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by

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Massive Neutrinos*

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Abstract

Most extensions of the standard $SU_2 \times U_1$ electroweak model predict nonzero neutrino masses. In this talk I summarize the theoretical expectations for neutrino mass in a variety of possible extensions and discuss their implications for various types of experiments.

1 Introduction

The question of whether the neutrino has a nonzero mass is one of the most important issues in both particle physics and astrophysics. In the minimal $SU_2 \times U_1$ model the neutrinos are predicted to be massless. However, extensions of the standard model involving new SU_2-singlet neutral fermions (the right-handed neutrino partners needed for Dirac mass terms) or new Higgs representations (to generate Majorana masses) allow non-zero masses. [1] In fact, most extensions of the standard model (e.g. most grand unified theories other than SU_5) involve one or both of these mechanisms.

Neutrino mass is also of great importance for astrophysics and cosmology. Masses in the 10 eV range could account for the dark matter of the universe, while masses $\leq 10^{-2} eV$ could resolve the Solar neutrino problem.

In this talk I will describe the complicated subject neutrino mass: the principle theoretical models and their implications will be described, and the experimental situation will be briefly summarized.

2 Weyl, Majorana, and Dirac Neutrinos.

For the weak interactions it is convenient to deal with Weyl two-component spinors ψ_L or ψ_R, each of which represents two physical degrees of freedom. The field ψ_L can annihilate

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a left-handed (L) particle or create a right-handed (R) antiparticle, while $\psi_L^\dagger$ annihilates a L-particle or creates an R-antiparticle. For a ψ_R field the roles of L and R are reversed. An ordinary four-component Dirac field ψ can be written as the sum $\psi = \psi_L + \psi_R$ of two Weyl fields, where ψ_L and ψ_R are just the chiral projections $\psi_{L,R} = P_{L,R}\psi$, with $P_{L,R} = (1 \pm \gamma_5)/2$.

Alternatively, one can consider Weyl fermions that do not have distinct partners of the opposite chirality. We will see below that such spinors correspond to particles that are either massless or carry no conserved quantum numbers.

In the free field limit a Weyl field ψ_L can be written as

$$\psi_L(x) = \sum_{\vec{p}} \left[b_L(\vec{p}) u_L(\vec{p}) e^{-ip\cdot x} + d_R^\dagger(\vec{p}) v_R(\vec{p}) e^{+ip\cdot x} \right], \tag{1}$$

where $\sum_{\vec{p}}$ represents $\int d^3\vec{p}/\sqrt{(2\pi)^3 2E}$. In(1), b_L and d_R are annihilation operators for L particles and R-antiparticles, respectively, and u_L and v_R are the corresponding (4-component) spinors satisfying $P_L u_L = u_L$, $P_L v_R = v_R$, $P_R u_L = P_R v_R = 0$. For a ψ_R spinor one simply interchanges L and R. Equation (1) differs from an ordinary (Dirac) free field in that there is no sum over spin.

It is apparent from (1) that each left-handed (right-handed) particle is necessarily associated with a right-handed (left-handed) antiparticle. The right-handed antiparticle [2] field ψ_R^c is not independent of ψ_L, but is closely related to $\psi_L^\dagger$. One has $\psi_R^c = C\bar{\psi}_L^T$, where C is the charge conjugation matrix, defined by $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Similarly, for a R-Weyl spinor, $\psi_L^c = C\bar{\psi}_R^T$. In the special case that ψ_L is the chiral projection $P_L\psi$ of a Dirac field ψ, ψ_R^c is just the R-projection $P_R\psi^c$ of the antiparticle field $\psi^c = C\bar{\psi}^T$.

If ψ_R and ψ_R^c both exist, they have the opposite values for all additive quantum numbers. Since the quarks and charged leptons carry conserved quantum numbers (e.g. color and electric charge), they must be Dirac fields - i.e. ψ_R and ψ_R^c must be distinct. The only quantum number associated with the neutrinos is lepton number, however, and it is conceivable that that is violated in nature. As we will see, that will allow for two very different possibilities for neutrino mass.

The known neutrinos of the first family are the left-handed electron neutrino ν_{eL} and its CP partner, the right-handed "antineutrino" $\nu_{eR}^c = C\bar{\nu}_{eL}^T$. These are associated with the e_L^- and e_R^+, respectively, in ordinary charged current weak interactions.

Mass terms always take left- and right-handed fields into each other. If one introduces a new field N_R (distinct from ν_R^c) and its CP conjugate $N_L^c = C\bar{N}_R^T$ into the theory, then one can write a Dirac (lepton number conserving) mass term

$$-L_{Dirac} = m_D \bar{\nu}_L N_R + h.c., \tag{2}$$

which connects N_R and ν_L. In this case ν_L, N_R, N_L^c and ν_R^c form a four component Dirac particle - i.e. one can define $\nu \equiv \nu_L + N_R$, $\nu^c = N_L^c + \nu_R^c = C\bar{\nu}^T$, so that $-L_{Dirac} = m_D\bar{\nu}\nu$. Clearly lepton number is conserved in this case, because there is no transition between ν and ν^c. In the free field limit the Dirac neutrino field ν has the canonical expression

$$\nu_{Dirac}(x) = \sum_{\vec{p}} \sum_{S=L,R} \left[b_S(\vec{p}) u_S(\vec{p}) e^{-ip\cdot x} + d_S^\dagger(\vec{p}) v_S(\vec{p}) e^{+ip\cdot x} \right], \tag{3}$$

Usually, the N_R is an $SU_2 \times U_1$ singlet, with m_D generated by an ordinary Higgs doublet, and $L = L_e + L_\mu + L_\tau$ is conserved in the three family generalization. This possibility is most similar to the way in which masses are generated for the other fermions (e^-, u, d, etc.) in the standard model, but it is difficult to understand why m_{ν_e} is so small in this case.

Another possibility [3] is that N_R is a known doublet neutrino, such as $\nu^c_{\tau R}$. This is a variation on the Konopinski-Mahmoud model. [4] Then ν_{eL}, $\nu^c_{\tau R}$, $\nu_{\tau L}$ and ν^c_{eR} can be combined to form a Dirac neutrino with $L_e - L_\tau$ conserved.

For the generalization of (2) to F fermion families one has

$$- L_{Dirac} = \bar{n}^0_L m_D N^0_R + h.c., \tag{4}$$

where m_D is an arbitrary [5] $F \times F$ mass matrix, and n^0_L and N^0_R are F-component vectors; thus $n^0_L = (n^0_{1L}\ n^0_{2L} \dots n^0_{FL})^T$, where n^0_{iL} are the "weak eigenstate" neutrinos - i.e. n^0_{iL} is associated with e^-_{iL} in weak transitions. The weak eigenstate neutrinos are related to the neutrinos n_{iL}, N_{iR} of definite mass by unitary transformations $n^0_L = V_L n_L$, $N^0_R = V_R N_R$, where V_L and V_R are $F \times F$ unitary matrices, determined by

$$V^\dagger_L m_D V_R = m_d = \text{diag}(m_1\ m_2 \cdots m_F) \tag{5}$$

and m_d is the diagonal matrix of physical neutrino masses. V_L and V_R can be determined by

$$V^\dagger_L m_D m^\dagger_D V_L = V^\dagger_R m^\dagger_D m_D V_R = m^2_d \tag{6}$$

($m_D m^\dagger_D$ and $m^\dagger_D m_D$ are Hermitian). In general V_L and V_R are unrelated. If there are no degeneracies then V_L and V_R are determined uniquely by (6) up to diagonal phase matrices; i.e. if $V_{L,R}$ satisfy (6) then so do $V_{L,R}K_{L,R}$, where $K_{L,R}$ are diagonal phase matrices associated with the unobservable phases of the n_{iL} and N_{jR} fields. Usually one chooses K_L to put V_L into a simple conventional form. Then K_R is determined by the requirement that m_d be real.

In the presence of neutrino mass the leptonic weak charged current becomes

$$J^{\mu\dagger}_W = (\bar{\nu}_e\ \bar{\nu}_\mu\ \bar{\nu}_\tau)\ V^\dagger_L \gamma^\mu (1+\gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} \tag{7}$$

so that $V^\dagger_L$ is just the analogue of the CKM quark mixing matrix. It describes the relative strengths [7] of the weak transition between the various charged leptons and neutrinos of definite mass.

In a Majorana (lepton number violating) mass term one avoids the need for a new fermion field by coupling the ν_L to its CP conjugate ν^c_R:

$$- L_M = \tfrac{1}{2} m \bar{\nu}_L \nu^c_R + h.c. = \tfrac{1}{2} m \bar{\nu}_L C \bar{\nu}^T_L + h.c. \tag{8}$$

L_M can be thought of as creating or annihilating two neutrinos, and violates lepton number by $\Delta L = \pm 2$. ν_L and ν^c_R can be combined to form a two component Majorana neutrino $\nu = \nu_L + \nu^c_R$, so that $-L_M = \frac{1}{2} m \bar{\nu}\nu$. One has $\nu = C\bar{\nu}^T$, i.e. a Majorana neutrino is its own antiparticle. In the free field limit ν is just

$$\nu(x) = \sum_{\vec{p}} \sum_{S=L,R} \left[b_S(\vec{p}) u_S(\vec{p}) e^{-ip\cdot x} + b^\dagger_S(\vec{p}) v_S(\vec{p}) e^{+ip\cdot x} \right], \tag{9}$$

i.e. it has the same form as for a free Dirac field (cf (3)) except that there is no distinction between b and d annihilation operators.

The Majorana mass m in (8) can be generated by the vacuum expectation value (VEV) of a new Higgs triplet [8] or as a higher order effective operator. Majorana masses are popular amongst theorists because they are so different from quark and lepton masses, and there is therefore the possibility of explaining why m_{ν_e} is so small (if it is non-zero).

For F fermion families, the Majorana mass term is

$$- L_M = \tfrac{1}{2} \bar{n}^0_L M n^{0c}_R + h.c. \tag{10}$$

where M is an $F \times F$ Majorana mass matrix and n^0_L and n^{0c}_R are F component vectors: i.e. $n^0_L = (n^0_{1L} \dots n^0_{FL})^T$, $n^{0c}_R = (n^{0c}_{1R} \dots n^{0c}_{FR})^T$, where n^0_{iL} and n^{0c}_{iR} are weak eigenstate neutrinos and "antineutrinos", related by

$$n^{0c}_{iR} = C \bar{n}^{0T}_{iL} \tag{11}$$

From (11) one can prove the identity $\bar{n}^0_{iL} n^{0c}_{jR} = \bar{n}^0_{jL} n^{0c}_{iR}$, from which it follows that the Majorana mass matrix M must be symmetric: $M = M^T$. Proceeding in analogy to the Dirac case, one can relate the n^0_{iL} and n^{0c}_{jR} to mass eigenstate neutrino fields by $n^0_L = U_L n_L$, $n^{0c}_R = U_R n^c_R$, where U_L and U_R are $F \times F$ unitary matrices chosen so that

$$U^\dagger_L M U_R = M_d = \text{diag}(m_1\ m_2\ \dots m_F), \tag{12}$$

where M_d is a diagonal matrix of Majorana mass eigenvalues. Unlike the Dirac case (for which m_D was an arbitrary matrix and V_L and V_R unrelated), the symmetry of M implies a relation between U_L and U_R, viz

$$U_L = U^*_R K^\dagger, \tag{13}$$

where K is unitary and symmetric. That is, just as in the Dirac case, U_L is determined from $U^\dagger_L M M^\dagger U_L = M^2_d$ to be of the form $U_L = \hat{U}_L K_L$, where K_L is a matrix of phases that can be chosen for convenience. U_R is then determined from (13), where K is chosen so that M_d is real and positive. If there are no degeneracies then K is just a matrix of phases. [9] One can always pick K_L such that $K = I$, but it is not always convenient to do so.

In terms of the mass eigenstates, (10) reduces to

$$- L_M = \tfrac{1}{2} \sum_{i=1}^{F} m_i \bar{n}_{iL} n^c_{iR} + h.c. = \tfrac{1}{2} \sum_{i=1}^{F} m_i \bar{n}_i n_i, \tag{14}$$

where $n_i = n_{iL} + n^c_{iR}$ is the i^{th} Majorana mass eigenstate. [10] Written in terms of the n_{iL}, the weak charged current assumes a form analogous to (7), with $U^\dagger_L$ replacing $V^\dagger_L$ to describe the leptonic mixing. [11]

There are several physical distinctions between Dirac and Majorana neutrinos. If the ν_e is Majorana, for example, one could have the sequence $\pi^+ \to e^+ \nu_e$ followed by $\nu_e p \to e^+ n$. The combined process violates lepton number by two units and is allowed for Majorana but not Dirac neutrinos. Similarly, a hypothetical heavy neutrino N would undergo the decays $N \to e^+ q_1 \bar{q}_2$ and $N \to e^- \bar{q}_1 q_2$ with equal rates if it is Majorana, while for a Dirac particle one would have $N \to e^- \bar{q}_1 q_2$, $N^c \to e^+ q_1 \bar{q}_2$ only [12]. There are differences due to Fermi statistics in the production of $\nu\nu$ (Majorana) or $\nu\nu^c$ (Dirac) pairs near threshold [13], and finally Majorana neutrinos cannot have electromagnetic form factors, such as magnetic moments [14].

It is important to keep in mind, however, that these distinctions must all disappear in the limit that the neutrino mass can be neglected. For $m_\nu \to 0$ the ν_R component of a Dirac neutrino decouples, and both Majorana and Dirac neutrinos reduce to Weyl two-component neutrinos - there is no difference between them. [15] In particular, lepton number conservation is reestablished smoothly as $m_\nu \to 0$ for a Majorana neutrino, because in that limit helicity - which is conserved up to corrections of order m_ν/E_ν - plays the role of an approximate lepton number. For example, the ν_e produced in $\pi^+ \to e^+\nu_e$ has $h_\nu = -1$ up to corrections of order $(m_\nu/E_\nu)^2$ (in rate), while the reaction $\nu_e p \to e^+ n$ has a cross section that is suppressed by $(m_\nu/E_\nu)^2$ for the wrong (negative) helicity.

In many models Dirac and Majorana mass terms are both present. For one doublet neutrino ν_L^0 (with $\nu_R^{0c} = C\bar{\nu}_L^{0T}$) and one new singlet N_R^0 (with $N_L^{0c} = C\bar{N}_R^{0T}$), for example, one could have the general mass term

$$-L = \tfrac{1}{2}\begin{pmatrix}\bar{\nu}_L^0 & \bar{N}_L^{0c}\end{pmatrix}\begin{pmatrix} m_t & m_D \\ m_D^T & m_S\end{pmatrix}\begin{pmatrix}\nu_R^{0c} \\ N_R^0\end{pmatrix} + h.c., \tag{15}$$

where $m_D = m_D^T$ is a Dirac mass generated by a Higgs doublet (analogous to (2)), m_t is a Majorana mass for ν_L^0 generated by a Higgs triplet or effective interaction (cf. (8)), and m_S is a Majorana mass for N_R^0, generated by a Higgs singlet or bare mass. Similarly, for F families (15) still holds provided one interprets ν_L^0 , N_L^{0c} , ν_R^{0c} , and N_R^0 as F component vectors, and m_t, m_D, and m_S as $F \times F$ matrices (with $m_t = m_t^T$, $m_S = m_S^T$). Then, (15) becomes simply

$$-L = \tfrac{1}{2}\bar{n}_L^0 M n_R^{0c} + h.c., \tag{16}$$

where $n_L^0 \equiv (\nu_L^0,\ N_L^{0c})^T$ and $n_R^{0c} \equiv (\nu_R^{0c},\ N_R^0)^T$ are $2F$ component vectors and M is the symmetric $2F \times 2F$ Majorana mass matrix in (15). Equation (16) can be diagonalized in exact analogy with (10-14), yielding finally

$$-L = \tfrac{1}{2}\sum_{i=1}^{2F} m_i \bar{n}_{iL} n_{iR}^c + h.c. \tag{17}$$

i.e. there are in general $2F$ Majorana neutrinos, related to n_L^0, n_R^{0c} by unitary transformations. Unlike the pure Majorana case, however, there is now mixing between particles with different weak interaction properties (e.g. $n_{iL} = (U_L^\dagger)_{ij} n_{jL}^0$ is a mixture of SU_2 doublets and singlets), which can have important consequences for neutrino oscillations [16] and decays.

It is instructive to see how the Dirac case ($m_t = m_S = 0$) emerges as a limiting case of (15). For a single family one has $M = m_D\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. Since M is Hermitian (for m_D real) one can diagonalize it by a unitary transformation U_L. One finds

$$U_L^\dagger M U_L = m_D\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{18}$$

with $U_L = \frac{1}{\sqrt{2}}\mathrm{diag}(1\ \ -1)$; i.e. the mass eigenstates are

$$\begin{aligned} n_{1L} &= \frac{1}{\sqrt{2}}(\nu_L^0 + N_L^{0c}) \\ n_{2L} &= \frac{1}{\sqrt{2}}(\nu_L^0 - N_L^{0c}) \\ n'_{1R} &= \frac{1}{\sqrt{2}}(\nu_R^{0c} + N_R) \\ n'_{2R} &= \frac{1}{\sqrt{2}}(\nu_R^{0c} - N_R). \end{aligned} \tag{19}$$

The negative mass eigenvalue in (18) can be removed by redefining [17] the right-handed fields $n_{1R} = n'_{1R}$, $n_{2R} = -n'_{2R}$. This is nothing more than taking $U_L^\dagger M U_R = m_d = m_D$ diag(1 1), where U_R is given by (13) with $K = \mathrm{diag}(1\ \ -1)$. Finally, the two Majorana states $n_1 = n_{1L} + n_{1R}^c$ and $n_2 = n_{2L} + n_{2R}^c$ are degenerate. We can therefore reexpress L in the new basis

$$\begin{aligned} \nu &\equiv \frac{1}{\sqrt{2}}(n_1 + n_2) = \nu_L^0 + N_R^0 \\ \nu^c &\equiv \frac{1}{\sqrt{2}}(n_1 - n_2) = N_L^{0c} + \nu_R^{0c}, \end{aligned} \tag{20}$$

yielding

$$\begin{aligned} -L &= \tfrac{1}{2} m_D(\bar{n}_{1L} n_{1R}^c + \bar{n}_{2L} n_{2R}^c) + h.c. \\ &= m_D \bar{\nu}_L^0 N_R^0 + h.c. \ = \ m_D \bar{\nu}\nu. \end{aligned} \tag{21}$$

This is just a standard Dirac mass term, with a conserved lepton number (i.e. no transition between ν and ν^c). A Dirac neutrino is therefore nothing but a pair of degenerate two-component Majorana neutrinos (n_1 and n_2), combined to form a 4-component neutrino with a conserved lepton number.

One sometimes refers to a pseudo-Dirac neutrino, which is just a Dirac neutrino to which is added as small lepton number-violating perturbation. For example, one could modify the Dirac mass to

$$M = \begin{pmatrix}\epsilon & m_D\\ m_D & 0\end{pmatrix}, \tag{22}$$

with $\epsilon << m_D$. One then finds two Majorana mass eigenstates $n_\pm$, with

$$\begin{aligned} n_{+L} &= n_{1L} + \frac{\epsilon}{4} n_{2L} \\ n_{-L} &= -\frac{\epsilon}{4} n_{1L} + n_{2L}, \end{aligned} \tag{23}$$

(n_{1L} and N_{2L} are defined in (19)), with masses $m_D \pm \frac{\epsilon}{2}$.

Other important special cases of (15) are considered below.

3 Models of Neutrino Mass

There are many models for neutrino mass [1], all of which have good and bad features. The major classes of models are listed in Table 1, along with the most natural scales for the neutrino masses and for $\langle m_{\nu_e}\rangle$, an effective mass relevant to neutrinoless double β decay.

Table 1: Models of neutrino mass, along with their most natural scales for the light neutrino masses.

Model	m_{ν_e}	$\langle m_{\nu_e}\rangle$	m_{ν_μ}	m_{ν_τ}
Dirac	$1-10\ MeV$	0	$100\ MeV - 1\ GeV$	$1-100\ GeV$
pure Majorana [8] (Higgs triplet)	arbitrary	m_{ν_e}	arbitrary	arbitrary
GUT seesaw [18,19] ($M \sim 10^{14}\ GeV$)	$10^{-11}\ eV$	m_{ν_e}	$10^{-6}\ eV$	$10^{-3}\ eV$
intermediate seesaw [20] ($M \sim 10^{9}\ GeV$)	$10^{-7}\ eV$	m_{ν_e}	$10^{-2}\ eV$	$10\ eV$
$SU_{2L} \times SU_{2R} \times U_1$ seesaw [21] ($M \sim 1\ TeV$)	$10^{-1}\ eV$	m_{ν_e}	$10\ KeV$	$1\ MeV$
light seesaw [22] ($M \ll 1\ GeV$)	$1-10\ MeV$	$\ll m_{\nu_e}$	-	-
charged Higgs [23]	$< 1\ eV$	$\ll m_{\nu_e}$	-	-

Dirac neutrinos are exactly like other fermions. They involve a conserved total lepton number (though the individual L_e, L_μ, and L_τ lepton numbers are violated by mixing in general) and therefore do not lead to neutrinoless double beta decay. The problem with Dirac neutrinos is that it is hard to understand why the neutrinos are so much lighter than the other fermions. In the standard model Dirac mass are generated by the vacuum expectation value (VEV) $v = \sqrt{2}\langle\varphi^0\rangle \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246\ GeV$ of the neutral component of a doublet of Higgs scalar fields. One has $m_D = h_\nu v$, where h_ν is the Yukawa coupling

$$L = -\sqrt{2}h_\nu(\bar{\nu}_L\ \bar{e}_L)\begin{pmatrix}\varphi^0\\ \varphi^-\end{pmatrix} N_R + h.c. \qquad (24)$$

of the neutrino to φ^0.

A ν_e mass in the 20 eV range would require an anomalously small Yukawa coupling $h_{\nu_e} \leq 10^{-10}$. Moreover, h_{ν_e} would have to be smaller by $m_{\nu_e}/m_e \leq 10^{-4}$ than the analogous Yukawa coupling for the electron. Of course, we do not understand the masses of the other fermions either (or why they range over at least five orders of magnitude), so it is hard to totally exclude the possibility that h_{ν_e} is simply small. Nevertheless, the possibility seems sufficiently ugly that it is hard to take seriously unless some mechanism (other than fine-tuning) for the smallness is proposed.

One possibility is that h_ν is actually zero to lowest order (tree level) due to some new symmetry, and that h_ν is only generated as a higher order correction (i.e. so that m_ν/m_e is some power of α.) This is a very attractive possibility, but no particularly compelling models to implement it have emerged. The idea has recently been resurrected in some superstring inspired models [24], which have difficulty incorporating the seesaw type ideas described below.

Majorana mass terms for the ordinary SU_2-doublet neutrinos involve a transition from

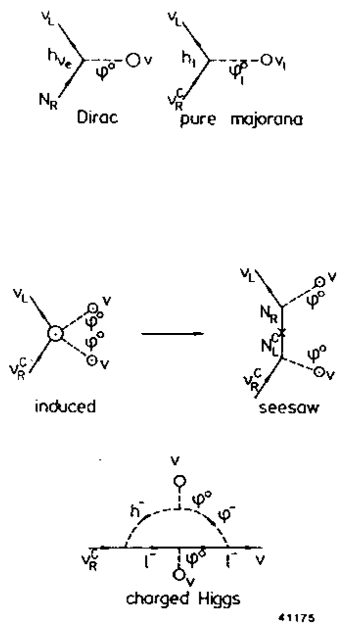

Figure 1: Dirac, pure Majorana, induced, and charged Higgs generated neutrino masses.

ν_R^c ($t_3 = -\frac{1}{2}$) into ν_L ($t_3 = +\frac{1}{2}$), and therefore must be generated by an operator transforming as a triplet under weak SU_2.

The simplest possibility is the Gelmini-Roncadelli model [8], in which one introduces a triplet of Higgs fields $\vec{\varphi}_t = (\varphi_t^0,\ \varphi_t^-,\ \varphi_t^{--})$ into the theory. The Yukawa coupling

$$\begin{aligned} L &= \tfrac{1}{2}h_t(\bar{\nu}_L\ \bar{e}_L)\vec{\tau}\cdot\vec{\varphi}_t\begin{pmatrix} e_R^c \\ -\nu_R^c\end{pmatrix} \\ &= \tfrac{1}{2}h_t(\bar{\nu}_L\ \bar{e}_L)\begin{pmatrix}\varphi_t^- & \sqrt{2}\varphi_t^0 \\ \sqrt{2}\varphi_t^{--} & -\varphi_t^-\end{pmatrix}\begin{pmatrix} e_R^c \\ -\nu_R^c\end{pmatrix} \end{aligned} \qquad (25)$$

then generates a Majorana mass $m_t = h_t v_t$ for the ν, where $v_t = \sqrt{2}\langle\varphi_t^0\rangle$ is the VEV of the Higgs triplet. Since both h_t and v_t are unknown the neutrino mass is unrelated to the other fermions and can in principle be arbitrarily small, at least at tree level.

However, small m_{ν_e} is not explained in such models - it is merely parametrized and in fact is almost as problematic as a Dirac mass. The weak neutral current (and W and Z masses) require [25] $v_t \leq 0.08v \sim 20\ GeV$. For v_t close to this limit one requires $h_t \leq 10^{-9}$, i.e. almost as bad a fine-tuning as the Dirac case. For $v_t \ll v$ one can tolerate more reasonable values for h_t, but then it is difficult to understand the large hierarchy in vacuum expectation values. One generally expects all non-zero VEV's to be comparable in magnitude unless fine-tunings are performed on the parameters in the Higgs potential. Even if one does this, higher order corrections are likely to upset the hierarchy. [26]

The VEV $\langle\varphi_t^0\rangle \neq 0$ necessarily violates lepton number conservation by two units (the Yukawa coupling in (25) does not by itself violate L because φ_t can be regarded as carrying two units of L). If the rest of the Lagrangian conserves L then lepton-number is spontaneously broken, and there will be an associated massless Goldstone boson, the triplet-Majoron. (This is the version of the model that is usually considered [8].) In this case limits based on stellar energy loss (carried off by Majorons) require [27] $v_t \leq 2 - 10\ KeV$. Implications of the Majoron for neutrino decay and annihilation, cosmology, and neutrinoless double beta decay will be mentioned below.

It is also possible introduce other couplings into the Higgs triplet model which explicitly break lepton number conservation, such as a cubic interaction between $\vec{\varphi}_t$ and two Higgs doublets. (This violates L since $\vec{\varphi}_t$ was assigned $L = 2$ to make (25) invariant). In that case all of the new scalar particles associated with φ_t become massive - i.e. there is no Majoron.

Another mechanism for introducing a Majorana mass is to consider the induced interaction (Fig. 1).

$$L_{eff} = \frac{1}{2}\frac{C}{M}(\bar{\nu}_L\ \bar{e}_L)\vec{\tau}\begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix} \cdot (\varphi^- \ \ -\varphi^0)\vec{\tau}\begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} \tag{26}$$

between two leptons and two Higgs doublets. The Higgs fields in (26) are arranged to transform as an SU_2 triplet, so L_{eff} is $SU_2 \times U_1$ invariant; however, L_{eff} is non-renormalizable, as is evidenced by the dimensional coupling C/M, where M is a mass. L_{eff} cannot therefore be an elementary coupling, but it could be an effective four-particle interaction induced [28] by new physics at some large mass scale M (just as the four-fermion weak interaction is a nonrenormalizable effective interaction that is really generated by W and Z exchange). When φ^0 is replaced by its vacuum expectation value, (26) yields an effective Majorana mass $m \sim Cv^2/M$, which is naturally small for $M \gg v$. For example, if (26) were somehow induced by quantum gravity one would expect $M \sim 10^{19}\ GeV$ (the Planck scale). Then for $C \sim 1$ one would have $m_\nu \sim 10^{-5}\ eV$.

The most popular realisation of this idea is the seesaw model, [18] in which the underlying physics is the exchange of a very heavy SU_2-singlet Majorana neutrino N_R^0, as indicated in Fig. 1. The seesaw model for one family is a special case of the general mass matrix in (15), in which m_D is a typical Dirac mass (typically assumed to be comparable to m_u or m_e for the first family) connecting ν_L^0 to a new SU_2-singlet N_R^0 and $m_S \gg m_D$ is a Majorana mass for N_R^0, presumably comparable to some new (large) physics scale. One typically assumes that $m_t = 0$ in the seesaw model, i.e. that there is not a Higgs triplet as well. [29] In that case, (15) yields two Majorana mass eigenstates n_1 and n_2 with

$$\begin{aligned} \nu_L^0 &= n_{1L}\cos\theta + n_{2L}\sin\theta \\ N_L^{0c} &= -n_{1L}\sin\theta + n_{2L}\cos\theta \\ \nu_R^{0c} &= -(n_{1R}^c\cos\theta + n_{2R}^c\sin\theta) \\ N_R^0 &= -n_{1R}^c\sin\theta + n_{2R}^c\cos\theta. \end{aligned} \tag{27}$$

The physical masses [30] are

$$\begin{aligned} m_1 &\simeq \frac{m_D^2}{m_S} \ll m_D \\ m_2 &\simeq m_S \end{aligned} \tag{28}$$

and the mixing angle is

$$\tan\theta = \left(\frac{m_1}{m_2}\right)^{1/2} \simeq \frac{m_D}{m_S} \ll 1. \tag{29}$$

Hence, one naturally obtains one very light neutrino, which is mainly the ordinary SU_2 doublet (ν_L^0, ν_R^{0c}), and one very heavy neutrino, which is mainly the singlet (N_L^{0c}, N_R^0).

If one does allow $m_t \neq 0$ (but $\ll m_S$) then there are still two Majorana neutrinos with masses $|a - \frac{m_D^2}{m_S}|$ and m_S, respectively, while $\theta \sim m_D/m_S \ll 1$ still holds. (The minus sign in ν_R^{0c} is removed if $a - \frac{m_D^2}{m_S}$ is positive). In this case, however, one loses the natural explanation of why m_1 is so small, unless m_t is itself induced by the underlying physics and is of the same order as m_D^2/m_S.

The seesaw model is easily generalized to F families. One then has the general $2F \times 2F$ Majorana mass matrix in (15). Assuming that the eigenvalues of m_S are all much larger than any of the components of m_D or m_t (if it is non-zero) one can calculate the eigenvalues and mixing matrices to leading order in m_S^{-1}. One finds that there are F light Majorana neutrinos (consisting of the F doublets (ν_L^0, ν_R^{0c}), up to corrections of order $m_D m_S^{-1}$ and F heavy Majorana neutrinos (consisting of the singlets (N_L^{0c}, N_R^0), to $O(m_D m_S^{-1})$). That is, one can write

$$\begin{aligned} \begin{pmatrix} \nu_L^0 \\ N_L^{0c} \end{pmatrix} &= U_L \begin{pmatrix} n_{lL} \\ n_{hL} \end{pmatrix} \\ \begin{pmatrix} \nu_R^{0c} \\ N_R^0 \end{pmatrix} &= U_R \begin{pmatrix} n_{lR}^c \\ n_{hR}^c \end{pmatrix}, \end{aligned} \tag{30}$$

where n_{lL} and n_{hL} are F component vectors of light and heavy Majorana mass eigenstates, respectively, and similarly for n_{lR}^c, n_{hR}^c. As usual, U_L and U_R are $2F \times 2F$ unitary matrices which diagonalize M in (15), viz

$$U_L^\dagger \begin{pmatrix} m_t & m_D \\ m_D^T & m_S \end{pmatrix} U_R = m_d = \begin{pmatrix} m_l & 0 \\ 0 & m_h \end{pmatrix}, \tag{31}$$

where m_l and m_h are diagonal $F \times F$ matrices of the F light and F heavy eigenvalues, respectively. To leading order in m_S^{-1} one can write $U_L^\dagger$ and U_R in block diagonal form

$$U_L^\dagger = K U_R^T = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \begin{pmatrix} A^T & -A^T m_D m_S^{-1} \\ D^T m_S^{-1\dagger} m_D^\dagger & D^T \end{pmatrix}, \tag{32}$$

where A^T and D^T are unitary (to leading order) $F \times F$ matrices defined by

$$\begin{aligned} m_l &= K_1 A^T (m_t - m_D m_S^{-1} m_D^T) A \\ m_h &= K_2 D^T m_S D \end{aligned} \tag{33}$$

i.e. the mass matrix for the light neutrinos is $m_t - m_D m_S^{-1} m_D^T$, which is diagonalized by A, while that for the heavy neutrinos is m_S, diagonalized by D. K_1 and K_2 are diagonal phase matrices which ensure that m_l and m_h are real and positive. We see from (30-33) that indeed there are F heavy states with masses of $O(m_S)$, and in the simplest case $m_t = 0$ there are F states which are naturally very light ($O(m_D^2 m_S^{-1})$). (For $m_t \neq 0$ one must separately assume m_t is small). Furthermore, the mixing between the light and heavy sectors is very small (of $O(m_D m_S^{-1})$), while the matrices A and D, which describe mixings within the two sectors, are in general arbitrary.

There are several classes of seesaw models [18], depending on the scale of m_S. In simple grand unified models one assumes that the scale is a typical GUT unification scale of around 10^{14} GeV. In many such models (e.g. SO_{10}) one has that the neutrino Dirac mass matrix m_D is the same as m_u/k where m_u is the u-quark mass matrix and $k \simeq 4.7$ represents the running of the Yukawa couplings between the GUT scale and low energies. If one makes the somewhat ad-hoc assumption that the matrix m_S is just $M_X I$, where $M_X \sim 10^{14}$ GeV is the unification scale and I is the identity matrix, one has (for $m_t = 0$) the light eigenvalues

$$m_{\nu_i} \sim \frac{m_{u_i}^2}{M_X k^2} \tag{34}$$

$\sim 10^{-11}$ eV, 10^{-6} eV, 10^{-3} eV, i.e. the neutrino masses are naturally expected to be extremely tiny, and to scale like the squares of the u, c, and t quark masses. (Equation (34) was computed for $m_{top} \sim 50$ GeV). Several caveats are in order: the assumption of $m_S \sim M_X I$ was quite arbitrary. One could easily imagine that the eigenvalues of m_S are smaller than M_X due to small Yukawa coupling couplings (increasing m_{ν_i}). Also, they need not be the same. For example, if the m_S eigenvalues followed the same family hierarchy as the ordinary fermions (i.e. $m_{S_i} \propto m_{u_i}$) then one would have m_{ν_i} scaling as m_{u_i} rather than $m_{u_i}^2$. (A similar linear hierarchy ensues in some variant GUTs in which m_S is zero at tree level but is generated at higher orders [31]. Of course, more complicated patterns for m_S and m_D (in (33)) are also possible. Furthermore, in many cases loop corrections to the (GUT) Higgs potential may induce [29] VEV's for Higgs representations that can yield a non-zero triplet terms m_t in (31). These are most likely to affect the smallest masses (e.g. m_{ν_e}). Equation (34) should therefore be regarded only as a typical order of magnitude.

If one does assume that $m_S = M_X I$, however, then m_u^2/M_X is diagonalized by the same transformations that diagonalize m_u. Since one also has equal electron and d-quark mass matrices (i.e. $m_e = m_d/k$) in most simple GUTs the final result is that flavor mixing in the lepton sector (analogous to (7)) is described by the same mixing matrix as the CKM quark mixing matrix. This result continues to hold [19] approximately for a far wider class of m_S than does the simple mass prediction in (34).

Lower mass scales for m_S imply larger values for the light neutrino masses (and generally less predictive power for m_D). Several authors [20] have suggested that the heavy Majorana scale could be the intermediate range $10^8 - 10^{12}$ GeV associated with invisible axions. For $m_{D_i} \sim m_{e_i}$ and $m_S \sim 10^9$ $GeV I$, for example, one obtains the values $\sim 10^{-7}$ eV, 10^{-2} eV, 10 eV for m_{ν_e}, m_{ν_μ}, m_{ν_τ}, respectively.

If m_S is in the several TeV range (as expected in some left-right symmetric [32] $SU_{2L} \times SU_{2R} \times U_1$, models [21], for example) one typically expects (for $m_{D_i} \sim m_{e_i}, m_S \propto I$) $m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}$ to have relatively large values 10^{-1} eV, 10 KeV, and 1 MeV, respectively. As we will see, such models run into severe cosmological difficulties unless the mass hierarchy is somehow modified or a fast decay mechanism is found for the ν_μ and ν_τ. Of course,

one could also have m_S much smaller than the $SU_{2L} \times SU_{2R} \times U_1$ scale (e.g. in the 10 $GeV - 100$ GeV range), with corresponding larger masses for the light neutrinos. Similar statements apply to models with extra Z bosons in the 100 $GeV - 10$ TeV range, which usually also have heavy Majorana neutrinos.

Finally, one can consider light seesaw models, in which typically $m_S \ll 1$ GeV. Such models are very artificial and abandon the principal advantages of the seesaw, because both m_D and m_S must be taken unnaturally small to obtain an acceptable ν_e mass. Their only virtue is that they yield strongly suppressed neutrinoless double beta decay rates, even though the neutrinos are Majorana.

Seesaw models were first introduced in GUT type models in which lepton number is explicitly violated by the gauge interactions. One can also consider non-gauge seesaw models [33] in which lepton number is spontaneously broken by the VEV of the Higgs field which generates m_S. Such models imply the existence of a massless Goldstone boson, the singlet-Majoron. [34] Unlike the triplet-Majoron in the Gelmini-Roncadelli model,[8] which can couple strongly to the ordinary neutrinos (coupling $\sim h_t$), the singlet-Majoron effectively decouples from ordinary particles. That is, it couples strongly to the heavy neutrino, with a coupling of order $m_D m_S^{-1}$ to off-diagonal $n_l n_h$ vertices, and with strength $(m_d m_S^{-1})^2$ to light neutrinos.

It is difficult to implement the seesaw model in most superstring inspired models, because there is no Higgs field available to generate a large m_S. It has been suggested [35] that m_S could be generated by a higher order effective operator, but such model may run into serious cosmological problems [36].

There have also been variant seesaw models constructed [37] in which the light neutrinos occur in degenerate pairs which can be combined from Dirac neutrinos with a conserved L.

Finally, I mention the charged Higgs models [23], in which small Majorana masses are generated by loop diagrams involving new charged Higgs bosons with explicit L-violating couplings (Fig. 1). Viable versions often lead to pseudo-Dirac neutrinos. The approximately conserved lepton number is typically $L_e - L_\mu + L_\tau$, for example, rather than L. The actual mass scale depends on unknown Yukawa couplings and masses.

4 Experimental Constraints

There are a number of excellent reviews [1] of the experimental status of neutrino mass. My major purpose in this section is to comment on the implications of the various theoretical models for the different types of experiments.

4.1 Kinematic Tests

Direct kinematic limits on the masses of the ν_e, ν_μ, and ν_τ are given in Table 2. The ITEP group [38] has long claimed evidence for a non-zero ν_e mass in the 20 eV range from tritium β decay, but this has not been confirmed by other groups, and in fact the Zurich-SIN measurement is on the verge of conflicting with the ITEP result. In addition the neutrinos from supernova 1987A observed by the Kamiokande [45] and IMB [46] experiments place upper limits in the 20 eV range on the ν_e mass (otherwise the arrival times of the detected

Table 2: Kinematic limits/values on neutrino masses.

$17\ eV < m_{\nu_e} < 40\ eV$	ITEP [38]
$m_{\nu_e} < 18\ eV$	Zurich [39]
$m_{\nu_e} < 27\ eV$	LANL [40]
$m_{\nu_e} < 32\ eV$	INS-Tokyo [41]
$m_{\nu_e} < O(20\ eV)$	SN1987A [42]
$m_{\nu_\mu} < 0.25\ MeV$	SIN [43]
$m_{\nu_\tau} < 50\ MeV$	ARGUS [44]

neutrinos would be spread out more than is observed), but it is hard to make this limit precise because it depends on the details of the neutrino emission [42].

A 20 eV neutrino mass is just in the range that would be most interesting cosmologically, so clearly it is essentially to resolve the situation. Hopefully, the current and next generation of tritium β decay experiments will be sensitive down to a few eV, but it is doubtful whether experiments of this type will ever be able to probe to much lower scales. As can be seen in Table 1, none of the models really predict m_{ν_e} in the 20 eV range (the $SU_{2L} \times SU_{2R} \times U_1$ models come closest), but most can accomodate masses in this range by fine-tuning parameters.

As can be seen in Table 2, the direct kinematic limits on m_{ν_μ} (from $\pi_{\mu 2}$ decay) and on m_{ν_τ} (from $\tau \to \nu_\tau + 5\pi$) are relatively weak. The experiments are extremely difficult (the mass scales being probed are very much smaller than the energies released in the decays), so it is unlikely that these measurements will improve by much more than a factor of two.

4.2 Heavy Neutrinos

There are many limits [1,47] on possible small admixtures of heavy neutrino states in the ν_e or ν_μ, including universality tests in nuclear β decay, searches for secondary peaks or distortions of the lepton spectra in β, π, and K decay, searches for the decay products of heavy neutrinos (e.g. $\nu_h \to \nu_e e^+ e^-$) produced in beam dumps, e^+e^- annihilation, or neutrino scattering. The limits on the mixing angle U_{ai}, $a = e$ or μ, where $\nu_a^0 = \sum_i U_{ai}\nu_i$ are quite impressive, especially for m_i in the range 10 MeV – 10 GeV, where they are comparable to the expectations in (29) of a seesaw model with $m_1 \sim 10\ eV$ and $m_2 = m_i$. The lower part of this range corresponds to the masses expected in the "light-seesaw" model (Table 1), while the 1 GeV – 1 TeV range is consistent with $SU_{2L} \times SU_{2R} \times U_1$ models. [21]

As has already been mentioned, heavy neutrinos in the $GeV - TeV$ range are likely to give too large m_{ν_μ} and m_{ν_τ} unless the typical seesaw hierarchy $m_{\nu_i} \propto m_{l_i}^n$ or $m_{u_i}^n$, $n = 1$ or 2, for the light neutrinos is avoided or new physics is invoked to ensure fast decays or annihilations for the ν_μ and ν_τ. On the other hand, if such new physics is present some of the limits (those based on decays) may no longer be valid, because in many cases the heavy neutrinos will decay rapidly into unobservable channels (e.g. $\nu_h \to \nu_l +$ Majoron) before reaching the detector.

4.3 Neutrino Oscillations

Neutrino oscillations are a beautiful example of a common quantum phenomenon: viz that if one starts at time $t = 0$ in a state that is not an energy eigenstate [48] then at later times it can oscillate into another (orthogonal) state. For example, suppose that the ν_e^0 and a second neutrino ν_a^0 (e.g. $\nu_a^0 = \nu_\mu^0$ or ν_τ^0) are mixtures of two mass eigenstates ν_1 and ν_2 with mixing angle θ. If at time $t = 0$ the weak eigenstate $\nu_e^0 = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$ is produced (e.g. in the process $\pi^+ \to \pi^0 e^+ \nu_e^0$) then at time t it will have evolved into the state

$$\begin{aligned} \nu_e^0(t) &= \cos\theta\ \nu_1 e^{-iE_1 t} + \sin\theta\ \nu_2 e^{-iE_2 t} \\ &\simeq \cos\theta\ \nu_1 e^{\frac{-im_1^2 t}{2p}} + \sin\theta\ \nu_2 e^{\frac{-im_2^2 t}{2p}}. \end{aligned} \tag{35}$$

In the second form I have assumed relativistic neutrinos $E_i = \sqrt{p^2 + m_i^2} \sim p + m_i^2/2p$ with definite momentum [49] $p >> m_i$, and have neglected an irrelevant overall phase $\exp(-ipt)$. The state $\nu_e^0(t)$ has a non-trivial overlap with ν_a^0. After traveling a distance $L \sim t$, there will be a probability

$$\begin{aligned} P(\nu_e \to \nu_a) &= |\langle \nu_a^0 \mid \nu_e^0(t)\rangle|^2 \\ &= \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4p}\right) \\ &= \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2(eV^2)L(m)}{p(MeV)}\right) \end{aligned} \tag{36}$$

that the state will have evolved into ν_a^0 (as can be observed in the process $\nu_a N \to e_a N'$, for example), and a probability $P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_a)$ that the state will remain a ν_e^0. In (36), $\Delta m^2 = m_1^2 - m_2^2$, and the last form is valid for Δm^2 in eV^2, L in m, and p in MeV. It is seen that the $\nu_e \to \nu_a$ probability depends on both the mixing angle θ and on $\Delta m^2 L/p$. For moderate values of the latter quantity the probability oscillates as a function of L and p, while for very large values the oscillations are averaged by a finite-sized detector or non-monochromatic source, (the second factor in (36) averages to 1/2). It is easy to generalize [1] (36) to the case that the initial neutrino is a mixture of more than two mass eigenstates, $\nu_a^0 = \sum_i U_{ai}\nu_i$. One obtains

$$P(\nu_e \to \nu_a) = \sum_i |U_{ei}U_{ai}^*|^2 + Re \sum_{i \neq j} U_{ei}U_{ai}^* U_{ej}^* U_{aj} e^{\frac{-i(m_i^2 - m_j^2)L}{2p}} \tag{37}$$

Neutrino oscillations can be searched for in (a) appearance experiments, in which one looks for the interactions of ν_a in a detector, and (b) disappearance experiments, in which one looks for a reduced ν_e flux. In both cases one can compare the observed counting rate with the expectation from known backgrounds (appearance) or from the expected flux (disappearance) as determined, for example, by measuring the electron spectrum from $n \to pe^-\bar{\nu}_e$ in reactor $\bar{\nu}_e$ oscillation experiments. A much cleaner technique is to search for actual oscillations in the appearance or disappearance probabilities as a function of L or p, such as by using two detectors at different distances form the source.

There are many limits on neutrino oscillations from accelerator experiments [1] (e.g. counter and emulsion experiments and beam dumps, searching for $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$ and

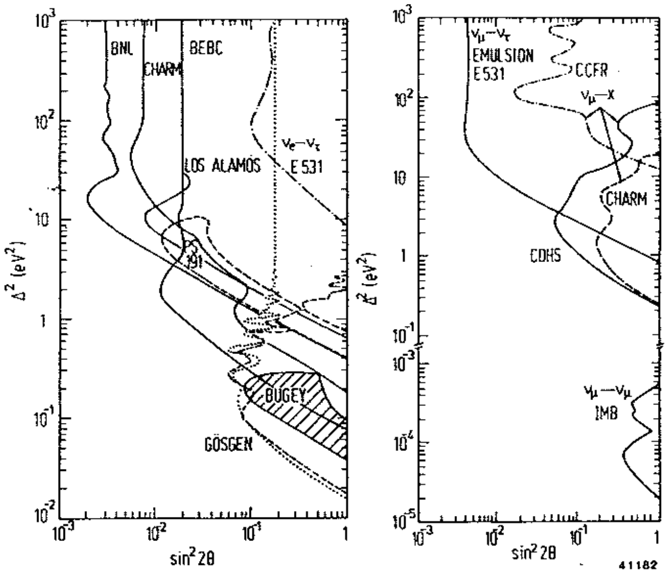

Figure 2: 90% *c.l.* limits on neutrino oscillations, from [55]. (a) $\nu_\mu \to \nu_e$ (BNL, CHARM, BEBC, Los Alamos, PS-191), $\nu_e \to \nu_\tau$ (E531), and $\bar{\nu}_e \to \nu_X$ (Bugey, Gösgen). (b) $\nu_\mu \to \nu_\tau$, ν_X, ν_μ. The Bugey [52] and PS-191 [54] regions are allowed by positive results. The other contours are exclusion plots (the regions to the right are excluded).

$\nu_e \to \nu_\tau$, as well as ν_μ disappearance), and reactors [1] ($\bar{\nu}_e$ disappearance), as well as on the oscillations of ν_μ produced in cosmic ray interactions in the atmosphere [50]. (Implications for the Solar neutrino problem are discussed below). The results of these searches [51] are summarized in Fig. 2. The Bugey reactor experiment [52] reports a positive signal for $\bar{\nu}_e$ disappearance, but their results are contradicted by the Gösgen experiment [53]. Similarly, the CERN PS-191 counter experiment [54] reports an excess of ν_e events in a ν_μ beam, but their signal is in conflict with several other $\nu_\mu \to \nu_e$ experiments [55]. Clearly, a clarification of the situation is essential .

From Fig. 2 it is clear that there are stringent limits on neutrino mixings for $|\Delta m^2|$ above $\simeq 1\ eV^2$. This should be contrasted with the suggested value $m_{\nu_e} \sim 17 - 40\ eV$ by the ITEP experiment [38]. If the ITEP result is correct then most likely the ν_e could not have any significant mixing with other neutrinos (the alternative possibility, that the ν_e is almost degenerate with another neutrino flavor so that $|\Delta m^2| << m^2_{\nu_e}$, seems rather contrived but cannot be excluded). A comparison of Fig. 2 with the expectations of various models (Table 1) suggests that $\nu_\mu \to \nu_\tau$ oscillations may be the most optimistic possibility for the future. Many of the seesaw-type models predict that the lepton mixing angles are roughly correlated with the corresponding quark mixing angles. This would suggest $\sin^2 2\theta \sim 10^{-4}$, 10^{-2}, 10^{-1} for $\nu_e \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow \nu_\tau$, and $\nu_e \leftrightarrow \nu_\mu$, respectively.

Oscillations between ordinary SU_2 doublet neutrinos ($\nu_e^0, \nu_\mu^0, \nu_\tau^0$, and possible fourth

family ν's), known as first class or flavor oscillations, occur for pure Dirac and pure Majorana neutrinos, as well as in the multi-family seesaw models. In models involving both Dirac and Majorana mass terms of comparable magnitude, however, there can be additional light neutrinos, and the mass eigenstates can have significant admixtures of both SU_2 doublets and singlets. In this case second class oscillations [16] can occur, in which the ordinary neutrinos oscillate into SU_2 singlets with negligible interactions. These "sterile" neutrinos are essentially undetectable, so second class oscillations can be observed [56] only in disappearance experiments. Of course, first and second class oscillations can occur simultaneously. For three families, for example, there could be oscillations between six Majorana neutrinos (3 doublets and 3 singlets).

Yet another possibility [57] are models in which the ordinary neutrinos have small mixings with heavy neutrinos. In that case the neutrinos actually produced in weak processes are the projections of the weak eigenstates onto the subspace of light or massless neutrinos. It can easily occur that the projections of the ν_e^0 and ν_μ^0, for example, are not orthogonal. The result is that a ν_μ^0 could produce an e^- in a subsequent reaction. Such a non-orthogonality would mimic the effects of oscillation appearance experiments, even if the masses of the light neutrinos are zero or negligible.

4.4 Cosmology

There are many limits on neutrino mass and decays from cosmology [58]. Ordinary light or massless neutrinos would have been produced by such weak processes as $e^+e^- \leftrightarrow \nu\nu^c$ in the early universe. As long as the weak reaction rate [59]

$$\Gamma_{weak} \sim \langle\sigma v\rangle n_T \sim G_F^2 T^5 \tag{38}$$

($\langle\sigma v\rangle \sim G_F^2 T^2$ is the thermally averaged cross section times relative velocity, and $n_T \sim T^3$ is the density of target particles, where T is the temperature) was large compared to the expansion rate $H \sim T^2/m_p$ (where $m_p = G_N^{-1/2} \sim 10^{19}\ GeV$ is the Planck scale) the number of neutrinos stayed in equilibrium. However, as soon as T dropped below the temperature $T_D \sim (G_F^2 m_P)^{-1/3} \simeq 3\ MeV$ for which $\Gamma_{weak} \sim H$, the weak rate became negligible and the neutrinos decoupled, i.e. effectively stopped interacting. According to most models these neutrinos should remain in the present universe, undisturbed from the first second of the big bang except for a redshifting of their momenta by the expansion of the universe. They are analogous to the $2.7°K$ microwave radiation (which decoupled later). If the neutrino masses are much less than 1 eV there should be $\simeq 50$ neutrinos/cm^3 of each type (ν_{eL}, ν_{eR}^c, etc) with momenta characterized by a thermal spectrum with temperature $\simeq 1.9°K$ ($10^{-4}\ eV$). Despite the large number of neutrinos ($\simeq 10^{10}$ per baryon) they are essentially impossible to detect [60] - [62] because their cross section $\sim G_F^2 E_\nu^2 \sim 10^{-62} cm^2$ is so low. [63]

The major cosmological bound is based on the energy density of the present universe. There are predicted to be so many relic neutrinos that even for a small mass in the 10 eV range they would be important. Limits on the energy density imply

$$\sum_i m_{\nu_i} < 40\ eV \tag{39}$$

where the sum extends over the light, stable (at least compared to the age of the universe) doublet neutrinos. Conversely, a neutrino with mass in this range would dominate the energy density and could account for the dark (missing) matter in galaxies and clusters [64]. In particular, for the ITEP value $m_{\nu_e} \sim (17-40)\ eV$, the ν_e would be an ideal candidate for the dark matter, but one would probably then have to find a mechanism to explain why the ν_e is the heaviest neutrino.

Similarly, the energy density associated with light or massless neutrinos for $T \sim T_D$ affects nucleosynthesis and leads to the limit $N_\nu \leq 4$ on the number of neutrino flavors with $m_\nu \leq 1\ MeV$. [65]

There are also a variety of constraints on unstable neutrinos. An ordinary doublet mass eigenstate neutrino ν_2 (with $m_{\nu_2} > m_{\nu_1}$) is expected to decay into

$$\begin{aligned} \nu_2 &\rightarrow \nu_1\gamma, \quad (m_{\nu_2} < 2m_e) \\ \nu_2 &\rightarrow \nu_1 e^+ e^-, \quad (2m_e < m_{\nu_2} < m_\mu + m_e). \end{aligned} \tag{40}$$

The first decay occurs at one loop, while the second occurs at tree level. Both decays are very slow for small m_{ν_2} and the decay products are detectable. There are a large variety of cosmological and astrophysical constraints [66] on m_{ν_2} and τ_{ν_2} from the present energy density, the growth of galaxies, the distortion of the $2.7^o K$ background radiation, the non-observation of the decay photons, supernovae, and nucleosynthesis and breakup. For reasonable mixing angles these limits exclude the range $40\ eV - (20-40)\ MeV$ for ordinary neutrinos [67] decaying according to (40). Combined with laboratory limits this implies [66,68] that the ν_μ and ν_τ (i.e. their dominant mass eigenstate components) should be lighter than $40\ eV$. In particular, this poses serious problems for the TeV scale seesaw model.

Most of the cosmological limits can be evaded if new physics is invoked to allow fast and invisible (except for the relativistic energy of the decay products) decays or annihilation for the heavy neutrinos. One possibility is the decay $\nu_2 \rightarrow 3\nu_1$. However, the rate for this mode from off-diagonal Z couplings [69] is too slow, while models in which the couplings of a Higgs triplet [70] (present in $SU_{2L} \times SU_{2R} \times U_1$) are arranged to allow a fast decay generally run into problems [71] with $\mu \rightarrow 3e$.

More promising are models in which $\nu_2 \rightarrow \nu_1 G$, where G is a Goldstone boson [72]-[74] associated with a spontaneously broken global symmetry. Likely examples are the case that G is a familon [72] (a Goldstone boson associated with a broken family symmetry) or a triplet-Majoron [73]. In fact, for triplet-Majorons one expects the annihilation process $\nu\nu \rightarrow MM$ (which begins when T drops below v_t) to have removed any relic neutrinos from the present universe [8]. In familon models some care must be taken to avoid unacceptably large flavor changing neutral current effects. The decay $\nu_2 \rightarrow \nu_1 M$ is too slow in the simpler versions of the singlet-Majoron model [74] to avoid cosmological problems.

The role of spontaneous L violation in Majoron models in reducing possible initial large lepton asymmetries to cosmologically interesting values at the time of nucleosynthesis is discussed in [75].

4.5 Double Beta Decay

Another important source of information on the ν_e mass (if it is Majorana) is neutrinoless double beta decay ($\beta\beta_{0\nu}$).

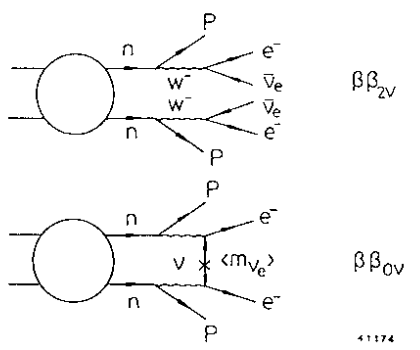

Figure 3: Diagrams for two neutrino ($\beta\beta_{2\nu}$) and neutrinoless ($\beta\beta_{0\nu}$) double beta decay.

First consider the lepton-number conserving two-neutrino ($\beta\beta_{2\nu}$) process $(Z,N) \rightarrow (Z+2, N-2)e^-e^-\nu_e^c\nu_e^c$, which can be thought of as two ordinary beta decays occurring in the same nucleus (Fig. 3). In the context of neutrino mass this process is mainly of interest as a calibration of the calculated nuclear matrix elements that are needed for the neutrinoless case. There has long been a two order of magnitude discrepancy between the predicted rates [76], e.g. for $^{130}Te \rightarrow {}^{130}Xe$, and indirect measurements by geochemical techniques [77]. Within the last year, however, this discrepancy has gone away. The geochemical measurements were confirmed by the first laboratory observation of double beta decay (at Irvine [78].) In addition, several groups [76] have found that previously neglected ground state correlation effects could suppress the matrix element by the required order of magnitude. Furthermore, there is no analogous uncertainty in the $\beta\beta_{0\nu}$ case.

The neutrinoless double beta decay process $(Z,N) \rightarrow (Z+2, N-2)e^-e^-$, which violates lepton number by two units, can proceed through the second diagram [79] in Fig. 3. In the absence of mixing the quantity $\langle m_{\nu_e}\rangle$, the effective Majorana neutrino mass, is

$$\langle m_{\nu_e}\rangle = \begin{cases} 0, & \text{Dirac neutrino} \\ m_{\nu_e}, & \text{unmixed Majorana neutrino} \end{cases} \tag{41}$$

Although the matrix element is proportional to $\langle m_{\nu_e}\rangle$, which is necessarily very small, $\beta\beta_{0\nu}$ has an enormous advantage in phase space over $\beta\beta_{2\nu}$ and could be observable for $\langle m_{\nu_e}\rangle$ in the eV range. Of course, the sum of the electron energies should be a sharp peak in $\beta\beta_{0\nu}$ (and a continuum for $\beta\beta_{2\nu}$), so the principal difficulty is controlling the background. [80,1] Currently, the most sensitive experiments are for $^{76}Ge \rightarrow {}^{76}Se\ e^-e^-$. No evidence for $\beta\beta_{0\nu}$ has been observed, [81] and the lower limit on the lifetime is [55] $\tau_{1/2} > 9 \times 10^{23}\ yr$ (68% $c.l.$). According to several calculations of the nuclear matrix elements [82] this implies $\langle m_{\nu_e}\rangle \leq 1\ eV$. However, a recent estimate by Engel et al. [83] yielded a much weaker limit $\langle m_{\nu_e}\rangle \leq 11\ eV$, so caution is advisable.

Even the largest value $\langle m_{\nu_e}\rangle \leq 11\ eV$ is smaller than the range $m_{\nu_e} \sim (17-40)\ eV$ suggested by the ITEP experiment. If the latter is correct the simplest possibility is that the ν_e is Dirac. Another possibility [84] is that the ν_e^0 is a mixture of Majorana mass eigenstate neutrinos. Then, $\langle m_{\nu_e}\rangle$ becomes

$$\langle m_{\nu_e}\rangle = \sum_i m_i U_{Lei}^2 \xi_i F(m_i, A), \tag{42}$$

where $m_i \geq 0$ is the physical mass of the i^{th} mass eigenstate, U_{Lei} is the mixing matrix element ($\nu_{eL}^0 = \sum_i U_{Lei}\nu_{iL}$) and $\xi_i = \pm 1$ is the CP parity of ν_{iL}. ξ_i is just K_{ii} in (13), and a negative value $\xi_i = -1$ means simply that the eigenvalue of M in (10) was negative before choosing K to redefine ν_R^c. In (42), $F(m_i, A)$ is a nucleus dependent propagator correction, [85] defined by

$$F(m_i, A) = \frac{\langle e^{-m_i r}/r\rangle}{\langle 1/r\rangle}. \tag{43}$$

It is ~ 1 for $m_i \ll 10\ MeV$. For $m_i \gg 10\ MeV$, $F(m_i, A) \ll 1$ (it falls as m_i^{-2}) and allows the possibility [86] of A dependence of $\langle m_{\nu_e}\rangle$.

Because of the possibility of negative contributions to $\langle m_{\nu_e}\rangle$ it is conceivable that there are cancellations so that $\langle m_{\nu_e}\rangle$ is much smaller than the mass of the dominant Majorana component of ν_e (e.g. $m_1 \sim (17-40)\ eV$). Such a cancellation is actually not so contrived as it might first appear. If all of the m_i are small enough that $F(m_i, A) = 1$ then from (12) $\langle m_{\nu_e}\rangle$ is just the M_{ee} component of the original Majorana mass matrix in (10). As we have seen, M_{ee} must be generated by a Higgs triplet and vanishes in many models. In fact, the light seesaw model of Table 1 automatically leads to $\langle m_{\nu_e}\rangle = 0$ for sufficiently small m_i. For two neutrinos, for example, $\langle m_{\nu_e}\rangle = m_1 \cos^2\theta - m_2 \sin^2\theta$, which vanishes by (28) and (29). However, the light seesaw model was devised just in order to give $\langle m_{\nu_e}\rangle = 0$. For seesaw models with more natural scales $m_2 \gg 10\ MeV$ one has that $F(m_i, A) \ll 1$ and $U_{e1} \sim 1$, so that $\langle m_{\nu_e}\rangle \sim m_{\nu_e}$. In most Majorana models, therefore, one expects $\langle m_{\nu_e}\rangle \sim m_{\nu_e}$ unless fine-tuned deviations from the seesaw formula are invoked.

Whether or not the cancellation of the terms in (42) is natural, one can consider whether it is phenomenologically viable. For two neutrinos, for example, the conditions $m_1 \sim 20\ eV$, and $\langle m_{\nu_e}\rangle \ll m_1$ imply

$$\tan^2\theta = \frac{m_1}{m_2 F(m_2, A)}, \tag{44}$$

where $m_1 \leq m_2$, $\tan^2\theta \leq 1$ since the ITEP experiment presumably measures the dominant component of ν_e. However, the reactor oscillation limits in Fig. 2 allow only two possibilities. One is that $m_1 \simeq m_2$, $0 \sim 45^0$. In that case ν_1 and ν_2 can be combined to form a Dirac neutrino (or pseudo-Dirac if the degeneracy is not exact), possibly with a non-canonical lepton number (such as $L_e - L_\mu + L_\tau$) conserved. Alternatively, one can have $m_2 \geq 450\ eV$. However, the various laboratory and cosmological limits exclude [22] almost all values of m_2 except for small windows around $40\ MeV$ and $2\ GeV$. Hence, if the ITEP results turn out to be correct they would almost certainly imply either (a) the ν_e is Dirac, or (b) there is new physics (such as a Majoron) that evades the cosmological bounds.

There are additional contributions to neutrinoless double beta decay in $SU_{2L}\times SU_{2R}\times U_1$ models [87]. Typically, such models contain additional charged $W_R^\pm$ bosons which couple to right-handed currents $\bar{e}_R\gamma^\mu N_R$, where N_R is a heavy Majorana neutrino. The exchange

of a N_R (rather than a ν_L in Fig. 3) yields a new contribution $M_N F(M_N, A)(M_{W_L}/M_{W_R})^4$ to $\langle m_{\nu_e}\rangle$, which sets non-trivial constraints [21] on M_N and M_{W_R}. Furthermore, mixed contributions involving one ordinary left-handed current $\bar{e}_L\gamma^\mu\nu_L$ and one right-handed current $\bar{e}_R\gamma^\mu N_R$ can yield contributions to to $0^+ \to 2^+$ decay amplitudes that are not directly proportional to a neutrino mass [88] However, the relevant amplitudes are of order [1,87]

$$\left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \theta,\ \zeta\theta, \tag{45}$$

where θ is a light-heavy neutrino mixing angle and ζ is the $W_L - W_R$ mixing angle. One typically expects $(M_{W_L}/M_{W_R})^2$ and ζ to be less than 10^{-3}. Since we expect $\theta \sim m_D/M_{W_R} \leq 10^{-4} - 10^{-5}$ in a typical TeV-seesaw, the expected values for the quantities in (45) are smaller than the experimental limits (of $\sim 10^{-6}$).

One typically has $\langle m_{\nu_e}\rangle \ll m_{\nu_e}$ for the charged Higgs models [23] because the antisymmetry of the relevant Yukawa coupling forces M_{ee} to vanish.

4.6 The Solar Neutrino Problem

For some years the event rate in the $^{37}Cl \to {}^{37}Ar$ Solar neutrino experiment [89] ($2.0 \pm 0.3\ SNU$ [90]) has been considerably below the prediction [91] $5.8 \pm 2.2\ SNU$ of the standard Solar model. The discrepancy has recently been confirmed by the Kamiokande group which reports [92] an upper limit on the ν_e flux (from $\nu_e e$ elastic scattering) that is less than half the expected event rate. One explanation for the discrepancy is the existence of vacuum oscillations of the ν_e into other neutrinos. These could be important for neutrino mass-squared differences [93] $\Delta m^2 \equiv m_1^2 - m_2^2$ as small as $\Delta m^2 \sim (10^{-11} - 10^{-10})\ eV^2$, but only if the mixing angles are large.

Another possibility [94] is that the ν_e is a Dirac particle with a magnetic moment in the range $\mu_{\nu_e} \sim (0.6 - 10) \times 10^{-10}\mu_B$. The ν_e spin could then process in the Solar magnetic field into a sterile right-handed ν_e, thus reducing the observed flux by a factor $\simeq 2$. The necessary value of μ_{ν_e} is barely consistent with laboratory limits [95] but is probably excluded by astrophysical constraints from nucleosynthesis and stellar cooling [96] (Table 3). The worst objection, however, is that the necessary μ_{ν_e} is unnaturally high. In the standard model with a Dirac mass one expects [97]

$$\mu_\nu = \frac{3G_F m_\nu m_e}{4\pi^2\sqrt{2}}\mu_B \sim 3\times 10^{-19}\left(\frac{m_\nu}{1eV}\right)\mu_B \tag{46}$$

which is many orders of magnitude too small. Non-standard models [98] can yield larger μ_ν, but to obtain a sufficiently large value appears highly contrived.

Other canonical explanations involve non-standard Solar models. The existing experiments are mainly sensitive to the relatively high energy (from $0.81\ MeV$ up to $14\ MeV$) neutrinos from ^8B decay. The flux of these ^8B neutrinos depends very sensitively on the temperature of the Solar core and could be changed significantly by modifications of the standard Solar model. Recently, there has been much attention to the possibility that weakly interacting massive particles (WIMPs), which could form the dark matter, could carry energy out of the Solar core and lower the central temperature slightly. [58] Less exotic modifications of the standard model are also possible.

Table 3: Limits on the neutrino magnetic moments. A value $\mu_{\nu_e} \sim (0.6 - 10) \times 10^{-10}\mu_B$ would be needed to resolve the Solar ν problem.

laboratory [95]	$\mu_{\bar{\nu}_e} < 1.5\times10^{-10}\mu_B$ $\mu_{\nu_\mu} < 9.5\times10^{-10}\mu_B$
Stellar cooling [96] ($\gamma\to\nu\bar{\nu}$)	$\mu_\nu < 0.8\times10^{-11}\mu_B$
Nucleosynthesis [96] (ν_R produced by spin precession)	$\mu_\nu < 0.5\times10^{-10}\mu_B$
Standard model [97] (Dirac mass)	$\mu_\nu \sim 3\times10^{-19}\left(\frac{m_\nu}{1\ eV}\right)\mu_B$

A $^{71}Ga \to {}^{71}Ge$ experiment could distinguish the nonstandard Solar model from the first two possibilities. Most of the expected ^{71}Ga event rate is from the low energy pp neutrinos, the flux of which can be inferred from the over-all Solar luminosity and is relatively insensitive to the temperature of the Solar core. The predicted ^{71}Ga event rate of $\simeq 107$ SNU can be reduced at most to around 78 SNU in most non-standard Solar models [91,99]. The traditional view has been that a flux lower than this would imply large vacuum oscillations, which would reduce the ^{71}Ga rate by a factor comparable to the ^{37}Cl event rate reduction for most oscillation parameters (e.g. to around 40 SNU).

Yet another possibility, i.e. that neutrinos decay between the Sun and the Earth, is all but excluded by the survival of neutrinos from supernova 1987A, except in some two-component models with large mixing angles. [100]

Recently, Mikheyev and Smirnov [101] have proposed an elegant new solution to the Solar neutrino problem, in which even tiny vacuum mixing angles can be amplified by the coherent interactions of ν_e with matter.

Considering $\nu_e \leftrightarrow \nu_\mu$ oscillations for definiteness, the vacuum oscillation equation in (35) can be described in terms of the weak basis states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ by

$$|\nu(t)\rangle = \nu_e(t)|\nu_e\rangle + \nu_\mu(t)|\nu_\mu\rangle, \tag{47}$$

where the coefficients satisfy the Schrödinger-like equation

$$i\frac{d}{dt}\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = M_0 \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix}, \tag{48}$$

with

$$M_0 = \begin{pmatrix} \frac{\Delta m^2}{2p}\cos 2\theta & \frac{-\Delta m^2}{4p}\sin 2\theta \\ \frac{-\Delta m^2}{4p}\sin 2\theta & 0 \end{pmatrix}, \tag{49}$$

where an irrelevant term proportional to the identity (which only affects the overall phase) has been dropped. Wolfenstein pointed out [102] that in the presence of matter, M_0 is replaced by the M', where

$$M' = M_0 + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} \tag{50}$$

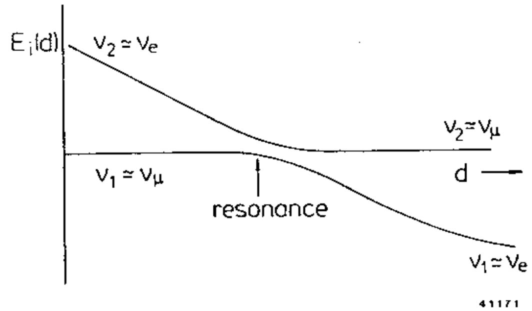

Figure 4: The energy eigenvalues of M' as a function of d, the distance from the center of the Sun.

and n_e is the density of electrons. The new term [102] - [104] is the effect of the coherent forward scattering amplitude for $\nu_e e^- \to \nu_e e^-$ via the charged current. The effects of neutral current scattering from e^-, p, and n have been neglected because they are the same for ν_e and ν_μ and only contribute to the overall phase. For $\Delta m^2 < 0$ (i.e. $m_{\nu_e} < m_{\nu_\mu}$ [104]) there is a critical density [105] $n_e^{crit} = -\Delta m^2 \cos 2\theta/(2\sqrt{2}G_F p)$ for which the diagonal elements of M' are equal (i.e. zero). At that density a resonance occurs, i.e. even a tiny off diagonal mixing term leads to large mixing effects.

In particular, if n_e in the Sun varies slowly an adiabatic approximation applies [101, 106]. ν_e's produced in the core of the Sun (where $n_e > n_e^{crit}$) correspond to the larger mass eigenstate ν_2 of M' (Fig. 4). Outside the Sun, on the other hand, the higher energy state ν_2 corresponds to ν_μ for $\Delta m^2 < 0$. Hence, if the variation of n_e with the distance from the center of the Sun is sufficiently slow, the initial ν_e will be adiabatically converted to ν_μ as they pass through the resonance.

A number of authors [99],[101],[106] - [108] have analyzed the implications of the Mikheyev-Smirnov-Wolfenstein (MSW) effect for the Solar neutrinos quantitatively. It is found that there are three classes of parameters which can explain the reduction of ^8B neutrinos observed in the ^{37}Cl experiment. These roughly form the sides of a triangle, as is illustrated schematically in Fig. 5. For solution (a) corresponding to $|\Delta m^2| \sim 5\times10^{-5}\ eV^2$, $\sin^2 2\theta \gtrsim 4\times10^{-4}$, the adiabatic hypothesis is valid and $\simeq 100\%$ conversion occurs. However, only the high energy ^8B neutrinos actually encounter a resonance layer (the central density is too low for the low energy neutrinos) and are converted. For this parameter range one expects little reduction in the counting rate for the gallium experiment (i.e. the effect is similar to non-standard Solar models in that respect).

For solution (b), extending down to $|\Delta m^2| \sim 10^{-8}\ eV^2$ one has [107] $|\Delta m^2|\sin^2 2\theta \sim 10^{-7.5}\ eV^2$. Here the adiabatic approximation starts to break down. All neutrino energies are affected, but the conversion probability is less than unity. For these solutions one expects a significant reduction in the gallium counting rate, similar to large vacuum oscillations or magnetic moments. Solution (c), corresponding to large vacuum mixing angles,

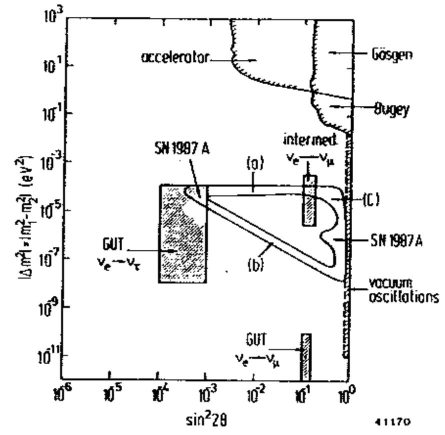

Figure 5: A schematic view of the regions in the Δm^2 - $\sin^2 2\theta$ plane which can explain the Solar neutrino problem via the Mikheyev-Smirnov-Wolfenstein (MSW) effect.

is an extension of the vacuum oscillation solution. In the middle of region (c) one expects a large day-night asymmetry in the ν_e counting rate due to MSW regeneration in the Earth at night [99].

The MSW solution is very elegant, but it severely complicates the task of sorting out which if any of the proposed solutions to the Solar ν problem is correct. It will take an ambitious program of experiments [99,109] to clarify the matter.

The first two events from SN1987A observed by the Kamiokande experiment [45] point back towards the supernova. They are consistent with ν_e from the initial neutronization burst, scattering via $\nu_e e^- \to \nu_e e^-$. However, they could also be $\bar{\nu}_e$ from the subsequent thermal burst, scattering via $\bar{\nu}_e p \to e^+ n$, which has a much larger cross section (and which produces an isotropic distribution of positrons). If they are indeed ν_e they are problematic for the MSW mechanism because one expects $\nu_e \to \nu_\mu$ conversions on the way out of the supernova. However, there are two parameter regions (shown in Fig. 5), which would still be consistent [110], corresponding respectively to incomplete conversion in the supernova and reconversion in the Earth. Unfortunately there is no way to determine whether the two events are really ν_e's.

The MSW mechanism is consistent with the expectation of GUT [19] and intermediate scale [20] seesaws. As is illustrated in Fig. 5 the predictions of the GUT seesaw are consistent with $\nu_e \leftrightarrow \nu_\tau$ conversions in the Sun. In this case, the mass ranges are too small to ever see any direct laboratory effects of neutrino mass. The intermediate scale seesaw could account for the Solar ν problem via $\nu_e \leftrightarrow \nu_\mu$ conversions. In that case, $\nu_\mu \leftrightarrow \nu_\tau$ oscillations could well be observable in the laboratory.

It is also possible that the Solar ν problem could be explained by small neutrino masses, but that neutrino appearance experiments might nevertheless yield positive signals due to

non-orthogonal neutrino states (induced by mixings between very light and very heavy neutrinos. [57])

5 Summary

- The question of whether the neutrinos have mass is vital for both particle physics and cosmology. However, there is at present no compelling evidence for neutrino mass. The ITEP result $m_{\nu_e} \sim (17 - 40)\ eV$ has not been confirmed by other experiments and is on the verge of being excluded. Although there are two positive indications of neutrino oscillations (with different parameters), these are contradicted by other experiments. The negative results suggest $m_{\nu_i} \simeq O(1\ eV)$ unless there are very small mixings or degeneracies. There are also stringent limits on incoherent mixing with heavy neutrinos.

- The MSW solution to the Solar neutrino problem would work for ν_μ or μ_τ in the $10^{-2}\ eV$ range, consistent with intermediate mass or GUT seesaws.

- The nonobservation of neutrinoless double beta decay implies $\langle m_{\nu_e}\rangle < 1 - 11\ eV$. If the ITEP result is correct this would most likely imply a Dirac neutrino or new physics to evade cosmological bounds.

- A ν mass in the 5 - 40 eV range would dominate the energy density of the universe and would be an excellent candidate for the dark matter, though other mechanisms would have to be invoked to explain the initial formation of galaxies. Conversely, the light stable neutrinos must be lighter than $\simeq 40\ eV$. A variety of astrophysical, laboratory, and cosmological bounds exclude unstable neutrinos up to $\sim$ 40 MeV (unless new physics is invoked for fast, invisible decays or annihilations), implying that $m_{\nu_\mu},\ m_{\nu_\tau} < 40\ eV$.

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[7] $V_L^\dagger$ contains F^2 real parameters, of which $F(F-1)/2$ are angles, analogous to the Cabibbo angle, which describe transitions between families and lead to such effects as neutrino oscillations. $2F-1$ parameters are unobservable phases, which can be removed by appropriate choices of phases for the n_{iL} and e_{iL}^- fields (corresponding phases must be chosen for the n_{iR} and e_{iR} to ensure real masses). The remaining $(F-1)(F-2)/2$ parameters are phases which could lead to CP violation in the leptonic sector.

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