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Mixing Between Ordinary and Exotic Fermions and Effects of Non-orthogonal Neutrinos†

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ABSTRACT

We have performed a comprehensive analysis of the limits on mixings between ordinary fermions and possible heavy exotic fermions. All exotic fermions which do not have non-standard color or charge assignments have been considered. A general formalism for describing such mixings is given. It is shown that a variety of constraints, including the relation between the W and Z masses and the Fermi constant, charged current universality, and flavor-diagonal neutral currents suffice to limit all directions in parameter space that are not excluded by the absence of flavor-changing neutral currents. The mixing between light and heavy neutrinos can also induce lepton number violation - even if the light neutrinos are massless - because the states actually produced in weak interaction processes are not orthogonal. We discuss the phenomenological effects of this non-orthogonality for neutrino "oscillation" experiments, constraints from universality, and neutrinoless double beta decay. We also consider the implications of the mixing limits on the mass scales of heavy exotic neutrinos.

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1. Introduction

Exotic fermions are present in many models which go beyond the standard model. Often these new fermions have non-standard $SU(2) \times U(1)$ assignments, and can manifest themselves in mixing with the known fermions. In this paper we present the results of a very general systematic analysis of limits on such mixings [1], using experimental data from the many high-precision charged current and flavor-diagonal neutral current experiments. We consider all new fermions which do not have exotic color or charge assignments. It will be seen that the limits on the mixings of ν_e, e^-, ν_μ, μ^-, u, and d are quite stringent, particularly when only one particle is allowed to mix at a time. Although the limits decrease by about an order of magnitude when all particles are fit simultaneously, we still obtain limits on mixings of the light particles of about $\sin^2\theta_{\text{mix}} \simeq 0.05$, and in the range $\sin^2\theta_{\text{mix}} \simeq$ 0.1-0.4 for the heavy particles.

Mixing with exotic fermions will usually also induce flavor changing neutral currents (FCNC). Most FCNC limits are extremely stringent. However, there exist certain directions in parameter space in which no FCNC are induced. We will show that the data from charged current and flavor-diagonal neutral current experiments can constrain these directions.

We consider 5 classes of new fermions [2]: (i) Sequential Fermions, with canonical (L-doublet, R-singlet) $SU(2) \times U(1)$ assignments; (ii) Mirror Fermions, with opposite (L-singlet, R-doublet) $SU(2) \times U(1)$ assignments; (iii) Vector Doublets, in which both the L- and R-handed components have the same $SU(2) \times U(1)$ quantum numbers (L-doublet, R-doublet); (iv) Vector Singlets, which have (L-singlet, R-singlet); (v) extra $SU(2)$-singlet Weyl Neutrinos. We consider all five possibilities simultaneously.

In developing the formalism for dealing with the mixings of ordinary and exotic fermions, we rediscovered a mechanism for generating both lepton flavor and total lepton number violation [3]. Briefly the idea is as follows. $SU(2)_W$ invariance requires that the weak eigenstates ν^0_{iL} must be orthogonal. However, in the presence of mixing between light and heavy neutrinos, the light components of ν^0_{iL} (which are what are relevant in most weak processes) need not be orthogonal, even if they are massless [4]. We explore the phenomenological implications of this effect for neutrino "oscillation" experiments, constraints from universality, and $\Delta L = \pm 2$ effects in neutrinoless double beta decay [5]. For a complete discussion of the effects, see Ref. 3.

In Section II we develop the formalism needed to describe both the general fermion mixing and the non-orthogonal neutrinos. In Section III we present the charged and neutral current data used for the mixing analysis. Because of space limitations, we do not discuss all of the experimental results. Rather, we give a few representative examples of how theoretical quantities get altered in the presence of mixing, and how these are related to the numbers actually measured, i.e. the experimental data. In section IV we present the results of the fit for the mixing angles. We also describe some of the phenomenological limits on the lepton number violating parameters from non-orthogonal neutrinos, and implications for the mass scales of the heavy neutrinos. We conclude with a discussion of the results.

2. Formalism

We begin by developing the formalism needed to describe both the mixing between ordinary and exotic fermions, and the phenomena associated with non-orthogonal neutrinos. We start first with the quarks and charged leptons (neutrinos are considered separately below).

We classify all left-handed particles which occur in $SU(2)_W$ doublets as "ordinary" (independent of whether they are associated with known or sequential families, or with vector doublets), and all left-handed $SU(2)$ singlets as "exotic" (mirror families, vector singlets). Similarly, all right-handed singlets are denoted as "ordinary", while all right-handed doublets are "exotic". It is convenient to arrange all left-handed (right-handed) particles of a given charge into a column vector, which we denote generically as ψ_L^0 (ψ_R^0). The superscript ^{0} refers to the weak interaction basis. We make a further decomposition into ordinary and exotic sectors:

$$\psi_L^0 = \begin{pmatrix} \psi_{OL}^0 \\ \psi_{EL}^0 \end{pmatrix} \quad , \quad \psi_R^0 = \begin{pmatrix} \psi_{OR}^0 \\ \psi_{ER}^0 \end{pmatrix} \quad , \tag{1}$$

where ψ_{OL}^0 and ψ_{EL}^0 are column vectors consisting of n_L ordinary fields and m_L exotic fields, respectively, with similar meanings for ψ_{OR}^0 and ψ_{ER}^0. The mass eigenstates can be similarly decomposed into light and heavy sectors:

$$\psi_L = \begin{pmatrix} \psi_{lL} \\ \psi_{hL} \end{pmatrix} \quad , \quad \psi_R = \begin{pmatrix} \psi_{lR} \\ \psi_{hR} \end{pmatrix} \quad . \tag{2}$$

Here, ψ_{lL} is a column vector of the n_L "light" fields, while ψ_{hL} consists of the m_L "heavy" fields. This ansatz suggests that the light states are mainly ψ_O^0, while the heavy states are primarily ψ_E^0. However, since the ordinary fields include sequential families, for example, the labels "light" and "heavy" should be taken as suggestive only.

The weak and mass eigenstates are related by

$$\psi_L^0 = U_L \psi_L \quad , \quad \psi_R^0 = U_R \psi_R \ , \tag{3}$$

where U_L and U_R are the matrices

$$U_a = \begin{pmatrix} A_a & E_a \\ F_a & G_a \end{pmatrix} \ , \quad a = L, R \tag{4}$$

(written in block form) which diagonalize the fermion mass matrix. We will refer to E_a and F_a as the light-heavy mixing matrices. From the unitarity of U_a, one has

$$\begin{matrix} A_a^\dagger A_a + F_a^\dagger F_a = I \\ A_a A_a^\dagger + E_a E_a^\dagger = I \end{matrix} \quad a = L, R \ . \tag{5}$$

Therefore the matrix A_a, which relates the ordinary states to the light mass eigenstates, is not unitary (the deviation is second order in light-heavy mixing). Many of the physical effects of mixing with exotic fermions are associated with the non-unitarity of A_a.

Using (3) and (4), the weak neutral current for the light mass eigenstates is

$$\frac{1}{2} J_Z^\mu = \bar{\psi}_{lL} \gamma^\mu t_3 A_L^\dagger A_L \psi_{lL} + \bar{\psi}_{lR} \gamma^\mu t_3 F_R^\dagger F_R \psi_{lR} - \sin^2 \theta_W J_{Em}^\mu \ , \tag{6}$$

where t_3 is the $SU(2)_W$ charge of the ψ_{OL}^0 (first term) and ψ_{ER}^0 (second term). Although the last term is flavor diagonal, the first two terms include flavor changing neutral currents (FCNC). The limits on FCNC are extremely stringent, and we therefore make the physical assumption that there are <u>no</u> FCNC involving the light fermions. With this assumption the matrices $A_L^\dagger A_L$ and $F_R^\dagger F_R$ are diagonal, and it can be shown that this implies that the matrices A_a and F_a $(a = L, R)$ can be written

$$A_a = \hat{A}_a c_a \ , \quad F_a = \hat{F}_a s_a \ , \tag{7}$$

where $\hat{A}_a$ is unitary ($\hat{A}_a^\dagger \hat{A}_a = \hat{A}_a \hat{A}_a^\dagger = I$), and $\hat{F}_a$ is unitary in the special case $n_a = m_a$, i.e. when the number of ordinary fields is the same as the number of exotic fields. (The case in which $n_a \neq m_a$ does not cause any particular trouble, and we refer the reader to Ref. 1 for details. For the rest of the analysis, however, we shall take $n_a = m_a$.) The diagonal matrices c_a and s_a are defined as

$$\begin{matrix} c_a = \mathrm{diag}\,(c_a^1 \ c_a^2 \ \dots \ c_a^{n_a}) \\ s_a = \mathrm{diag}\,(s_a^1 \ s_a^2 \ \dots \ s_a^{n_a}) \end{matrix} \ , \quad a = L, R \ , \tag{8}$$

where $c_a^i \equiv \cos \theta_a^i$ and $s_a^i \equiv \sin \theta_a^i$. θ_L^i and θ_R^i will be interpreted as light-heavy mixing angles. Therefore, using (7) and (8), $A_L^\dagger A_L = c_L^2$, $A_R^\dagger A_R = c_R^2$, $F_L^\dagger F_L = s_L^2$, and $F_R^\dagger F_R = s_R^2$. It can also be shown that a weak eigenstate basis can be chosen such that $\hat{A}_a = I$ and $\hat{F}_a = I$. In the following we will always choose $\hat{A}_a^e = I$, $\hat{F}_a^e = I$. One can also choose the quark weak basis such that, for example, $\hat{A}_L^u = I$ <u>or</u> $\hat{A}_L^d = I$, but, in general, not both simultaneously. A rough interpretation of (7) is that there are no FCNC if and only if the light-heavy mixing is restricted to distinct pairs of states, i.e. each light state mixes with its own heavy state, with mixing angle θ_L^i or θ_R^i.

From (6) and (7), the neutral current for the light quarks and charged leptons is

$$\frac{1}{2}J_Z^\mu = \sum_i \overline{\psi}_{iL}\gamma^\mu\left(t_3 {c_L^i}^2 - \sin^2\theta_W q\right)\psi_{iL} + \overline{\psi}_{iR}\gamma^\mu\left(t_3 {s_R^i}^2 - \sin^2\theta_W q\right)\psi_{iR} \ , \qquad (9)$$

where q is the charge of ψ_i, and we have dropped the subscript l. The ${c_L^i}^2$ term represents a non-universal reduction of the strength of the normal neutral current, due to mixing with L-singlets, and the ${s_R^i}^2$ is an induced right-handed current.

Similarly, the hadronic weak charged current for the light states is

$$\frac{1}{2}J_W^{\mu\,\dagger} = \overline{u}_L\gamma^\mu V_L d_L + \overline{u}_R\gamma^\mu V_R d_R \ , \qquad (10)$$

where u_L and d_L are n-component column vectors of the light mass eigenstate left-handed quarks of charge $\frac{2}{3}$ and $-\frac{1}{3}$, respectively, and similarly for u_R, d_R. In (10), $V_L = A_L^{u\,\dagger} A_L^d$ is the apparent Kobayashi-Maskawa-Cabibbo (KMC) quark mixing matrix [6], which is non-unitary in the presence of mixing with exotic fermions. It is related to the true (unitary) KMC matrix by $V_L = c_L^u \hat{V}_L c_L^d$, where $\hat{V}_L = \hat{A}_L^{u\,\dagger}\hat{A}_L^d$. We always use the term true to refer to underlying quantities (hatted) and apparent to refer to the directly measured quantities (unhatted). Similarly $V_R = F_R^{u\,\dagger}F_R^d$ is the induced right-handed (non-unitary) mixing matrix, related to the true (unitary) mixing matrix by $V_R = s_R^u \hat{V}_R s_R^d$, where $\hat{V}_R = \hat{F}_R^{u\,\dagger}\hat{F}_R^d$. V_R is of second order in light-heavy mixing, since both the u_i and d_j must mix into the R-doublet.

The neutrinos must be treated separately because of the possibility of Majorana masses and because there are no experimental constraints on FCNC in the neutrino sector.

It is convenient to denote left (L) and right (R)-handed neutrinos by n_L and n_R^c, respectively. The two are not independent but are related by $n_R^c = C\bar{n}_L^T$, where C is the charge conjugation matrix, i.e. n_L and n_R^c are essentially CP conjugates.

If there are no exotic electric charges then there are only three possible $SU(2)\times U(1)$ assignments of the weak eigenstate L-neutrinos

$$\begin{pmatrix} n_{OL}^0 \\ e_L^{0-} \end{pmatrix}, \quad \begin{pmatrix} e_L^{0+} \\ n_{EL}^0 \end{pmatrix}, \quad n_{SL}^0 \ . \qquad (11)$$

The n_{OL}^0 are the "ordinary" $SU(2)$ doublets, and the n_{EL}^0 are "exotic" neutrinos, which are related by CP to right-handed doublets. The third type are the exotic $SU(2)$ singlets, denoted n_{SL}^0. In the presence of general Majorana mass terms, the n_{OL}^0, n_{EL}^0, and n_{SL}^0 can all mix with each other. In analogy to (1) we arrange all of the weak eigenstate neutrinos into a vector

$$n_L^0 = \begin{pmatrix} n_{OL}^0 \\ n_{EL}^0 \\ n_{SL}^0 \end{pmatrix} \ , \quad n_R^{0\,c} = \begin{pmatrix} n_{OR}^{0^c} \\ n_{ER}^{0^c} \\ n_{SR}^{0^c} \end{pmatrix} \ , \qquad (12)$$

where n_{OL}^0, n_{EL}^0, and n_{SL}^0 are themselves vectors containing the ordinary, exotic, and singlet neutrinos, respectively.

We assume that the mass eigenstate neutrinos are all either "massless" (i.e. with masses too small to be kinematically relevant) or heavy. We write

$$n_L = \begin{pmatrix} n_{lL} \\ n_{hL} \end{pmatrix} \ , \quad n_R^c = \begin{pmatrix} n_{lR}^c \\ n_{hR}^c \end{pmatrix} \ , \qquad (13)$$

where the n_{lL} are the "massless" neutrinos and the n_{hL} the heavy neutrinos. The weak and mass eigenstate neutrinos are related by a unitary transformation

$$n_L^0 = U_L n_L \ , \quad n_R^{0^c} = U_R n_R^c \ , \qquad (14)$$

where $U_L = U_R^*$ [5]. As in (4), we write U_L in block form as

$$U_L = \begin{pmatrix} A_L & E_L \\ F_L & G_L \\ H_L & J_L \end{pmatrix} \ . \qquad (15)$$

The unitarity of U_L implies

$$\begin{aligned} A_L^\dagger A_L + F_L^\dagger F_L + H_L^\dagger H_L &= I \\ A_L A_L^\dagger + E_L E_L^\dagger &= I \ , \end{aligned} \qquad (16)$$

so that A_L is not unitary. Unlike the quarks and charged leptons, there is no evidence to justify assuming that $A_L^\dagger A_L$ is diagonal. One can, however, accomplish much the same purpose by summing over the flavors of unobserved final neutrinos in weak processes.

The leptonic charged current is (dropping the subscript l):

$$\frac{1}{2}J_W^{\mu\,\dagger} = \sum_{ia}\left[\bar{n}_{iL}\gamma^\mu\left(A_L^{\nu\,\dagger}\right)_{ia} c_L^{e_a} e_{aL} + \bar{n}_{iR}^c\gamma^\mu\left(F_R^{\nu\,\dagger}\right)_{ia} s_R^{e_a} e_{aR}\right] \ . \qquad (17)$$

The first term in J_W^μ represents the non-universal reduction in strength of the left-handed charged current due to the neutrino and electron mixings, while the second is a right-handed current (RHC) induced by the mixing of the light neutrinos and charged leptons into heavy right-handed doublets.

Writing $\Gamma(e_a \to n_i)$ to represent any weak decay involving the $e_a \to n_i$ transition (e.g. $K^+ \to e_a^c n_i$), the decay rate is changed relative to the normal rate Γ_0 by a factor

$$\frac{1}{\Gamma_0}\sum_i \Gamma(e_a \to n_i) = (c_L^{e_a})^2 \sum_i |\left(A_L^{\nu\,\dagger}\right)_{ia}|^2 = (c_L^{e_a})^2 \left(A_L^\nu A_L^{\nu\,\dagger}\right)_{aa} \equiv (c_L^{e_a})^2 (c_L^{\nu_a})^2 \ , \quad (18)$$

where we have neglected the $O(s^4)$ RHC in (17), and where we have summed over the flavor of the final neutrino. In (18) we have introduced the effective neutrino mixing angles $(c_L^{\nu_a})^2 = \cos^2\theta_L^{\nu_a} \equiv \left(A_L^\nu A_L^{\nu\dagger}\right)_{aa}$. The normalized state produced in (18) is

$$|n_{aL}\rangle \equiv \frac{\sum_i \left(A_L^\dagger\right)_{ia} |n_{iL}\rangle}{c_L^{\nu_a}} , \tag{19}$$

i.e. a coherent superposition of states n_{iL}. Since the n_{iL} are degenerate (massless), n_{aL} does not change in time except for an irrelevant overall phase. The states n_{aL} and n_{bL} with $a \neq b$ are non-orthogonal. One has

$$\lambda_{ba}^L \equiv \langle n_{bL}|n_{aL}\rangle c_L^{\nu_a} c_L^{\nu_b} = \left(A_L A_L^\dagger\right)_{ba} , \tag{20}$$

where in general $\lambda_{ba}^L \neq 0$. From (16), $\lambda_{ba}^L = -\left(E_L E_L^\dagger\right)_{ba}$, i.e. the λ_{ba}^L are of second order in light-heavy mixing.

The non-orthogonality of the neutrinos can lead to lepton flavor violation when one considers the subsequent interactions of the neutrinos produced in (18). These neutrinos can rescatter in a target via the charged current to produce either the "right" lepton e_{aL} or the "wrong" lepton e_{bL}. The cross sections relative to σ_0, the normal weak cross section in the absence of mixing, are

$$\begin{aligned} \frac{1}{\sigma_0}\sigma(n_{aL} \to e_{aL}) &= (c_L^{e_a})^2 (c_L^{\nu_a})^2 \\ \frac{1}{\sigma_0}\sigma(n_{aL} \to e_{bL}) &= \frac{(c_L^{e_b})^2}{(c_L^{\nu_a})^2}|\lambda_{ba}^L|^2 . \end{aligned} \tag{21}$$

The non-orthogonality of neutrinos can also lead to total lepton number violation because the RHC in (17) allows decays into "wrong" helicity neutrinos (with a highly suppressed rate ($O(s^4)$) relative to normal weak). The state produced in such a decay (e.g. $K^- \to e_{aR}^- \bar{n}_{aL}$) is

$$|\bar{n}_{aL}\rangle \equiv \frac{\sum_i \left(F_L^\dagger\right)_{ia} |n_{iR}^c\rangle}{s_R^{\nu_a}} , \tag{22}$$

where the bar distinguishes this state from n_{aL} in (19). (If L were conserved, n_{aL} and $\bar{n}_{aL}$ would be leptons and antileptons, respectively.) The $s_R^{\nu_a}$ in (22) are defined as $(s_R^{\nu_a})^2 = \sin^2\theta_R^{\nu_a} \equiv \left(F_R F_R^\dagger\right)_{aa} = \sum_i |\left(F_R^\dagger\right)_{ia}|^2$. Since these neutrinos are also non-orthogonal, non-zero values for

$$\beta_{ba}^L \equiv \langle n_{bL}|\bar{n}_{aL}\rangle c_L^{\nu_b} s_R^{\nu_a} = \left(A_L F_L^\dagger\right)_{ba} = \left(F_L A_L^\dagger\right)_{ab}^* \tag{23}$$

indicate total lepton number violation: i.e. $\beta_{ba}^L \neq 0$ describes the overlap of "neutrino" and "antineutrino" states.

The weak neutral current for the light neutrinos is

$$\frac{1}{2}J_Z^\mu = \frac{1}{2}\bar{n}_L\gamma^\mu \left[A_L^{\nu\dagger} A_L^\nu - F_L^{\nu\dagger} F_L^\nu\right] n_L . \tag{24}$$

The $A_L^\dagger A_L$ and $F_L^\dagger F_L$ terms are the neutral currents of n_{OL}^0 and n_{EL}^0, respectively. From (19) and (24) the cross section for neutral current rescattering of the n_{aL}, summed over the unobserved final neutrino flavor, is

$$\frac{1}{\sigma_0}\sum_i \sigma(n_{aL} \to n_{iL}) = \frac{\left[A_L^\nu \left(A_L^{\nu\dagger} A_L^\nu - F_L^{\nu\dagger} F_L^\nu\right)^2 A_L^{\nu\dagger}\right]_{aa}}{(c_L^{\nu_a})^2} . \tag{25}$$

The cross section depends not only on the mixing angle, but on which neutrinos are involved in the mixing. Fortunately, to second order in light-heavy mixing, it can be shown that

$$\frac{1}{\sigma_0}\sum_i \sigma(n_{aL} \to n_{iL}) \to 1 - \Lambda_a (s_L^{\nu_a})^2 + O(s^4) , \tag{26}$$

where Λ_a is an auxiliary parameter that can vary between 0 and 4, depending on the particles involved in the mixing. For the case of mixing with only one type of heavy neutrino one has

$$\Lambda_a = \begin{cases} 0, & \text{heavy sequential (or vector doublet) in } n_{OL}^0 \\ 2, & \text{singlet in } n_{SL}^0 \\ 4, & \text{exotic doublet } (\Delta L = \pm 2) \text{ in } n_{EL}^0 . \end{cases} \tag{27}$$

As expected, the neutral current is unaffected by mixing with a heavy sequential or vector doublet neutrino.

3. Experimental Constraints

In this section we describe some of the experimental constraints on mixing. True (underlying) quantities are denoted with a hat, while apparent (measured) quantities, which differ by mixing effects, are unhatted. We work to second order in light-heavy mixing except where otherwise stated. As stated in the introduction, we do not present all the data used, but instead describe in detail the effects of mixing on both the theoretical expressions and experimental data for a few examples.

(a) M_W, M_Z

The W and Z masses provide an absolute prediction of the strength of the weak interactions in the absence of exotic mixings. One has [7]

$$M_W = \frac{\hat{A}_0}{\sin\theta_W (1 - \Delta r)^{\frac{1}{2}}} , \quad M_Z = \frac{M_W}{\cos\theta_W} . \tag{28}$$

In (28), $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$ is the renormalized weak angle, $\hat{G}_\mu$ is the (true) Fermi constant, $\hat{A}_0 = \left(\pi\alpha/\sqrt{2}\hat{G}_\mu\right)^{\frac{1}{2}}$, and Δr is a radiative correction parameter predicted to be 0.0713 ± 0.0013 [7]. $\hat{G}_\mu$ differs from the value $G_\mu = 1.16637 \times 10^{-5}$ GeV^{-2} measured in muon decay by

$$G_\mu = \hat{G}_\mu c_L^e c_L^{\nu_e} c_L^\mu c_L^{\nu_\mu} \ , \tag{29}$$

because the effective four-fermion interaction is reduced in strength by the factors c_L^i. Hence, $\hat{A}_0$ is reduced from its canonical value of $A_0 = (\pi\alpha/\sqrt{2}G_\mu)^{\frac{1}{2}} = 37.281$ GeV to

$$\hat{A}_0 = A_0 \left[c_L^e c_L^{\nu_e} c_L^\mu c_L^{\nu_\mu}\right]^{\frac{1}{2}} \ . \tag{30}$$

Therefore, to $O(s^2)$, the theoretical expression for M_W is

$$M_W = \frac{A_0}{\sin\theta_W\,(1-\Delta r)^{\frac{1}{2}}}\left[1 - \frac{1}{4}\left[(s_L^e)^2 + (s_L^{\nu_e})^2 + (s_L^\mu)^2 + (s_L^{\nu_\mu})^2\right]\right] \ , \tag{31}$$

with M_Z still given by $M_W/\cos\theta_W$. We use the correlated average of the UA1 [8] and UA2 [9] values of M_W and M_Z:

$$M_W = 80.9 \pm 1.4 \text{ GeV} \ , \quad M_Z = 91.5 \pm 1.7 \text{ GeV} \ , \quad \text{correlation} = .75 \ . \tag{32}$$

$\sin^2\theta_W$ is determined by fitting to the data simultaneously with the mixing angles.

(b) Charged Current Universality

Universality tests constrain the relative strengths of weak amplitudes, and therefore the relative sizes of left-handed mixing angles.

The apparent KMC mixing angles V_{ui} (we suppress the subscript L) give the following universality relation [6]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9979 \pm 0.0021 \ , \tag{33}$$

in excellent agreement with the prediction of unity for three family universality. $|V_{ud}|$ is determined from superallowed β decay and $|V_{us}|$ from K_{l3} (mainly K_{e3}) and the vector parts of hyperon decay. Hence, they depend only on hadronic vector currents, which are largely free of theoretical uncertainties. ($|V_{ub}|$ suffers from considerable theoretical uncertainty, but for all reasonable values is too small to be of relevance for universality.)

V_{ud} is extracted from the β decay data by dividing the measured coefficient $G_\mu V_{ud}$ of the vector current by G_μ from μ decay. However, in the presence of mixing the actual coefficient of the (hadronic) vector part of the 4-Fermi interaction is

$$G_\mu V_{ud} = \hat{G}_\mu c_L^e c_L^{\nu_e}\left[\hat{V}_{Lud} c_L^u c_L^d + \hat{V}_{Rud} s_R^u s_R^d\right] \ , \tag{34}$$

where the second term is the induced RHC from (10). Using (29) one finds the relation

$$V_{ud} = \frac{\hat{V}_{Lud} c_L^u c_L^d + \hat{V}_{Rud} s_R^u s_R^d}{c_L^\mu c_L^{\nu_\mu}} \ . \tag{35}$$

Similarly, the apparent and true values of V_{us} are related by

$$V_{us} = \frac{\hat{V}_{Lus} c_L^u c_L^s + \hat{V}_{Rus} s_R^u s_R^s}{c_L^\mu c_L^{\nu_\mu}} \ . \tag{36}$$

Combining (33), (35), (36) and the unitarity relation

$$\sum_{i=1}^{n} |\hat{V}_{Lui}|^2 = 1 \ , \tag{37}$$

one obtains

$$\begin{aligned} 1 - \sum_{i=1}^{3} |V_{ui}|^2 &= (s_L^u)^2 - (s_L^\mu)^2 - (s_L^{\nu_\mu})^2 + \sum_{i=4}^{n} |V_{ui}|^2 \\ &\quad + |V_{ud}|^2\left((s_L^d)^2 - 2\mathrm{Re}(\kappa_{ud})\right) + |V_{us}|^2\left((s_L^s)^2 - 2\mathrm{Re}(\kappa_{us})\right) \\ &= 0.0021 \pm 0.0021 \ . \end{aligned} \tag{38}$$

In (38) we have replaced $\hat{V}_{Lij}$ with the measured V_{ij} in the coefficients of $O(s^2)$ terms, allowed for possible mixings with extra sequential or vector doublets, ignored the negligible modifications to V_{ub} and V_{ui}, $i \geq 4$, due to exotic mixing, and introduced the symbol

$$\kappa_{ij} = s_R^i s_R^j \frac{\hat{V}_{Rij}}{\hat{V}_{Lij}} \ . \tag{39}$$

The universality constraint is extremely stringent. Unlike the W and Z masses, however, the mixing terms do not all have the same sign so there is a possibility of cancellations. s_L^e and $s_L^{\nu_e}$ do not enter because β, K_{e3}, hyperon, and μ decay are all modified in the same way by c_L^e and $c_L^{\nu_e}$.

In addition to these, we have included the constraints from e-μ-τ universality, induced right-handed leptonic and hadronic currents, and D^0 semi-leptonic decays. Flavor-diagonal neutral current processes such as deep inelastic neutrino scattering, neutrino-electron scattering, and weak-electromagnetic interference (atomic parity violation, polarized eD asymmetry) have also been included. We refer the reader to Ref. 1 for details.

4. Results and Discussion

Using charged current and flavor-diagonal neutral current data, we find that there are enough constraints to simultaneously determine $\sin^2\theta_W$, to constrain the mixings $(s_{L,R}^i)^2$

of all of the known L and R-fermions, to limit the mixings $|V_{ui}|^2$ and $|V_{ci}|^2$ with sequential or vector doublets, and to limit all leptonic and hadronic induced right handed charged currents except $b \leftrightarrow c, u$. In Figures 1-3 we present the results of (a) allowing only one parameter to vary; (b) allowing all parameters to vary simultaneously, and (c) specific models (which generally lead to limits between those of (a) and (b)). The three specific models are (a) The E_6 model with all fermions assigned to 27-plets. No assumption is made concerning the Higgs structure, so all three kinds of neutrino mixing are possible. (b) Mirror fermions with a Hermitian mass matrix and Dirac neutrinos. (We consider $\Lambda = 2$ only). In this case the fermion mass matrices are diagonalized by the same unitary matrices for L and R so that $s^i_L = s^i_R$ for neutrinos, charged leptons, and quarks. (c) A model in which there are exotic R-doublets involving massless neutrinos. This could come about in a mirror model in which the neutrinos are massless in both the ordinary and mirror sectors, for example. Since the R-doublet neutrinos are already massless, no neutrino mixings are required.

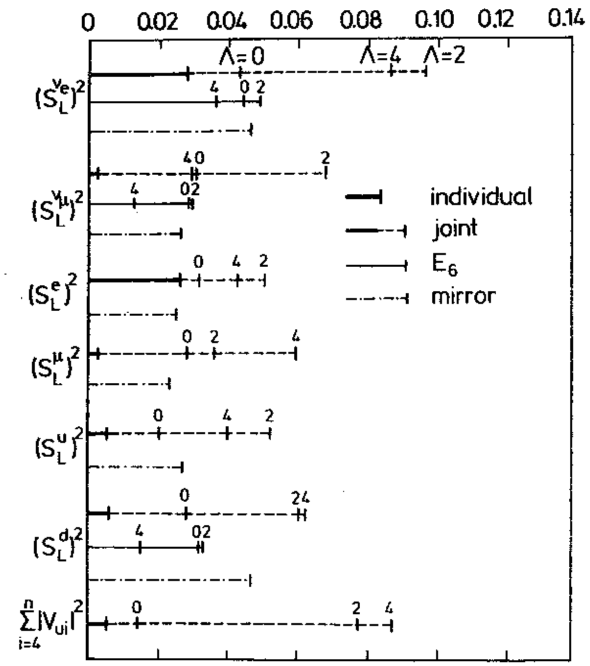

Fig 1: 90 % c.l. upper limits on mixings for the light left-handed fermions, for individual fits (heavy solid lines), joint fits to all mixings (dashed lines), E_6 models (solid lines), and the Hermitian mirror model (dot-dash line). The values of Λ_μ are indicated.

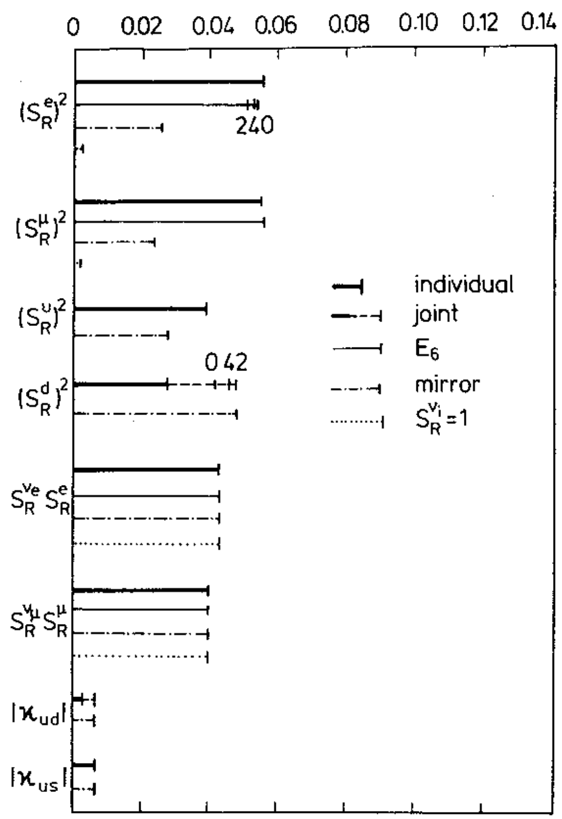

Fig 2: 90 % c.l. upper limits on mixings for light right-handed fermions.

Figures 1-2 contain the limits on the mixings of the light fermions. The extremely stringent individual limits on $(s^{\nu_\mu}_L)^2$, $(s^\mu_L)^2$, $(s^d_L)^2$, $(s^u_L)^2$, and on the mixing $\sum_{i=4}^n |V_{ui}|^2$ with extra L-doublets are from the universality relation (33), while the much weaker limits on $(s^{\nu_e}_L)^2$ and $(s^e_L)^2$ are from e-μ universality. In the joint fits the universality constraints can be partially evaded by cancellations. However, the combined effect of the neutral current and universality is to restrict $(s^{\nu_e}_L)^2$, $(s^{\nu_\mu}_L)^2$, $(s^i_{L,R})^2$, $i = e, \mu, u, d$, and $\sum_{i=4}^n |V_{ui}|^2$ to the few percent level. For the E_6 model, since not all kinds of exotic quarks and charged leptons are present, the limits are somewhat stronger than in the general case. In the Hermitian mirror model, limits on the s^i_R are somewhat better than for the general case in which s_L^i and s_R^i are unrelated. The model with $s_R^{\nu_i} = 1$ yields very stringent limits on $(s_R^e)^2$ and $(s_R^\mu)^2$.

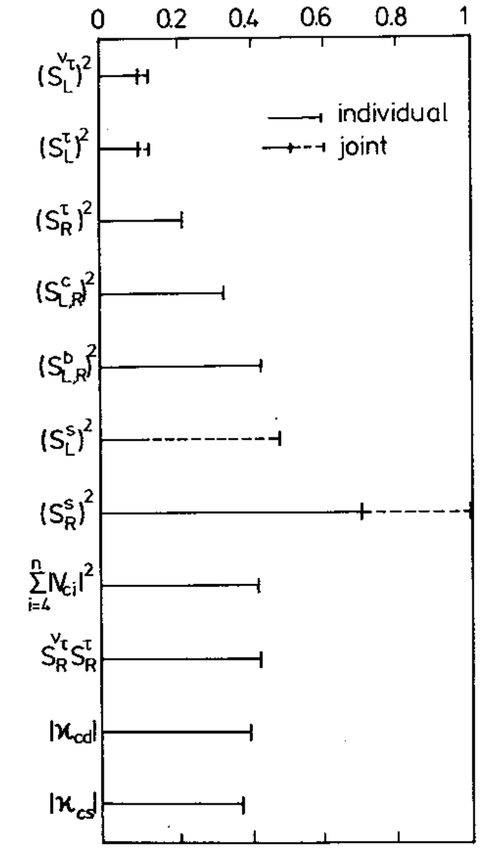

Fig 3: 90 % c.l. limits on mixings for the s, b, c, and τ.

Figure 3 shows the limits on the s, c, b, and τ mixings and induced RHC. These limits are much weaker than and largely decoupled from those in Figures 1-2.

We now turn to a discussion of a few of the phenomenological implications of non-orthogonal neutrinos. (We refer the reader to Ref. 3 for a complete analysis.) As noted in section II, the non-orthogonality of the neutrinos leads to a non-zero cross section for the process $n_{aL} \to e_{bL}$ (eqn (21)). Therefore the sequence $K^+ \to e^+_{aR} n_{aL}$ followed by $n_{aL} N \to e^-_{bL} X$ would mimic the effect of $n_{aL} \to n_{bL}$ in neutrino oscillation appearance

experiments, except that the apparent "oscillation" probability

$$P(n_a \to n_b) = |\lambda_{ba}^L|^2 \simeq |\langle n_{bL} | n_{aL} \rangle|^2 \tag{40}$$

is independent of the neutrino energy or distance travelled. That is, if the light mass eigenstates n_{iL} are really massless (or extremely light) then n_{aL} is independent of time. One would not see any characteristic oscillation signatures, only an excess of e_b^- events above background. The limits on the λ_{ba}^L from searches for neutrino oscillations in appearance experiments are given in Table 1. Non-orthogonal neutrinos do not lead to significant effects in neutrino disappearance experiments.

The most stringent limits on the λ_{ba}^L follow from unitarity constraints. These constraints do not involve lepton number violation at all. From the unitarity of U_L one has

$$|\lambda_{ba}^L| \le \sqrt{(s_L^{\nu_a})^2 (s_L^{\nu_b})^2}\ , \tag{41}$$

which follows by combining the Schwarz inequality for $\left(E_L E_L^\dagger\right)_{ba}$ with the unitarity relation (16). The corresponding limits on $|\lambda_{ba}^L|$ from (41) are presented in Table 1 for two cases: (a) only one mixing $(s_L^{\nu_a})^2$ is allowed to be non-zero at a time, and (b) all mixings are allowed to be non-zero simultaneously. It is seen that the latter constraints are much weaker.

Source	$\lvert\lambda_{e\mu}^L\rvert$	$\lvert\lambda_{e\tau}^L\rvert$	$\lvert\lambda_{\mu\tau}^L\rvert$
ν appearance	0.041	0.27	0.045
unitarity (single parameter)	0.0083	0.054	0.015
unitarity (simultaneous fit)	0.082	0.11	0.095
$\mu \not\to e\gamma$	9.5×10^{-4}		
quadratic seesaw	$2.5 \times 10^{-7}\ x^2$	$2.5 \times 10^{-6}\ x^2$	$2.5 \times 10^{-4}\ x^2$
linear seesaw	$5 \times 10^{-4}\ x$	$1.6 \times 10^{-3}\ x$	$0.016x$

Table 1: 90% c.l. limits on lepton flavor violating parameters and expectations for the quadratic and linear seesaw models. x is defined as 20 GeV$/M$, where M is the heavy neutrino mass.

In addition to producing the wrong lepton flavor, as in (40), non-orthogonal neutrinos can also lead to $\Delta L = \pm 2$ processes. The neutrinos n_{aL} and n_{aR}^c produced in $K^+ \to e_a^+ n_a$ can also rescatter into e_b^+ (with $b = a$ or $b \neq a$). The total probability for the this process is

$$P(n_a \to \bar{n}_b) \simeq (s_R^{e_b})^2\, |\beta_{ab}^L|^2 + r\, (s_R^{e_a})^2\, |\beta_{ba}^L|^2 \tag{42}$$

relative to the normal $K^+ \to e_a^+ n_a$, $n_a \to e_a^-$, where the two terms are associated with intermediate n_{aL} and n_{aR}^c, respectively, and where $r = \sigma_0 (\nu_{aR}^c \to e_{aR}^c) / \sigma_0 (\nu_{aL} \to e_{aL})$ is the ratio of antineutrino to neutrino cross sections in the absence of mixing. From a search for the process $\bar{\nu}_\mu e^- \to \mu^- \nu$ [10], a limit on $s_R^\mu|\beta_{\mu\mu}^L| < 0.19$ has been set.

A limit on $s_R^e|\beta_{ee}^L|$ can be obtained from neutrinoless double beta decay ($\beta\beta_{0\nu}$). We list all the limits on the $s_R^a\beta_{ba}^L$ in Table 2.

quantity	experimental limit	quadratic seesaw	linear seesaw	source
$\lvert s_R^e\beta_{ee}^L\rvert$	4×10^{-7}	$2.5\times10^{-9}\ x^2$	$4\times10^{-5}x$	$\beta\beta_{0\nu}$
$\lvert s_R^\mu\beta_{\mu\mu}^L\rvert$	0.19	$2.5\times10^{-5}\ x^2$	$5\times10^{-3}x$	$\bar{\nu}_\mu e \not\to \mu^-\nu$
$\lvert s_R^e\beta_{\mu e}^L\rvert$	0.045	$2.5\times10^{-7}\ x^2$	$5\times10^{-4}x$	μ decay
	0.041			$\nu_\mu \not\to e^+$
$\lvert s_R^\mu\beta_{e\mu}^L\rvert$	0.048	$2.5\times10^{-5}\ x^2$	$5\times10^{-3}x$	μ decay
	0.071			$\nu_\mu \not\to e^+$
$\lvert s_R^\tau\beta_{\mu\tau}^L\rvert$	0.54	$2.5\times10^{-3}\ x^2$	$0.05x$	τ decay
$\lvert s_R^\tau\beta_{e\tau}^L\rvert$	0.43	$2.5\times10^{-3}\ x^2$	$0.05x$	τ decay

Table 2: 90 % c.l. limits on $\Delta L = \pm 2$ parameters, and expectations for the quadratic and linear seesaw models; $x = 20$ GeV$/M$, where M is a heavy neutrino mass.

It is interesting to compare the limits on the λ_{ba}^L and $s_R^a\beta_{ba}^L$ with theoretical expectations. Typically these quantities will depend on m_a^0 and M, where m_a^0 is the typical light-heavy mixing term for the a^{th} family and M is the typical heavy lepton mass. The values of the m_a^0 for a broad class of models will fall between the following two cases: (a) The quadratic seesaw. In this case the quark and charged lepton masses m_a are generated by the normal Higgs mechanism, with $m_a^0 = O(m_a)$. (b) The linear seesaw. In this situation the direct quark and charged lepton masses are zero for some reason. They then acquire masses m_a of $O(m_a^{0^2}/M)$ by light-heavy mixing. Hence, one expects

$$\lambda_{ab}^L \sim \left(\frac{m_a m_b}{M^2}\right)^\sigma \ , \quad s_R^{e_a}\beta_{ba}^L \sim \left(\frac{m_a^2}{M^2}\right)^\sigma \ , \qquad (43)$$

where $\sigma = 1$ or $1/2$ for the quadratic and linear seesaws, respectively. We choose $m_a =$ 1 MeV, 100 MeV, and 1 GeV for the typical quark and charged lepton masses of the three known families. The predictions of these two simple models for $M \sim 20$ GeV are listed in Tables 1 and 2.

Similarly, one expects $(s^a)^2 \sim (m_a^2/M^2)^\sigma$ for the exotic mixings. For the quadratic seesaw the limits in Figures 1-3 correspond to very small values of M, while for the linear seesaw there is sensitivity up to $M \sim O(50$ GeV).

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