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by

P. Langacker
Deutsches Elektronen Synchrotron DESY, Hamburg

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Massive Neutrinos in Gauge Theories*

Paul Langacker[†]
Deutsches Elektronen Synchrotron, DESY
D-2000 Hamburg 52, Fed. Rep. Germany

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Abstract

The present status of several aspects of neutrino physics are summarized, including the weak interactions of neutrinos, neutrino counting, and the theoretical expectations for and experimental constraints on neutrino mass.

1 Introduction

Neutrinos have long been amongst the most important probes of the fundamental interactions. In the last fifteen years, in particular, neutrinos have helped establish the standard $SU_2 \times U_1$ electroweak model as correct to first approximation, have been important probes of the structure of the nucleon and of the strong interactions, and have set stringent limits on new physics beyond the standard model. Furthermore, the question of whether the neutrino has a nonzero mass is one of the most important issues in both particle physics and astrophysics: most extensions of the standard model predict a nonzero mass at some level. Masses in the 10 eV range could account for the dark matter of the universe, while masses $\leq 10^{-2} eV$ could resolve the Solar neutrino problem.

In this talk I will describe several aspects of neutrino physics starting with the weak interactions of neutrinos. It will be seen that both the charged and neutral current processes are very well described by the standard model. I will then turn to the question of neutrino counting: indirect evidence leaves little doubt as to the existence of the τ-neutrino, while a number of laboratory and cosmological constraints strongly suggest that the number of neutrinos (with mass $<< \frac{M_Z}{2}$ is less than 0(3-5)). Finally, I will consider the complicated subject neutrino mass: the principle theoretical models and their implications will be described, and the experimental situation will be briefly summarized.

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†Permanent address: Department of Physics, University of Pennsylvania, Philadelphia, PA 19104.

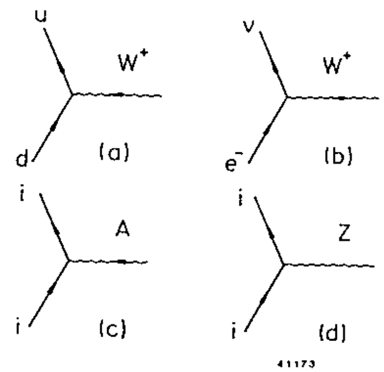

Figure 1: Charged current, QED, and neutral current interactions. The vertex factors are (a) $-\frac{ig}{2\sqrt{2}}\gamma_\mu(1+\gamma^5)V_{ud}$, (b) $-\frac{ig}{2\sqrt{2}}\gamma_\mu(1+\gamma^5)$, (c) $-ieq_i\gamma_\mu$, and (d) $-\frac{ig}{2\cos\theta_W}\gamma_\mu\left[t_{3L}(i)(1+\gamma^5)-2\sin^2\theta_W q_i\right]$. V_{ud} is an element of the quark mixing matrix, q_i is the electric charge of fermion i (in units of e), and $t_{3L}(i)$, the eigenvalue of the third generator of SU_2, is $+\frac{1}{2}$ for (u,ν) and $-\frac{1}{2}$ for (d,e^-).

2 The Weak Interactions of Neutrinos

The Glashow-Weinberg-Salam standard electroweak model [1] is based on the gauge group $SU_2 \times U_1$, with gauge couplings g and g' for the two factors and gauge bosons $(W^\pm, W^0, B)$. It incorporates the Fermi theory of the charged current weak interactions [2] and quantum electrodynamics (QED), and successfully predicted the existence and properties of a new neutral current interaction (Fig. 1). The charged and neutral current interactions are mediated by the massive gauge bosons $W^\pm$ and Z, respectively, while QED is mediated by the massless photon A, where

$$\begin{aligned} A &\equiv \cos\theta_W B + \sin\theta_W W^0 \\ Z &\equiv -\sin\theta_W B + \cos\theta_W W^0 \end{aligned} \quad (1)$$

In (1), $\theta_W \equiv \tan^{-1}(g'/g)$ is the weak angle. e, the positron electron charge, is related by

$$e = g\sin\theta_W \quad (2)$$

The W and Z masses are predicted in terms of $\sin^2\theta_W$, which can be determined independently from deep inelastic neutrino scattering. One has

$$\begin{aligned} M_W &= \frac{A_o}{\sin\theta_W(1-\Delta r)^{\frac{1}{2}}} \\ M_Z &= \frac{M_W}{\cos\theta_W} \end{aligned} \quad (3)$$

Table 1: The measured W and Z masses (in GeV), compared with the theoretical expectations [5] from deep inelastic scattering with and without radiative corrections. (The radiative corrections include Δr from (3) as well as to the value of $\sin^2\theta_W$ extracted from experiment).

	M_W	M_Z
UA1 + UA2	80.9 ± 1.4	91.9 ± 1.8
prediction (with radiative corrections)	80.2 ± 1.1	91.6 ± 0.9
prediction (without radiative corrections)	75.9 ± 1.0	87.1 ± 0.7

where $A_o = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.281\ GeV$. Δr is a higher order correction, mainly due to A, W, and Z self-energy diagrams. It is predicted to be 0.0713 ± 0.0013 for top quark and Higgs boson masses of 45 GeV and 100 GeV, respectively, while $\Delta r \to 0$ for $m_t \sim 245\ GeV$. The predictions of (3) are in striking agreement with the data from the UA1 [3] and UA2 [4] groups at CERN, and even provide a rough confirmation of the radiative corrections [5] (Table 1). The production cross sections, couplings, and angular distributions (= spin) are also in agreement with expectations.

2.1 The Charged Current

The weak charged current interaction is described by the coupling

$$L = -\frac{g}{2\sqrt{2}}(J^{\mu}_W W^-_{\mu} + J^{\mu\dagger}_W W^+_{\mu}) \qquad (4)$$

between the massive $W^{\pm}$ bosons and the charged fermion current J^{μ}_W, given (for massless neutrinos), by

$$J^{\mu\dagger}_W = (\bar{u}\ \bar{c}\ \bar{t})\ \gamma^{\mu}(1+\gamma^5)V \begin{pmatrix} d \\ s \\ b \end{pmatrix} + (\bar{\nu}_e\ \bar{\nu}_{\mu}\ \bar{\nu}_{\tau})\ \gamma^{\mu}(1+\gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix}. \qquad (5)$$

The weak charged current is purely $V - A$, which means that it involves only the left-chiral $(1+\gamma^5)$ projections of the quark and lepton fields [6]. In (5),

$$V \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad (6)$$

is the unitary Cabibbo-Kobayashi-Maskawa [7] (CKM) quark mixing matrix, which is due to the mismatch between the weak interactions and the quark mass matrix. V_{ij} describes

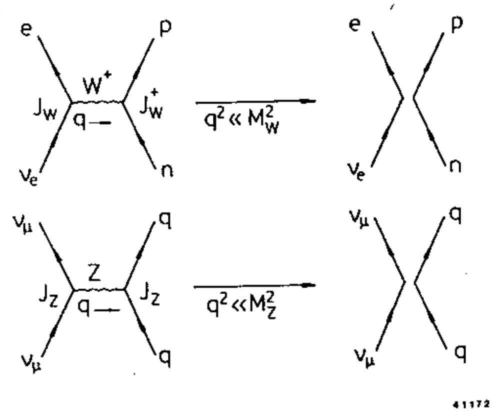

Figure 2: The effective four-fermion interactions for $\nu_e n \to e^- p$ and $\nu_{\mu} q \to \nu_{\mu} q$.

the relative amplitude for the transition $d_j \to u_i$. Experimentally,

$$V \simeq \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\theta_c^2), \qquad (7)$$

where $\sin\theta_c \simeq 0.23$ is the sine of the Cabibbo angle. For massless neutrinos there is no analogue of V in the leptonic current. Since there are no mass terms to define the neutrino flavours one can simply define ν_e as the state produced in weak transitions involving the electron, etc. For momenta small compared to M_W, weak charged current processes can be described by an effective four-fermion interaction (Fig. 2) $-L^{CC}_{eff} = \frac{G_F}{\sqrt{2}} J^{\mu\dagger}_W J_{W\mu}$, where the fermi constant G_F is given (to lowest order) by

$$G_F = \frac{\sqrt{2}g^2}{8M_W^2} = 1.16637\times10^{-5}GeV^{-2}. \qquad (8)$$

The numerical value is determined from muon decay.

The standard model predictions for the weak charged current have been extensively tested in a variety of processes [8]. In particular, there have been many precise tests in the purely leptonic sector (which is free from any uncertainties from the strong interactions), including μ and τ decay and $\nu_{\mu} e \to \mu^- \nu_e$ scattering. In a recent model-independent analysis of muon decay and inverse decay data, Fetscher, Gerber, and Johnson[9] have considered the most general local derivative-free four-fermion interaction for muon decay, assuming only Lorentz invariance, separately conserved electron and muon lepton numbers [10], and massless neutrinos. They found that the data uniquely require $V - A$ couplings for the leptonic interactions (Fig. 3 and Table 2). The other invariants, involving $V + A$ as well as scalar, pseudoscalar, and tensor operators are all required to be small, with stringent limits on the coefficients of all operators except the scalar interaction involving left-chiral e and μ.

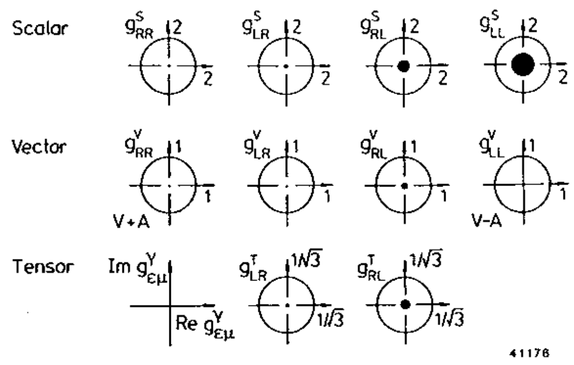

Figure 3: Values of scalar, vector, and tensor interactions in muon decay, as determined by Fetscher et al [9]. The subscripts refer to the chiralities of the e and μ, respectively.

Table 2: Limits on the branching ratios $Q^{\gamma}_{\epsilon\mu}$ for muon decay via scalar ($\gamma = S$), vector ($\gamma = V$), and tensor ($\gamma = T$) interactions, from Fetscher et al.[9] ϵ and μ are the chiralities of the e and μ, respectively. The $Q^{\gamma}_{\epsilon\mu}$ are related to the couplings in Fig. 3 by $Q^{\gamma}_{\epsilon\mu} = \lambda_\gamma |g^{\gamma}_{\epsilon\mu}|^2$. where $\lambda_S = \frac{1}{4}$, $\lambda_V = 1$, and $\lambda_T = 3$.

Quantity	Limit (90% c.l.)
$Q^S_{RR} + Q^V_{RR}$	< 0.002
$Q^S_{LR} + Q^V_{LR} + Q^T_{LR}$	< 0.008
$Q^S_{RL} + Q^V_{RL} + Q^T_{RL}$	< 0.04
$Q^S_{LL} + Q^V_{LL}$	> 0.95
Q^S_{LL}	< 0.21
Q^V_{LL}	> 0.79

One way of seeing to what extent pure $V - A$ is required is provided by a series of measurements of polarized μ^+ decay asymmetries at TRIUMF [11]. They find that the mass of W_R, a hypothetical gauge boson coupling to right-chiral ($V + A$) current [12] in μ decay, must exceed 400 GeV, in contrast to the ordinary W (coupling to $V - A$) mass of 80.9 ± 1.4 GeV. The same results can be used [9] to infer that $1 - |h_{\nu_\mu}| < 0.0032$, where h_{ν_μ} is the helicity of ν_μ produced in $\pi_{\mu 2}$ decays. This is in striking agreement with the $V - A$ prediction [13] of $h_{\nu_\mu} = -1$.

Similarly, L^{CC}_{eff} has been extensively tested in a variety of semi-leptonic decay processes, such as β, hyperon, π, K, c, and b decays. The results are in impressive agreement with the predictions of the standard model. In particular, the $V - A$ nature of the charged current interaction and the relative strength of the various weak processes, as predicted in (5), are quantitatively confirmed. For example, from μ, β, K, hyperon, and b decays one can extract the CKM matrix elements $|V_{ud}|$, $|V_{us}|$, and $|V_{ub}|$. One finds [14]

$$|V^2_{ud}| + |V^2_{us}| + |V^2_{ub}| = 0.9979 \pm 0.0021, \tag{9}$$

in remarkable agreement with the expectation of unity from universality [15] (i.e. from the unitarity of V).

The leptonic and semi-leptonic data combined leave little room for any deviation from the standard model in charged current processes. In particular, we can be certain that neutrinos are produced by (almost) canonical $V - A$ interactions in weak decays.

The semi-leptonic charged current interaction has also been extensively tested [16] in neutrino scattering processes such as quasi-elastic $\nu_\mu n \to e^- p$ and deep inelastic $\nu_\mu N \to \mu^- X$. These processes are more useful as probes of the hadron than of the neutrino. They have been very useful in testing the QCD-improved proton model and in measuring the relative amount of u and d quarks, antiquarks, and strange quarks in the proton, as well as in determining the CKM elements V_{cd} and V_{cs}. L^{CC}_{eff} has also been qualitatively tested in $|\Delta S| = 1$ nonleptonic decays and $\Delta S = 0$ parity violating interference effects, but in these cases hadronic uncertainties obscure the interpretation of the experiments. Higher order weak effects have been semi-quantitatively tested in the $K_L - K_S$ mass difference, the CP-violating parameters ϵ and ϵ' observed in K decays [17,18], and, recently, in the $B^0_d \leftrightarrow \bar{B}^0_d$ oscillations observed by the ARGUS collaboration [19] at DESY.

2.2 The Neutral Current

The weak neutral current interaction is

$$L = -\frac{g}{2\cos\theta_W} J^\mu_Z Z_\mu, \tag{10}$$

where

$$\begin{aligned} J^\mu_Z &= \tfrac{1}{2}\bar{u}\gamma^\mu(1+\gamma^5)u - \tfrac{1}{2}\bar{d}\gamma^\mu(1+\gamma^5)d \\ &+ \tfrac{1}{2}\bar{\nu}\gamma^\mu(1+\gamma^5)\nu - \tfrac{1}{2}\bar{e}\gamma^\mu(1+\gamma^5)e \\ &- 2\sin^2\theta_W J^\mu_{EM} \end{aligned} \tag{11}$$

(+ heavy fermion terms), and

$$J^\mu_{EM} = \sum_i q_i\, \bar{\psi}_i\gamma^\mu\psi_i = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d - \bar{e}\gamma^\mu e + \cdots \tag{12}$$

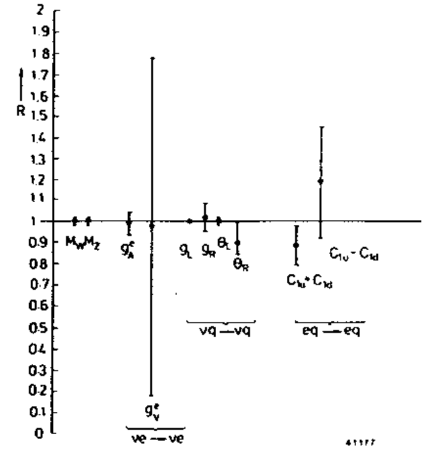

Figure 4: Experimental values of the W and Z masses and the neutral current couplings, relative to the standard model predictions for the global best fit value $\sin^2\theta_W = 0.230$ (the value of g_L should be regarded as the major determinant of $\sin^2\theta_W$ rather than a prediction). $C_{1i}, i = u, d$ are the coefficients in $-L_{eff}^{NC}$ of the parity-violating eq interaction $\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\mu\gamma^5 e\bar{q}_i\gamma^\mu q_i$. The other quantities are defined in the text. The error bars on g_V^e are large only because the predicted value (-0.045) is so small.

is the (purely vector) electromagnetic current. Z couples to both left and right-chiral fermions, but with different strength. For low momenta compared to M_Z, (10) implies the effective four-fermion interaction

$$-L_{eff}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}, \tag{13}$$

The neutral current interaction has been observed and quantitatively tested in a wide variety of weak processes, including deep inelastic $\overset{(-)}{\nu}_\mu N$ scattering from isoscalar and proton targets, elastic $\overset{(-)}{\nu}_\mu p$ scattering, coherent $\nu N \to \nu\pi^0 N$ scattering, elastic $\overset{(-)}{\nu}_i e$ ($i = e, \mu$) scattering, and $e^+e^- \to$ hadrons. In addition, weak-electromagnetic interference has been studied in polarized eD and μC scattering, atomic parity violation, and forward-backward asymmetries in $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$, $c\bar{c}$, and $b\bar{b}$. All processes are in excellent agreement with the standard model predictions, as can be seen in Fig. 4 and Table 3. Combined with the W and Z masses the standard model is quantitatively confirmed over an enormous momentum range, $10^{-6}\ GeV^2 < |Q^2| < 10^4\ GeV^2$. It is almost certainly correct to first approximation.

Let us now examine the neutral current interactions of neutrinos in more detail. It is convenient to write the terms in $-L_{eff}^{NC}$ relevant to ν-hadron processes in a form that is

Table 3: Values of the model independent neutral current parameters, compared with the standard model prediction for $\sin^2\theta_W = 0.230$. Correlations are not given for the neutrino-hadron couplings because of the non-Gaussian χ^2 distributions. However, the neutrino-hadron constraints are accurately represented by the ranges of the variable g_i^2 and θ_i, $i = L, R$, which are very weakly correlated.

Quantity	Experimental Value	Standard Model Prediction	Correlation	
$\epsilon_L(u)$	$0.339 \pm .017$	0.345		
$\epsilon_L(d)$	$-0.429 \pm .014$	-0.427		
$\epsilon_R(u)$	$-0.172 \pm .014$	-0.152		
$\epsilon_R(d)$	$-0.011^{+.081}_{-.057}$	0.076		
g_L^2	0.2996 ± 0.0044	0.301		
g_R^2	0.0298 ± 0.0038	0.029		
θ_L	2.47 ± 0.04	2.46		
θ_R	$4.65^{+0.48}_{-0.32}$	5.18		
g_A^e	$-0.498 \pm .027$	-0.503		-0.08
g_V^e	$-0.044 \pm .036$	-0.045		
C_{1u}	-0.249 ± 0.071	-0.191	-0.98	-0.88
C_{1d}	0.381 ± 0.064	0.340		0.88
$C_{2u} - \frac{1}{2}C_{2d}$	0.19 ± 0.37	-0.039		

valid in an arbitrary gauge theory (assuming massless left-handed neutrinos). One has

$$-L^{\nu H} = \frac{G_F}{\sqrt{2}}\bar{\nu}\gamma^{\mu}(1+\gamma^5)\nu \left\{\sum_i [\epsilon_L(i)\,\bar{q}_i\gamma_\mu(1+\gamma^5)q_i + \epsilon_R(i)\,\bar{q}_i\gamma_\mu(1-\gamma^5)q_i]\right\}, \qquad (14)$$

where in the standard model [20]

$$\begin{aligned}
\epsilon_L(u) &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \\
\epsilon_L(d) &= -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \\
\epsilon_R(u) &= -\frac{2}{3}\sin^2\theta_W \\
\epsilon_R(d) &= +\frac{1}{3}\sin^2\theta_W
\end{aligned} \qquad (15)$$

It is also convenient to define the variables

$$\begin{aligned}
g_L^2 &\equiv \epsilon_L(u)^2 + \epsilon_L(d)^2 \simeq \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \\
g_R^2 &\equiv \epsilon_R(u)^2 + \epsilon_R(d)^2 \simeq \frac{5}{9}\sin^4\theta_W,
\end{aligned} \qquad (16)$$

and

$$\theta_i \equiv \tan^{-1}(\epsilon_i(u)/\epsilon_i(d)),\ i = L \text{ or } R \qquad (17)$$

At present the most precise determinations of $\sin^2\theta_W$ are from deep inelastic neutrino scattering from (approximately) isoscalar targets. The ratio $R_\nu \equiv \sigma^{NC}_{\nu N}/\sigma^{CC}_{\nu N}$ of neutral to charged current cross sections has been measured to 1% accuracy by the CDHS [21] and CHARM [22] collaborations, so it is important to obtain theoretical expressions for R_ν and $R_{\bar{\nu}} \equiv \sigma^{NC}_{\bar{\nu}N}/\sigma^{CC}_{\bar{\nu}N}$ (as functions of $\sin^2\theta_W$) to comparable accuracy. Fortunately, most of the uncertainties concerning the strong interactions (as well as neutrino spectra) cancel in the ratio. For neutral current parameters in the vicinity of the standard model $\simeq 90\%$ of R_ν can be predicted from isospin alone [23]. The remaining 10% (from such effects as quark mixing and the s sea) is strongly constrained by independent measurements involving deep inelastic e, μ, and charged-current ν scattering, including dimuon production, and can be estimated to the necessary (10%) accuracy.

A simple zeroth order approximation (ignoring quark mixing, the s and c sea, and certain tiny higher twist effects) is

$$\begin{aligned}
R_\nu &= g_L^2 + g_R^2 r \\
R_{\bar{\nu}} &= g_L^2 + \frac{g_R^2}{r},
\end{aligned} \qquad (18)$$

where $r \equiv \sigma^{CC}_{\bar{\nu}N}/\sigma^{CC}_{\nu N}$ is the ratio of $\bar{\nu}$ and ν charged current cross sections, which can be measured directly. (In the simple parton model, ignoring hadron energy cuts, $r \simeq (\frac{1}{3}+\epsilon)/(1+\frac{1}{3}\epsilon)$, where $\epsilon \sim 0.125$ is the ratio of the fraction of the nucleon's s momentum carried by antiquarks to that carried by quarks. i.e. $\epsilon \equiv (\bar{U}+\bar{D})/(U+D)$, where $U \equiv \int_o^1 xu(x)dx$ is the first moment of the u quark distribution.) In practice, (18) must be corrected for quark mixing, the s and c seas, c quark threshold effects (which mainly

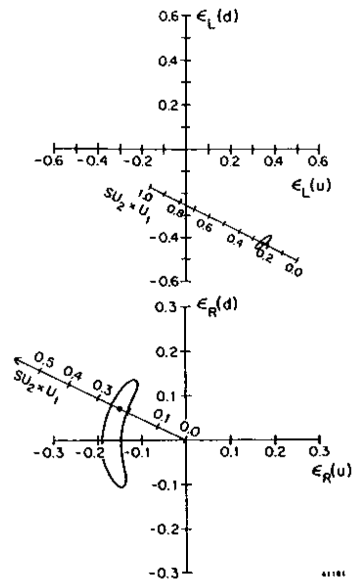

Figure 5: Allowed regions at 90% c.l. for the (weak) model independent νq parameters $\epsilon_i(u)$ and $\epsilon_i(d)$, $i = L$ or R and the predictions of the standard model as a function of $\sin^2\theta_W$.

affect σ^{CC} - these turn out to be the largest theoretical uncertainty), non-isoscalar target effects, $W - Z$ propagator differences, and radiative corrections (which lower the extracted value of $\sin^2\theta_W$ by ~ 0.009.). Details of the neutrino spectra, experimental cuts, x and Q^2 dependence of structure functions, and longitudinal structure functions enter only at the level of these corrections and therefore lead to very small uncertainties. Altogether, the theoretical uncertainty is $\Delta\sin^2\theta_W \sim \pm 0.005$, which would be very hard to improve in the future.

There are also a number of measurements [24] of deep inelastic $\overset{(-)}{\nu}_\mu$ scattering from non-isoscalar targets, which are useful for determining the isospin structure of the neutral current interaction. [25] The most recent result (from BEBC [26]) determines the ratio of neutral to charged current cross sections to around 7% accuracy for both ν_μ and $\bar{\nu}_\mu$.

The differential cross sections for elastic $\overset{(-)}{\nu}_\mu p \to \overset{(-)}{\nu}_\mu p$ scattering have been precisely measured in the BNL E734 experiment [27]. Four groups [24] have measured the cross section for coherent $\nu N \to \nu\pi^0 N$, for which the hadronic matrix elements can be estimated fairly reliably [28] using PCAC.

From these results [29] the neutrino-hadron couplings can be determined uniquely and (for the left-handed couplings) precisely. The extracted couplings, shown in Fig. 5 and Table 3, are in impressive agreement with the standard model predictions.

Similarly, for an arbitrary gauge theory with massless left-handed neutrinos, the four-fermion interaction for $\overset{(-)}{\nu}_\mu e$ scattering is

$$-L^{\nu e} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1+\gamma^5)\nu_\mu \; \bar{e}\gamma_\mu (g_V^e + g_A^e \gamma^5) e \tag{19}$$

(for $\overset{(-)}{\nu}_e e$ the charged current contribution must be included). In the standard model

$$\begin{aligned} g_V^e &= -\tfrac{1}{2} + 2\sin^2\theta_W \\ g_A^e &= -\tfrac{1}{2}, \end{aligned} \tag{20}$$

up to radiative corrections.

The laboratory cross section for $\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\nu}_\mu e$ elastic scattering is

$$\frac{d\sigma_{\nu_\mu,\bar{\nu}_\mu}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi}\left[(g_V^e \pm g_A^e)^2 + (g_V^e \mp g_A^e)^2(1-y)^2 - (g_V^{e2} - g_A^{e2})\frac{y m_e}{E_\nu}\right], \tag{21}$$

where the upper (lower) sign refers to $\nu_\mu(\bar{\nu}_\mu)$, $y \equiv T_e/E_\nu$ (which runs from 0 to $(1+\frac{m_e}{2E_\nu})^{-1}$) is the ratio of the kinetic energy of the recoil electron to the incident $\overset{(-)}{\nu}$ energy, and $G_F^2 m_e/2\pi = 4.31 \times 10^{-42}\ cm^2/GeV$. For $E_\nu \gg m_e$ this yields a total cross section

$$\begin{aligned} \sigma &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(g_V^e \pm g_A^e)^2 + \frac{1}{3}(g_V^e \mp g_A^e)^2\right] \\ &\sim \frac{G_F^2 m_e E_\nu}{2\pi}\left\{\begin{array}{ll} 1 - 4\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W, & \nu_\mu e \\ \frac{1}{3} - \frac{4}{3}\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W, & \bar{\nu}_\mu e \end{array}\right. \end{aligned} \tag{22}$$

The most accurate leptonic measurements [30,31] of $\sin^2\theta_W$ are from the ratio $R \equiv \sigma_{\nu_\mu e}/\sigma_{\bar{\nu}_\mu e}$, in which many of the systematic uncertainties cancel. Radiative corrections, which are small compared to the precision of present experiments, increase the extracted $\sin^2\theta_W$ by $\simeq 0.002$.

The $\bar{\nu}_e e$ cross section was measured a decade ago at the Savannah River reactor [32], while $\nu_e e \to \nu_e e$ has been measured recently at Los Alamos [33]. These are not nearly so precise as the $\overset{(-)}{\nu}_\mu e$ measurements, but are interesting because they involve both neutral and charged current contributions. (The cross sections for $\overset{(-)}{\nu}_e e$ may be obtained from (21) by replacing $g_{V,A}^e$ by $g_{V,A}^{e\prime} \equiv g_{V,A}^e + 1$, where the 1 is due to the charged current.)

In fact, the Los Alamos result strongly supports destructive interference ($g_A^e < 0$) between the two amplitudes and rules out constructive interference ($g_A^e > 0$).

The results of the various reactions [5] for the νe couplings are shown in Fig. 6. The $\bar{\nu}_\mu e$ data alone allow four solutions (which differ by $g_i^e \leftrightarrow -g_i^e$ and $g_V^e \leftrightarrow g_A^e$). The reactor $\bar{\nu}_e e$ results eliminate C, while the Los Alamos $\nu_e e$ experiment eliminates solutions C and D. The remaining two solutions (axial dominant (A) and vector dominant (B)) are consistent with all νe data. However, solution (B) is eliminated by the $e^+e^- \to \mu^+\mu^-$ forward-backward asymmetry under the (now very reasonable) assumption that the neutral current is dominated by the exchange of a single Z. The remaining solution (A) is in excellent agreement with the standard model prediction, as can be seen in Table 3.

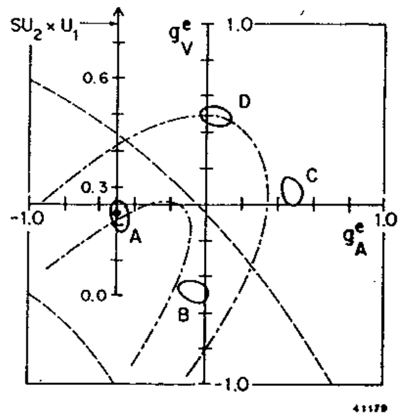

Figure 6: Allowed regions (90% c.l.) for the νe parameters g_V^e and g_A^e, for $\bar{\nu}_\mu e$ (solid lines), reactor $\bar{\nu}_e e$ (dot-dash), and $\nu_e e$ (dash).

The ν - hadron and νe interactions are therefore uniquely determined and are consistent with the standard model within uncertainties. Similar statements hold for the e-hadron and e^+e^- couplings [5]. Having established the standard model couplings as correct to first approximation, the neutral current and boson mass results can be used to test the standard model more stringently and to set limits on possible new physics.

The values of $\sin^2\theta_W$ and, equivalently, M_Z (using (3)) determined from various processes are shown in Table 4 and Fig. 7. They are in impressive agreement with each other, reconfirming the quantitative success of the standard model. The best fit to all data yields [34] $\sin^2\theta_W = 0.230 \pm 0.0048$ and $M_Z = 92.0 \pm 0.7\ GeV$, where the errors include full statistical, systematic, and theoretical uncertainties.

As can be seen in Fig. 7 consistency of the various $\sin^2\theta_W$ values (especially those obtained from deep inelastic νN and the W, Z masses) depends sensitively on the top quark mass, which enters the radiative corrections. In fact, one can use these results to set an upper limit [5] $m_t < 200\ GeV$ (90% c.l.), with similar limits applying to the splitting between the masses of possible fourth generation fermions. Similarly, the deep inelastic neutrino data can be combined with the W and Z masses to determine Δr in (3). One finds [5] $\Delta r = 0.077 \pm 0.037$, in excellent agreement with the value 0.0713 ± 0.0013 predicted for $m_t = 45\ GeV$ and $M_H = 100\ GeV$, and providing a rough test of the theory at the level of radiative corrections (see also Table 1).

The best fit value of $\sin^2\theta_W \equiv 1 - \frac{M_W^2}{M_Z^2}$ corresponds to the modified minimal subtraction value [35]

$$\sin^2\hat{\theta}_W(M_W) = 0.228 \pm 0.0044 \tag{23}$$

This is larger by $\simeq 2.5\ \sigma$ than the prediction $0.214^{+0.003}_{-0.004}$ of minimal SU_5 (for $\Lambda_{\overline{MS}}^{(4)} = 150^{+150}_{-75}\ MeV$) and other "great desert" models. Similar conclusions hold for all values of m_t and M_H, as can be seen in Fig. 8. Of course, the simplest grand unified theories

Table 4: Determination of $\sin^2\theta_W$ and M_Z (in GeV) from various reactions. The central values of all fits assume $m_t = 45\ GeV$ and $M_H = 100\ GeV$ in the radiative corrections. Where two errors are shown the first is experimental and the second (in square brackets) is theoretical, computed assuming 3 fermion families, $m_t < 100\ GeV$, and $M_H < 1\ TeV$. In the other cases the theoretical and experimental uncertainties are combined.

Reaction	$\sin^2\theta_W$	M_Z
Deep inelastic (isoscalar)	0.233 ± .003 ± [.005]	91.6 ± 0.4 ± [0.8]
$\overset{(-)}{\nu}_\mu p \to \overset{(-)}{\nu}_\mu p$	0.210 ± .033	95.0 ± 5.2
$\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\nu}_\mu e$	0.223 ± .018 ± [.002]	93.0 ± 2.7
W, Z	0.228 ± .007 ± [.002]	92.3 ± 1.1
Atomic parity violation	0.209 ± .018 ± [.014]	95.1 ± 3.9
SLAC eD	0.221 ± .015 ± [.013]	93.3 ± 2.7
μC	0.25 ± .08	89.6 ± 9.7
All data	0.230 ± 0.0048	92.0 ± 0.7

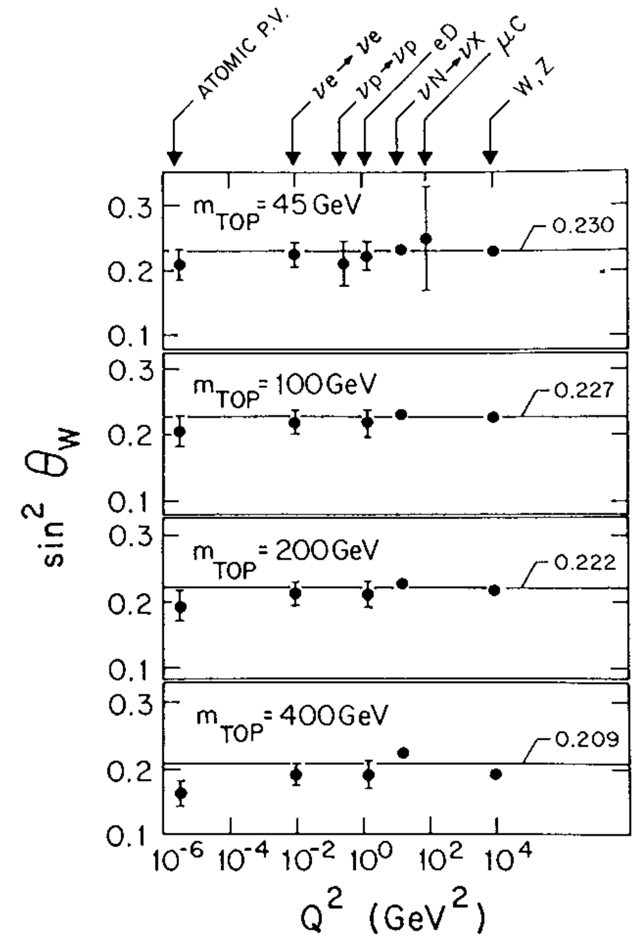

Figure 7: (a) $\sin^2\theta_W$ for various reactions as a function of the typical Q^2, determined for $m_t = 45\ GeV$. The best fit line $\sin^2\theta_W = 0.230$ is also shown. (b-d) $\sin^2\theta_W$ values determined for $m_t = 100$, 200, and 400 GeV.

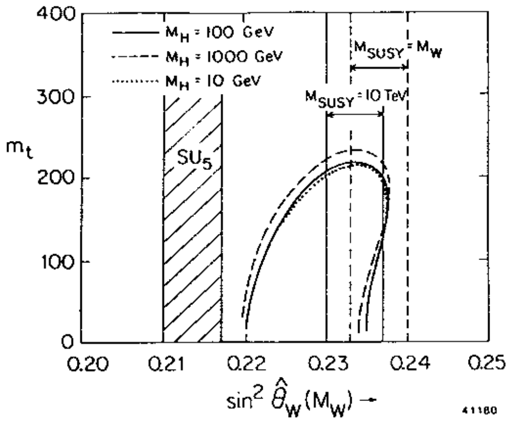

Figure 8: Allowed regions (90% $c.l.$) in $\sin^2 \hat{\theta}_W(M_W)$and m_t for fixed values of M_H. Also shown are the predictions of ordinary and supersymmetric GUTs, assuming no new thresholds between M_W or M_{SUSY} and the unification scale.

(GUTs) have been excluded for some time by the nonobservation of proton decay [36], but the additional evidence is welcome, especially since variations on the simplest GUTs can yield much longer lifetimes.

The fact that the $\sin^2 \hat{\theta}_W(M_W)$value in (23) is close to but not identical with the SU_5 prediction can be taken as a hint that the basic ideas of GUTs may be roughly correct, but that there is additional structure in the desert. For example, (23) is closer to (but still somewhat below) the prediction of the simplest supersymmetric GUTs. (Typically $0.237^{+0.003}_{-0.004}$ for $M_{SUSY} \sim M_W$, decreasing by ~ 0.003 for $M_{SUSY} \sim 10\ TeV$). The agreement is better for larger m_t (Fig. 8). Similarly, SO_{10} models [36] with three stages of symmetry breaking can be compatible with (23).

The neutral current data can be used to place rather stringent constraints on certain deviations from the standard model, such as the existence of Higgs triplets with significant vacuum expectation values [5], or the mixing between ordinary and exotic fermions [10]. The $e^+e^- \to b\bar{b}$ forward-backward asymmetry [37] excludes all topless models not involving exotic quarks. Many extensions of the standard model predict the existence of additional Z bosons [5], which could conceivably be light enough to be experimentally relevant. Some limits on the masses M_2 and mixing angle θ between the new and ordinary Z are shown for a class of E_6 models in Fig. 9. These neutral current limits are somewhat more stringent [38] than limits from direct searches $\bar{p}p \to Z_2 + X$, $Z_2 \to l^+l^-$ at the $S\bar{p}pS$ except for a small region in β near the Z_η. Nevertheless, the limits (typically 120 – 300 GeV) are still relatively weak. In constrast, there is a non-rigorous but plausible lower limit [39] from the $K_L - K_S$ mass difference of several TeV on the mass of the new charged bosons in many $SU_{2L} \times SU_{2R} \times U_1'$ models. This situation will presumably change in the near future: for example, the FNAL $\bar{p}p$ collider should be sensitive to bosons up to around 400 GeV

Table 5: Limits on the number N_ν of neutrino flavors and the mass ranges to which they apply. The laboratory limits are at 90% $c.l.$

N_ν	mass range	source	reference
$N_\nu \geq 2$	-	direct	
$N_\nu \geq 3$	-	τ properties	
$N_\nu \leq 4$	$m_\nu < 1\ MeV$	nucleosynthesis	[40]
$N_\nu \leq 6$	$m_\nu < O(MeV)$	SN1987A energetics	[41]
$N_\nu \leq$ { 7.5 (ASP); 4.9 (combined)	$m_\nu < O(5\ GeV)$	$e^+e^- \not\to \gamma\nu\bar{\nu}$	[42]
$N_\nu \leq$ { 5, $m_t < 40\ GeV$; 3, $m_t > 50\ GeV$	$m_\nu < O(40\ GeV)$	R	[43]

and the SSC would be sensitive up to several TeV.

3 Neutrino Counting

Constraints on the number of neutrino flavors are listed in Table 5.

There is direct laboratory proof for the existence of only two neutrinos, ν_e and ν_μ. However, indirect evidence leaves little doubt as to the separate existence of the ν_τ. If there were no ν_τ then, up to mixing effects, the τ_L^- would have to be a singlet under SU_2 transformations. Including mixing, the two left-handed lepton doublets and one charged singlet would be

$$\begin{pmatrix} \nu_1 \\ U_{1i}e_i^- \end{pmatrix}_L \begin{pmatrix} \nu_2 \\ U_{2i}e_i^- \end{pmatrix}_L U_{3i}e_{iL}^- \tag{24}$$

where $(e_1, e_2, e_3)_L \equiv (e, \mu, \tau)_L$ and U is a unitary matrix. However, one knows that the μ and e weak interactions are canonical - there is little room for mixing with an SU_2 singlet. From μ, β, K, and hyperon decays and the W mass one can show [10]

$$|U_{13}|, |U_{23}| < 0.05 \tag{25}$$

(this is confirmed by the absence of $\tau^- \to \mu^-\mu^-\mu^+$ decays). On the other hand, the τ lifetime [44] $\tau_\tau = (3.07 \pm 0.09) \times 10^{-13}$ sec, which agrees at least roughly with the value $(2.87 \pm 0.06) \times 10^{-13}$sec expected if ν_τ exists, implies

$$|U_{13}|^2 - |U_{23}|^2 = 0.94 \pm 0.04, \tag{26}$$

in clear conflict with (25). An independent argument is that A^τ, the axial vector coupling of τ in the weak neutral current, is determined from the e^+e^- forward-backward asymmetry to be $A^\tau = -0.46 \pm 0.05$. This is in agreement with the value $-\frac{1}{2}$ expected if the τ_L^- is in an SU_2 doublet with its own partner (ν_τ), and disagrees with the value (zero) expected if τ_L^- and τ_R^- are in SU_2 singlets. [45] Hence, the ν_τ almost certainly exists, but it would nevertheless be desirable to observe it directly.

There are several upper limits on the number of neutrinos with normal weak interactions. An upper limit of $N_\nu \leq 4$ neutrino flavors with masses $\leq 1\ MeV$ is determined by nucleosynthesis [40] (the abundance primordial ^4He). Extra neutrino flavors [46] would

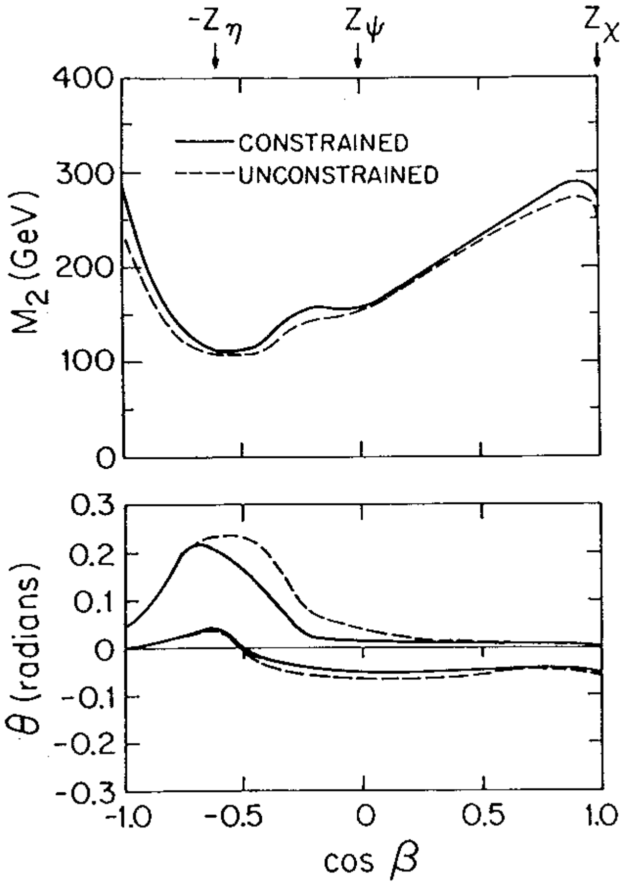

Figure 9: Lower limits on M_2 and allowed θ range (both at 90% $c.l.$) for an E_6 boson $Z(\beta) = \cos\beta\ Z_\chi + \sin\beta\ Z_\psi$, where Z_χ and Z_ψ refer to the breaking patterns $SO_{10} \to SU_5 \times U_{1\chi}$ and $E_6 \to SO_{10} \times U_{1\psi}$, respectively, and $Z_\eta = -Z(\pi - \tan^{-1}\sqrt{\frac{5}{3}})$ occurs in many superstring models. Constrained and unconstrained refer to whether or not it is assumed that SU_2 breaking is due to Higgs doublets only.

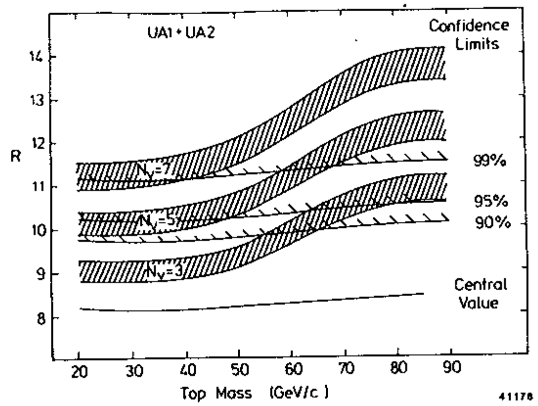

Figure 10: The value of R (27) as a function of N_ν and m_t, and the experimental results form UA1 and UA2.

cause the universe to expand faster, causing the $\nu_e n \leftrightarrow e^- p$ reactions to freeze out earlier (when there are more neutrons), leading to too much ^4He.

Limits can also be set from the cross section for $e^+e^- \to \gamma\nu\bar{\nu}$ (with only the photon observed), which effectively sums the number of neutrinos. The ASP experiment at PEP obtained [42] $N_\nu < 7.5$. Combined with cross section limits from MAC and CELLO this implies $N_\nu < 4.9$ (90% $c.l.$) sensitive to masses less than several GeV. Finally, the Z width increases by 170 MeV for each new neutrino with mass $\leq 40\ GeV$. Indirect limits on Γ_Z already exist from the ratio

$$R = \frac{\sigma_{\bar{p}p\to W} B_{W\to l\nu}}{\sigma_{\bar{p}p\to Z} B_{Z\to l^+l^-}} = \frac{\sigma_{\bar{p}p\to W}}{\sigma_{\bar{p}p\to Z}} \frac{\Gamma_{W\to l\nu}}{\Gamma_{Z\to l^+l^-}} \frac{\Gamma_Z}{\Gamma_W}. \qquad (27)$$

Using the measured R and theoretical values for the cross section ratio and leptonic widths one determines Γ_Z/Γ_W, which is sensitive to both N_ν and the t quark mass. Recent estimates [43] typically yield $N_\nu \leq 5$ for $m_t \leq 40\ GeV$ and $N_\nu \leq 3$ for $m_t \geq 50\ GeV$, (the larger m_t range is favored by $B - \bar{B}$ oscillations [19] and the non-observation [47] of the t by UA1), and incidentally suggest the upper limit $m_t \leq 65\ GeV$. These limits are suggestive but should be viewed with caution. As can be seen in Fig. 10 the bounds essentially disappear if one increases the uncertainty in either R itself or the cross section ratio.

Future direct measurements of Γ_Z at SLC and LEP should ultimately yield a precision of $\Delta\Gamma_Z \simeq 35\ MeV$, which is equivalent to an uncertainty [48] of $\Delta N_\nu \sim 0.2$. It should be possible to obtain an independent measurement of $\Gamma_{Z\to\nu\bar{\nu}}$ accurate to $\simeq 50\ MeV$ by measuring $e^+e^- \to \gamma Z \to \gamma\nu\bar{\nu}$ above the Z pole.

4 Neutrino Mass

In the minimal $SU_2 \times U_1$ model the neutrinos are predicted to be massless. However, extensions of the standard model involving new SU_2-singlet neutral fermions (the right-handed neutrino partners needed for Dirac mass terms) or new Higgs representations (to generate Majorana masses) allow non-zero masses. [49] In fact, most extensions of the standard model (e.g. most grand unified theories other than SU_5) involve one or both of these mechanisms. Furthermore, non-zero masses could have important implications for the missing Solar neutrinos and/or the missing (dark) matter of the universe.

4.1 Weyl, Majorana, and Dirac Neutrinos.

For the weak interactions it is convenient to deal with Weyl two-component spinors ψ_L or ψ_R, each of which represents two physical degrees of freedom. The field ψ_L can annihilate a left-handed (L) particle or create a right-handed (R) antiparticle, while $\psi_L^\dagger$ annihilates a L-particle or creates an R-antiparticle. For a ψ_R field the roles of L and R are reversed. An ordinary four-component Dirac field ψ can be written as the sum $\psi = \psi_L + \psi_R$ of two Weyl fields, where ψ_L and ψ_R are just the chiral projections

$$\psi_{L,R} = P_{L,R}\psi, \tag{28}$$

with $P_{L,R} = (1 \pm \gamma_5)/2$.

Alternatively, one can consider Weyl fermions that do not have distinct partners of the opposite chirality. We will see below that such spinors correspond to particles that are either massless or carry no conserved quantum numbers.

In the free field limit a Weyl field ψ_L can be written as

$$\psi_L(x) = \sum_{\vec{p}} \left[b_L(\vec{p}) u_L(\vec{p}) e^{-ip\cdot x} + d_R^\dagger(\vec{p}) v_R(\vec{p}) e^{+ip\cdot x} \right], \tag{29}$$

where $\sum_{\vec{p}}$ represents $\int d^3\vec{p}/\sqrt{(2\pi)^3 2E}$. In(29), b_L and d_R are annihilation operators for L particles and R-antiparticles, respectively, and u_L and v_R are the corresponding (4-component) spinors satisfying $P_L u_L = u_L$, $P_L v_R = v_R$, $P_R u_L = P_R v_R = 0$. For a ψ_R spinor one simply interchanges L and R. Equation (29) differs from an ordinary (Dirac) free field in that there is no sum over spin.

It is apparent from (29) that each left-handed (right-handed) particle is necessarily associated with a right-handed (left-handed) antiparticle. The right-handed antiparticle [50] field ψ_R^c is not independent of ψ_L, but is closely related to $\psi_L^\dagger$. One has

$$\psi_R^c = C\bar{\psi}_L^T, \tag{30}$$

where C is the charge conjugation matrix, defined by $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Similarly, for a R-Weyl spinor, $\psi_L^c = C\bar{\psi}_R^T$. In the special case that ψ_L is the chiral projection $P_L\psi$ of a Dirac field ψ, ψ_R^c is just the R-projection $P_R\psi^c$ of the antiparticle field $\psi^c = C\bar{\psi}^T$.

If ψ_R and ψ_R^c both exist, they have the opposite values for all additive quantum numbers. Since the quarks and charged leptons carry conserved quantum numbers (e.g. color and electric charge), they must be Dirac fields - i.e. ψ_R and ψ_R^c must be distinct. The only quantum number associated with the neutrinos is lepton number, however, and it is

conceivable that that is violated in nature. As we will see, that will allow for two very different possibilities for neutrino mass.

The known neutrinos of the first family are the left-handed electron neutrino ν_{eL} and its CP partner, the right-handed "antineutrino" $\nu_{eR}^c = C\bar{\nu}_{eL}^T$. These are associated with the e_L^- and e_R^+, respectively, in ordinary charged current weak interactions.

Mass terms always take left- and right-handed fields into each other. If one introduces a new field N_R (distinct from ν_R^c) and its CP conjugate $N_L^c = C\bar{N}_R^T$ into the theory, then one can write a Dirac (lepton number conserving) mass term

$$-L_{Dirac} = m_D \bar{\nu}_L N_R + h.c., \tag{31}$$

which connects N_R and ν_L. In this case ν_L, N_R, N_L^c and ν_R^c form a four component Dirac particle - i.e. one can define $\nu \equiv \nu_L + N_R$, $\nu^c = N_L^c + \nu_R^c = C\bar{\nu}^T$, so that

$$-L_{Dirac} = m_D \bar{\nu}\nu. \tag{32}$$

Clearly lepton number is conserved in this case, because there is no transition between ν and ν^c. In the free field limit the Dirac neutrino field ν has the canonical expression

$$\nu_{Dirac}(x) = \sum_{\vec{p}} \sum_{S=L,R} \left[b_S(\vec{p}) u_S(\vec{p}) e^{-ip\cdot x} + d_S^\dagger(\vec{p}) v_S(\vec{p}) e^{+ip\cdot x} \right], \tag{33}$$

Usually, the N_R is an $SU_2 \times U_1$ singlet, with m_D generated by an ordinary Higgs doublet, and $L = L_e + L_\mu + L_\tau$ is conserved in the three family generalization. This possibility is most similar to the way in which masses are generated for the other fermions (e^-, u, d, etc.) in the standard model, but it is difficult to understand why m_{ν_e} is so small in this case.

Another possibility [51] is that N_R is a known doublet neutrino, such as $\nu_{\tau R}^c$. This is a variation on the Konopinski-Mahmoud model. [52] Then ν_{eL}, $\nu_{\tau R}^c$, $\nu_{\tau L}$ and ν_{eR}^c can be combined to form a Dirac neutrino with $L_e - L_\tau$ conserved.

For the generalization of (31) to F fermion families one has

$$-L_{Dirac} = \bar{n}_L^0 m_D N_R^0 + h.c., \tag{34}$$

where m_D is an arbitrary [53] $F\times F$ mass matrix, and n_L^0 and N_R^0 are F-component vectors: thus $n_L^0 = (n_{1L}^0\ n_{2L}^0 \dots n_{FL}^0)^T$, where n_{iL}^0 are the "weak eigenstate" neutrinos - i.e. n_{iL}^0 is associated with e_{iL}^- in weak transitions. The weak eigenstate neutrinos are related to the neutrinos n_{iL}, N_{iR} of definite mass by unitary transformations

$$\begin{aligned} n_L^0 &= V_L n_L \\ N_R^0 &= V_R N_R. \end{aligned} \tag{35}$$

V_L and V_R are $F \times F$ unitary matrices, determined by

$$V_L^\dagger m_D V_R = m_d = \mathrm{diag}(m_1\ m_2 \cdots m_F) \tag{36}$$

where m_d is the diagonal matrix of physical neutrino masses. V_L and V_R can be determined by

$$V_L^\dagger m_D m_D^\dagger V_L = V_R^\dagger m_D^\dagger m_D V_R = m_d^2 \tag{37}$$

($m_D m_D^\dagger$ and $m_D^\dagger m_D$ are Hermitian). In general V_L and V_R are unrelated. If there are no degeneracies then V_L and V_R are determined uniquely by (37) up to diagonal phase matrices; i.e. if $V_{L,R}$ satisfy (37) then so do $V_{L,R}K_{L,R}$, where $K_{L,R}$ are diagonal phase matrices associated with the unobservable phases of the n_{iL} and N_{jR} fields. Usually one chooses K_L to put V_L into a simple conventional form. Then K_R is determined by the requirement that m_d be real.

V_L modifies the leptonic weak charged current in (5) to [54]

$$J_W^{\mu\dagger} = (\bar{\nu}_e\ \bar{\nu}_\mu\ \bar{\nu}_\tau)\ V_L^\dagger \gamma^\mu (1+\gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} \qquad (38)$$

so that $V_L^\dagger$ is just the analogue of the CKM quark mixing matrix. It describes the relative strengths [55] of the weak transition between the various charged leptons and neutrinos of definite mass.

In a Majorana (lepton number violating) mass term one avoids the need for a new fermion field by coupling the ν_L to its CP conjugate ν_R^c:

$$\begin{aligned} -L_M &= \tfrac{1}{2} m \bar{\nu}_L \nu_R^c + h.c. \\ &= \tfrac{1}{2} m \bar{\nu}_L C \bar{\nu}_L^T + h.c. \end{aligned} \qquad (39)$$

L_M can be thought of as creating or annihilating two neutrinos, and violates lepton number by $\Delta L = \pm 2$. ν_L and ν_R^c can be combined to form a two component Majorana neutrino $\nu = \nu_L + \nu_R^c$, so that $-L_M = \frac{1}{2} m \bar{\nu}\nu$. From (30) we see that $\nu = C\bar{\nu}^T$, i.e. a Majorana neutrino is its own antiparticle. In the free field limit ν is just

$$\nu(x) = \sum_{\vec{p}} \sum_{S=L,R} \left[b_S(\vec{p}) u_S(\vec{p}) e^{-ip\cdot x} + b_S^\dagger(\vec{p}) v_S(\vec{p}) e^{+ip\cdot x} \right], \qquad (40)$$

i.e. it has the same form as for a free Dirac field (cf (33)) except that there is no distinction between b and d annihilation operators.

The Majorana mass m in (39) can be generated by the vacuum expectation value (VEV) of a new Higgs triplet [56] or as a higher order effective operator. Majorana masses are popular amongst theorists because they are so different from quark and lepton masses, and there is therefore the possibility of explaining why m_{ν_e} is so small (if it is non-zero).

For F fermion families, the Majorana mass term is

$$-L_M = \tfrac{1}{2} \bar{n}_L^0 M n_R^{0c} + h.c. \qquad (41)$$

where M is an $F \times F$ Majorana mass matrix and n_L^0 and n_R^{0c} are F component vectors: i.e. $n_L^0 = (n_{1L}^0 \ldots n_{FL}^0)^T$, $n_R^{0c} = (n_{1R}^{0c} \ldots n_{FR}^{0c})^T$, where n_{iL}^0 and n_{iR}^{0c} are weak eigenstate neutrinos and "antineutrinos", related by

$$n_{iR}^{0c} = C\bar{n}_{iL}^{0T} \qquad (42)$$

From (42) one can prove the identity

$$\bar{n}_{iL}^0 n_{jR}^{0c} = \bar{n}_{jL}^0 n_{iR}^{0c}, \qquad (43)$$

from which it follows that the Majorana mass matrix M must be symmetric: $M = M^T$. Proceeding in analogy to the Dirac case, one can relate the n_{iL}^0 and n_{jR}^{0c} to mass eigenstate neutrino fields by

$$\begin{aligned} n_L^0 &= U_L n_L \\ n_R^{0c} &= U_R n_R^c, \end{aligned} \qquad (44)$$

where U_L and U_R are $F \times F$ unitary matrices chosen so that

$$U_L^\dagger M U_R = M_d = \mathrm{diag}(m_1\ m_2\ \ldots m_F), \qquad (45)$$

where M_d is a diagonal matrix of Majorana mass eigenvalues. Unlike the Dirac case (for which m_D was an arbitrary matrix and V_L and V_R unrelated), the symmetry of M implies a relation between U_L and U_R, viz

$$U_L = U_R^* K^\dagger, \qquad (46)$$

where K is unitary and symmetric. That is, just as in the Dirac case, U_L is determined from

$$U_L^\dagger M M^\dagger U_L = M_d^2 \qquad (47)$$

to be of the form $U_L = \hat{U}_L K_L$, where K_L is a matrix of phases that can be chosen for convenience. U_R is then determined from (46), where K is chosen so that M_d is real and positive. If there are no degeneracies then K is just a matrix of phases. [57] One can always pick K_L such that $K = I$, but it is not always convenient to do so.

In terms of the mass eigenstates, (41) reduces to

$$\begin{aligned} -L_M &= \tfrac{1}{2} \sum_{i=1}^{F} m_i \bar{n}_{iL} n_{iR}^c + h.c. \\ &= \tfrac{1}{2} \sum_{i=1}^{F} m_i \bar{n}_i n_i, \end{aligned} \qquad (48)$$

where $n_i = n_{iL} + n_{iR}^c$ is the i^{th} Majorana mass eigenstate. [58] Written in terms of the n_{iL}, the weak charged current assumes a form analogous to (38), with $U_L^\dagger$ replacing $V_L^\dagger$ to describe the leptonic mixing. [59]

There are several physical distinctions between Dirac and Majorana neutrinos. If the ν_e is Majorana, for example, one could have the sequence $\pi^+ \to e^+\nu_e$ followed by $\nu_e p \to e^+ n$. The combined process violates lepton number by two units and is allowed for Majorana but not Dirac neutrinos. Similarly, a hypothetical heavy neutrino N would undergo the decays $N \to e^+ q_1 \bar{q}_2$ and $N \to e^- \bar{q}_1 q_2$ with equal rates if it is Majorana, while for a Dirac particle one would have $N \to e^- \bar{q}_1 q_2$, $N^c \to e^+ q_1 \bar{q}_2$ only [60]. There are differences due to Fermi statistics in the production of $\nu\nu$ (Majorana) or $\nu\nu^c$ (Dirac) pairs near threshold [61], and finally Majorana neutrinos cannot have electromagnetic form factors, such as magnetic moments [62].

It is important to keep in mind, however, that these distinctions must all disappear in the limit that the neutrino mass can be neglected. For $m_\nu \to 0$ the ν_R component of a Dirac neutrino decouples, and both Majorana and Dirac neutrinos reduce to Weyl two-component neutrinos - there is no difference between them. [63] In particular, lepton number conservation is reestablished smoothly as $m_\nu \to 0$ for a Majorana neutrino, because in that limit helicity - which is conserved up to corrections of order m_ν/E_ν - plays the role of an approximate lepton number. For example, the ν_e produced in $\pi^+ \to e^+\nu_e$ has $h_\nu = -1$ up to corrections of order $(m_\nu/E_\nu)^2$ (in rate), while the reaction $\nu_e p \to e^+ n$ has a cross section that is suppressed by $(m_\nu/E_\nu)^2$ for the wrong (negative) helicity.

In many models Dirac and Majorana mass terms are both present. For one doublet neutrino ν_L^0 (with $\nu_R^{0c} = C\bar{\nu}_L^{0T}$) and one new singlet N_R^0 (with $N_L^{0c} = C\bar{N}_R^{0T}$), for example, one could have the general mass term

$$-L = \tfrac{1}{2}\begin{pmatrix}\bar{\nu}_L^0 & \bar{N}_L^{0c}\end{pmatrix}\begin{pmatrix} m_t & m_D \\ m_D^T & m_S \end{pmatrix}\begin{pmatrix}\nu_R^{0c} \\ N_R^0\end{pmatrix} + h.c., \tag{49}$$

where $m_D = m_D^T$ is a Dirac mass generated by a Higgs doublet (analogous to (31)), m_t is a Majorana mass for ν_L^0 generated by a Higgs triplet or effective interaction (cf. (39)), and m_S is a Majorana mass for N_R^0, generated by a Higgs singlet or bare mass. Similarly, for F families (49) still holds provided one interprets ν_L^0 , N_L^{0c} , ν_R^{0c} , and N_R^0 as F component vectors, and m_t, m_D, and m_S as $F \times F$ matrices (with $m_t = m_t^T$, $m_S = m_S^T$). Then, (49) becomes simply

$$-L = \tfrac{1}{2}\bar{n}_L^0 M n_R^{0c} + h.c., \tag{50}$$

where $n_L^0 \equiv (\nu_L^0,\ N_L^{0c})^T$ and $n_R^{0c} \equiv (\nu_R^{0c},\ N_R^0)^T$ are $2F$ component vectors and

$$M = \begin{pmatrix} m_t & m_D \\ m_D^T & m_S \end{pmatrix} \tag{51}$$

is a symmetric $2F \times 2F$ Majorana mass matrix. Equation (50) can be diagonalized in exact analogy with (41-48), yielding finally

$$-L = \tfrac{1}{2}\sum_{i=1}^{2F} m_i \bar{n}_{iL} n_{iR}^c + h.c. \tag{52}$$

i.e. there are in general $2F$ Majorana neutrinos, related to n_L^0, n_R^{0c} by unitary transformations similar to (44). Unlike the pure Majorana case, however, there is now mixing between particles with different weak interaction properties (e.g. $n_{iL} = (U_L^\dagger)_{ij} n_{jL}^0$ is a mixture of SU_2 doublets and singlets), which can have important consequences for neutrino oscillations [64] and decays.

It is instructive to see how the Dirac case ($m_t = m_S = 0$) emerges as a limiting case of (49). For a single family one has

$$M = m_D\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{53}$$

Since M is Hermitian (for m_D real) one can diagonalize it by a unitary transformation U_L. One finds

$$U_L^\dagger M U_L = m_D\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{54}$$

with $U_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; i.e. the mass eigenstates are

$$\begin{aligned} n_{1L} &= \frac{1}{\sqrt{2}}(\nu_L^0 + N_L^{0c}) \\ n_{2L} &= \frac{1}{\sqrt{2}}(\nu_L^0 - N_L^{0c}) \\ n'_{1R} &= \frac{1}{\sqrt{2}}(\nu_R^{0c} + N_R) \\ n'_{2R} &= \frac{1}{\sqrt{2}}(\nu_R^{0c} - N_R). \end{aligned} \tag{55}$$

The negative mass eigenvalue in (54) can be removed by redefining [65] the right-handed fields $n_{1R} = n'_{1R}$, $n_{2R} = -n'_{2R}$. This is nothing more than taking

$$U_L^\dagger M U_R = m_d = m_D\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{56}$$

where U_R is given by (46) with $K = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Finally, the two Majorana states $n_1 = n_{1L} + n_{1R}^c$ and $n_2 = n_{2L} + n_{2R}^c$ are degenerate. We can therefore reexpress L in the new basis

$$\begin{aligned} \nu &\equiv \frac{1}{\sqrt{2}}(n_1 + n_2) = \nu_L^0 + N_R^0 \\ \nu^c &\equiv \frac{1}{\sqrt{2}}(n_1 - n_2) = N_L^{0c} + \nu_R^{0c}, \end{aligned} \tag{57}$$

yielding

$$\begin{aligned} -L &= \tfrac{1}{2} m_D(\bar{n}_{1L} n_{1R}^c + \bar{n}_{2L} n_{2R}^c) + h.c. \\ &= m_D \bar{\nu}_L^0 N_R^0 + h.c. \\ &= m_D \bar{\nu}\nu. \end{aligned} \tag{58}$$

This is just a standard Dirac mass term, with a conserved lepton number (i.e. no transition between ν and ν^c). A Dirac neutrino is therefore nothing but a pair of degenerate two-component Majorana neutrinos (n_1 and n_2), combined to form a 4-component neutrino with a conserved lepton number.

Similarly, the Dirac limit for F families ($m_t = m_S = 0$ in (51)), can be obtained by choosing

$$U_L = \frac{1}{\sqrt{2}}\begin{pmatrix} V_L & V_L \\ V_R^* & -V_R^* \end{pmatrix} \tag{59}$$

and $K = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, where V_L and V_R are the $F \times F$ unitary matrices that diagonalize m_D (in (36). One then obtains

$$U_L^\dagger\begin{pmatrix} 0 & m_D \\ m_D^T & 0 \end{pmatrix} U_R = \begin{pmatrix} m_d & 0 \\ 0 & m_d \end{pmatrix}, \tag{60}$$

so that one obtains F pairs of degenerate Majorana neutrinos, which can be combined into F Dirac neutrinos.

One sometimes refers to a pseudo-Dirac neutrino, which is just a Dirac neutrino to which is added as small lepton number-violating perturbation. For example, for $F = 1$ one could modify the Dirac mass in (53) to

$$M = \begin{pmatrix} \epsilon & m_D \\ m_D & 0 \end{pmatrix}, \tag{61}$$

with $\epsilon << m_D$. One then finds two Majorana mass eigenstates $n_\pm$, with

$$\begin{aligned} n_{+L} &= n_{1L} + \frac{\epsilon}{4} n_{2L} \\ n_{-L} &= -\frac{\epsilon}{4} n_{1L} + n_{2L}, \end{aligned} \tag{62}$$

(n_{1L} and N_{2L} are defined in (55)), with masses $m_D \pm \frac{\epsilon}{2}$.

Other important special cases of (51) are considered below.

4.2 Models of Neutrino Mass

There are many models for neutrino mass [49], all of which have good and bad features. The major classes of models are listed in Table 6, along with the most natural scales for the neutrino masses and for $\langle m_{\nu_e} \rangle$, an effective mass relevant to neutrinoless double β decay.

Dirac neutrinos are exactly like other fermions. They involve a conserved total lepton number (though the individual L_e, L_μ, and L_τ lepton numbers are violated by mixing in general) and therefore do not lead to neutrinoless double beta decay. The problem with Dirac neutrinos is that it is hard to understand why the neutrinos are so much lighter than the other fermions. In the standard model Dirac mass are generated by the vacuum expectation value (VEV) $v = \sqrt{2}\langle \varphi^0 \rangle \simeq (\sqrt{2} G_F)^{-1/2} \simeq 246\ GeV$ of the neutral component of a doublet [72] of Higgs scalar fields. One has

$$m_D = h_\nu v, \tag{63}$$

where h_ν is the Yukawa coupling

$$L = -\sqrt{2} h_\nu (\bar{\nu}_L\ \bar{e}_L) \begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} N_R + h.c. \tag{64}$$

of the neutrino to φ^0.

A ν_e mass in the 20 eV range would require an anomalously small Yukawa coupling $h_{\nu_e} \leq 10^{-10}$. Moreover, h_{ν_e} would have to be smaller by $m_{\nu_e}/m_e \leq 10^{-4}$ than the analogous Yukawa coupling for the electron. Of course, we do not understand the masses of the other fermions either (or why they range over at least five orders of magnitude), so it is hard to totally exclude the possibility that h_{ν_e} is simply small. Nevertheless, the possibility seems sufficiently ugly that it is hard to take seriously unless some mechanism (other than fine-tuning) for the smallness is proposed.

One possibility is that h_ν is actually zero to lowest order (tree level) due to some new symmetry, and that h_ν is only generated as a higher order correction (i.e. so that m_ν/m_e

Table 6: Models of neutrino mass, along with their most natural scales for the light neutrino masses.

Model	m_{ν_e}	$\langle m_{\nu_e} \rangle$	m_{ν_μ}	m_{ν_τ}
Dirac	$1 - 10\ MeV$	0	$100\ MeV - 1\ GeV$	$1 - 100\ GeV$
pure Majorana [56] (Higgs triplet)	arbitrary	m_{ν_e}	arbitrary	arbitrary
GUT seesaw [66,67] ($M \sim 10^{14}\ GeV$)	$10^{-11}\ eV$	m_{ν_e}	$10^{-6}\ eV$	$10^{-3}\ eV$
intermediate seesaw [68] ($M \sim 10^9\ GeV$)	$10^{-7}\ eV$	m_{ν_e}	$10^{-2}\ eV$	$10\ eV$
$SU_{2L} \times SU_{2R} \times U_1$ seesaw [69] ($M \sim 1\ TeV$)	$10^{-1}\ eV$	m_{ν_e}	$10\ KeV$	$1\ MeV$
light seesaw [70] ($M \ll 1\ GeV$)	$1 - 10\ MeV$	$\ll m_{\nu_e}$	-	-
charged Higgs [71]	$< 1\ eV$	$\ll m_{\nu_e}$	-	-

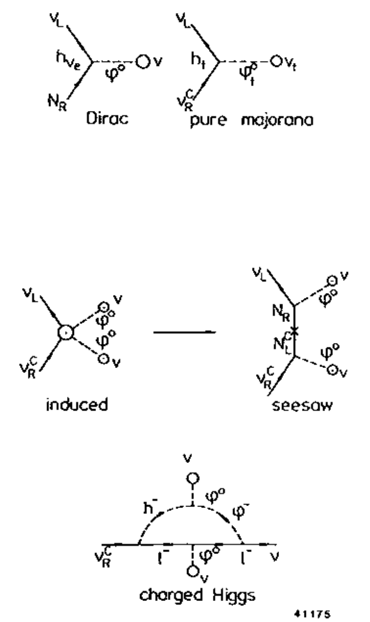

Figure 11: Dirac, pure Majorana, induced, and charged Higgs generated neutrino masses.

is some power of α.) This is a very attractive possibility, but no particularly compelling models to implement it have emerged. The idea has recently been resurrected in some superstring inspired models [73], which have difficulty incorporating the seesaw type ideas described below.

Majorana mass terms for the ordinary SU_2-doublet neutrinos involve a transition from ν_R^c ($t_3 = -\frac{1}{2}$) into ν_L ($t_3 = +\frac{1}{2}$), and therefore must be generated by an operator transforming as a triplet under weak SU_2.

The simplest possibility is the Gelmini-Roncadelli model [56], in which one introduces a triplet of Higgs fields $\vec{\varphi}_t = (\varphi_t^0, \varphi_t^-, \varphi_t^{--})$ into the theory. The Yukawa coupling

$$\begin{aligned} L &= \tfrac{1}{2}h_t(\bar{\nu}_L\ \bar{e}_L)\vec{\tau}\cdot\vec{\varphi}_t \begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix} \\ &= \tfrac{1}{2}h_t(\bar{\nu}_L\ \bar{e}_L)\begin{pmatrix} \varphi_t^- & \sqrt{2}\varphi_t^0 \\ \sqrt{2}\varphi_t^{--} & -\varphi_t^- \end{pmatrix}\begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix} \end{aligned} \tag{65}$$

then generates a Majorana mass

$$m_t = h_t v_t \tag{66}$$

for the ν, where $v_t = \sqrt{2}\langle\varphi_t^0\rangle$ is the VEV of the Higgs triplet. Since both h_t and v_t are unknown the neutrino mass is unrelated to the other fermions and can in principle be arbitrarily small, at least at tree level.

However, small m_{ν_e} is not explained in such models - it is merely parametrized and in fact is almost as problematic as a Dirac mass. The weak neutral current (and W and Z masses) require [5] $v_t \le 0.08v \sim 20\ GeV$. For v_t close to this limit one requires $h_t \le 10^{-9}$, i.e. almost as bad a fine-tuning as the Dirac case. For $v_t \ll v$ one can tolerate more reasonable values for h_t, but then it is difficult to understand the large hierarchy in vacuum expectation values. One generally expects all non-zero VEV's to be comparable in magnitude unless fine-tunings are performed on the parameters in the Higgs potential. Even if one does this, higher order corrections are likely to upset the hierarchy. [74]

The VEV $\langle\varphi_t^0\rangle \neq 0$ necessarily violates lepton number conservation by two units (the Yukawa coupling in (65) does not by itself violate L because φ_t can be regarded as carrying two units of L). If the rest of the Lagrangian conserves L then lepton-number is spontaneously broken, and there will be an associated massless Goldstone boson, the triplet-Majoron. (This is the version of the model that is usually considered [56].) In this case limits based on stellar energy loss (carried off by Majorons) require [75] $v_t \lesssim 2 - 10\ KeV$. Implications of the Majoron for neutrino decay and annihilation, cosmology, and neutrinoless double beta decay will be mentioned below.

It is also possible introduce other couplings into the Higgs triplet model which explicitly break lepton number conservation, such as a cubic interaction between $\vec{\varphi}_t$ and two Higgs doublets. (This violates L since $\vec{\varphi}_t$ was assigned $L = 2$ to make (65) invariant). In that case all of the new scalar particles associated with φ_t become massive - i.e. there is no Majoron.

Another mechanism for introducing a Majorana mass is to consider the induced interaction (Fig. 11).

$$L_{eff} = \frac{1}{2}\frac{C}{M}(\bar{\nu}_L\ \bar{e}_L)\vec{\tau}\begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix}\cdot(\varphi^-\ \ -\varphi^0)\vec{\tau}\begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} \tag{67}$$

between two leptons and two Higgs doublets. The Higgs fields in (67) are arranged to transform as an SU_2 triplet, so L_{eff} is $SU_2 \times U_1$ invariant; however, L_{eff} is non-renormalizable, as is evidenced by the dimensional coupling C/M, where M is a mass. L_{eff} cannot therefore be an elementary coupling, but it could be an effective four-particle interaction induced [76] by new physics at some large mass scale M (just as the four-fermion weak interaction is a nonrenormalizable effective interaction that is really generated by W and Z exchange). When φ^0 is replaced by its vacuum expectation value, (67) yields an effective Majorana mass $m \sim Cv^2/M$, which is naturally small for $M \gg v$. For example, if (67) were somehow induced by quantum gravity one would expect $M \sim 10^{19}\ GeV$ (the Planck scale). Then for $C \sim 1$ one would have $m_\nu \sim 10^{-5}\ eV$.

The most popular realisation of this idea is the seesaw model, [66] in which the underlying physics is the exchange of a very heavy SU_2-singlet Majorana neutrino N_R^0, as indicated in Fig. 11. The seesaw model for one family is a special case of the general mass matrix in (49), in which m_D is a typical Dirac mass (typically assumed to be comparable to m_u or m_e for the first family) connecting ν_L^0 to a new SU_2-singlet N_R^0 and $m_S \gg m_D$ is a Majorana mass for N_R^0, presumably comparable to some new (large) physics scale. One typically assumes that $m_t = 0$ in the seesaw model, i.e. that there is not a Higgs triplet as well. [77] In that case, (49) yields two Majorana mass eigenstates n_1 and n_2 with

$$\begin{aligned} \nu_L^0 &= n_{1L}\cos\theta + n_{2L}\sin\theta \\ N_L^{0c} &= -n_{1L}\sin\theta + n_{2L}\cos\theta \\ \nu_R^{0c} &= -(n_{1R}^c\cos\theta + n_{2R}^c\sin\theta) \\ N_R^0 &= -n_{1R}^c\sin\theta + n_{2R}^c\cos\theta. \end{aligned} \tag{68}$$

The physical masses [78] are

$$\begin{aligned} m_1 &\simeq \frac{m_D^2}{m_S} \ll m_D \\ m_2 &\simeq m_S \end{aligned} \tag{69}$$

and the mixing angle is

$$\tan\theta = \left(\frac{m_1}{m_2}\right)^{1/2} \simeq \frac{m_D}{m_S} \ll 1. \tag{70}$$

Hence, one naturally obtains one very light neutrino, which is mainly the ordinary SU_2 doublet (ν_L^0, ν_R^{0c}), and one very heavy neutrino, which is mainly the singlet (N_L^{0c}, N_R^0).

If one does allow $m_t \neq 0$ (but $\ll m_S$) then there are still two Majorana neutrinos with masses $|a - \frac{m_D^2}{m_S}|$ and m_S, respectively, while $\theta \sim m_D/m_S \ll 1$ still holds. (The minus sign in ν_R^{0c} is removed if $a - \frac{m_D^2}{m_S}$ is positive). In this case, however, one loses the natural explanation of why m_1 is so small, unless m_t is itself induced by the underlying physics and is of the same order as m_D^2/m_S.

The seesaw model is easily generalized to F families. One then has the general $2F \times 2F$ Majorana mass matrix in (51). Assuming that the eigenvalues of m_S are all much larger than any of the components of m_D or m_t (if it is non-zero) one can calculate the eigenvalues and mixing matrices to leading order in m_S^{-1}. One finds that there are F light Majorana neutrinos (consisting of the F doublets (ν_L^0, ν_R^{0c}), up to corrections of order $m_D m_S^{-1}$ and F

heavy Majorana neutrinos (consisting of the singlets (N_L^{0c}, N_R^0), to $O(m_D m_S^{-1})$). That is, one can write

$$\begin{aligned} \begin{pmatrix} \nu_L^0 \\ N_L^{0c} \end{pmatrix} &= U_L \begin{pmatrix} n_{lL} \\ n_{hL} \end{pmatrix} \\ \begin{pmatrix} \nu_R^{0c} \\ N_R^0 \end{pmatrix} &= U_R \begin{pmatrix} n_{lR}^c \\ n_{hR}^c \end{pmatrix}, \end{aligned} \tag{71}$$

where n_{lL} and n_{hL} are F component vectors of light and heavy Majorana mass eigenstates, respectively, and similarly for n_{lR}^c, n_{hR}^c. As usual, U_L and U_R are $2F \times 2F$ unitary matrices which diagonalize M in (51), viz

$$U_L^\dagger \begin{pmatrix} m_t & m_D \\ m_D^T & m_S \end{pmatrix} U_R = m_d = \begin{pmatrix} m_l & 0 \\ 0 & m_h \end{pmatrix}, \tag{72}$$

where m_l and m_h are diagonal $F \times F$ matrices of the F light and F heavy eigenvalues, respectively. To leading order in m_S^{-1} one can write $U_L^\dagger$ and U_R in block diagonal form

$$U_L^\dagger = K U_R^T = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \begin{pmatrix} A^T & -A^T m_D m_S^{-1} \\ D^T m_S^{-1\dagger} m_D^\dagger & D^T \end{pmatrix}, \tag{73}$$

where A^T and D^T are unitary (to leading order) $F \times F$ matrices defined by

$$\begin{aligned} m_l &= K_1 A^T (m_t - m_D m_S^{-1} m_D^T) A \\ m_h &= K_2 D^T m_S D \end{aligned} \tag{74}$$

i.e. the mass matrix for the light neutrinos is $m_t - m_D m_S^{-1} m_D^T$, which is diagonalized by A, while that for the heavy neutrinos is m_S, diagonalized by D. K_1 and K_2 are diagonal phase matrices which ensure that m_l and m_h are real and positive. We see from (71-74) that indeed there are F heavy states with masses of $O(m_S)$, and in the simplest case $m_t = 0$ there are F states which are naturally very light ($O(m_D^2 m_S^{-1})$). (For $m_t \neq 0$ one must separately assume m_t is small). Furthermore, the mixing between the light and heavy sectors is very small (of $O(m_D m_S^{-1})$), while the matrices A and D, which describe mixings within the two sectors, are in general arbitrary.

There are several classes of seesaw models [66], depending on the scale of m_S. In simple grand unified models one assumes that the scale is a typical GUT unification scale of around 10^{14} GeV. In many such models (e.g. SO_{10}) one has that the neutrino Dirac mass matrix m_D is the same as m_u/k where m_u is the u-quark mass matrix and $k \simeq 4.7$ represents the running of the Yukawa couplings between the GUT scale and low energies. If one makes the somewhat ad-hoc assumption that the matrix m_S is just $M_X I$, where $M_X \sim 10^{14}$ GeV is the unification scale and I is the identity matrix, one has (for $m_t = 0$) the light eigenvalues

$$m_{\nu_i} \sim \frac{m_{u_i}^2}{M_X k^2} \tag{75}$$

$\sim 10^{-11}$ eV, 10^{-6} eV, 10^{-3} eV, i.e. the neutrino masses are naturally expected to be extremely tiny, and to scale like the squares of the u, c, and t quark masses. (Equation (75) was computed for $m_{top} \sim 50$ GeV). Several caveats are in order: the assumption of

$m_S \sim M_X I$ was quite arbitrary. One could easily imagine that the eigenvalues of m_S are smaller than M_X due to small Yukawa coupling couplings (increasing m_{ν_i}). Also, they need not be the same. For example, if the m_S eigenvalues followed the same family hierarchy as the ordinary fermions (i.e. $m_{S_i} \propto m_{u_i}$) then one would have m_{ν_i} scaling as m_{u_i} rather than $m_{u_i}^2$. (A similar linear hierarchy ensues in some variant GUTs in which m_S is zero at tree level but is generated at higher orders [79,36]. Of course, more complicated patterns for m_S and m_D (in (74)) are also possible. Furthermore, in many cases loop corrections to the (GUT) Higgs potential may induce [77] VEV's for Higgs representations that can yield a non-zero triplet terms m_t in (72). These are most likely to affect the smallest masses (e.g. m_{ν_e}). Equation (75) should therefore be regarded only as a typical order of magnitude.

If one does assume that $m_S = M_X I$, however, then m_u^2/M_X is diagonalized by the same transformations that diagonalize m_u. Since one also has equal electron and d-quark mass matrices (i.e. $m_e = m_d/k$) in most simple GUTs the final result is that flavor mixing in the lepton sector (analogous to (38)) is described by the same mixing matrix as the CKM quark mixing matrix. This result continues to hold [67] approximately for a far wider class of m_S than does the simple mass prediction in (75).

Lower mass scales for m_S imply larger values for the light neutrino masses (and generally less predictive power for m_D). Several authors [68] have suggested that the heavy Majorana scale could be the intermediate range $10^8 - 10^{12}$ GeV associated with invisible axions. For $m_{D_i} \sim m_{e_i}$ and $m_S \sim 10^9$ $GeVI$, for example, one obtains the values $\sim 10^{-7}$ eV, 10^{-2} eV, 10 eV for m_{ν_e}, m_{ν_μ}, m_{ν_τ}, respectively.

If m_S is in the several TeV range (as expected in some left-right symmetric [80] $SU_{2L} \times SU_{2R} \times U_1$, models [69], for example) one typically expects (for $m_{D_i} \sim m_{e_i}, m_S \propto I$) $m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}$ to have relatively large values 10^{-1} eV, 10 KeV, and 1 MeV, respectively. As we will see, such models run into severe cosmological difficulties unless the mass hierarchy is somehow modified or a fast decay mechanism is found for the ν_μ and ν_τ. Of course, one could also have m_S much smaller than the $SU_{2L} \times SU_{2R} \times U_1$ scale (e.g. in the 10 GeV – 100 GeV range), with corresponding larger masses for the light neutrinos. Similar statements apply to models with extra Z bosons in the 100 GeV – 10 TeV range, which usually also have heavy Majorana neutrinos.

Finally, one can consider light seesaw models, in which typically $m_S \ll 1$ GeV. Such models are very artificial and abandon the principal advantages of the seesaw, because both m_D and m_S must be taken unnaturally small to obtain an acceptable ν_e mass. Their only virtue is that they yield strongly suppressed neutrinoless double beta decay rates, even though the neutrinos are Majorana.

Seesaw models were first introduced in GUT type models in which lepton number is explicitly violated by the gauge interactions. One can also consider non-gauge seesaw models [81] in which lepton number is spontaneously broken by the VEV of the Higgs field which generates m_S. Such models imply the existence of a massless Goldstone boson, the singlet-Majoron. [82] Unlike the triplet-Majoron in the Gelmini-Roncadelli model,[56] which can couple strongly to the ordinary neutrinos (coupling $\sim h_t$), the singlet-Majoron effectively decouples from ordinary particles. That is, it couples strongly to the heavy neutrino, with a coupling of order $m_D m_S^{-1}$ to off-diagonal $n_l n_h$ vertices, and with strength $(m_d m_S^{-1})^2$ to light neutrinos.

It is difficult to implement the seesaw model in most superstring inspired models, because there is no Higgs field available to generate a large m_S. It has been suggested [83] that m_S could be generated by a higher order effective operator, but such model may run into serious cosmological problems [84].

There have also been variant seesaw models constructed [85] in which the light neutrinos occur in degenerate pairs which can be combined from Dirac neutrinos with a conserved L.

Finally, I mention the charged Higgs models [71], in which small Majorana masses are generated by loop diagrams involving new charged Higgs bosons with explicit L-violating couplings (Fig. 11). Viable versions often lead to pseudo-Dirac neutrinos. The approximately conserved lepton number is typically $L_e - L_\mu + L_\tau$, for example, rather than L. The actual mass scale depends on unknown Yukawa couplings and masses.

4.3 Experimental Constraints

There are a number of excellent reviews [49] of the experimental status of neutrino mass. My major purpose in this section is to comment on the implications of the various theoretical models for the different types of experiments.

4.4 Kinematic Tests

Direct kinematic limits on the masses of the ν_e, ν_μ, and ν_τ are given in Table 7. The ITEP group [86] has long claimed evidence for a non-zero ν_e mass in the 20 eV range from tritium β decay, but this has not been confirmed by other groups, and in fact the Zurich-SIN measurement is on the verge of conflicting with the ITEP result. In addition the neutrinos from supernova 1987A observed by the Kamiokande [93] and IMB [94] experiments place upper limits in the 20 eV range on the ν_e mass (otherwise the arrival times of the detected neutrinos would be spread out more than is observed), but it is hard to make this limit precise because it depends on the details of the neutrino emission [90].

A 20 eV neutrino mass is just in the range that would be most interesting cosmologically, so clearly it is essentially to resolve the situation. Hopefully, the current and next generation of tritium β decay experiments will be sensitive down to a few eV, but it is doubtful whether experiments of this type will ever be able to probe to much lower scales. As can be seen in Table 6, none of the models really predict m_{ν_e} in the 20 eV range (the $SU_{2L} \times SU_{2R} \times U_1$ models come closest), but most can accomodate masses in this range by fine-tuning parameters.

As can be seen in Table 7, the direct kinematic limits on m_{ν_μ} (from $\pi_{\mu 2}$ decay) and on m_{ν_τ} (from $\tau \to \nu_\tau + 5\pi$) are relatively weak. The experiments are extremely difficult (the mass scales being probed are very much smaller than the energies released in the decays), so it is unlikely that these measurements will improve by much more than a factor of two.

4.5 Heavy Neutrinos

There are many limits [49,95] on possible small admixtures of heavy neutrino states in the ν_e or ν_μ, including universality tests in nuclear β decay, searches for secondary peaks or distortions of the lepton spectra in β, π, and K decay, searches for the decay products

Table 7: Kinematic limits/values on neutrino masses.

$17\ eV < m_{\nu_e} < 40\ eV$	ITEP [86]
$m_{\nu_e} < 18\ eV$	Zurich [87]
$m_{\nu_e} < 27\ eV$	LANL [88]
$m_{\nu_e} < 32\ eV$	INS-Tokyo [89]
$m_{\nu_e} < O(20\ eV)$	SN1987A [90]
$m_{\nu_\mu} < 0.25\ MeV$	SIN [91]
$m_{\nu_\tau} < 50\ MeV$	ARGUS [92]

of heavy neutrinos (e.g. $\nu_h \to \nu_e e^+ e^-$) produced in beam dumps, e^+e^- annihilation, or neutrino scattering [96]. The limits on the mass m_i versus mixing angle U_{ai}, $a = e$ or μ, where

$$\nu_a^0 = \sum_i U_{ai}\nu_i \tag{76}$$

are shown in Fig. 12.

It is seen that the constraints on $|U_{ai}|^2$ are quite impressive, especially for m_i in the range 10 MeV − 10 GeV, where they are comparable to the expectations in (70) of a seesaw model with $m_1 \sim 10\ eV$ and $m_2 = m_i$. The lower part of this range corresponds to the masses expected in the "light-seesaw" model (Table 6), while the 1 GeV − 1 TeV range is consistent with $SU_{2L} \times SU_{2R} \times U_1$ models. [69]

Also, most models with extra Z bosons in the 100 GeV − 1 TeV range predict [98] the existence of heavy SU_2-singlet Majorana or Dirac neutrinos [95,99,100]. The extra Z's typically couple to these new neutrinos and other exotic fermions much more strongly than to the ordinary fermions. Future hadron colliders should therefore be able to extend the search for heavy neutrinos via

$$\begin{aligned} \overset{(-)}{p}\, p &\to Z' \to N N^c \\ \overset{(-)}{p}\, p &\to W_R \to N l \end{aligned} \tag{77}$$

into the several hundred GeV range. The subsequent decays of the N's should be a superb probe of underlying physics. In models with just an extra Z, for example, the N is expected to decay due to mixing with the light neutrinos. The N can then decay [95,99] via virtual W or Z exchange [101] into such modes as $3\nu_1$, $\nu_1 l^+ l^-$, $\nu_1 q\bar{q}$, and $\nu_1 e^+\mu^-$. On the other hand, in $SU_{2L} \times SU_{2R} \times U_1$ models the N will generally decay via virtual W_R exchange [100], and for the lightest N the decay should usually be into $l^\pm q\bar{q}$. Moreover, the decay modes should easily establish whether the heavy neutrino is Majorana or Dirac, because in the former case the decays $N \to l^+\bar{q}q$ and $N \to l^- q\bar{q}$ would be equally likely [60] (though with different angular distributions).

It is of course also possible that a heavy neutrino could simply be a massive 4^{th} generation neutrino.

As has already been mentioned, heavy neutrinos in the $GeV - TeV$ range are likely to give too large m_{ν_μ} and m_{ν_τ} unless the typical seesaw hierarchy $m_{\nu_i} \propto m_{l_i}^n$ or $m_{u_i}^n$, $n = 1$ or 2, for the light neutrinos is avoided or new physics is invoked to ensure fast decays or annihilations for the ν_μ and ν_τ. On the other hand, if such new physics is present some

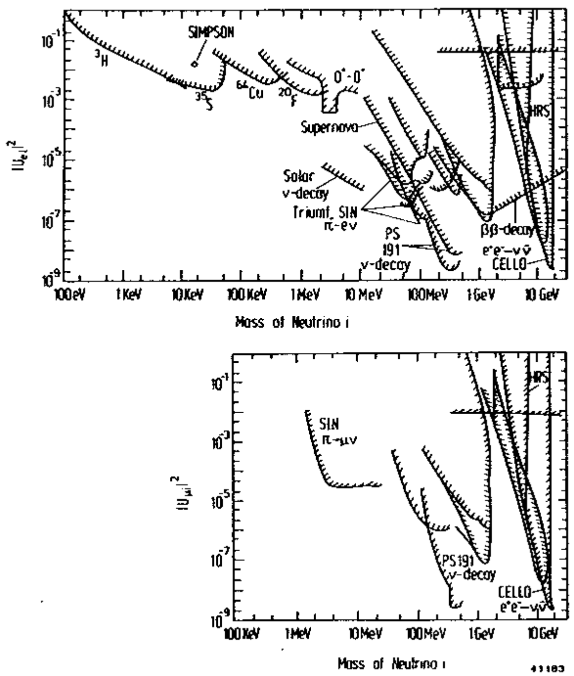

Figure 12: Limits on the mass and mixing of heavy neutrinos, from [97].

of the limits in Fig. 12 (those based on decays) may no longer be valid, because in many cases the heavy neutrinos will decay rapidly into unobservable channels (e.g. $\nu_h \to \nu_l +$ Majoron) before reaching the detector.

4.6 Neutrino Oscillations

Neutrino oscillations are a beautiful example of a common quantum phenomenon: viz that if one starts at time $t = 0$ in a state that is not an energy eigenstate [102] then at later times it can oscillate into another (orthogonal) state. For example, suppose that the ν_e^0 and a second neutrino ν_a^0 (e.g. $\nu_a^0 = \nu_\mu^0$ or ν_τ^0) are mixtures of two mass eigenstates ν_1 and ν_2 with mixing angle θ:

$$\begin{aligned} \nu_e^0 &= \cos\theta\ \nu_1 + \sin\theta\ \nu_2 \\ \nu_a^0 &= -\sin\theta\ \nu_1 + \cos\theta\ \nu_2 \end{aligned} \tag{78}$$

If at time $t = 0$ the weak eigenstate ν_e^0 is produced (e.g. in the process $\pi^+ \to \pi^0 e^+ \nu_e^0$) then at time t it will have evolved into the state

$$\begin{aligned} \nu_e^0(t) &= \cos\theta\ \nu_1 e^{-iE_1 t} + \sin\theta\ \nu_2 e^{-iE_2 t} \\ &\simeq \cos\theta\ \nu_1 e^{\frac{-im_1^2 t}{2p}} + \sin\theta\ \nu_2 e^{\frac{-im_2^2 t}{2p}}. \end{aligned} \tag{79}$$

In the second form I have assumed relativistic neutrinos $E_i = \sqrt{p^2 + m_i^2} \sim p + m_i^2/2p$ with definite momentum [103] $p >> m_i$, and have neglected an irrelevant overall phase $\exp(-ipt)$. The state $\nu_e^0(t)$ has a non-trivial overlap with ν_a^0. After traveling a distance $L \sim t$, there will be a probability

$$\begin{aligned} P(\nu_e \to \nu_a) &= |\langle \nu_a^0 \mid \nu_e^0(t)\rangle|^2 \\ &= \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4p}\right) \\ &= \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2(eV^2)L(m)}{p(MeV)}\right) \end{aligned} \tag{80}$$

that the state will have evolved into ν_a^0 (as can be observed in the process $\nu_a N \to \epsilon_a N'$, for example), and a probability

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_a) \tag{81}$$

that the state will remain a ν_e^0. In (80), $\Delta m^2 = m_1^2 - m_2^2$, and the last form is valid for Δm^2 in eV^2, L in m, and p in MeV. It is seen that the $\nu_e \to \nu_a$ probability depends on both the mixing angle θ and on $\Delta m^2 L/p$. For moderate values of the latter quantity the probability oscillates as a function of L and p, while for very large values the oscillations are averaged by a finite-sized detector or non-monochromatic source, (the second factor in (80) averages to 1/2). It is easy to generalize [49] (80) to the case that the initial neutrino is a mixture of more than two mass eigenstates, as in (76). One obtains

$$P(\nu_e \to \nu_a) = \sum_i |U_{ei} U_{ai}^*|^2 + Re \sum_{i \neq j} U_{ei} U_{ai}^* U_{ej}^* U_{aj} e^{\frac{-i(m_i^2 - m_j^2)L}{2p}} \tag{82}$$

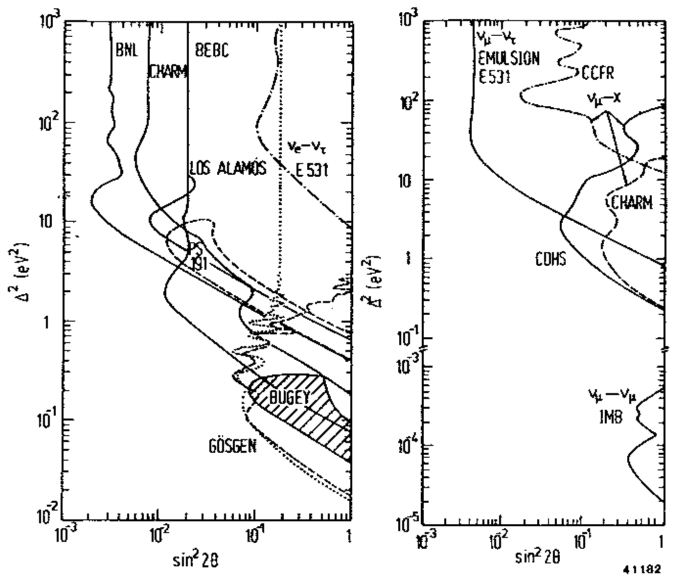

Figure 13: 90% *c.l.* limits on neutrino oscillations, from [97]. (a) $\nu_\mu \to \nu_e$ (BNL, CHARM, BEBC, Los Alamos, PS-191), $\nu_e \to \nu_\tau$ (E531), and $\bar{\nu}_e \to \nu_X$ (Bugey, Gösgen). (b) $\nu_\mu \to \nu_\tau$, ν_X, ν_μ. The Bugey [106] and PS-191 [108] regions are allowed by positive results. The other contours are exclusion plots (the regions to the right are excluded).

Neutrino oscillations can be searched for in (a) appearance experiments, in which one looks for the interactions of ν_a in a detector, and (b) disappearance experiments, in which one looks for a reduced ν_e flux. In both cases one can compare the observed counting rate with the expectation from known backgrounds (appearance) or from the expected flux (disappearance) as determined, for example, by measuring the electron spectrum from $n \to pe^-\bar{\nu}_e$ in reactor $\bar{\nu}_e$ oscillation experiments. A much cleaner technique is to search for actual oscillations in the appearance or disappearance probabilities as a function of L or p, such as by using two detectors at different distances form the source.

There are many limits on neutrino oscillations from accelerator experiments [49] (e.g. counter and emulsion experiments and beam dumps, searching for $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$, as well as ν_μ disappearance), and reactors [49] ($\bar{\nu}_e$ disappearance), as well as on the oscillations of ν_μ produced in cosmic ray interactions in the atmosphere [104]. (Implications for the Solar neutrino problem are discussed below). The results of these searches [105] are summarized in Fig. 13. The Bugey reactor experiment [106] reports a positive signal for $\bar{\nu}_e$ disappearance, but their results are contradicted by the Gösgen experiment [107]. Similarly, the CERN PS-191 counter experiment [108] reports an excess of ν_e events in a ν_μ beam, but their signal is in conflict with several other $\nu_\mu \to \nu_e$ experiments [97]. Clearly, a clarification of the situation is essential .

From Fig. 13 it is clear that there are stringent limits on neutrino mixings for $|\Delta m^2|$

above $\simeq 1\ eV^2$. This should be contrasted with the suggested value $m_{\nu_e} \sim 17 - 40$ eV by the ITEP experiment [86]. If the ITEP result is correct then most likely the ν_e could not have any significant mixing with other neutrinos (the alternative possibility, that the ν_e is almost degenerate with another neutrino flavor so that $|\Delta m^2| << m^2_{\nu_e}$, seems rather contrived but cannot be excluded). A comparison of Fig. 13 with the expectations of various models (Table 6) suggests that $\nu_\mu \to \nu_\tau$ oscillations may be the most optimistic possibility for the future. Many of the seesaw-type models predict that the lepton mixing angles are roughly correlated with the corresponding quark mixing angles. This would suggest $\sin^2 2\theta \sim 10^{-4}$, 10^{-2}, 10^{-1} for $\nu_e \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow \nu_\tau$, and $\nu_e \leftrightarrow \nu_\mu$, respectively.

Oscillations between ordinary SU_2 doublet neutrinos ($\nu_e^0, \nu_\mu^0, \nu_\tau^0$, and possible fourth family ν's), known as first class or flavor oscillations, occur for pure Dirac and pure Majorana neutrinos, as well as in the multi-family seesaw models. In models involving both Dirac and Majorana mass terms of comparable magnitude, however, there can be additional light neutrinos, and the mass eigenstates can have significant admixtures of both SU_2 doublets and singlets. In this case second class oscillations [64] can occur, in which the ordinary neutrinos oscillate into SU_2 singlets with negligible interactions. These "sterile" neutrinos are essentially undetectable, so second class oscillations can be observed [109] only in disappearance experiments. Of course, first and second class oscillations can occur simultaneously. For three families, for example, there could be oscillations between six Majorana neutrinos (3 doublets and 3 singlets).

Yet another possibility [110] are models in which the ordinary neutrinos have small mixings with heavy neutrinos. In that case the neutrinos actually produced in weak processes are the projections of the weak eigenstates onto the subspace of light or massless neutrinos. It can easily occur that the projections of the ν_e^0 and ν_μ^0, for example, are not orthogonal. The result is that a ν_μ^0 could produce an e^- in a subsequent reaction. Such a non-orthogonality would mimic the effects of oscillation appearance experiments, even if the masses of the light neutrinos are zero or negligible.

4.7 Cosmology

There are many limits on neutrino mass and decays from cosmology [111]. Ordinary light or massless neutrinos would have been produced by such weak processes as $e^+e^- \leftrightarrow \nu\nu^c$ in the early universe. As long as the weak reaction rate [112]

$$\Gamma_{weak} \sim \langle\sigma v\rangle n_T \sim G_F^2 T^5 \quad (83)$$

($\langle\sigma v\rangle \sim G_F^2 T^2$ is the thermally averaged cross section times relative velocity, and $n_T \sim T^3$ is the density of target particles, where T is the temperature) was large compared to the expansion rate $H \sim T^2/m_p$ (where $m_p = G_N^{-1/2} \sim 10^{19}\ GeV$ is the Planck scale) the number of neutrinos stayed in equilibrium. However, as soon as T dropped below the temperature

$$T_D \sim (G_F^2 m_P)^{-1/3} \simeq 3\ MeV \quad (84)$$

for which $\Gamma_{weak} \sim H$, the weak rate became negligible and the neutrinos decoupled, i.e. effectively stopped interacting. According to most models these neutrinos should remain in the present universe, undisturbed from the first second of the big bang except for a redshifting of their momenta by the expansion of the universe. They are analogous to the

2.7°K microwave radiation (which decoupled later). If the neutrino masses are much less than 1 eV there should be $\simeq$ 50 neutrinos/cm^3 of each type (ν_{eL}, ν^c_{eR}, etc) with momenta characterized by a thermal spectrum with temperature $\simeq 1.9°K$ (10^{-4} eV). Despite the large number of neutrinos ($\simeq 10^{10}$ per baryon) they are essentially impossible to detect [113] - [115] because their cross section $\sim G_F^2 E_\nu^2 \sim 10^{-62} cm^2$ is so low. [116]

The major cosmological bound is based on the energy density of the present universe. There are predicted to be so many relic neutrinos that even for a small mass in the 10 eV range they would be important. Limits on the energy density imply

$$\sum_i m_{\nu_i} < 40\ eV \tag{85}$$

where the sum extends over the light, stable (at least compared to the age of the universe) doublet neutrinos. Conversely, a neutrino with mass in this range would dominate the energy density and could account for the dark (missing) matter in galaxies and clusters [117]. In particular, for the ITEP value $m_{\nu_e} \sim (17 - 40)$ eV, the ν_e would be an ideal candidate for the dark matter, but one would probably then have to find a mechanism to explain why the ν_e is the heaviest neutrino.

Similarly, the energy density associated with light or massless neutrinos for $T \sim T_D$ affects nucleosynthesis and leads to the limit $N_\nu \leq 4$ (section III).

There are also a variety of constraints on unstable neutrinos. An ordinary doublet mass eigenstate neutrino ν_2 (with $m_{\nu_2} > m_{\nu_1}$) is expected to decay into

$$\begin{aligned} \nu_2 &\rightarrow \nu_1\gamma, \quad (m_{\nu_2} < 2m_e) \\ \nu_2 &\rightarrow \nu_1 e^+ e^-, \quad (2m_e < m_{\nu_2} < m_\mu + m_e). \end{aligned} \tag{86}$$

The first decay occurs at one loop, while the second occurs at tree level. Both decays are very slow for small m_{ν_2} and the decay products are detectable. There are a large variety of cosmological and astrophysical constraints [118] on m_{ν_2} and τ_{ν_2} from the present energy density, the growth of galaxies, the distortion of the 2.7°K background radiation, the non-observation of the decay photons, supernovae, and nucleosynthesis and breakup. For reasonable mixing angles these limits exclude the range 40 eV – (20 – 40) MeV for ordinary neutrinos [119] decaying according to (86). Combined with laboratory limits this implies [118,120] that the ν_μ and ν_τ (i.e. their dominant mass eigenstate components) should be lighter than 40 eV. In particular, this poses serious problems for the TeV scale seesaw model.

Most of the cosmological limits can be evaded if new physics is invoked to allow fast and invisible (except for the relativistic energy of the decay products) decays or annihilation for the heavy neutrinos. One possibility is the decay $\nu_2 \rightarrow 3\nu_1$. However, the rate for this mode from off-diagonal Z couplings [121] is too slow, while models in which the couplings of a Higgs triplet [122] (present in $SU_{2L} \times SU_{2R} \times U_1$) are arranged to allow a fast decay generally run into problems [123] with $\mu \rightarrow 3e$.

More promising are models in which $\nu_2 \rightarrow \nu_1 G$, where G is a Goldstone boson [124]-[127] associated with a spontaneously broken global symmetry. Likely examples are the case that G is a familon [124] (a Goldstone boson associated with a broken family symmetry) or a triplet-Majoron [125]. In fact, for triplet-Majorons one expects the annihilation process $\nu\nu \rightarrow MM$ (which begins when T drops below v_t) to have removed any relic neutrinos from

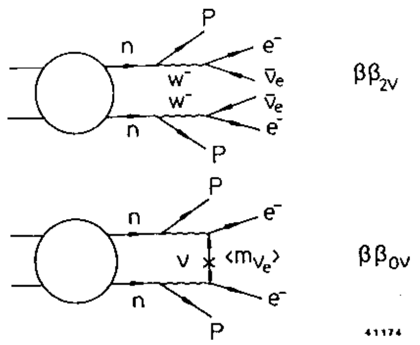

Figure 14: Diagrams for two neutrino ($\beta\beta_{2\nu}$) and neutrinoless ($\beta\beta_{0\nu}$) double beta decay.

the present universe [56]. In familon models some care must be taken to avoid unacceptably large flavor changing neutral current effects. The decay $\nu_2 \rightarrow \nu_1 M$ is too slow in the simpler versions of the singlet-Majoron model [126] to avoid cosmological problems.

The role of spontaneous L violation in Majoron models in reducing possible initial large lepton asymmetries to cosmologically interesting values at the time of nucleosynthesis is discussed in [128].

4.8 Double Beta Decay

Another important source of information on the ν_e mass (if it is Majorana) is neutrinoless double beta decay ($\beta\beta_{0\nu}$).

First consider the lepton-number conserving two-neutrino ($\beta\beta_{2\nu}$) process $(Z, N) \rightarrow (Z+2, N-2)e^-e^-\nu_e^c\nu_e^c$, which can be thought of as two ordinary beta decays occurring in the same nucleus (Fig. 14). In the context of neutrino mass this process is mainly of interest as a calibration of the calculated nuclear matrix elements that are needed for the neutrinoless case. There has long been a two order of magnitude discrepancy between the predicted rates [129], e.g. for $^{130}Te \rightarrow {}^{130}Xe$, and indirect measurements by geochemical techniques [130]. Within the last year, however, this discrepancy has gone away. The geochemical measurements were confirmed by the first laboratory observation of double beta decay (at Irvine [131].) In addition, several groups [129] have found that previously neglected ground state correlation effects could suppress the matrix element by the required order of magnitude. Furthermore, there is no analogous uncertainty in the $\beta\beta_{0\nu}$ case.

The neutrinoless double beta decay process $(Z, N) \rightarrow (Z+2, N-2)e^-e^-$, which violates lepton number by two units, can proceed through the second diagram [132] in Fig. 14. In the absence of mixing the quantity $\langle m_{\nu_e} \rangle$, the effective Majorana neutrino mass, is

$$\langle m_{\nu_e} \rangle = \begin{cases} 0, & \text{Dirac neutrino} \\ m_{\nu_e}, & \text{unmixed Majorana neutrino} \end{cases} \tag{87}$$

Although the matrix element is proportional to $\langle m_{\nu_e} \rangle$, which is necessarily very small,

$\beta\beta_{0\nu}$ has an enormous advantage in phase space over $\beta\beta_{2\nu}$ and could be observable for $\langle m_{\nu_e}\rangle$ in the eV range. Of course, the sum of the electron energies should be a sharp peak in $\beta\beta_{0\nu}$ (and a continuum for $\beta\beta_{2\nu}$), so the principal difficulty is controlling the background. [133,49] Currently, the most sensitive experiments are for $^{76}Ge \to {}^{76}Se\, e^- e^-$. No evidence for $\beta\beta_{0\nu}$ has been observed, [134] and the lower limit on the lifetime is [97] $\tau_{1/2} > 9 \times 10^{23}\ yr$ (68% c.l.). According to several calculations of the nuclear matrix elements [135] this implies $\langle m_{\nu_e}\rangle \leq 1\ eV$. However, a recent estimate by Engel et al. [136] yielded a much weaker limit $\langle m_{\nu_e}\rangle \leq 11\ eV$, so caution is advisable.

Even the largest value $\langle m_{\nu_e}\rangle \leq 11\ eV$ is smaller than the range $m_{\nu_e} \sim (17-40)\ eV$ suggested by the ITEP experiment. If the latter is correct the simplest possibility is that the ν_e is Dirac. Another possibility [137] is that the ν_e^0 is a mixture of Majorana mass eigenstate neutrinos, as in (44). Then, $\langle m_{\nu_e}\rangle$ becomes

$$\langle m_{\nu_e}\rangle = \sum_i m_i U_{Lei}^2 \xi_i F(m_i, A), \tag{88}$$

where $m_i \geq 0$ is the physical mass of the i^{th} mass eigenstate, U_{Lei} is the mixing matrix element ($\nu_{eL}^0 = \sum U_{Lei}\nu_{iL}$) and $\xi_i = \pm 1$ is the CP parity of ν_{iL}. ξ_i is just K_{ii} in (46), and a negative value $\xi_i = -1$ means simply that the eigenvalue of M in (41) was negative before choosing K to redefine ν_R^c. In (88), $F(m_i, A)$ is a nucleus dependent propagator correction, [138] defined by

$$F(m_i, A) = \frac{\langle e^{-m_i r}/r\rangle}{\langle 1/r\rangle}. \tag{89}$$

It is ~ 1 for $m_i \ll 10\ MeV$. For $m_i \gg 10\ MeV$, $F(m_i, A) \ll 1$ (it falls as m_i^{-2}) and allows the possibility [139] of A dependence of $\langle m_{\nu_e}\rangle$.

Because of the possibility of negative contributions to $\langle m_{\nu_e}\rangle$ it is conceivable that there are cancellations so that $\langle m_{\nu_e}\rangle$ is much smaller than the mass of the dominant Majorana component of ν_e (e.g. $m_1 \sim (17-40)\ eV$). Such a cancellation is actually not so contrived as it might first appear. If all of the m_i are small enough that $F(m_i, A) = 1$ then from (45) $\langle m_{\nu_e}\rangle$ is just the M_{ee} component of the original Majorana mass matrix in (41). As we have seen, M_{ee} must be generated by a Higgs triplet and vanishes in many models. In fact, the light seesaw model of Table 6 automatically leads to $\langle m_{\nu_e}\rangle = 0$ for sufficiently small m_i. For two neutrinos, for example, $\langle m_{\nu_e}\rangle = m_1 \cos^2\theta - m_2 \sin^2\theta$, which vanishes by (69) and (70). However, the light seesaw model was devised just in order to give $\langle m_{\nu_e}\rangle = 0$. For seesaw models with more natural scales $m_2 \gg 10\ MeV$ one has that $F(m_i, A) \ll 1$ and $U_{e1} \sim 1$, so that $\langle m_{\nu_e}\rangle \sim m_{\nu_e}$. In most Majorana models, therefore, one expects $\langle m_{\nu_e}\rangle \sim m_{\nu_e}$ unless fine-tuned deviations from the seesaw formula are invoked.

Whether or not the cancellation of the terms in (88) is natural, one can consider whether it is phenomenologically viable. For two neutrinos, for example, the conditions $m_1 \sim 20\ eV$, and $\langle m_{\nu_e}\rangle \ll m_1$ imply

$$\tan^2\theta = \frac{m_1}{m_2 F(m_2, A)}, \tag{90}$$

where $m_1 \leq m_2$, $\tan^2\theta \leq 1$ since the ITEP experiment presumably measures the dominant component of ν_e. However, the reactor oscillation limits in Fig. 13 allow only two possibilities. One is that $m_1 \simeq m_2$, $\theta \sim 45^0$. In that case ν_1 and ν_2 can be combined

to form a Dirac neutrino (or pseudo-Dirac if the degeneracy is not exact), possibly with a non-canonical lepton number (such as $L_e - L_\mu + L_\tau$) conserved. Alternatively, one can have $m_2 \geq 450\ eV$. However, the various laboratory and cosmological limits exclude [70] almost all values of m_2 except for small windows around 40 MeV and 2 GeV. Hence, if the ITEP results turn out to be correct they would almost certainly imply either (a) the ν_e is Dirac, or (b) there is new physics (such as a Majoron) that evades the cosmological bounds.

There are additional contributions to neutrinoless double beta decay in $SU_{2L}\times SU_{2R}\times U_1$ models [140]. Typically, such models contain additional charged $W_R^\pm$ bosons which couple to right-handed currents $\bar{e}_R\gamma^\mu N_R$, where N_R is a heavy Majorana neutrino. The exchange of a N_R (rather than a ν_L in Fig. 14) yields a new contribution $M_N F(M_N, A)(M_{W_L}/M_{W_R})^4$ to $\langle m_{\nu_e}\rangle$, which sets non-trivial constraints [69] on M_N and M_{W_R}. Furthermore, mixed contributions involving one ordinary left-handed current $\bar{e}_L\gamma^\mu\nu_L$ and one right-handed current $\bar{e}_R\gamma^\mu N_R$ can yield contributions to to $0^+ \to 2^+$ decay amplitudes that are not directly proportional to a neutrino mass [141] However, the relevant amplitudes are of order [49,140]

$$\left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \theta, \ \ \zeta\theta, \tag{91}$$

where θ is a light-heavy neutrino mixing angle and ζ is the $W_L - W_R$ mixing angle. One typically expects $(M_{W_L}/M_{W_R})^2$ and ζ to be less than 10^{-3}. Since we expect $\theta \sim m_D/M_{W_R} \leq 10^{-4} - 10^{-5}$ in a typical TeV-seesaw, the expected values for the quantities in (91) are smaller than the experimental limits (of $\sim 10^{-6}$).

One typically has $\langle m_{\nu_e}\rangle \ll m_{\nu_e}$ for the charged Higgs models [71] because the antisymmetry of the relevant Yukawa coupling forces M_{ee} to vanish.

4.9 The Solar Neutrino Problem

For some years the event rate in the $^{37}Cl \to {}^{37}Ar$ Solar neutrino experiment [142] ($2.0 \pm 0.3\ SNU$ [143]) has been considerably below the prediction [144] $5.8 \pm 2.2\ SNU$ of the standard Solar model. The discrepancy has recently been confirmed by the Kamiokande group which reports [145] an upper limit on the ν_e flux (from $\nu_e e$ elastic scattering) that is less than half the expected event rate. One explanation for the discrepancy is the existence of vacuum oscillations of the ν_e into other neutrinos. These could be important for neutrino mass-squared differences [146] $\Delta m^2 \equiv m_1^2 - m_2^2$ as small as $\Delta m^2 \sim (10^{-11} - 10^{-10})\ eV^2$, but only if the mixing angles are large.

Another possibility [147] is that the ν_e is a Dirac particle with a magnetic moment in the range $\mu_{\nu_e} \sim (0.6-10)\times 10^{-10}\mu_B$. The ν_e spin could then process in the Solar magnetic field into a sterile right-handed ν_e, thus reducing the observed flux by a factor $\simeq 2$. The necessary value of μ_{ν_e} is barely consistent with laboratory limits [148] but is probably excluded by astrophysical constraints from nucleosynthesis and stellar cooling [149] (Table 8). The worst objection, however, is that the necessary μ_{ν_e} is unnaturally high. In the standard model with a Dirac mass one expects [150]

$$\mu_\nu = \frac{3G_F m_\nu m_e}{4\pi^2\sqrt{2}}\mu_B \sim 3\times 10^{-19}\left(\frac{m_\nu}{1eV}\right)\mu_B \tag{92}$$

Table 8: Limits on the neutrino magnetic moments. A value $\mu_{\nu_e} \sim (0.6 - 10) \times 10^{-10}\mu_B$ would be needed to resolve the Solar ν problem.

laboratory [148]	$\mu_{\bar{\nu}_e} < 1.5\times10^{-10}\mu_B$
	$\mu_{\nu_\mu} < 9.5\times10^{-10}\mu_B$
Stellar cooling [149] ($\gamma\to\nu\bar{\nu}$)	$\mu_\nu < 0.8\times10^{-11}\mu_B$
Nucleosynthesis [149] (ν_R produced by spin precession)	$\mu_\nu < 0.5\times10^{-10}\mu_B$
Standard model [150] (Dirac mass)	$\mu_\nu \sim 3\times10^{-19}\left(\frac{m_\nu}{1\ eV}\right)\mu_B$

which is many orders of magnitude too small. Non-standard models [151] can yield larger μ_ν, but to obtain a sufficiently large value appears highly contrived.

Other canonical explanations involve non-standard Solar models. The existing experiments are mainly sensitive to the relatively high energy (from 0.81 MeV up to 14 MeV) neutrinos from ^8B decay. The flux of these ^8B neutrinos depends very sensitively on the temperature of the Solar core and could be changed significantly by modifications of the standard Solar model. Recently, there has been much attention to the possibility that weakly interacting massive particles (WIMPs), which could form the dark matter, could carry energy out of the Solar core and lower the central temperature slightly. [111] Less exotic modifications of the standard model are also possible.

A $^{71}Ga \to\ ^{71}Ge$ experiment could distinguish the nonstandard Solar model from the first two possibilities. Most of the expected ^{71}Ga event rate is from the low energy pp neutrinos, the flux of which can be inferred from the over-all Solar luminosity and is relatively insensitive to the temperature of the Solar core. The predicted ^{71}Ga event rate of $\simeq 107$ SNU can be reduced at most to around 78 SNU in most non-standard Solar models [144,152]. The traditional view has been that a flux lower than this would imply large vacuum oscillations, which would reduce the ^{71}Ga rate by a factor comparable to the ^{37}Cl event rate reduction for most oscillation parameters (e.g. to around 40 SNU).

Yet another possibility, i.e. that neutrinos decay between the Sun and the Earth, is all but excluded by the survival of neutrinos from supernova 1987A, except in some two-component models with large mixing angles. [153]

Recently, Mikheyev and Smirnov [154] have proposed an elegant new solution to the Solar neutrino problem, in which even tiny vacuum mixing angles can be amplified by the coherent interactions of ν_e with matter.

Considering $\nu_e \leftrightarrow \nu_\mu$ oscillations for definiteness, the vacuum oscillation equation in (79) can be described in terms of the weak basis states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ by

$$|\nu(t)\rangle = \nu_e(t)|\nu_e\rangle + \nu_\mu(t)|\nu_\mu\rangle, \tag{93}$$

where the coefficients satisfy the Schrödinger-like equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} = M_0\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix}, \tag{94}$$

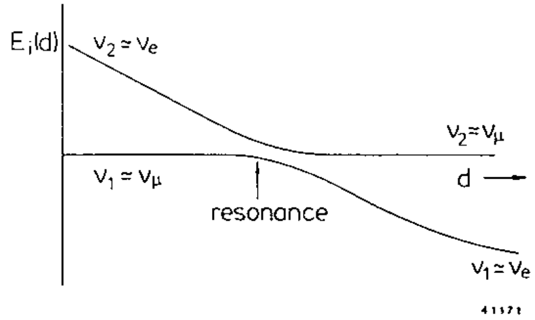

Figure 15: The energy eigenvalues of M' as a function of d, the distance from the center of the Sun.

with

$$M_0 = \begin{pmatrix} \frac{\Delta m^2}{2p}\cos 2\theta & \frac{-\Delta m^2}{4p}\sin 2\theta \\ \frac{-\Delta m^2}{4p}\sin 2\theta & 0 \end{pmatrix}, \tag{95}$$

where an irrelevant term proportional to the identity (which only affects the overall phase) has been dropped. Wolfenstein pointed out [155] that in the presence of matter, M_0 is replaced by the M', where

$$M' = M_0 + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} \tag{96}$$

and n_e is the density of electrons. The new term [155] - [157] is the effect of the coherent forward scattering amplitude for $\nu_e e^- \to \nu_e e^-$ via the charged current. The effects of neutral current scattering from e^-, p, and n have been neglected because they are the same for ν_e and ν_μ and only contribute to the overall phase. For $\Delta m^2 < 0$ (i.e. $m_{\nu_e} < m_{\nu_\mu}$ [157]) there is a critical density [158] $n_e^{crit} = -\Delta m^2 \cos 2\theta/(2\sqrt{2}G_F p)$ for which the diagonal elements of M' are equal (i.e. zero). At that density a resonance occurs, i.e. even a tiny off diagonal mixing term leads to large mixing effects.

In particular, if n_e in the Sun varies slowly an adiabatic approximation applies [154, 159]. ν_e's produced in the core of the Sun (where $n_e > n_e^{crit}$) correspond to the larger mass eigenstate ν_2 of M' (Fig. 15). Outside the Sun, on the other hand, the higher energy state ν_2 corresponds to ν_μ for $\Delta m^2 < 0$. Hence, if the variation of n_e with the distance from the center of the Sun is sufficiently slow, the initial ν_e will be adiabatically converted to ν_μ as they pass through the resonance.

A number of authors [152],[154],[159] - [161] have analyzed the implications of the Mikheyev-Smirnov-Wolfenstein (MSW) effect for the Solar neutrinos quantitatively. It is found that there are three classes of parameters which can explain the reduction of ^8B neutrinos observed in the ^{37}Cl experiment. These roughly form the sides of a triangle, as is illustrated schematically in Fig. 16. For solution (a) corresponding to $|\Delta m^2| \sim 5\times10^{-5}\ eV^2$, $\sin^2 2\theta \geq 4\times10^{-4}$, the adiabatic hypothesis is valid and $\simeq 100\%$ conversion

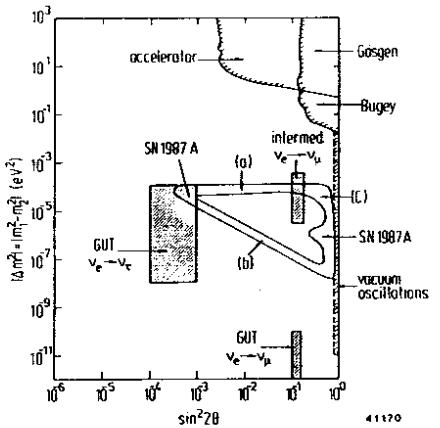

Figure 16: A schematic view of the regions in the Δm^2 - $\sin^2 2\theta$ plane which can explain the Solar neutrino problem via the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

occurs. However, only the high energy ^8B neutrinos actually encounter a resonance layer (the central density is too low for the low energy neutrinos) and are converted. For this parameter range one expects little reduction in the counting rate for the gallium experiment (i.e. the effect is similar to non-standard Solar models in that respect).

For solution (b), extending down to $|\Delta m^2| \sim 10^{-8}\ eV^2$ one has [160] $|\Delta m^2| \sin^2 2\theta \sim 10^{-7.5}\ eV^2$. Here the adiabatic approximation starts to break down. All neutrino energies are affected, but the conversion probability is less than unity. For these solutions one expects a significant reduction in the gallium counting rate, similar to large vacuum oscillations or magnetic moments. Solution (c), corresponding to large vacuum mixing angles, is an extension of the vacuum oscillation solution. In the middle of region (c) one expects a large day-night asymmetry in the ν_e counting rate due to MSW regeneration in the Earth at night [152].

The MSW solution is very elegant, but it severely complicates the task of sorting out which if any of the proposed solutions to the Solar ν problem is correct. It will take an ambitious program of experiments [152,162] to clarify the matter.

The first two events from SN1987A observed by the Kamiokande experiment [93] point back towards the supernova. They are consistent with ν_e from the initial neutronization burst, scattering via $\nu_e e^- \to \nu_e e^-$. However, they could also be $\bar{\nu}_e$ from the subsequent thermal burst, scattering via $\bar{\nu}_e p \to e^+ n$, which has a much larger cross section (and which produces an isotropic distribution of positrons). If they are indeed ν_e they are problematic for the MSW mechanism because one expects $\nu_e \to \nu_\mu$ conversions on the way out of the supernova. However, there are two parameter regions (shown in Fig. 16), which would still be consistent [163], corresponding respectively to incomplete conversion in the supernova and reconversion in the Earth. Unfortunately there is no way to determine whether the

two events are really ν_e's.

The MSW mechanism is consistent with the expectation of GUT [67] and intermediate scale [68] seesaws. As is illustrated in Fig. 16 the predictions of the GUT seesaw are consistent with $\nu_e \leftrightarrow \nu_\tau$ conversions in the Sun. In this case, the mass ranges are too small to ever see any direct laboratory effects of neutrino mass. The intermediate scale seesaw could account for the Solar ν problem via $\nu_e \leftrightarrow \nu_\mu$ conversions. In that case, $\nu_\mu \leftrightarrow \nu_\tau$ oscillations could well be observable in the laboratory.

It is also possible that the Solar ν problem could be explained by small neutrino masses, but that neutrino appearance experiments might nevertheless yield positive signals due to non-orthogonal neutrino states (induced by mixings between very light and very heavy neutrinos. [110])

5 Summary

- The predictions of the standard $SU_2 \times U_1$ model for the W and Z, the charged current, and the neutral current interaction are qualitatively confirmed. In particular, the charged and neutral current interactions of the neutrino are correctly described by the standard model to excellent precision. Furthermore, neutrino interactions are superb probes of the strong interactions and of possible new physics.

- Indirect arguments indicate that the ν_τ must exist. Nucleosynthesis constraints imply that there are no more than 4 neutrinos with masses $\leq 1\ MeV$. e^+e^- and $\bar{p}p$ constraints imply $\leq (3-5)$ neutrinos with masses up to $\sim 40\ GeV$.

- The question of whether the neutrinos have mass is vital for both particle physics and cosmology. However, there is at present no compelling evidence for neutrino mass. The ITEP result $m_{\nu_e} \sim (17-40)\ eV$ has not been confirmed by other experiments and is on the verge of being excluded. Although there are two positive indications of neutrino oscillations (with different parameters), these are contradicted by other experiments. The negative results suggest $m_{\nu_i} \simeq O(1\ eV)$ unless there are very small mixings or degeneracies. There are also stringent limits on incoherent mixing with heavy neutrinos.

- The MSW solution to the Solar neutrino problem would work for ν_μ or μ_τ in the $10^{-2}\ eV$ range, consistent with intermediate mass or GUT seesaws.

- The nonobservation of neutrinoless double beta decay implies $\langle m_{\nu_e} \rangle < 1 - 11\ eV$. If the ITEP result is correct this would most likely imply a Dirac neutrino or new physics to evade cosmological bounds.

- A ν mass in the 5 - 40 eV range would dominate the energy density of the universe and would be an excellent candidate for the dark matter, though other mechanisms would have to be invoked to explain the initial formation of galaxies. Conversely, the light stable neutrinos must be lighter than $\simeq 40\ eV$. A variety of astrophysical, laboratory, and cosmological bounds exclude unstable neutrinos up to $\sim$ 40 MeV (unless new physics is invoked for fast, invisible decays or annihilations), implying that m_{ν_μ}, $m_{\nu_\tau} < 40\ eV$.

Acknowledgement

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