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QCD Calculation of $K^0 - \overline{K}^0$ Mixing from Three-Point Function Sum Rules

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Abstract

We calculate the B parameter for $K^0 - \overline{K}^0$ mixing in the framework of QCD sum rules for a three-point function involving pseudoscalar currents, and contrast our results with other calculations. We find $B = 0.5 \pm 0.1 \pm 0.2$, where the first error reflects uncertainties in the various QCD parameters and the second one is an estimate of uncalculated three-loop radiative and higher order quark mass corrections.

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I. Introduction

The knowledge of the various matrix elements involving the local four-quark operators which appear in the calculation of different processes in kaon physics, e.g., $K^0 - \overline{K}^0$ mixing, K decays, CP violation, etc., is known to be very important to test the standard model [1] - [3]. A quantity of particular significance is the value of the matrix element entering the transition amplitude for $K^0 - \overline{K}^0$ mixing. Some qualitative features of the above processes may be understood on the basis of current algebra, isospin symmetry, etc. However, when it comes to quantitative predictions, these are subject to large uncertainties. The need for a more accurate knowledge of these matrix elements has led to the development of new theoretical approaches. Presently, there are three main frameworks: lattice gauge theory simulations [4], QCD sum rules either combined with chiral perturbation theory [5] - [7] or based on three-point functions, [8] - [10], and the $1/N_c$ expansion [11] - [13].

In the case of $K^0 - \overline{K}^0$ mixing, the problem reduces in the end to a calculation of the matrix element of the local four-quark $\Delta S = 2$ operator. Results are usually given in terms of the parameter B which measures the departure from the simple vacuum saturation ($B_{v.s.} = 1$). Preliminary results from lattice calculations suggest $B < 1$ but with large errors [4]. The authors of [5] have obtained $B \simeq 1/3$ from QCD sum rules, while the $1/N_c$ expansion gives $B \simeq 0.7$ but within a truncated theory. The approach based on three-point function QCD sum rules, in spite of some unsettled issues concerning their analyticity properties [14], represents a viable alternative. Unfortunately, available predictions are scattered over the wide range $B \simeq 0.4 - 1.5$ [8] - [10]. Although some authors have used three-point functions involving axial-vector currents [8] [9] and others [10] pseudoscalar currents, one is to expect consistent results, at least within error bars. The reason for such a discrepancy does not appear obvious and, furthermore, it seems that some of these calculations are in contradiction with each other as we shall discuss later.

Given the impact of the B parameter on our present understanding of the standard model, and in an attempt to resolve some of the existing discrepancies, in this paper we reexamine this issue in the framework of three-point function QCD sum rules.

The paper is organized as follows. In Section II we give some general definitions and discuss the saturation of the three-point function. The hadronic and the QCD parametrizations are derived in Sections III and IV, respectively. In Section V we obtain the solutions to the Laplace transform sum rules which determine the B parameter. Finally, Section VI is devoted to conclusions and a brief general discussion of the results.

II. Saturation of the Three-point Function

Following [10], we choose to work with a three-point function involving two pseudoscalar currents, in addition to the $\Delta S = 2$ four-quark operator. This choice has the advantage that loop corrections are suppressed, thus reducing uncertainties from perturbative terms and the continuum, while the dominant contribution is proportional to the quark condensate $< 0|\bar{q}q|0 >$. This three-point function reads

$$\Psi(p,p') = i^2 \int d^4x e^{ip\cdot x} \int d^4y e^{-ip'\cdot y} < 0|T(j_5(x)\mathcal{O}_{\Delta S=2}(0)j_5(y)|0 > \quad (1)$$

where

$$j_5(x) =: \bar{d}(x) i\gamma_5 s(x) : \quad (2)$$

is the pseudoscalar current which couples to the kaon according to

$$< 0|j_5(0)|K >= \frac{f_K m_K^2}{m_s + m_d} \quad (3)$$

and $f_K = 1.22$, $f_\pi = 161$ MeV.The local operator $\mathcal{O}_{\Delta S=2}$, stemming from the short distance expansion of the box diagram, is given by

$$\mathcal{O}_{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma^\mu(1-\gamma_5)d]. \quad (4)$$

The matrix element of this operator between kaon states reads

$$< \overline{K}^0|\mathcal{O}_{\Delta S=2}|K^0 >= 2(1+\frac{1}{N_c})f_K^2 m_K^2 B(\mu^2), \quad (5)$$

where

$$B(\mu^2) = \hat{B}[\alpha_s(\mu^2)]^{2/9}, \quad (6)$$

and $\hat{B}$ is μ independent quantity. $B = B_{v.s.}$=1 if vacuum saturation is used. Notice that the dependence on the renormalization scale μ cancels with the corresponding μ dependence of the Wilson coefficient appearing in the off-diagonal $K^0 - \overline{K}^0$ matrix element.

To make contact with the $1/N_c$ expansion of [13], one may rewrite (6) as

$$B = \frac{3}{4} + \frac{3}{4N_c} + O(\frac{1}{N_c})_{nonfact}, \quad (7)$$

so that the leading term is $B_{lead} = 0.75$, the second term in (7) being the factorizable $1/N_c$ correction, and the third term the nonfactorizable $1/N_c$ contribution.

The idea behind the large N_c expansion originates from the hope that the true expansion parameter could be not exactly $1/N_c = 1/3$, but rather something like $1/4\pi N_c$ or even $1/4\pi N_c^2$, as it happens in QED where an expansion in the coupling constant e becomes in reality an expansion in $\alpha = e^2/4\pi$ [15]. Unfortunately, eq.(7) clearly shows that the factorizable subleading term is already a large 33% correction, which makes this expansion doubtful, at least in the sense of [15]. Nevertheless, it has been noticed recently in connection with charm decays [3] that although $1/N_c$ corrections appear large by themselves, there are large cancellations among them which approximately restore the validity of large N_c results. In principle, this feature would also be tested in the calculation performed.

Turning to $\Psi(p,p')$, the phenomenological expression may be obtained by saturating $\Psi(p,p')$ with the lowest intermediate kaon states, i. e.,

$$\begin{aligned}\Psi(p,p')_{hadr} &= (\frac{f_K m_K^2}{m_s+m_d})^2 \Delta_K(p'^2) < \overline{K}^0(p')|O_{\Delta S=2}|K^0(p) > \Delta_K(p^2) \\ &\quad + \text{higher states} + \text{continuum},\end{aligned} \tag{8}$$

where

$$\Delta_K(p^2) = \frac{1}{m_K^2 - p^2}, \tag{9}$$

is the free kaon propagator. Since, in general, the kaons are off mass shell in the matrix element appearing in (8) , one should write

$$< \overline{K}^0|O_{\Delta S=2}|K^0 >= 2(1+\frac{1}{N_c})f_K^2(p\cdot p')B. \tag{10}$$

The factor $p\cdot p'$ will eventually get canceled as it will also appear in the QCD expression for $\Psi(p,p')$.

In previous calculations [8]-[10] the contribution from higher states in (8) was absorbed in the continuum and eventually moved to recombine with its counterpart appearing in the QCD expression for $\Psi(p,p')$. We shall argue here that this is not a good approximation as far as the chiral symmetry breaking contribution from the $K'(1400)$ is concerned. This kaon radial excitation term should be taken into account explicitly and, effectively, it leads to corrections to the free kaon propagator (9).

III. Hadronic Corrections to $\Psi(p,p')$

We now discuss the hadronic corrections to the kaon propagator , i. e., the continuum integral in

$$\Delta_K(Q^2) = \frac{1}{m_K^2+Q^2} + \int_{t_0}^{\infty} \frac{dt}{t+Q^2}\rho_K(t), \tag{11}$$

where $t_0 = (m_K + 2m_\pi)^2$. It has been pointed out in [16] that, as a matter of principle, the pseudoscalar meson radial excitation contributions to the kaon (pion) propagator cannot be neglected , as in some cases they are of the same order in the quark masses as the ground state term and in other cases they induce non-negligible chiral symmetry breaking corrections. As a matter of practice, these corrections have the welcome feature of stabilizing light quark mass and vacuum condensate predictions from QCD sum rules [16]-[18]. It is easy to show the matter of principle by considering, e.g., the two- point function associated to the axial-vector current divergences

$$\psi_5(q^2) = i \int d^4x e^{iq\cdot x} < 0|T(\partial^\mu A_\mu(x)\partial^\nu A_\nu^\dagger(0))|0 >, \tag{12}$$

where $\partial^\mu A_\mu$ has charged kaon (pion) quantum numbers. The hadronic spectral function is given by

$$\frac{1}{\pi} Im\psi_5(t) = f_K^2 m_K^4 [\delta(t - m_K^2) + \rho_K(t)]. \tag{13}$$

Just for the sake of argument let us concentrate on the first K radial excitation and make a zero-width approximation to $\rho_K(t)$, which then becomes

$$\rho_K^0(t) = \frac{f_{K'}^2 m_{K'}^4}{f_K^2 m_K^4} \delta(t - m_{K'}^2). \tag{14}$$

Since the K radial excitations do not become Goldstone bosons in the chiral limit, $f_{K'}$ must vanish linearly in the quark masses and the coefficient of the delta function in Eq. (14) is of order $O(1)$.

As it stands, the parametrization (14) is highly unrealistic since, for all practical purposes, $f_{K'}$ is not measurable. In addition, the $K'(1460)$ has a sizable width , $\Gamma_{K'} \simeq$250 MeV. As first pointed out in [16], it is possible to get rid of $f_{K'}$ by imposing the threshold behavior which follows from the effective chiral Lagrangian realization of QCD at long distances. Working in the chiral limit for simplicity, one has [19]

$$\lim_{t\to 0} \frac{1}{\pi} Im\psi_5(t) \to \frac{2}{3(16\pi^2 f_\pi^2)^2} f_K^2 m_K^4 t. \tag{15}$$

Although the full expression away from the chiral limit is now available [18], we shall use eq.(15) as in the end the various corrections tend to compensate. Imposing this threshold behavior on a Breit-Wigner resonance form, one finds that

$$\frac{1}{\pi} Im\psi_5(t) = f_K^2 m_K^4 [\delta(t - m_K^2) + a^2(1 + \gamma^2) \frac{t}{(t - m_{K'}^2)^2 + m_{K'}^2 \Gamma_{K'}^2}], \tag{16}$$

where

$$a^2 \equiv \frac{2}{3(16\pi^2 f_\pi^2)^2} m_{K'}^4 \tag{17}$$

and

$$\gamma \equiv \frac{\Gamma_{K'}}{m_{K'}}. \tag{18}$$

The accuracy of this parametrization may be assessed by computing the strange quark mass, and then comparing the result with the value obtained from an estimate of $m_u + m_d$ [18] together with the current algebra ratio [20] $m_s/(m_u + m_d) = 13\pm 1$, leading to $m_s(1GeV^2) =$ 199$\pm$ 33 MeV. Using eq.(16) together with Laplace transform or finite energy QCD sum rules for $\psi_5''(q^2)$, one obtains $m_s(1GeV^2) =$ 170 - 240 MeV, in very good agreement with the above value. The uncertainty in m_s is due to uncertainties in the values of the vacuum condensates and to the kind of sum rule being used.

We now consider the following two-point function:

$$\Pi_{5\mu}(p) = i\int d^4x e^{ip\cdot x} < 0|T(j_5(x)A_\mu(0))|0 > \\ = ip_\mu \Pi_5(p^2), \tag{19}$$

which contributes to the factorizable piece of $\Psi(p,p')$, as discussed in the next section. The hadronic parametrization of the two-point function $\Pi_5(t)$ may be obtained directly from eq.(16) using the relation

$$\frac{1}{\pi}Im\Pi_5(t) = \frac{1}{m_s+m_d}\frac{1}{t}\frac{1}{\pi}Im\psi_5(t). \tag{20}$$

Notice that contrary to $\psi_5(t)$, the two-point function $\Pi_5(t)$ does not vanish in the chiral limit. In fact, in this limit one obtains the PCAC relation

$$f_K^2 m_K^2 = -(m_s+m_d) < 0|\bar{s}s+\bar{d}d|0 > . \tag{21}$$

Chiral symmetry breaking corrections to this result, arising from K radial excitations, have been extracted in[17] and found to be large.Their contribution to the three-point function $\Psi(p,p')$ may be written as

$$\Psi(p,p')_{kaon} = (p\cdot p')2(1+\frac{1}{N_c})B\frac{f_K^4 m_K^4}{m_s^2}[\frac{1+C_K(p^2)}{m_K^2-p^2}][\frac{1+C_K(p'^2)}{m_K^2-p'^2}], \tag{22}$$

where

$$C_K(t) = m_K^2(m_K^2-t)a^2(1+\gamma^2)\int_0^{s_0}\frac{ds}{(s-t)}\frac{1}{[(s-m_{K'}^2)^2+m_{K'}^2\Gamma_{K'}^2]}, \tag{23}$$

and one should keep in mind that the threshold behavior $s^{-1}[s-(m_K+2m_\pi)^2]$ of the integrand in (23) has been approximated by its chiral limit value of unity. As pointed out earlier, the various kinematical corrections tend to compensate, so that in the end eq.(23) is a very good approximation to the full expression.

Concerning the contributions from higher resonances and from the hadronic continuum, we follow the established practice and approximate them by the asymptotic freedom expression for $\Psi(s,s')$ starting at some threshold $s_0 > m_{K'}^2$, i. e.,

$$\Psi(s,s')_{hadr} = \Psi(s,s')_{kaon} + \theta(s-s_0)\theta(s'-s_0)\Psi(s,s')_{A.F.}, \tag{24}$$

where $Im\Psi(p,p')_{A.F.}$ entering the dispersion relation for $\Psi(p,p')$ is given, to leading order in m_s and α_s, by

$$\frac{1}{\pi^2}Im\Psi(p,p')_{A.F.} = 2(1+\frac{1}{N_c})\frac{9}{64\pi^4}m_s^2(p\cdot p'). \tag{25}$$

IV. The Three-point Function $\Psi(p,p')$ in QCD

A theoretical expression for $\Psi(p,p')$has been calculated in QCD, to leading order in m_s and α_s, by the authors of [10]. We have repeated this calculation to have an independent check, and have been able to reproduce their results except for the gluon condensate contribution to $\Pi_{5\mu}$ whose correct Wilson coefficient is obtained below.

In the framework of the Operator Product Expansion with power corrections to asymptotic freedom [21] [22], the expression for $\Psi(p,p')$ may be decomposed into two contributions: one without color flow through the four-quark vertex in Fig.1 (factorizable contribution) and the other with color flow (nonfactorizable contribution), i.e.,

$$\Psi(p,p')_{QCD} = \Psi(p,p')_{fact} + \Psi(p,p')_{nonfact}. \tag{26}$$

The piece $\Psi(p.p')_{fact}$ is simply given in terms of the product

$$\Psi(p,p')_{fact} = 2(1+\frac{1}{N_c})\Pi_{5\mu}(p)\Pi_5^{\mu}(-p'), \tag{27}$$

where $\Pi_{5\mu}(p)$ is defined by (19). To leading order in α_s and m_s we obtain

$$\Pi_{5\mu}(p) = ip_\mu[-\frac{3}{8\pi^2}m_s\ln(\frac{-p^2}{\mu^2}) + \frac{1}{p^2}(<\bar{d}d> + <\bar{s}s>) - \frac{1}{8}\frac{m_s}{p^4} < \frac{\alpha_s}{\pi}G^2 >], \tag{28}$$

which differs from [10] in the last term (these authors have $-1/12$ instead of $-1/8$). In order to compute the gluon contribution, one has to use the massive quark propagator. However, as is well known [21][22], in the case of light quarks the operator $m_q\bar{q}q$ also contributes to the two-gluon matrix element (to the same order in α_s) and these two effects must be distinguished. In fact, the Wilson coefficient of the gluon condensate may be written as [21][22]

$$C_G(p) = C'_G(p,m_1,m_2) + \frac{1}{12}[C_{m_1}(p) + C_{m_2}(p)], \tag{29}$$

where $C'_G(p,m_1,m_2)$ is the Wilson coefficient of $< \frac{\alpha_s}{\pi}G^2 >$ computed directly using the full massive propagators, i.e., including all vacuum gluon field contributions, and $C_{m_j}(p)$ are the coefficients of the quark condensates $< m_j\bar{q}_jq_j >$. For the first term we obtain

$$\Pi_{5\mu}(p)|_{C'_G} = i\frac{p_\mu}{p^4}(m_1+m_2)\frac{1}{12} < \frac{\alpha_s}{\pi}G^2 > \hat{C}_G, \tag{30}$$

where

$$\hat{C}_G = -\frac{1}{2} - \frac{m_1}{m_2} - \frac{m_2}{m_1} + O(\frac{m_j^2}{p^2}). \tag{31}$$

Next, to compute C_{m_1} and C_{m_2}, one has to include contributions of order $m_q^2 < \bar{q}q >$ to $\Pi_{5\mu}$. We find that

$$\Pi_{5\mu}(p)|_{m^2<\bar{q}q>} = i\frac{p_\mu}{p^4}[(m_2^2+\frac{1}{2}m_1m_2-\frac{1}{2}m_1^2) < \bar{q}_1q_1 > +(m_1^2+\frac{1}{2}m_1m_2-\frac{1}{2}m_2^2) < \bar{q}_2q_2 >], \quad (32)$$

which on account of (29) leads to

$$\Pi_{5\mu}(p)|_{C_{m_1}+C_{m_2}} = i\frac{p_\mu}{p^4}\frac{1}{12} < \frac{\alpha_s}{\pi}G^2 > (m_1\frac{m_1}{m_2}+m_2\frac{m_2}{m_1}). \quad (33)$$

Finally, adding (30) to (33) gives the total result for the gluon condensate term

$$\Pi_{5\mu}(p^2)|_{<G^2>} = -i\frac{p_\mu}{p^4}(m_1+m_2)\frac{1}{8} < \frac{\alpha_s}{\pi}G^2 > . \quad (34)$$

Notice that potential mass singularities of the form m_1/m_2 and m_2/m_1 in eq.(31) are exactly canceled by corresponding terms in (33). It is important to stress that the procedure of setting *ab initio* one of the quark masses to zero and then using the dimensional regularization prescription to regulate mass singularities does not lead to the correct answer [23]. One should first compute the corrections (33) for massive quarks, add it to the direct piece (30), and in the end set one of the quark masses to zero in (34).

It is possible to obtain the complete expression (28) more economically by means of a Ward identity relating $\Pi_5(p^2)$ to the nontransverse invariant function $\mathcal{D}_5(p^2)$ defined through

$$\begin{aligned} \mathcal{T}_5^{\mu\nu}(p) &= i\int d^4x e^{ip\cdot x} < 0|T(A^\mu(x)A^\nu(0)^\dagger)|0 > \\ &= (-\eta^{\mu\nu}p^2+p^\mu p^\nu)\mathcal{T}_5(p^2)+\eta^{\mu\nu}\mathcal{D}_5(p^2), \end{aligned} \quad (35)$$

where $\mathcal{D}_5(p^2)$ is known in QCD at the two-loop level and with power corrections up to dimension $d = 6$ [24]. We briefly describe this procedure as it serves as a check of the correctness of expression (28). To leading order in α_s and in the quark masses, $\mathcal{D}_5(p^2)$ is given by [24]

$$\mathcal{D}_5(p^2) = -\frac{3}{8\pi^2}(m_s+m_d)^2\ln(\frac{-p^2}{\mu^2})+\frac{1}{p^2}(m_s+m_d)(<\bar{s}s+\bar{d}d>)-\frac{(m_s+m_d)^2}{p^4}\frac{1}{8}<\frac{\alpha_s}{\pi}G^2> . \quad (36)$$

The Ward identity relating $\Pi_5(p^2)$ to $\mathcal{D}_5(p^2)$ can be easily obtained by multiplying eq.(35) by, e.g., p_ν and integrating by parts. In this way one finds that $\Pi_5(p^2)$ is obtained from dividing (36) by (m_s+m_d); the result (28) then follows.

Turning to the nonfactorizable contributions to $\Psi(p,p')$, we are in agreement with the results of [10] to leading order in α_s and m_s for the power corrections involving vacuum condensates of dimension $d \leq 10$. We quote this expression for the sake of completeness:

$$\begin{aligned}\Psi(p,p')_{nonfact} &= -\frac{m_s^2}{16\pi^2} < \frac{\alpha_s}{\pi}G^2 > \frac{p\cdot p'}{p^2p'^2}[\ln(\frac{-p^2}{\mu^2})\ln(\frac{-p'^2}{\mu^2}) + (\ln(\frac{-p^2}{\mu^2})-1)(\ln(\frac{-p'^2}{\mu^2})-1)] \\ &+\frac{m_s}{4\pi^2} < g\bar{q}\sigma^{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}q > \frac{p\cdot p'}{p^2p'^2}[\ln(\frac{-p^2}{\mu^2})+\ln(\frac{-p'^2}{\mu^2})] \\ &-\frac{m_s}{6} < \frac{\alpha_s}{\pi}G^2 >< \bar{q}q > \frac{p\cdot p'}{p^2p'^2}[\frac{1}{p^2}\ln(\frac{-p'^2}{\mu^2})+\frac{1}{p'^2}\ln(\frac{-p^2}{\mu^2})] \\ &+\frac{2}{3} < \bar{q}q >< g\bar{q}\sigma^{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}q > \frac{p\cdot p'}{p^2p'^2}(\frac{1}{p^2}+\frac{1}{p'^2}) \\ &-\frac{4\pi^2}{9} < \frac{\alpha_s}{\pi}G^2 >< \bar{q}q >^2 \frac{p\cdot p'}{p^4p'^4} - \frac{13}{288}| < g\bar{q}\sigma^{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}q > |^2\frac{p\cdot p'}{p^4p'^4}. \end{aligned} \tag{37}$$

V. Results for the B parameter

In order to optimize the determination of the B parameter we shall work with the Laplace transform of the three-point function $\Psi(p,p')$, i.e.,

$$\Psi(M_1^2,M_2^2) = \frac{1}{\pi^2}\int_0^\infty ds e^{-s/M_1^2}\int_0^\infty ds' e^{-s'/M_2^2} Im\Psi(s,s'). \tag{38}$$

In this case, the sum rule determining B is

$$\Psi(M_1^2,M_2^2)_{hadr} \doteq 2(1+\frac{1}{N_c})[\Pi_5^\mu(M_1^2)\Pi_{5\mu}(M_2^2)]_{QCD} + \Psi(M_1^2,M_2^2)|^{nonfact}_{QCD}, \tag{39}$$

where

$$\begin{aligned}\Psi(M_1^2,M_2^2)|_{hadr} &\doteq 2(1+\frac{1}{N_c})B\frac{f_K^4 m_K^4}{m_s^2}\frac{e^{-m_K^2/M_1^2}}{M_1^2}\frac{e^{-m_K^2/M_2^2}}{M_2^2}[1+C_K(M_1^2)][1+C_K(M_2^2)] \\ &+2(1+\frac{1}{N_c})\frac{9}{64\pi^4}m_s^2 e^{-s_0/M_1^2}e^{-s_0/M_2^2}, \end{aligned} \tag{40}$$

and

$$C_K(M^2) = e^{m_K^2/M^2}m_K^2 a^2(1+\gamma^2)\int_0^{s_0} e^{-s/M^2}\frac{ds}{(s-m_{K'}^2)^2+m_{K'}^2\Gamma_{K'}^2}. \tag{41}$$

The QCD expressions on the r.h.s. of (39) are easily obtained by applying the familiar rules of Laplace transformation [21], [25] to Eqs. (28) and (37).

Following the established practice in QCD sum rule calculations, we shall seek for sum rule windows in the two-parameter space (M_1^2, M_2^2) inside which we expect duality between the hadronic l.h.s. of (39) and the QCD r.h.s. This duality will take place only for certain values of the B parameter which will then become the prediction from this method. This strategy differs from that used in [10] where the first QCD term on the r.h.s of (39) was saturated with the kaon-pole contribution to the hadronic piece of the three-point function. Although it is true that we have computed the QCD expressions to leading order in m_s and α_s, we find no reason to discriminate between the factorizable and the nonfactorizable pieces.

In other words, if one were to mistrust the former, there is no reason not to mistrust the latter. At the end we shall make a qualitative estimate of these systematic uncertainties associated with radiative and higher order quark mass corrections.

We now turn to our choice of values for the various QCD parameters entering Eqs. (28) and (37). For the strange quark mass we use the most recent result [18]

$$\begin{aligned} \hat{m}_s &= 288 \pm 48 MeV, \\ \overline{m}_s(1GeV^2) &= 199 \pm 33 MeV, \end{aligned} \tag{42}$$

where $\overline{m}_s$ and $\hat{m}_s$ are the running and invariant masses, respectively. The value of the gluon condensate is somewhat controversial (for recent reviews see, e.g.,[26]). We shall here use the conservative range

$$< \frac{\alpha_s}{\pi} G^2 >= (0.012 - 0.03) GeV^4, \tag{43}$$

which includes the so-called standard value [21],[27] (lower bound), as well as higher values obtained recently from e^+e^- [28] and τ -lepton decay data [29]. The mixed quark-gluon vacuum condensate may be parametrized as

$$< g\overline{q}\sigma^{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}q >= M_0^2 < \overline{q}q >, \tag{44}$$

where $M_0^2 \simeq 0.4 GeV^2$ [30] and $M_0^2 \simeq (0.6 - 1.0) GeV^2$ [31], both from baryon sum rules, $M_0^2 \simeq 0.3 GeV^2$ [32] from charmonium, $M_0^2 \simeq 1 GeV^2$ [33] from a lattice calculation, and $M_0^2 \simeq (0.4 - 0.6) GeV^2$ [34] from an analysis of the D and B mesons. We shall give results for a choice: $M_0^2 \simeq (0.4 - 0.6) GeV^2$ as well as for the extreme value $M_0^2 = 1 GeV^2$. To reduce the uncertainties, we shall absorb m_s into $< \overline{q}q >$ and use the renormalization group invariant $< m_s\overline{q}q >$. Even neglecting the small SU(3) vacuum symmetry breaking, so that $< \overline{d}d > \simeq < \overline{s}s >$ [22],[35], the quantity $< m_s\overline{q}q >$ is not known a priori. Assuming that it is given by kaon-PCAC is a dangerous approximation, as shown by explicit calculations of chiral symmetry breaking corrections [17]. For $< m_s\overline{q}q >$ we shall therefore use the value extracted from QCD sum rules. This result, however, depends on m_s since $< m_s\overline{q}q >$ is given in terms of the difference between a hadronic integral and a QCD expression proportional to $\hat{m}_s^2$ [17]. For instance, using the central value of $\hat{m}_s$ in (42), one finds

$$\begin{aligned} < m_s\overline{q}q > &\simeq -1.65 \times 10^{-3} GeV^4, \\ (\hat{m}_s &= 288 MeV), \end{aligned} \tag{45}$$

while changes in $\hat{m}_s$ within errors modify the above result by $20 - 30\%$; the smaller $\hat{m}_s$ the larger $< m_s\overline{q}q >$. This correlation between $\hat{m}_s$ and $< m_s\overline{q}q >$ will have the welcome feature of softening the dependence of B on m_s. Notice that, naively, B would depend on m_s^2, which is subject to a large uncertainty. Finally, the value of Λ_{QCD} , which appears as the renormalization scale in the Laplace transform $\Psi(M_1^2, M_2^2)|_{QCD}^{nonfact}$ in (39), will be fixed to $\Lambda_{QCD} = 100 MeV$.

With the above choice of values for the QCD parameters, and using the one-loop expression for the running quark-mass $\overline{m}_s$, we have solved the QCD sum rules (39) for the B parameter and searched for sum rule windows in the Laplace parameters M_1^2 and M_2^2. Some representative results are shown in Figs. 2-4, for $M_1^2 = M_2^2 = M^2$ and $s_0 = 3GeV^2$. Cases where $M_1^2 \neq M_2^2$ and s_0 has different values are qualitatively and quantitatively comparable with these results. Figure 2 shows the sensitivity of B to the value of the gluon condensate, which was allowed to change in the range (43), with all other parameters fixed, i.e., $M_0^2 = 0.6GeV^2$, $\hat{m}_s = 288MeV$ and $< m_s\bar{q}q >$ as in Eq.(45). The dependence of B on M_0^2 is illustrated in Fig.3 for $M_0^2 = 0.4GeV^2$ (curve a), $M_0^2 = 0.6GeV^2$ (curve b), and $M_0^2 = 1GeV^2$ (dashed curve). All other parameters are fixed as for Fig.2 except for the gluon condensate, which was taken as $< \frac{\alpha_s}{\pi}G^2 >= 0.012GeV^4$. Figure 4 shows the maximum (curve a) and minimum (dashed curve) values of B obtained from $M_0^2 = 0.4GeV^2, < \frac{\alpha_s}{\pi}G^2 >= 0.012GeV^4$ and $M_0^2 = 1GeV^2, < \frac{\alpha_s}{\pi}G^2 >= 0.03GeV^4$, respectively, with $\hat{m}_s = 288MeV^2$ and$< m_s\bar{q}q >$ as in Eq.(45). Allowing m_s and $< m_s\bar{q}q >$ to change within errors, one obtains results for B which are inside the maximum and minimum values shown in Fig.4.

The results above may be summarized as $B = 0.5 \pm 0.1$. However, there is an additional systematic uncertainty arising from radiative and higher order quark mass corrections. Using the two-loop results from [24] for the factorizable piece of the three-point function and computing the next order in m_s in Eq.(28), we estimate the overall uncertainty in B, from the uncalculated three-loop corrections to the nonfactorizable piece of $\Psi(p,p')$, to be at the level of 40%. This uncertainty includes the implicit μ dependence of our result. Three-loop radiative corrections would account for the explicit μ dependence of the form given in eq.(6). This is the same as in [5]-[7] where perturbative radiative corrections restored the μ dependence. However, this source of uncertainty is only modest since a variation of μ in the range $\mu \simeq$ 0.5-1.0 GeV produces a change in $B(\mu)$ of less than 8%. Our final result then becomes

$$B = 0.5 \pm 0.1 \pm 0.2, \tag{46}$$

where the first error reflects the uncertainties in the various QCD parameters, and the second is an educated guess of radiative and higher order quark mass effects. In view of the rather large overall uncertainty in B, we find it unjustified to go beyond the present level of accuracy in QCD and to attempt the formidable task of three-loop calculations for the nonfactorizable contributions.

VI. Discussion and Conclusions

We begin by contrasting our result for the B parameter , eq.(46), with the values reported by the authors of [8]-[10]. Using a three-point function involving axial-vector currents, together with $\mathcal{O}_{\Delta S=2}$, Chetyrkin *et al.*[8] have obtained

$$B = 1.2 \pm 0.1. \tag{47}$$

On the other hand, choosing exactly the same three-point function Decker [9] obtained quite a different result

$$B = 0.55 \pm 0.15. \tag{48}$$

The reason for such a flagrant discrepancy has been partially explained in [9]. Chetyrkin *et al.* [8] attributed *only* the perturbative term in the factorizable piece of the three-point function to the vacuum saturation result $B_{v.s.} = 1$. This is incorrect. In fact, a part of the nonperturbative corrections calculated in [8] contributes to $B_{v.s.}$. Actually, it is the *whole* factorizable QCD piece (and not only the free quark contribution as stated in [9]) which should build up the vacuum saturation value $B_{v.s.} = 1$. In any case, a distinction between perturbative and nonperturbative factorizable parts has no physical meaning, as far as the B parameter is concerned. The relative importance of these two parts depends on the choice of the currents in the three-point function. This relative "weighting" is the motivation behind the choice of pseudoscalar currents in [10]. For axial-vector currents one should then accept (48) , and not (47), as the answer. However, we should point out that the value (48) was obtained in [9] using the on-mass-shell expression (5). Since the kaons are definitely off-mass-shell in (8), the actual prediction for B could be somewhat different from (48) if the correct expression (10) is used in the calculation.

Our determination of the B parameter presented here is closely related to that of [10]. However, we differ from these authors in two important points, to wit. (*i*) On the hadronic side, we have explicitly included chiral-symmetry breaking corrections to the kaon propagator arising from the radial excitation $K'(1400)$. Past experience shows that these corrections are quite important and, furthermore , that they should not be absorbed in the hadronic continuum. (*ii*) On the QCD side, we have first corrected the result of [10] for the Wilson coefficient of the gluon condensate in $\Pi_{5\mu}(p)$, eq.(34). Next, instead of saturating $\Psi(p,p')|^{fact}_{QCD}$, eq.(27), by kaon states we have used its QCD expression. This is the standard procedure in QCD sum rule applications where one seeks for duality between a given hadronic parametrization and a QCD expression for a two- or a three-point function (see eq.(39)). In this way we have treated factorizable and nonfactorizable contributions on the same footing , and absorbed the overall uncertainty due to uncalculated radiative and higher order mass corrections in the final error bars.

It should be stressed that we have found these overall uncertainties to be significantly larger than those claimed in [8],[9], and in [10], i.e.,

$$B = 0.84 \pm 0.08. \tag{49}$$

In fact, we have estimated $\Delta B \simeq 0.3$ which then makes our prediction (46) consistent with (48) and (49) within errors.

As a byproduct of our calculation we confirm the general expectation [3], [13] from the $1/N_c$ expansion that factorizable and nonfactorizable $1/N_c$ contributions tend to cancel each other to some extent. Quantitatively, though, we are not able to state this very precisely because of the inherent uncertainties. However, we do confirm the conjecture [3] that non-factorizable contributions should significantly decrease the value of the B parameter from the vacuum saturation result $B_{v.s.} = 1$. In this connection, our prediction (46) is consistent, within errors, with that from an $1/N_c$ analysis [13], giving

$$B_{1/N_c} = 0.7 \pm 0.07. \tag{50}$$

In our opinion, however, the above error is most likely an underestimate. Notice that the calculation of the B parameter in this framework is closely related to the calculation of $K^+ \to \pi^+\pi^0$ and should therefore have a comparable accuracy. However, the uncertainties quoted for $K^+ \to \pi^+\pi^0$ [12] are considerably larger than those in (50). In any case, one should wait to see how the prediction (50) changes when one goes beyond the truncated theory (in this context, see also the discussion in [36]).

With a realistic estimate of the uncertainties in three-point function QCD sum rule calculations of the B parameter, as in Eq.(46), we may safely compare our result with that from a two-point function analysis [5], i.e.,

$$B = 0.33 \pm 0.09. \tag{51}$$

A similar answer has also been obtained earlier from PCAC and chiral perturbation theory [37]-[38]. It should be noticed that the same techniques leading to the prediction (51) in [5] reproduce the closely related amplitude for $K^+ \to \pi^+\pi^0$ almost exactly [6]. However, the $\Delta I = 1/2$ rule remains unexplained in this approach; its explanation could well lie outside the short distance local operator.[4]. This is at variance with the results from the $1/N_c$ analysis of [12]- [13] which claim an explanation of this problem. More effort is needed to understand this discrepancy.

In our view, it appears difficult to achieve a sizable reduction of the error in eq.(46) to compete with the accurate two-point function analysis of [5]. We then conclude that in the framework of QCD sum rules, three-point function calculations of the B parameter support the result (51), but cannot claim the same level of accuracy.

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[4]In this respect there are some interesting attempts to explain the $\Delta I = 1/2$ rule outside the conventional approach; see [39]-[40].

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References

[1] A. Buras, in: Proc. of the Inter. Europhysics Conf. on High Energy Physics, Bari, July 1985, Eds. L. Nitti and G. Preparata, p. 1035.

[2] E. de Rafael, in: Proc. of the Int. Conf. on A European Hadron Facility, Ed. Th. Walcher, (North Holland, Amsterdam , 1987), p. 143.

[3] M. A. Shifman,in:Proc. Int. Symp. on Lepton and Photon Interactions at High Energies, Hamburg, 1987 (to be published).

[4] N. Cabibbo, G. Martinelli, R. Petronzio, Nucl. Phys. **B244**, 381, (1984); R. C. Brower, G. Maturana, M. B. Gavela, R. Gupta, Phys. Rev. Lett. **53**, 1318 (1984); C. Bernard, G. Hockney, A. Soni, T. Draper, Phys. Rev. Lett. **55**, 2770 (1985).

[5] A. Pich and E de Rafael, Phys. Lett. **158B**, 477 (1985).

[6] B. Guberina, A. Pich and E. de Rafael, Phys. Lett. **163B**, 198 (1985).

[7] A. Pich, B. Guberina and E. de Rafael, Nucl. Phys. **B277**, 197 (1986); A. Pich and E. de Rafael, Phys. Lett. **189B**, 369 (1987).

[8] K. G. Chetyrkin, A. L. Kataev, A. B. Krasulin, A. A. Pivovarov, Phys.Lett. **174B**, 104 (1986).

[9] R. Decker, Nucl. Phys. **B277**, 661 (1986).

[10] L. J. Reinders and S. Yazaki, Nucl. Phys. **B288**, 789 (1987).

[11] D. Tadić and J. Trampetić, Phys. Lett. **114B**, 179 (1982); A.J. Buras and J.-M. Gerard, Nucl. Phys. **B264**, 371 (1986); R.S. Chivukula, J.M. Flynn and H. Georgi, Phys. Lett. **171B**, 453 (1986).

[12] W.A. Bardeen, A.J. Buras and J.-M. Gerard, Phys. Lett. **180B**, 133 (1986). MPI-PAE/PTh 45/86.

[13] W.A. Bardeen, A.J. Buras and J.-M. Gerard, MPI-PAE/PTh 55/86 (to be published in Phys. Lett.), MPI-PAE/PTh 22/87.

[14] N. S. Craigie, J. Stern, Nucl. Phys. **B216**, 209 (1983).

[15] E. Witten, Nucl. Phys. **B160**, 57 (1979).

[16] C. A. Dominguez, Z. Phys. C-Particles and Fields **26**, 269 (1984).

[17] C. A. Dominguez, M. Kremer, N. A. Papadopoulos, K. Schilcher, Z. Phys. C-Particles and Fields **27**, 481 (1985).

[18] C. A. Dominguez, E. de Rafael, Ann. Phys.(N.Y.) **174**, 372 (1987).

[19] H. Pagels and A. Zepeda, Phys. Rev. **D5**, 3262 (1972).

[20] J. Gasser, H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[21] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Fortsch. Phys. **32**, 585 (1984).

[22] L.J.Reinders, H.R.Rubinstein, S.Yazaki, Phys.Rep. **127**,1 (1985).

[23] S. C. Generalis, D. J. Broadhurst, Phys. Lett. **139B**, 85 (1984); E. Bagan, J. I. Latorre, P. Pascual, Z. Phys. C- Particles and Fields **32**, 43 (1986).

[24] C. Becchi, S. Narison, E. de Rafael, F. J. Yndurain, Z.Phys. C-Particles and Fields **8**, 335 (1981).

[25] S. Narison, E. de Rafael, Phys. Lett **103B**, 57 (1981).

[26] R. A. Bertlman, in: Proc. Intern. Symp. on Multiparticle Dynamics , Eds. M. Markytan, W. Majerotto, J. Mac Naughton (World Scientific , Singapore, 1987); C. A. Dominguez, in: Quarks, Gluons and Hadronic Matter, Eds. R. Viollier, N. Warner (World Scientific, Singapore, 1987).

[27] B. Guberina, R. Meckbach, R. D. Peccei, R. Rückl, Nucl. Phys. **B184**, 476 (1981).

[28] R. A. Bertlmann, C. A. Dominguez, M. Loewe, M. Perrottet, E. de Rafael, CERN Report (to appear).

[29] C. A. Dominguez, J. Sola, DESY Report No. DESY-87-119 (1987).

[30] Y. Chung, H. G. Dosch, M. Kremer, D. Schall, Z. Phys. C-Particles and Fields **25**, 151 (1984).

[31] V. M. Belyaev, B. L. Ioffe, Sov. Phys. JETP **56**, 493 (1982).

[32] S. N. Nikolaev, A. V. Radyushkin, Sov. J. Nucl. Phys. **39**, 91 (1984).

[33] M. Kremer, G. Schierholz, Phys. Lett. **B194**, 283 (1987).

[34] C. A. Dominguez, N. Paver, Phys. Lett. **B197**, 423 (1987).

[35] S. Narison, N. Paver, D. Treleani, Nuovo Cimento **A74**, 347 (1983), C. A. Dominguez, M. Loewe, Phys. Rev. **D31**, 2930 (1985); N. Paver, Riazuddin, M. D. Scadron, Phys. Lett. **B197**, 430 (1987).

[36] J. Bijnens, B. Guberina, DESY Report No. DESY-87-065 (1987).

[37] J. F. Donoghue, E. Golowich, B. R. Holstein, Phys. Lett. **B119**, 412 (1982).

[38] J. Bijnens, H. Sonoda, M. Wise, Phys. Rev. Lett. **53**,2367 (1984).

[39] H. Galić, Phys. Rev. **D24**, 2441, 3000 (1981).

[40] B. Stech, Phys. Rev. **D** (to be published); Heidelberg Report No. HD-THEP-87-7 (1987); Invited talk at the San Miniato Topical Seminar on Heavy Flavors (1987), Heidelberg Report No. HD-THEP-87-18 (1987).

Figure captions

1. Diagrammatic representation of the three-point function , Eq. (1).

2. Results for the B parameter as a function of $M_1^2 = M_2^2 \equiv M^2$, for $\hat{m}_s = 288 MeV$, $< m_s \bar{q}q >= -1.65 \times 10^{-3} GeV^4$, $M_0^2 = 0.6 GeV^2$, and $< \frac{\alpha_s}{\pi} G^2 >= 0.012 GeV^4$ (curve a), and $0.03 GeV^4$ (curve b).

3. Results for the B parameter as a function of $M_1^2 = M_2^2 \equiv M^2$ for $\hat{m}_s = 288 MeV$, $< m_s \bar{q}q >= -1.65 \times 10^{-3} GeV^4$, $< \frac{\alpha_s}{\pi} G^2 >= 0.012 GeV^4$, and $M_0^2 = 0.4 GeV^2$ (curve a), $0.6 GeV^2$ (curve b) , and $1 GeV^2$ (dashed curve).

4. Results for the B parameter as a function of $M_1^2 = M_2^2 \equiv M^2$ for $\hat{m}_s = 288 MeV$, $< m_s \bar{q}q >= -1.65 \times 10^{-3} GeV^4$, and the three pairs of values $M_0^2 = 0.4 GeV^2$ and $< \frac{\alpha_s}{\pi} G^2 >= 0.012 GeV^4$ (curve a), $0.6 GeV^2$ and $0.03 GeV^4$ (curve b), and $1.0 GeV^2$ and $0.03 GeV^4$ (dashed curve).

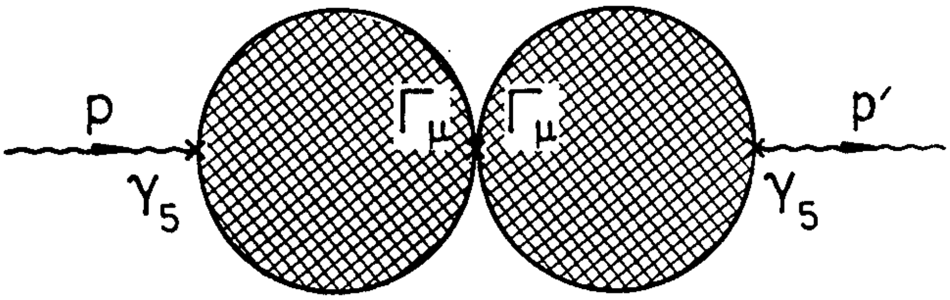


Fig. 1

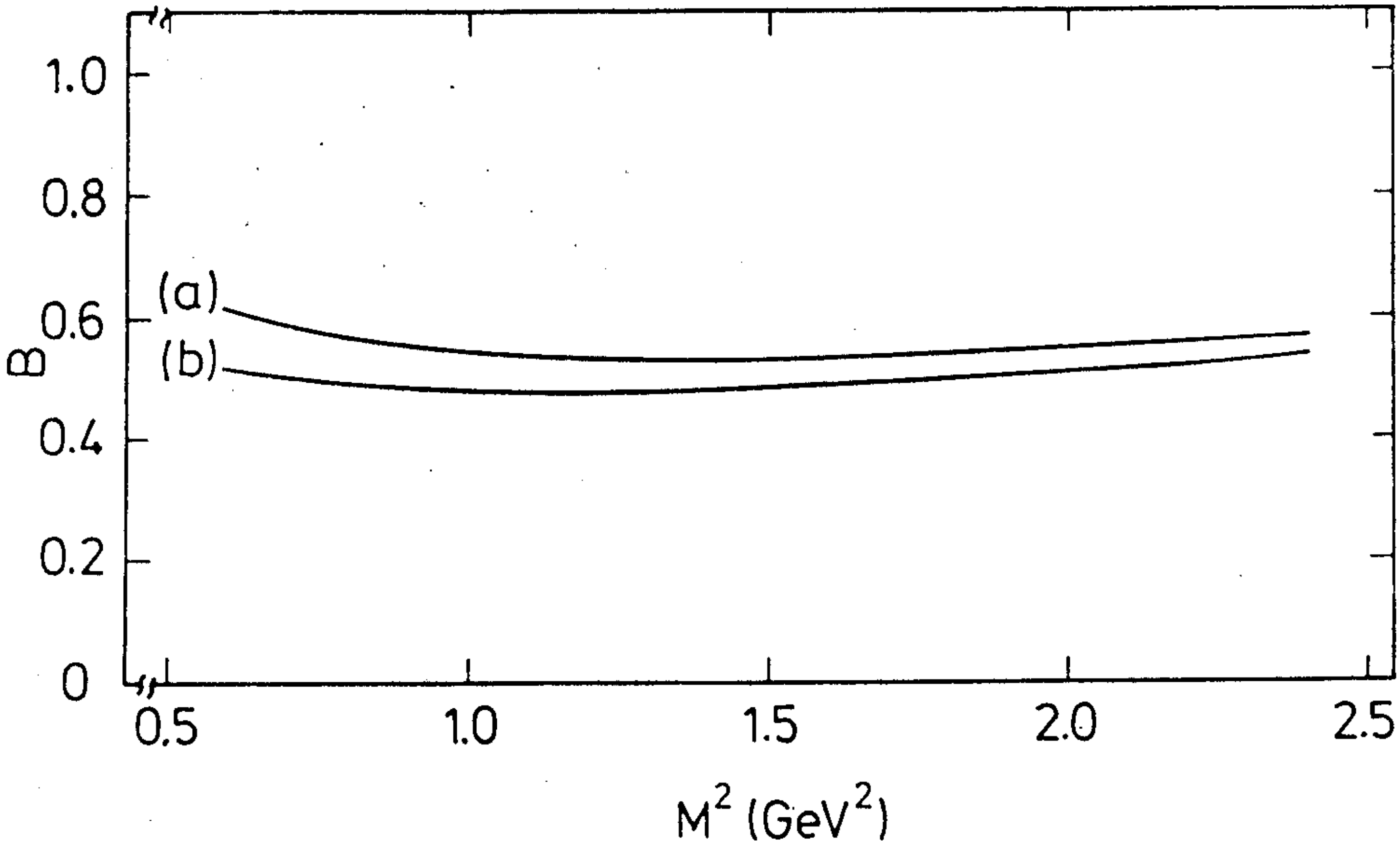


Fig. 2

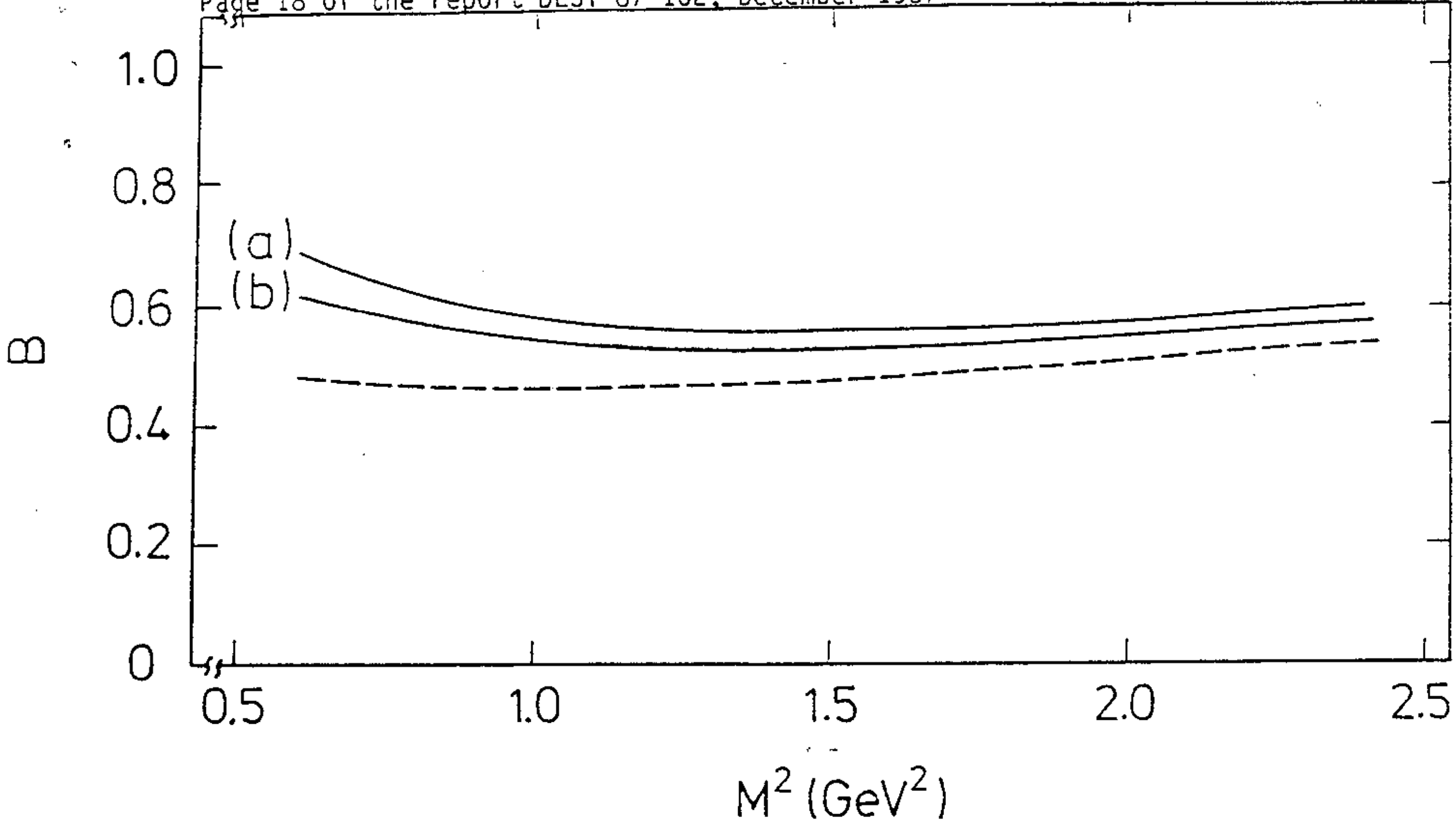
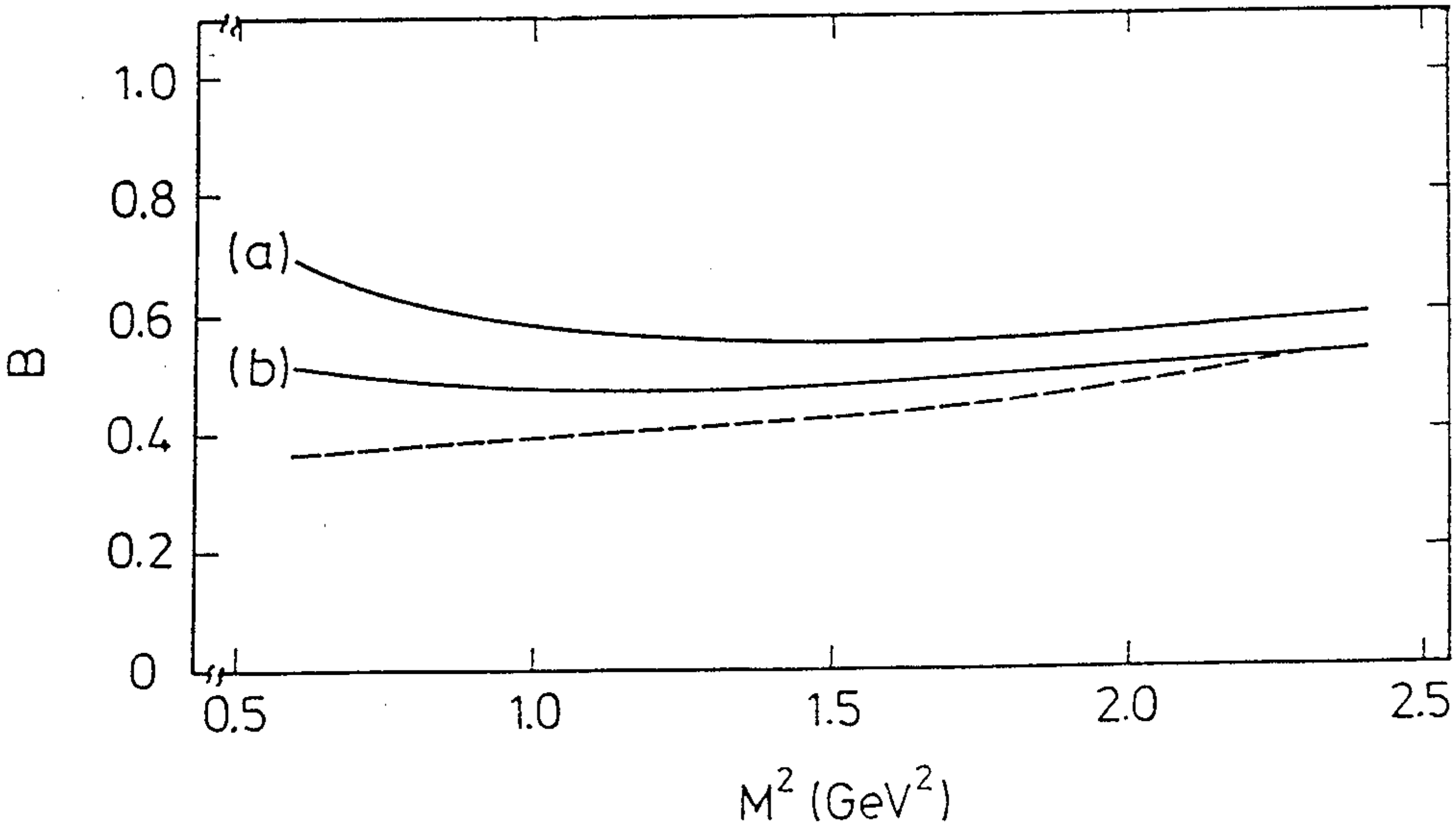

1.0
0.8
0.6
0.4
0.2
0
B
(a)
(b)
0.5
1.0
1.5
2.0
2.5
M^2 (GeV^2)

Fig. 3


1.0
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B
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(b)
0.5
1.0
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2.0
2.5
M^2 (GeV^2)

Fig. 4