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SPECULATIONS ON THE ORIGIN OF FLAVOR

by

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Abstract

I discuss some recent ideas on family generation, pointing out both their appealing features and their weaknesses. The necessity of some new experimental hints to choose among these options is stressed.

[*] Invited talk given at the Zuoz Spring School, Zuoz, Switzerland, April 5 – 15, 1988. To appear in the School's proceedings.

1 The Flavor Box

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[illegible] in a series of approximate steps [illegible]. As a first step, imagine turning off all gauge and Yukawa couplings in the standard model Lagrangian. In this approximation the Lagrangian just contains a number of kinetic energy terms for the fermions in the theory and thus possesses a very large global symmetry. It is convenient to describe the fermions in terms of only left handed fields, so called Weyl spinors [2]. Because one can rotate all the 48 Weyl spinors describing the quarks and leptons into each other, the flavor symmetry of the model is $U(48)$. The fermions in the theory, in this approximation, transform according to the fundamental representation of $U(48)$ and one has

$$\psi = \begin{pmatrix} \nu_e \\ \vdots \\ \vdots \end{pmatrix} \to U\psi$$

$$\mathcal{L}_{SM\, g_i = 0, f_{ij} = 0} \to \mathcal{L}_{SM\, g_i = 0, f_{ij} = 0} \quad (1)$$

Pictorially, the situation is described by Fig. 1.

As the next step, imagine turning on the gauge coupling constants but still keeping vanishing Yukawa couplings. In this case the flavor degrees of freedom are much more organized and the overall global symmetry has decreased. Now only the fields having the same $SU(3) \times SU(2) \times U(1)$ quantum numbers, but belonging to different families, can be rotated into each other. For instance, the lepton doublets can be mixed with one another

$$L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix} \to U_{ij}^L L_j \quad (2)$$

However, it is no longer possible to mix freely an electron with a muon without doing the same for their corresponding neutrinos. Since there are six basic entities under $SU(3) \times SU(2) \times U(1)$ [L_i, Q_i, [illegible]] [3] the global symmetry is $U(3)^6$. Under this symmetry,

$$\mathcal{L}_{SM\, f_{ij} = 0} \to \mathcal{L}_{SM\, f_{ij} = 0} \quad (3)$$

The flavor organization at this stage is sketched schematically in Fig. 2.

[1] In what follows I shall suppose that there are only three families of quarks and leptons, which include also right handed neutrinos. My discussion, however, can be trivially modified, in most instances, if there were more families or if the right handed neutrino states did not exist.

[2] Right handed fermions are connected to the appropriate charge conjugate left handed fermions: $e_R \sim e_L^c$ and so need not be considered separately.

[3] Here [illegible] is a color quantum number and capital letters indicate an $SU(2)$ doublet.

Before discussing some of the ideas suggested to answer these questions, it is worthwhile discussing briefly two related points. One of these points is concerned directly with the way we split up the flavor question, which needs some justification. The other addresses the issue of possible family symmetries.

Our three stage analysis of the flavor problem assumed that the sequential limit: $\Gamma_{ij} \to 0$; $g_i \to 0$ made sense. This is in fact so, but only because the fermions that enter in the standard model are chiral under $SU(3) \times SU(2) \times U(1)$. Thus no gauge invariant mass terms exist for them - with the exception of the right handed neutrinos, which are gauge singlets. The Pauli principle implies that a possible mass term for Weyl spinors

$$\mathcal{L}^{Mass}_{ab} = -m\psi_a^T C\psi_b \tag{5}$$

must be symmetrical in the group indices a and b, since it is antisymmetrical in the Lorentz indices. Clearly, therefore, no $SU(3) \times SU(2) \times U(1)$ singlet product of the basic building blocks is possible, except for a term containing two ν^c. Thus all fermions - with the possible exception of the right handed neutrinos, which could decouple by becoming very massive - are massless until $SU(2) \times U(1)$ is spontaneously broken. Hence, it is perfectly sensible to consider a limit in which the Yukawa coupling vanish, since in this limit there is indeed more symmetry in the theory. If the fermions of the theory had been in real representations, whether they coupled to Higgs bosons or not would have been of no particular relevance.

Our discussion assumed that on turning on the Yukawa couplings, the only remaining global symmetries of the theory are B and L. It is possible, of course, that the Yukawa couplings are such that some family symmetry exists. A local or a global family invariance is not excluded, provided this group is broken at a sufficiently large scale [e]. In either case, flavor changing transitions are suppressed. For the local family symmetry case, all gauge mediated flavor changing processes are proportional to M_F^{-2}, with M_F being the mass of the relevant family gauge boson. Hence for M_F big enough, no contradictions with experiment will arise ($M_F \geq 100 TeV$ will suffice, for most purposes). In the case of a spontaneously broken global family symmetry, one must worry about flavor changing decays involving the emission of the Goldstone boson associated with the breakdown of this symmetry - the Familon [1]. However, these amplitudes are also suppressed by the scale parameter Λ_{fam}, associated to the family group breakdown. For Λ_{fam} sufficiently big, all experimental constraints are rendered moot.

The existence of a family group, either local or global, has, nevertheless, implications for the fermion mass matrices, since the relevant Higgs couplings must respect these family symmetries. Two simple examples of possible family symmetries are shown in the flavor boxes of Fig. 4. Case a) has a vector like family group and hence G_{fam} can be a local symmetry group. Case b) is an axial family group, so that G_{fam} can only be a global symmetry of the theory. Since the Yukawa couplings of a doublet Higgs field ϕ always involve a fermion and an antifermion field, it follows that under G_{fam}:

$$\begin{aligned} \phi &\sim \mathbf{8} \oplus \mathbf{1} \quad Case\ a) \\ \phi &\sim \mathbf{6} \oplus \bar{\mathbf{3}} \quad Case\ b) \end{aligned} \tag{6}$$

The second possibility above offers a better starting point for constructing approximate

[e] An exact family symmetry is clearly ruled out phenomenologically

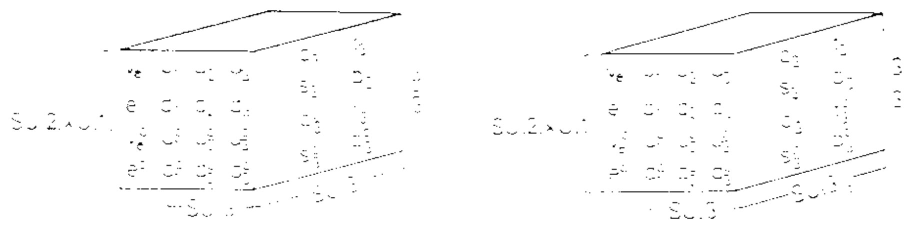

Figure 4: Examples of family symmetries. In both cases $G_{fam} = SU(3)$, but in Case a) the fermions transform as a $\mathbf{3}$ while the antifermions transform as a $\bar{\mathbf{3}}$, while in b) all excitations transform as a $\mathbf{3}$

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$$M = M_0 \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} \tag{7}$$

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2 Three Routes to Families

I want to discuss in this Section some speculative ideas on the origin of the quark and lepton families. In view of the preceeding discussion, this is probably a bit too narrow a viewpoint to take on the issue of flavor. It could well be that it is wrong only to focus on families, without asking at the same time why one has the specific quark and leptons one has in each family. At any rate, I will begin by looking at a variety of mechanisms which have been suggested for giving a generational repetition. However, later on I will try to address also the issues of quantum numbers and mass generation, associated with each of the family generation mechanisms proposed.

To my knowledge, there are three broad classes of suggestions in the literature for obtaining families. The replication of fermionic states occurs because:

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1 The Flavor Box

Flavor is an old problem. Already in the 1930's the elementary particles of the day, the electron and the proton, were joined by flavor partners: the muon, the (electron) neutrino and the neutron. The 1940's and 50's brought into the scene the strange particles, but the flavor explosion did not stop there. Today we are able to ask the question of flavor in a more compact way: why are there three (or more?) families of quarks and leptons? However, we still do not have any satisfactory answer to this fundamental question, but only rather tentative speculative ideas. My task here will be to discuss some of these speculations, exposing some of the virtues and faults of the ideas currently being proposed for the origin of flavor.

When thinking of flavor, it is useful to approach the standard model in a series of approximate steps [1]. As a first step, imagine turning off all gauge and Yukawa couplings in the standard model Lagrangian. In this approximation, the Lagrangian just contains a number of kinetic energy terms for the fermions in the theory and thus possesses a very large global symmetry. It is convenient to describe the fermions in terms of only left handed fields, so called Weyl spinors [2] Because one can rotate all the 48 Weyl spinors describing the quarks and leptons into each other, the flavor symmetry of the model is $U(48)$. The fermions in the theory, in this approximation, transform according to the fundamental representation of $U(48)$ and one has

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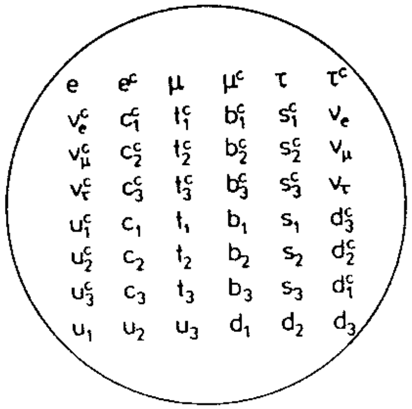

Figure 1: The $U(48)$ flavor box

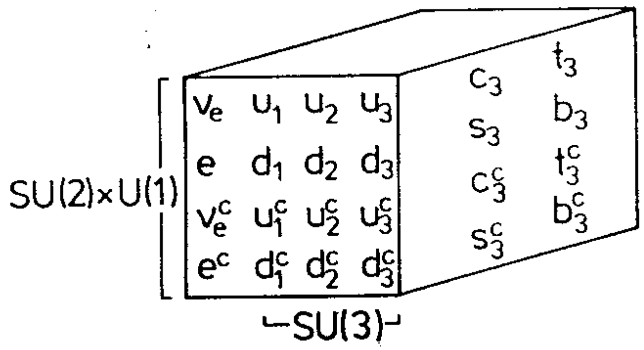

Figure 2: The $U(3)^6$ flavor box

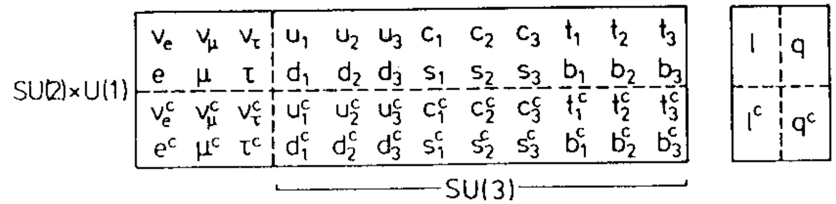

Figure 3: The final flavor box, which only has B and L global symmetries.

When the Yukawa interactions are turned on, the standard model is left, in general, with two global $U(1)$ symmetries, corresponding to Baryon and Lepton number conservation [4]. The flavor box shown in Fig. 3 is now highly organized. Except for the local $SU(3) \times SU(2) \times U(1)$ transformations, the standard model Lagrangian is invariant only under separate transformations for all the quarks and lepton fields [5].

$$l \rightarrow e^{i\alpha_L} l \quad ; \quad l^c \rightarrow e^{-i\alpha_L} l^c$$
$$q \rightarrow e^{i\alpha_B} q \quad ; \quad q^c \rightarrow e^{-i\alpha_B} q^c \qquad (4)$$

The above discussion makes it clear that there are really three separate flavor problems to solve. Namely:

1. What physics gives the Yukawa interactions which break the global $U(3)^6$ symmetry to just B and L, and generates, thereby, all the fermion masses and mixings we observe?

2. What is the origin of the family repetitions we see? That is, why is it that after gauging the standard model group there is a leftover $U(3)^6$ symmetry?

3. Who ordered $U(48)$? That is, what principle underlies the existence of all the quarks and leptons we see?

[4] This is actually only true at the classical level. At the quantum level B+L has an anomaly, so the only remaining global symmetry is B-L

[5] If no right handed neutrinos are present, the neutrinos are massless and there is actually a separate $U(1)$ symmetry for each lepton flavor

Before discussing some of the ideas suggested to answer these questions, it is worthwhile discussing briefly two related points. One of these points is concerned directly with the way we split up the flavor question, which needs some justification. The other addresses the issue of possible family symmetries.

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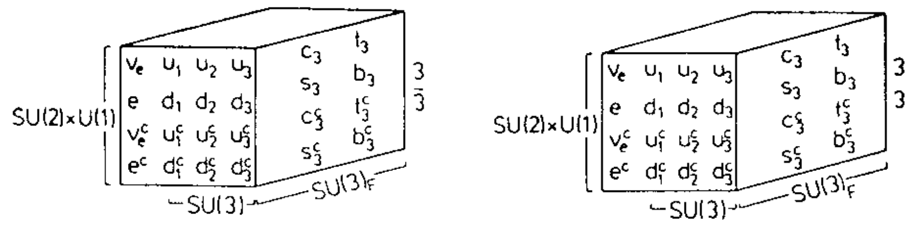

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To my knowledge, there are three broad classes of suggestions in the literature for obtaining families. The replication of fermionic states occurs because:

1. there is an underlying theory whose dynamics forces certain bound state replications

2. chaotic dynamics at short distances produces some cloning of fundamental states

3. families are connected with topological properties of higher dimensional compactification

I will try to briefly describe each of these ideas in turn, in a critical fashion.

2.1 Dynamical Family Generation

The party line here is that quarks and leptons are composite and family repetition is a result of the preon dynamics. Although this is an attractive idea, and a very good slogan, no realistic models exist in practice. That is, there are models where one indeed generates family replications, of even the right quantum numbers. However, in all models, the quarks and leptons are massless and one really has not yet found realistic dynamical ideas for having these states acquire the rich spectrum of masses they exhibit in nature.

Because quarks and leptons appear very pointlike, when probed with the energies of today's experiments, the scale of compositeness is certainly quite high, with present bounds putting it above a few TeV [4]. If $\Lambda_c \geq TeV$, preon dynamics must be such that some approximately massless fermionic bound states - the quarks and leptons, with $m_f << \Lambda_c$ - can appear in the spectrum. For this to happen, the preon models must be endowed with certain chiral symmetries, which are preserved in the binding. Since one knows that chiral symmetries in vector like theories, like QCD, are spontaneously broken [5], the natural supposition is that the preonic theory is based on some purely chiral gauge theory. An example of such a theory, is provided by an $SU(N)$ gauge theory with $N+4$ Weyl preons transforming according to the fundamental representation of $SU(N)$ and one preon transforming according to the complex conjugate of the two-rank symmetrical tensor representation of this group [7].

In chiral gauge theories one can give theoretical arguments, based on the 't Hooft anomaly matching conditions [6] and on complementarity [7], for the spectrum of the $m=0$ bound states. By means of these arguments, therefore, one can establish if some of these theories lead to family repetitions. The difficult question remains, however, how to get, eventually, some of these massless bound states to acquire some small mass. There are many preon models and it is not my intention here to survey them. Rather, I shall pick an illustrative model, due to Geng and Marshak [8], to exemplify both how families are generated and various issues one encounters when one tries to generate mass.

The Geng Marshak preon model [8] is based on an $SU(16)$ gauge theory, with 20 Weyl preons in the fundamental and one preon transforming according to the $\overline{\mathbf{136}}$ representation. Obviously this is an enormous gauge group and there are many preonic degrees of freedom, but the structure is dictated by wanting to get families in an efficient way [8] This model has an $SU(20) \times U(1)$ global chiral symmetry, but one can show, by using complementarity, that only a symmetry group $G = SU(16) \times SU(4) \times U(1)$ survives the binding. Here the $SU(4)$ group acts as a family group and 4 out of the 20 Weyl preons act essentially as carriers of family information.

[7]This preon assignment guarantees that there are no $SU(N)$ gauge anomalies at the preon level
[8]The quark and lepton question, who ordered $U(48)$, is here replaced by who ordered $U(456)$!

The massless bound states of the model can be found from the 't Hooft anomaly matching conditions to consist of two kind of states which transform under G as:

$$\mathbf{f} \sim (\mathbf{16},\mathbf{4})_1 \quad ; \quad \mathbf{f}' \sim (\mathbf{120},\mathbf{1})_2 \tag{8}$$

The $\mathbf{f}'$ fermions are a set of extra states, which must eventually be removed from the spectrum. However, the $\mathbf{f}$ fermions have all the characteristics of 4 families of quarks and leptons. This can be seen as follows. The bound states in this model are constructed schematically as

$$F_{ij} \sim [\Box_i \Box_j (\Box\Box)^*] \tag{9}$$

where i,j are indices denoting the 20 flavor preons. One sees, therefore, that for the states $\mathbf{f}$ one can think of the family quantum number as being just an extra label, provided by the effective scalar field

$$\Phi_j = [\Box_j (\Box\Box)^*] \tag{10}$$

transforming according to the complex conjugate of the fundamental representation of the $SU(16)$ gauge group. Hence, provided one picks 16 of the 20 preons to have the quantum numbers of the quarks and leptons, the model naturally produces four replicas of them at the bound state level. The effective scalar Φ_j just serves to bleach out all the $SU(16)$ gauge quantum numbers of these 16 preons.

The via crucis for all composite models is always provided by mass generation. I would like to argue that a possible solution to this difficult problem may be provided by the concept of irrelevant condensates [9]. The idea, put succintly, is that perhaps when the $SU(3) \times SU(2) \times U(1)$ gauge interactions are turned on, it may be possible for condensates to form, which would not otherwise have formed. These condensates can break $SU(2) \times U(1)$ and give masses to the, otherwise, massless bound state quarks and leptons. An example of a possible irrelevant condensate in the Geng Marshak model is the vacuum expectation value of

$$< I_c > = < \Box_i \Box_j (\Box\Box)^* \Box_i \Box_j (\Box\Box)^* > \tag{11}$$

Clearly, before gauging, $< I_c > = 0$, if we want the global symmetry group G to survive. However, after gauging, the fact that one can have preons which are both triplets and antitriplet of color may be enough to trigger the operators in I_c to condense. One would not expect, in this case, the size of the I_c condensate to be determined solely by Λ_c, since Λ_{QCD} should also play a role. Because what drives these condensates are color-anticolor pairing, just like in QCD, one would guess very roughly (and probably too naively!) that

$$< I_c > \sim \Lambda^3_{QCD} \Lambda^6_c << \Lambda^9_c \tag{12}$$

The presence of condensates which break $SU(2) \times U(1)$ at a scale much less than Λ_c can give small fermion masses to the erstwhile massless bound states. However, although this is a plausible scenario for generating small quark and lepton masses, it is extremely difficult to be quantitative. In addition, there are a number of issues associated with mass generation which are hard to avoid. Generally speaking, there will be flavor changing neutral processes proceeding at too large a rate and a whole host of pseudo Goldstone bosons to contend with. Furthermore, in most models, unwanted states - like the $\mathbf{f}'$ states of Eq. (8) - appear and must be removed to large masses. Hence, although compositeness provides a way to obtain family replications, the theoretical price one must pay still seems exhorbitant, for what one really gets out of present day models. Sadly, there is no agreed upon composite paradigm.

2.2 Families out of Chaos

Quite a different approach to the family question has been pursued by H.B. Nielsen and his collaborators, for a number of years now [10]. The key idea in their proposal is the notion that there is both order and chaos at short distances. Order is provided by imagining that there exists some kind of a lattice structure at very short distances ($a \leq M^{-1}_{Planck}$) and some fundamental lattice gauge theory. The symmetry group for this theory may be very large, but this is not important since one assumes that the dynamics in this lattice is random, so that this large group will rapidly break down. The Hamiltonian for the theory consists of sums of contributions from the various plaquettes, but each contribution is presumed to have random coefficients. Since these coefficients have both positive and negative signs, the minimum of the Hamiltonian is no longer given by having all plaquettes assume the unit element in the group. Thus the large group will spontaneously collapse!

One would imagine that, given the random Hamiltonian, the group collapse would be total. However, Brene and Nielsen [11] have argued that this total collapse can be avoided if the non unit elements assumed in the vacuum commuted with all the elements in the group [9]. So one may be left over with some surviving group, provided that this group had a non trivial center. Brene and Nielsen argue, furthermore, that the center needs to be connected, to guarantee that a degree of continuity in the lattice is maintained, typified by the requirement of fulfilling Bianchi's identity for neighboring plaquettes.

The simplest surviving group that obeys Brene and Nielsen's condition is $S[U(3) \times U(2)]$, which is just the standard model group with the extra condition that $Det\ U(3)\ Det\ U(2) = 1$. This condition precisely fixes the charges of the fermions, so that they obey the usual triality rule, giving the quarks one-third integral charge. Reflecting the above charge quantization, the center of this group is isomorphic to $U(1)$. I find it remarkable that one of the results of this chaotic behaviour at short distance, apparently, is to select out the Standard Model, with the right connection between quark and lepton charges, as the simplest surviving theory!

Even if one could argue, somehow, that random dynamics selects only the simplest surviving group, one would still have to argue that what really survives is just one factor of $S[U(3) \times U(2)]$. Bennett, Nielsen and Picek [12] have tried to supply this extra step recently, by appealing to the concept of confusion. Since, personally, I find this idea rather confusing, let me try to paraphrase their arguments in terms which I can understand. Imagine having N_f copies of $S[U(3) \times U(2)]$ as the surviving group, with the associated fermions in N_f complex representations, each containing only one full family of quarks and leptons, for one of the N_f gauge groups. Since there is no way to tell which gauge group is which (confusion), the product group collapses (once more!) to the diagonal subgroup of $(S[U(3) \times U(2)])^{N_f}$, which is just $S[U(3) \times U(2)]$. The N_f complex representations of the product group, in this process, just become N_f repetitions of one family of quarks and

[9] That is, they belonged to the center of the group

leptons. So through repetition and confusion, families have been generated out of chaos!

Given the very qualitative nature of the arguments used, one should view all of the above with a great deal of skepticism. However, there is one semiquantitative result which emerges which, if it is not fortuitous, is very remarkable. In the process of confusion, all the N_f different gauge fields are identified with each other. Hence the emerging effective coupling constants are an average of that of each of the separate groups. More precisely, by identifying the gauge field's kinetic energies, one has, for each of the gauge couplings:

$$\frac{1}{g^2} = \sum_i \frac{1}{g_i^2} \tag{13}$$

This equation, per se, is not useful unless one makes some assumption on what the g_i's are. A very interesting result emerges if one identifies [12] [13] the g_i's to be the critical coupling constants where, on the lattice, the transition between the strong and weak coupling phase occurs [10]. Then one has, from (13)

$$\frac{1}{g^2} = N_f \frac{1}{g^2_{crit}} \tag{14}$$

Given a value for g^2_{crit} and the assumption that all the chaotic behaviour occurs around the Planck scale, one has then fixed the value of the standard model coupling constants at this scale. These values are related to the measured values of the gauge couplings at low energy by the renormalization group, with the numerical result depending on the number of generations. Hence in principle, these ideas constrain the number of families!

The above analysis has been carried out in detail by Bennett, Nielsen and Picek [13] in a recent paper, obtaining - what else! - $N_f = 3$. I shall not repeat their analysis here, but just give the relevant numbers for the colour coupling constant, to give a flavor [11] of what is going on. The mean field value [14] for g_{crit} gives

$$[\frac{1}{g^2_{crit}}]_{U(3)} = 2.43 \tag{15}$$

This value agrees well with the Monte Carlo results of Creutz and Moriarty [15] of 2.29 ± 0.16, for this quantity. Hence

$$\alpha_3^{-1}(M_{Planck}) = 2\pi[\frac{2}{g^2}]_{U(3)} = 2\pi N_f[\frac{2}{g^2_{crit}}]_{U(3)} \simeq 14.5 N_f \tag{16}$$

On the other hand, using $\alpha_3^{-1}(M_W) = 8.3 \pm 0.6$, the renormalization group implies

$$\alpha_3^{-1}(M_{Planck}) = \alpha_3^{-1}(M_W) + \frac{1}{2\pi}(11 - \frac{4N_f}{3}) ln \frac{M_{Planck}}{M_W} \simeq 52, \tag{17}$$

where the numerical value holds for 3 families.

I find these results great fun, but I have a very hard time judging their reliability. What is clear to me, is that this approach ought to be critically examined and explored by a larger fraction of the theoretical community than it is at present. The open questions here are legions. A good one to start out with is: if the lattice is real at M_{Planck}, why is there Lorentz invariance?

[10] Since these values are dependent on the lattice action one uses, it seems to me that there are some conceptual issues here which still need to be resolved

[11] Forgive the unintended pun!

2.3 Families from Geometry

The final way to get family repetitions, which I would like to mention, is the one that is the most popular these days, as the result of all the work on superstrings. However, the idea was suggested earlier, in connection with the higher dimensional compactification present in Kaluza Klein theories [16] [17]. The geometry associated with the compactification of the extra dimension provides index theorems for chiral fermions, and the number of families is related to these topological indices.

The general idea is simple to understand. Consider chiral fermions in d-dimensional space time [12]. They obey, by definition, a massless Dirac equation

$$\Gamma^a D_a \psi = 0 \tag{18}$$

where $\Gamma^a D_a$ is the Dirac operator in the appropriate background field, provided by the gauge and gravitational fields. Suppose $(d-4)$ dimensions compactify, then the Dirac equation (18) reads

$$(\gamma^\mu D_\mu + \Gamma^a D_a)\psi = 0 \tag{19}$$

where the second term has indices in the compact dimensions. Obviously $\Gamma^a D_a$ acts precisely as a four-dimensional mass term. To get 4-dimensional chiral fermions, therefore, ψ must obey a chirality constraint in the compact space

$$\Gamma^a D_a \psi = 0 \tag{20}$$

which eliminates this effective mass term. The number of solutions of Eq. (20) is a function of the compact space geometry and, hence, so is the number of families which ensue. Furthermore, in general, the number of solutions of an equation like (20) is determined by intrinsic properties of the compact space. So the number of families is seen, in this approach, to be closely related to topological properties of the space which compactifies

A well known example of family generation via higher dimensional compactification is the Calabi-Yau compactification of the ten dimensional $E_8 \times E_8$ superstring [18]. Here the $d = 10$ chiral fermions are the gauginos sitting in the adjoint of E_8, whose decomposition in terms of the $SU(3) \times E_6$ maximal subgroup reads:

$$\mathbf{248} = (\mathbf{1},\mathbf{78}) \oplus (\mathbf{3},\mathbf{27}) \oplus (\bar{\mathbf{3}},\overline{\mathbf{27}}) \oplus (\mathbf{8},\mathbf{1}) \tag{21}$$

The six dimensional compact space is a Calabi Yau space, which has an $SU(3)$ holonomy, so that the chiral zero modes must have non trivial $SU(3)$ properties. Hence the fermion fields which obey the chiral constraint (20) in the compact space transform as $(\mathbf{3},\mathbf{27})$ or as a $(\bar{\mathbf{3}},\overline{\mathbf{27}})$ representation. One expects, therefore, as a result of the compactification, to obtain a certain number N_f of **27**'s, plus δ pairs of **27** and $\overline{\mathbf{27}}$. Both δ and N_f are determined by topological properties of the Calabi Yau space. In particular, N_f, which because it details the number of unpaired **27**'s is just the number of families, is given by half the Euler number [18]. In this case, not only does one have an elegant reason for the number of families produced, but also the families obtained have the "right stuff". In the **27** dimensional representation of E_6 sits the 16 dimensional representation of $SO(10)$, with the quantum numbers of a complete family of quarks and leptons:

$$\mathbf{27} = \mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1} \tag{22}$$

[12] Chiral fermions exist in d=2 mod 4 dimensions. They obey a Majorana condition in d=2 mod 8

The additional states are vector-like with respect to $SU(3) \times SU(2) \times U(1)$ and, hopefully, decouple, as do the **27** and $\overline{\mathbf{27}}$ paired states.

Besides relating families to topological indices, there is a further advantage associated with higher dimensional compactification. Namely, that the Yukawa couplings in four dimensions are a very natural byproduct of the gauge couplings in the $d > 4$ theory. After all, a gauge field A^α with index $\alpha = a$ in the compact directions, corresponds to a scalar field in $d = 4$. Unfortunately, one cannot really compute the Yukawa couplings since that would really entail knowing the full normal mode decomposition of the fermion and gauge fields in compact space. However, in a number of instances, one can infer partial information for a given compact space, such as the vanishing of various Yukawa couplings, or particular discrete symmetries among various couplings [19] [20].

Although family generation via dimensional reduction is a very pretty way to get repetitions, one should emphasize that at this stage there is large arbitrariness. In a Kaluza Klein theory one does not really know a priory what gauge group or dimension to start with. Even with superstrings, the Calabi Yau compactification is just one possible example of a four dimensional theory coming from the heterotic string. After the initial superstring enthusiasm about uniqueness, it has been realized that, although the string theory may be unique, its ground states are myriad. [21]. Thus, also here, although replications are undoubtedly being generated, we seem to lack predictability.

3 The Yukawa Lifeline

The three speculations for families I have discussed (Compositeness, Randomness and Geometry) have different answers for the broader questions in the flavor box, concerning why we have the quarks and leptons that we see and what physics gives the mass patterns observed. The chaotic approach, to my mind, gives the most pleasing answer to the question of quark and lepton quantum numbers: The standard model is selected by random dynamics and the quarks and leptons survive since they are the simplest chiral representation of the standard model. Compactification also provides an adequate answer to this question, but it is less unique. Basically, what this approach gives is that the quarks and leptons lie in some GUT representation. Compositeness, unfortunately, provides no really direct clue to the quark and lepton quantum numbers. At best, in models where these states share some common constituents [22], compositeness interrelates their quantum numbers. But often, as in the example I discussed, it introduces more fundamental constituents than quarks and leptons! I should caution, however, not to dismiss compositeness as a suggestion, even if it cannot "explain" why we have the particular quarks and leptons we see. After all, postulating strange quarks did not "explain" strangeness, but it was progress, nevertheless.

With respect to the question of mass generation the situation regarding these speculations is somewhat reversed. Randomness fares here the worst, since Yukawa couplings are not very naturally included in a Planck lattice. Such interactions can be latticized, of course, but who ordered them? Furthermore, after the chaotic short distance behaviour, there appears to be no mechanism to trigger any $SU(2) \times U(1)$ breakdown, since everything else appears to be smooth. Compositeness, is undoubtedly a better starting point, in principle, for generating masses. However, even here one must find mechanisms - like that of irrelevant condensates - to break all the chiral symmetries one carefully built into the theory. The geometrical approach, finally, is probably the one which starts with a built in advantage, since Yukawa couplings naturally ensue in these theories. However, even here the question of $SU(2) \times U(1)$ breakdown needs to be resolved.

In the chaotic approach to flavor, things are still in an embryonic stage and it is difficult to make very precise criticisms. In the other approaches, the unsolved problems are perhaps more obvious. I find it worthwhile listing some of these problems here, as a guide to what still needs to be done, before one can hope to really know the origin of flavor. I begin with some open problems in the compactification approach to flavor

3.1 Compactification problems

There are two main problems here and many, many problems of detail. The main problems are

1. Since one does not know what compact manifold K one is really dealing with, it is obviously impossible to really compute Yukawa couplings at the Planck scale. Today one can make educated guesses about K, but one does not have any hard physics reasons for picking one manifold over another

2. Although Yukawa couplings are, in principle, computable for a given K, to actually generate masses one must still be able to generate the scale $\Lambda_F \simeq 250GeV$ characteristic of the breakdown of $SU(2) \times U(1)$. Unfortunately, all compactification schemes in existence are a one scale theory with $M_{compact} \sim M_{Planck}$. To my knowledge, there are no believable dynamical mechanisms proposed so far, which from the Planck scale are able to generate a much smaller scale - like the Fermi scale, Λ_F- without putting this scale in from outside

The attitude towards these open problems varies from physicist to physicist. Real pessimists argue that it will be both impossible to find the right manifold and to generate the Fermi scale. I take a more middle of the road attitude. I do not worry so much about K but I do worry how a small scale can ever appear. Superstring enthusiasts imagine that the flavor problem is in the bag. However, even these zealots must sort out plenty of "little" problems still! Perhaps the more interesting of these concern the pattern of discrete symmetries, arising in certain specific compactification schemes. Here there has been some progress, both in better understanding the compactification [23] and in exploiting the discrete symmetries, plus some assumed pattern of symmetry breaking, to obtain interesting mass matrix patterns [24]. I mention particularly this issue since, without specific additional symmetries, in general the models which emerge from higher dimensional and/or superstring compactification are beset with potential fatal flaws, connected to flavor mixing. In particular, many models, without these protective symmetries, would have catastrophic baryon number violation.

3.2 Compositeness Problems

The problems here are, in some sense, even more serious since they are so easily identifiable. I have already briefly mentioned two of them earlier on, concerning pseudo Goldstone

bosons and flavor changing neutral currents (FCNC). Not having explicit Yukawa couplings always means that the models one constructs have very large approximate global symmetries, some of which are then broken down spontaneously. So one expects many pseudo Goldstone bosons in these theories. These present a challenge, since one must be sure to arrange that the mass they obtain, because of the presence of the gauge interactions, is above the PEP/PETRA bounds, of roughly 20 GeV. This is not always achievable, especially for those states which only get a mass via weak gauging.

The issue of FCNC is also a general bugaboo for these models. Since one always expects residual 4-Fermi interactions among the bound state fermions (scaled by Λ_c^{-2}), after mass is generated there will be allowed flavor changing transitions. The trick here is to construct models where one can sufficiently suppress these transitions. The most natural way to do so is to just make Λ_c very big. If $\Lambda_c \geq 10 - 100TeV$, all FCNC bounds are not a challenge. However, what is a challenge is to get small masses, or a small Fermi scale, out of a theory whose natural scale is very high.

Besides these main problems, also here there are a number of more technical issues that need to be resolved. I will indicate one of these side problems, which has been of recent interest to me. In general, even if one is able to give masses to the chirally protected states, it may be difficult to avoid a near degeneracy in the masses of the charge $\frac{2}{3}$ and charge $-\frac{1}{3}$ quarks. Let me explain briefly this point. A very good feature of the Higgs doublet breaking in the standard model is that it predicts that $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$. This prediction can be traced to the existence of an extra symmetry in the Higgs potential, which is 0(4) symmetric rather than just $SU(2) \times U(1)$ symmetric. This feature is retained in composite models by building in an $SU(2) \times SU(2) \to SU(2)$ symmetry breaking pattern in the condensates which cause the standard model breakdown [13]. Unfortunately, one cannot readily prevent this extra symmetry from being felt by the quarks and leptons. There is no explicit Yukawa couplings which break this custodial symmetry. In the absence of these Yukawa lifelines, the quark mass matrices will exhibit an approximate $SU(2)$ symmetry, unless this symmetry can be "laundered" away through a chain of effects, as is done in extended technicolor models [25].

4 Concluding Remarks

It is my opinion that the speculations I have discussed on the origin of flavor are not going to go beyond the level of being speculations, until we find some falsehood in the standard model dogma that:

- There are no other excitations but 3(?) complete families of quarks and leptons and a (light) Higgs boson
- There are no FCNC
- Gauge universality is manifested in the unitarity of the Cabibbo Kobayashi Maskawa mixing matrix

[13]This $SU(2) \times SU(2)$ symmetry exists, for instance, in the condensate $< I_c >$, which I discussed for the Geng Marshak model

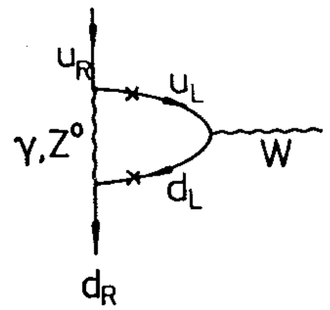

Figure 5: Graph inducing a $u_R d_R W$ coupling in the standard model

We need experimental input badly and of an accuracy that can really check the radiative effects in the electroweak theory.

Let me illustrate this last point, with a small example. In lowest order in the electroweak theory there is no coupling of a u_R and a d_R with a W boson. This coupling is induced at $O(\alpha)$ by the graph in Fig. 5, but it is highly suppressed since it requires mass insertions. Thus the induced coupling is of $O(\alpha \frac{m_3 m_4}{M_W^2})$. If quarks are composite, it is possible that their right handed components are not inert to the interactions which break $SU(2) \times U(1)$ [14]. In this case one may well get an induced $u_R d_R W$ coupling of order $\frac{\Lambda_F}{\Lambda_c}$ [26]. These are small effects, but they may be perhaps measurable.

The above is just an example of some unexpected flavor effects "beyond" the standard model. We must be on the lookout for such phenomena. The answer to the flavor riddle really lies more in future experiments than, in theoretical speculations. Without some experimental indications, it is doubtful that pure thought will resolve the riddle of flavor. So my plea, to my experimentalist friends, is simple:
PLEASE FIND SOMETHING ANOMALOUS IN THE FLAVOR SECTOR

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