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A Chiral Anomaly Channel in τ-Decay II

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Abstract: The decay rate for $\tau \to \nu\eta\pi\pi$ is calculated from an amplitude that has the chiral anomaly term as low-energy constraint and has realistic vector-meson pole continuation to physical energies.

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I. Introduction

Last year the HRS and Crystal Ball collaborations presented preliminary evidence for $\tau \to \eta X$ at the few percent level /1/. Both groups observed an enhancement at the η mass in an inclusive $\gamma\gamma$ mass spectrum. It was pointed out by Gilman /2/ that a one-prong η decay mode of a few percent is not consistent with other measurements. Later HRS reanalysed their data and reported their η signal was only consistent with the $\tau \to \nu\eta\pi$ mode /3/. This decay can only occur via a second class current which is forbidden in the Standard Model. In the meantime the Crystal Ball collaboration have also reanalysed their data and found their signal completely disappears under more stringent selection criteria, leading to upper limits for various η decay modes /4/. For example, their upper limit for $\tau \to \nu\eta\pi$ obtained from two independent analysis (inclusive /5/ and exclusive /4/) is now 0.3%. Less stringent limits were reported by the Argus /6/ and the Cello collaboration /7/ (the upper limits are 1.3% and 1.4% respectively). This rules out the decay $\tau \to \nu\eta\pi$ as the prime candidate for the inclusive η branching fraction. In the Standard Model, one would expect the $\tau \to \nu\eta\pi\pi$ to be the largest τ decay mode involving the η meson.

Some years ago we calculated the decay rate for $\tau^- \to \nu\eta\pi^-\pi^0$ using an effective Lagrangian approach for pseudoscalar mesons only /8/. The soft-meson limit of the decay amplitude was fixed by the chiral anomaly /9/. The amplitude was extrapolated to the physical region of phase space by multiplying with vector-meson-resonance factors. We considered several possibilities. In the $\eta\pi\pi$ channel we introduced the ρ(770) and ρ'(1600) resonances whereas the $\pi\pi$ channel was enhanced by the ρ(770) only. The results for the branching fraction B ranged from $3.4 \cdot 10^{-4}$ to $3.9 \cdot 10^{-4}$ depending on assumptions about the resonances in the $\eta\pi\pi$ channel.

Similar calculations were done for the decays η and $\eta' \to \pi^+\pi^-\gamma$, K_{e4} and $\omega \to \pi^+\pi^-\pi^0$ /9/. The decay $\eta \to \pi^+\pi^-\gamma$ proceeds through the same amplitude as $\tau^- \to \nu\eta\pi^-\pi^0$ except that the current is rotated in isospin space and that the τ decay amplitude is sensitive to higher vector-meson resonances as for example the $\rho'(1600)$ in the $\eta\pi\pi$ channel. F1) For these decays the continuation to higher energies was modified in a later publication /12/. This modification perhaps best can be explained taking the amplitude for $\eta \to \pi^+\pi^-\gamma$ as an example. Let G(0) be the low-energy theorem for the vector form factor at $\pi\pi$ momentum transfer u = 0. Then in /9/ we assumed that G(u) is given by

$$G(u) = G(0)\,(m^2 - im\Gamma)/(m^2 - im\Gamma - u) \qquad (1)$$

where m and Γ are the mass and width of the ρ-meson pole. In (1) the residue at the pole is determined on the one hand by G(0) which in turn depended on the pion decay constant F_π (in our normalization F_π = 186 MeV) and on the other hand by the $\rho\pi\pi$ coupling constant g and the $\rho\eta\gamma$ coupling which via vector dominance is related to the $\eta\gamma\gamma$ coupling and the latter to F_π again. The exact relation between g and F_π which emerged was $(F_\pi g)^2 = \frac{4}{3}m^2$, which is similar but not identical to the experimentally well-satisfied Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation $(F_\pi g)^2 = 2m^2$ /13/. In (1) the residue at the ρ-pole has not the correct strength if compared with measured coupling constants. In /12/ we left open the relation among the pion decay constant and the strong-interaction parameters of the ρ meson and took for the $\eta \to \pi^+\pi^-\gamma$ amplitude G(u) that we are using as an example the following ansatz

$$G(u) = G(0)\left[(1-3\alpha) + 3\alpha(m^2 - im\Gamma)/(m^2 - im\Gamma - u)\right] \qquad (2)$$

where

$$\alpha = (gF_\pi/2m)^2 = g_{\omega\rho\pi}\,\pi^2 F_\pi^3/3m^2 \qquad (3)$$

G(u) is now independent of α at the low-energy point u = 0. If $\alpha = \frac{1}{2}$ the residue at the ρ pole now has a strength in agreement with the KSRF relation, which roughly agrees with the experimental information on F_π and g. The value α = 0.55 best fits the modern parameters of the pion decay width and the ρ-meson parameters. $\alpha = \frac{1}{3}$ corresponds to our original ansatz (1) used in /9/. The vector-meson dominance scheme as given by (2) has the property that it respects the low-energy constraint coming from the chiral anomaly and has the empirically correct residue at the ρ pole. It differs from the simple resonance-saturation ansatz (1) by the presence of a low energy "contact" term that allows the theory to obey the chiral anomaly theorem. In the framework of generalized vector-meson dominance the contact term (1-3α) stands for the sum of higher mass resonance terms whose masses are large compared to u and whose couplings are unknown F2). Based on (2) with α = 0.55 we obtained in /12/ good results for $\eta \to \pi^+\pi^-\gamma$, $\eta' \to \pi^+\pi^-\gamma$ and the K_{14} form factors. Our comparison of the calculated decay rates with experimental data supported the need for the current algebra "contact" terms.

In this short note we apply the chiral-invariant vector-meson-dominance scheme as elucidated for the $\eta \to \pi^+\pi^-\gamma$ amplitude above to the special decay amplitude $\tau \to \nu\eta\pi^-\pi^0$. As in our earlier work /8/ we use the vector-meson-dominance scheme also for the $\eta\pi\pi$ channel. By allowing a separate contact term also in this channel we are able to compare with the results of a recent calculation by Braaten, Oakes and Sze-Mau Tze /15/. These authors based their analysis of the $\tau^- \to \nu\eta\pi^-\pi^0$ decay on a general low-energy effective Lagrangian for pseudoscalar and vector-mesons. The parameters in the Lagrangian were determined from vector-meson decay data in such a way that the τ decay amplitude obeyed the chiral anomaly constraint.

With this extended chiral-invariant vector-meson-dominance scheme we hope to have at hand a reasonably general ansatz for the decay amplitude of $\tau^- \to \nu \eta \pi^- \pi^0$ to allow a better estimate of the branching ratio as compared to our earlier work /8/. In the next section we explain our ansatz for the τ decay amplitude and present the results for various choices of coupling parameters, in particular for the ρ'.

II. The Decay $\tau \to \nu\eta\pi\pi$

The basic formulas for calculating partial widths for semihadronic τ decays were given in our earlier work /8/. Adopting the notation of this reference the partial width for $\tau^- \to \nu \eta \pi^- \pi^0$ is calculated from

$$\Gamma = \frac{G^2 \cos^2\theta_c}{8\,(2\pi)^2}\, m_\tau^5 \int_{W_{min}}^{1} dW\,(1-W)^2(1+2W)\,A \tag{4}$$

where $W^2 = q^2/m_\tau^2$. The coefficient A as defined in /8/ is obtained from

$$A = \frac{1}{144\pi^6}\left(\frac{m_\tau}{F_\pi}\right)^6 J(q^2) \tag{5}$$

The integral $J(q^2)$ is now modified in the following way

$$J(q^2) = \int_{(m_1+m_2)^2}^{(|q|-m_1)^2} \frac{ds}{q^2} \int_{(m_1+m_2)^2}^{(|q|-m_1)^2} \frac{dt}{q^2}\, \theta(\Delta)$$

$$\cdot \left| 1-\beta-\gamma+(1-3\alpha)\beta y + (\gamma+3\alpha\beta y)\,x \right|^2 \tag{6}$$

with

$$x = \frac{m_\rho^2 - i m_\rho \Gamma_\rho}{m_\rho^2 - i m_\rho \Gamma_\rho - u} \tag{7}$$

and

$$y = \frac{g_{\rho\pi\pi}}{g_\rho}\,\frac{m_\rho^2}{m_\rho^2 - i m_\rho \Gamma_\rho - q^2} + \frac{g'_{\rho\pi\pi}}{g_{\rho'}}\,\frac{m_{\rho'}^2}{m_{\rho'}^2 - i m_{\rho'} \Gamma_{\rho'} - q^2} \tag{8}$$

Here $m_1 = m_\pi$ and $m_2 = m_\eta$ and Δ is the usual triangle function /9/. x and y denote the resonance enhancement factors in the $\pi\pi$ and the $\eta\pi\pi$ channels respectively. When we go to the threshold $u = q^2 = 0$ we have the pure chiral anomaly prediction, provided

$$\frac{g_{\rho\pi\pi}}{g_\rho} + \frac{g'_{\rho\pi\pi}}{g_{\rho'}} = 1 \tag{9}$$

For $\alpha = \frac{1}{3}$, $\beta = 1$ and $\gamma = 0$ we recover our old ansatz in /8/ with no contact terms in the u and q^2 channel. For $\beta = 1$, $\gamma = 0$ and arbitrary α the vector-meson dominance factor y for the current can be factorized. This corresponds to the usual vector-meson dominance assumption for electro-weak currents. This ansatz was also used to describe the decays $\eta, \eta' \to \pi^+\pi^-\gamma$, $\omega \to \pi^+\pi^-\pi^0$ and the K_{l4} decay in /12/. There we obtained satisfactory results for α = 0.55. We have added the terms with parameters β and γ to account for a direct coupling of the weak current to the final state $\eta\pi\pi$ without going through resonances in the q² and u channel (term proportional to $(1-\beta-\gamma)$ and/or a direct coupling of the weak current to the state $\rho\eta$, i.e. a coupling of $\rho\eta$ that does not go through a resonance in the q² channel. Such direct couplings have been assumed in /15/ in the framework of an effective Lagrangian involving vector and pseudoscalar mesons. In that work the parameters α, β and γ were fixed by fitting them to decay data for $\rho^- \to \pi^-\gamma$, $\omega \to \pi^+\pi^-\pi^0$, $\omega \to \pi^0\gamma$ and $\omega \to \pi^0\mu^+\mu^-$. Without having a ρ' resonance in the current channel they obtained: $\alpha = 0.45 \pm 0.30$, $\beta = 1.77 \pm 0.35$ and $\gamma = -0.90 \pm 0.90$. For later use we take as representative values $\alpha = 0.45$, $\beta = 1.77$, $\gamma = -0.90$. The choice $\alpha = 0.55$, $\beta = 1$, $\gamma = 0$ with no direct current couplings is still roughly inside their error margin. In our more phenomenological approach we interprete the terms which do not have the vector-meson dominance factor y as coming from still higher mass resonances in the current channel. The remaining parameters concerning the ρ and ρ' meson are taken from PDG /16/: m_ρ = 0.77 GeV, Γ_ρ = 0.153 GeV, $m_{\rho'}$ = 1.59 GeV, $\Gamma_{\rho'}$ = 0.26 GeV and $B(\rho' \to \pi\pi)$ = 0.23 . From this we obtain: $g_{\rho\pi\pi}$ = 6.11 and $g_{\rho'\pi\pi}$ = 2.43. For g_ρ we take the value g_ρ = 4.99 corresponding to $\Gamma(\rho \to e^+e^-)$ = 6.89 KeV /16/. Then $g_{\rho'}$ is determined from (9), which gives $g_{\rho'}$ = - 10.8. This leads to the partial width $\Gamma(\rho' \to e^+e^-)$ = 3.0 KeV as compared to the experimental value $\Gamma(\rho' \to e^+e^-)$ = (7.5 ± 1.5) KeV /16/. This partial width gives $g_{\rho'}$ = - 6.88 which produces a 13% violation of the constraint relation (9).

We have calculated the branching ratio B using (4) - (8) under the following assumptions: (i) we take α = 0.55, β = 1, γ = 0 (no direct coupling of the current to $\eta\pi\pi$) and (a) no ρ', i.e. $g_{\rho\pi\pi} = g_\rho$, (b) $g_{\rho'\pi\pi}$ = 2.43, $g_{\rho'}$ = - 10.8, (c) $g_{\rho'\pi\pi}$ = 2.43, $g_{\rho'}$ = - 6.88, and (ii) α = 0.45, β = 1.77, γ = - 0.90 and (a), (b), (c) as under (i). The results together with the input parameters are summarized in the Table. The results in our Table show that the ρ' pole is very important. The first line can be compared to our old calculation corresponding to $\alpha = \frac{1}{3}$. It shows that an increased α produces a larger branching ratio. The old result /8/ with these parameters except $\alpha = \frac{1}{3}$ would have been B = 0.0045%. The result in the fourth line can be compared with the result of Braaten et al. /15/. It agrees with their result if we take into account that they considered a mixing angle of $\theta_p = -10^\circ$ which produces an increase of the partial width by the factor 1.48. The sensitivity to the ρ' parameters is seen by comparing the results in the second and third and the fifth and sixth line respectively. The branching fration for $\tau^- \to \nu\eta\pi^-\pi^0$ was also estimated recently by Pich /17/ using an approach similar to our old paper /8/, but including only the ρ'(1600) resonance in the $\eta\pi\pi$ channel. Our results indicate that the ρ is an important effect in the $\eta\pi\pi$ channel and should not be neglected. An estimate reported in the second publication of /17/ where also a combination of ρ(770) and ρ'(1600) resonances is considered is $B(\tau^- \to \nu\eta\pi^-\pi^0) \sim$ 0.2-0.3% consistent with our results in the Table. Now fixing $g_{\rho'\pi\pi}$ in accordance with the $\rho' \to \pi\pi$ decay data we have variied - $g_{\rho'}$. Decreasing $|g_{\rho'}|$ by 60% changes B between 70 and 90%. Although the factor $|g_{\rho'\pi\pi}/g_{\rho'}|$ in (8) is only of the order of 0.2 compared to $g_{\rho\pi\pi}/g_\rho$ the ρ' pole makes a substantial contribution since the pole is in the physical region of the decay and the total ρ' width is not too large. A larger total ρ' width as compared to the 260 MeV we have taken would make B smaller. As a last point we note that the η is not a pure octet state as we have assumed so far. The canonical singlet-octet mixing angle of $\theta_p = -11^\circ$ leads to an increase of the branching ratios B in the Table by 1.57. A mixing angle of $\theta_p = -20^\circ$, sometimes also advocated in the literature /18/ even produces B's larger by the factor 2.03. This would bring some of our predicted values laready above the experimental upper bound. The best bound so far comes from the inclusive analysis of the Crystal Ball collaboration /5/. The bound reported is $B < 0.9\%$. On the other hand the best agreement between theory and experimental data for the rate $\eta \to \pi^+\pi^-\gamma$ is for $\theta_p = 0°$ /12/ (for $\alpha = 0.55$, for the other choice of α, β, γ in the Table it is similar). This means that if we normalize the τ-branching ratio with the $\eta \to \pi^+\pi^-\gamma$ rate the results in the Table obtained for $\theta_p = 0°$ are the more realistic ones. Even then the largest branching ratio obtained is equal to the best experimental upper bound showing that in the near future when better data might become available, the $\tau \to \nu\eta\pi^-\pi^0$ decay rate can be used to test models like those presented here.

The ansatz in (6) has also been used to calculate the partial decay width for $\rho' \to \eta\pi^+\pi^-$. We must multiply the amplitude in (6) by $g_{\rho'}(m_{\rho'}^2 - i m_{\rho'}\Gamma_{\rho'} - q^2)$ (see also /9/). For $\alpha = 0.55$, $\beta = 1$, $\gamma = 0$ we obtain $\Gamma(\rho' \to \eta\pi\pi) = 25.2$ MeV as compared to $\Gamma(\rho' \to \eta\pi\pi) = 8.93$ MeV for $\alpha = \frac{1}{3}$, $\beta = 1$, $\gamma = 0$ /9/. For $\alpha = 0.45$, $\beta = 1.77$ and $\gamma = -0.99$ $\Gamma(\rho' \to \eta\pi\pi) = 20.7$ MeV. The experimental value is $\Gamma(\rho' \to \eta\pi\pi) = (18 \pm 5)$ MeV for a fixed total width $\Gamma_{\rho'} = 260$ MeV. Taking into account that $\Gamma_{\rho'}$ has an error of $\pm$ 100 MeV the agreement is satisfactory for all cases considered.

Footnotes:

F1) A possible $\rho'(1250)$ resonance in the $\eta\pi\pi$ channel is excluded by the $e^+e^- \to \eta\pi^+\pi^-$ measurements /10,11/.

F2) For an application of such higher resonances to vector meson decays in the framework of the dual model see /14/.

Table

α	β	γ	$g_{\rho\pi\pi}$	g_ρ	$g_{\rho'\pi\pi}$	$-g_{\rho'}$	B(%)
0.55	1.0	0.0	6.11	6.11	0	-	0.010
0.55	1.0	0.0	6.11	4.99	2.43	10.8	0.18
0.55	1.0	0.0	6.11	4.99	2.43	6.88	0.35
0.45	1.77	- 0.90	6.11	6.11	0	-	0.094
0.45	1.77	- 0.90	6.11	4.99	2.43	10.8	0.51
0.45	1.77	- 0.90	6.11	4.99	2.43	6.88	0.88

The branching ratio B in % for $\tau \to \nu \eta \pi^- \pi^0$ as predicted by various chiral invariant vector-dominance models. The parameters α, β and γ determine the decay amplitude in (6).

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