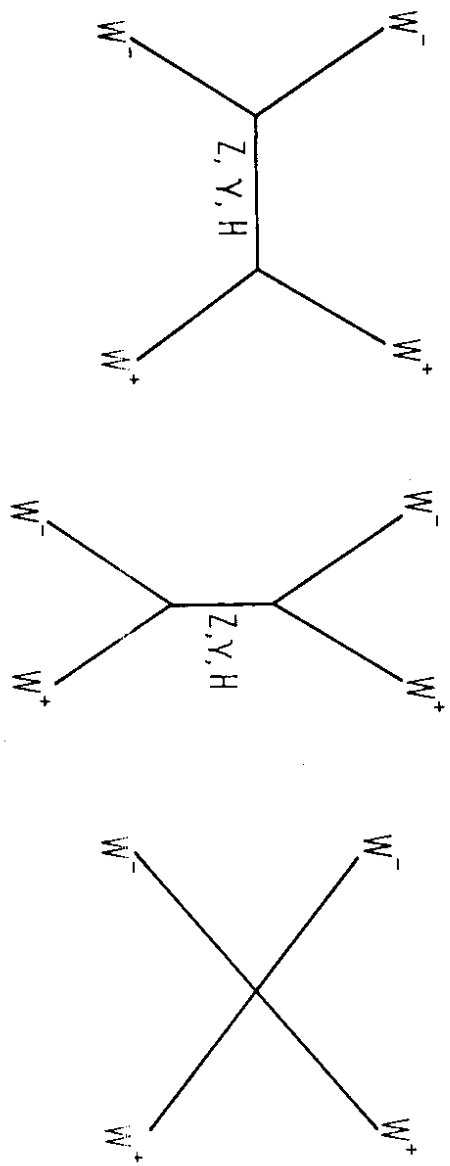

Fig. 1

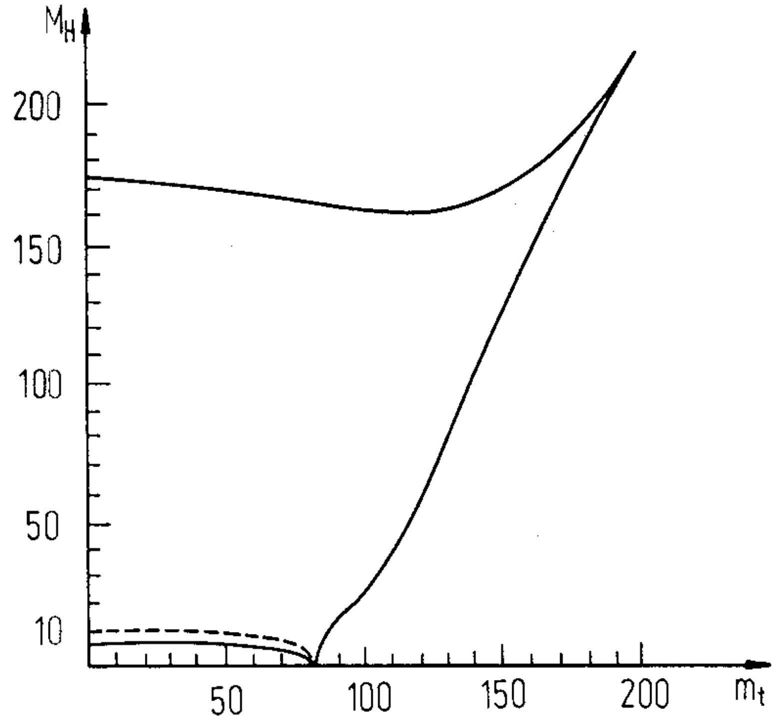

Fig. 2

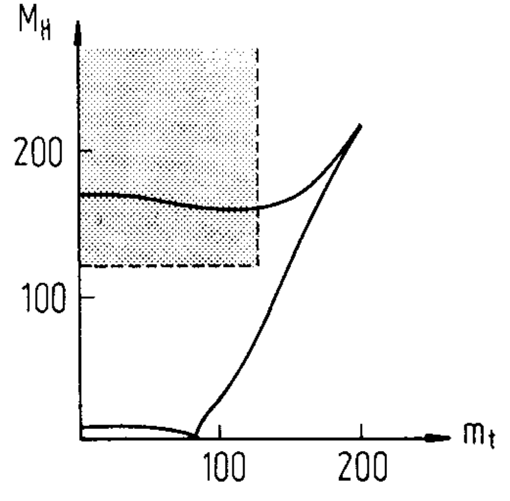

Fig. 3

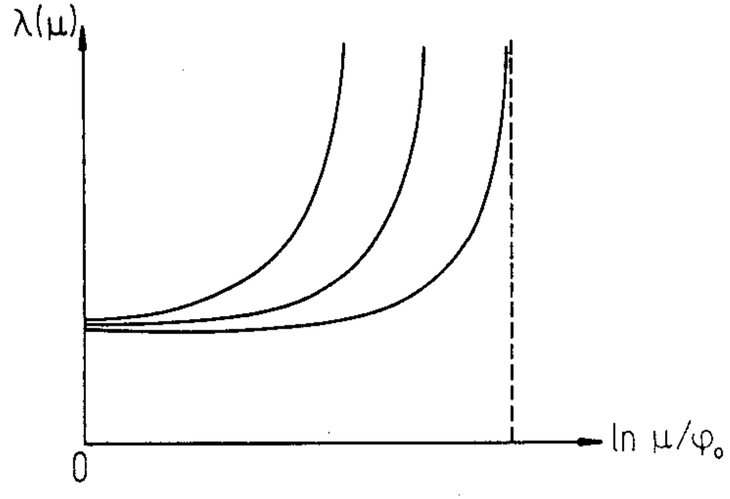

Fig. 4

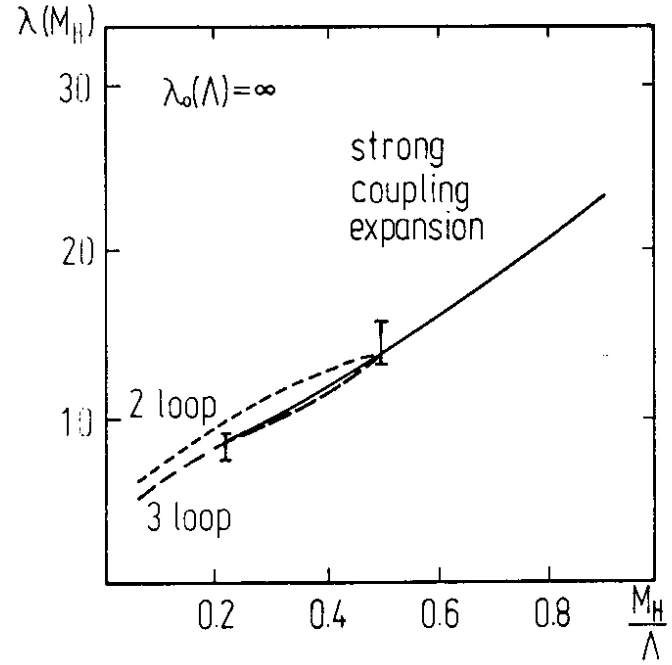

Fig. 5

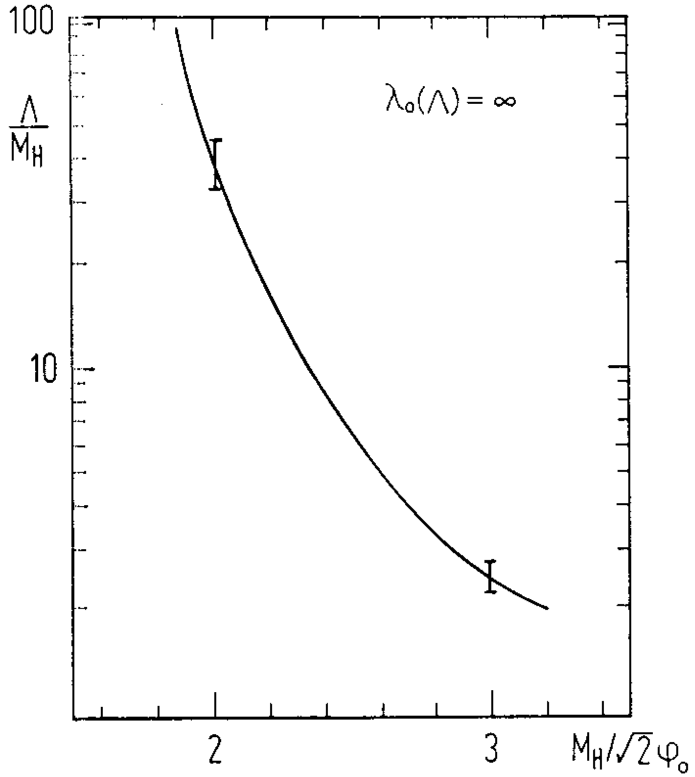

Fig. 6

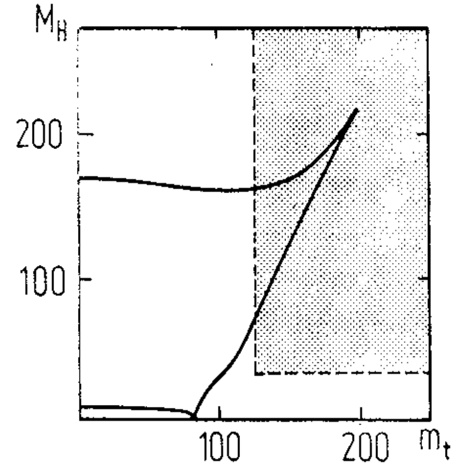

Fig. 7

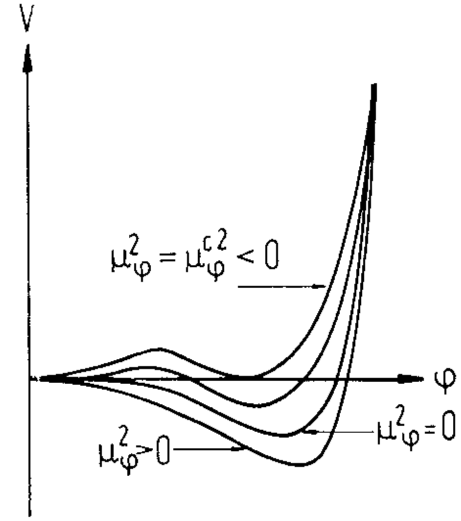

Fig. 8

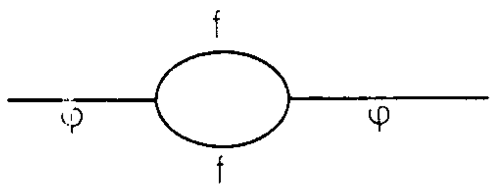

Fig. 9

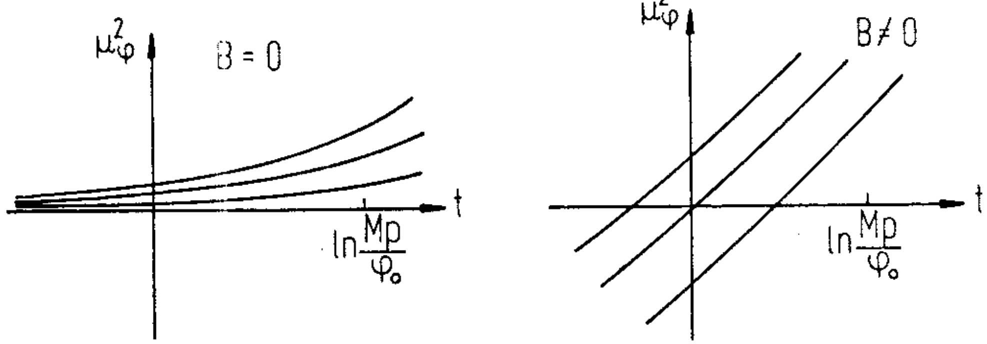

Fig. 10

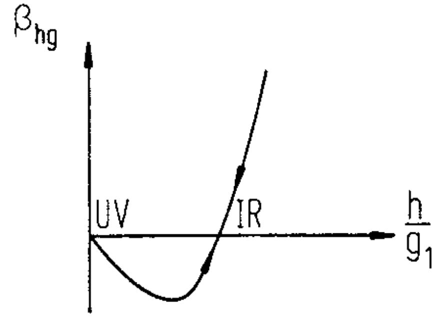

Fig. 11

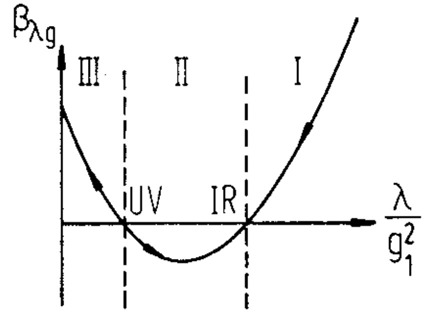

Fig. 12

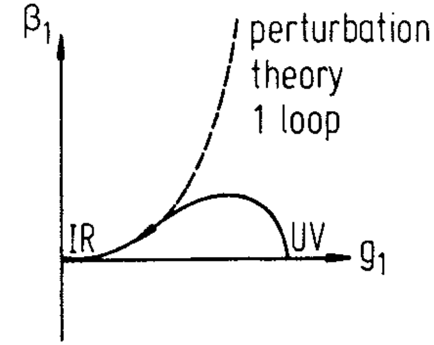

Fig. 13