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THE WEAK NEUTRAL CURRENT

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## THE WEAK NEUTRAL CURRENT\*

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### Abstract

The status and implications of the weak neutral current experiments are briefly reviewed.

It is now generally accepted that the standard electroweak model [1] is correct to an excellent first approximation, and almost all activity in the field is devoted to searching for or predicting the form of new physics beyond the standard model. It is sometimes useful, however, to step back and reexamine how well the standard model is really tested, and how stringent the constraints on possible new physics really are.

Ever since the discovery of weak neutral current-induced neutrino scattering events by the Gargamelle collaboration [2] at CERN in 1973, the weak neutral current (combined with the  $W$  and  $Z$  masses) has provided one of the most important quantitative tests of the standard model. The first set of neutrino scattering experiments in the mid 1970's confirmed the existence of the neutral current and that its form was compatible with the predictions of the  $SU_2 \times U_1$  model. A second generation of experiments [3], completed around 1980, probed the neutral current couplings of the  $\bar{\nu}_\mu^{(-)}$ ,  $e$ ,  $u$ , and  $d$  quantitatively in neutrino-hadron (deep inelastic scattering,  $\bar{\nu}_p^{(-)}$  elastic scattering, exclusive and inclusive pion production, and

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deuteron dissociation),  $\nu_{\mu}^{(-)}e$  and  $\bar{\nu}_{e}e$  scattering, and weak-electromagnetic interference effects in polarized  $eD$  asymmetries and parity violation in heavy atoms. The experiments obtained accurate values for  $\sin^2 \theta_W$  (for comparison with grand unification) and for  $\rho \equiv M_W^2 / (M_Z^2 \cos^2 \theta_W)$ , which is sensitive to the  $SU_2 \times U_1$  properties of the Higgs fields in the theory. In addition, it was found that the neutral current couplings relevant to  $\nu_{\mu}u$  and  $\nu_{\mu}d$  interactions were *uniquely* determined - in agreement with the standard model predictions - as were the  $cu$  and  $cd$  couplings relevant to parity violation in heavy atoms: *i.e.* the interactions were those of the standard model (plus possible small perturbations), and not those of some completely different gauge theory. The  $\nu e$  couplings were determined to within a two-fold ambiguity.

A third generation of experiments<sup>1</sup>, completed in the last few years, has extended the results in several ways. Many more reactions and quantities have been studied, including the  $W$  and  $Z$  masses, coherent pion production in  $\nu N \rightarrow \nu \pi^0 N$ ,  $\nu_e e$  scattering, precise measurements of atomic parity violation in cesium, asymmetries in polarized  $\mu C$  and quasi-elastic  $e Be$  scattering, forward-backward asymmetries in  $e^+ e^- \rightarrow e^+ e^-$ ,  $\mu^+ \mu^-$ ,  $\tau^+ \tau^-$ ,  $c\bar{c}$ , and  $b\bar{b}$ , as well as weak contributions to the total cross section for  $e^+ e^- \rightarrow \mu^+ \mu^-$ ,  $\tau^+ \tau^-$ , and hadrons. Not only do these probe a great variety of couplings, they extend over an enormous kinematic range (from  $\sim 10^{-6} \text{ GeV}^2$  to  $10^4 \text{ GeV}^2$ .) Finally, the third generation of experiments are very precise (to  $\sim 1\%$  accuracy for the deep inelastic neutrino scattering experiments). Because of these improvements, the recent experiments allow precise quantitative tests of the tree level structure of the standard model, and even a rough test of the radiative corrections. Combined with charged current data it is now possible to prove directly from the data that the  $SU_2 \times U_1$  assignments of all of the known fermions (*i.e.* left-handed doublets and right-handed singlets) are unique [6]. The standard model really is correct to first approximation (at least for the tree level gauge couplings of the known fermions) down to a distance scale of some  $10^{-16} \text{ cm}$ . Furthermore, the data set an upper limit on the top quark mass and place significant limits on many possible types of new physics beyond the standard model, with a sensitivity extending up to several hundred  $\text{GeV}$ .

A fourth generation of experiments of much greater sensitivity will be running in the next few years. These include ultra-precise measurements of  $M_Z$  (and later a less precise measurement of  $M_W$ ) and of forward-backward and polarization asymmetries at LEP and SLC, a new  $\nu_{\mu}e$  experiment at CERN which will

<sup>1</sup>For complete sets of references see [4,5]

measure  $\sin^2 \theta_W$  to  $\pm 0.005$  (and a later Los Alamos experiment accurate to  $\pm 0.002$ ) in an environment free from theoretical uncertainties, and improved atomic parity violation experiments in cesium at Paris and Boulder which should be accurate to 1% (including uncertainties from the atomic wave functions). Collectively, these experiments will yield an order of magnitude increase in the sensitivity of the tests of the electroweak model, allowing precise tests of the radiative corrections and probing many types of new physics up to the 500 GeV – 1 TeV range. These will be an important complement to direct searches for new particles at high energy colliders.

About five years ago U. Amaldi, A. Böhm, L. S. Durkin, A. K. Mann, W. J. Marciano, A. Sirlin, H. H. Williams, and I decided that it would be useful to form an ongoing collaboration to collect and systematically study all neutral current results. In particular we wanted to provide a systematic critical analysis of the experiments, apply the best possible theoretical analysis (with similar experiments treated uniformly), and to derive realistic theoretical uncertainties for the derived quantities. Our goals were to extract a global average value for  $\sin^2 \theta_W$  with meaningful uncertainties for comparison with the predictions of grand unification, test the standard model at the level of radiative corrections, and to search for and set limits on possible deviations from the standard model. The results of the first phase of this project, based on the third generation of experiments, was completed about a year ago [4]. A similar analysis, with results in reasonable agreement, was performed by Costa et al [5]. Here, I will summarize the conclusions of these and some related studies.

### Formalism

The neutral current interaction in the standard  $SU_2 \times U_1$  model is given by  $L_{NC} = -g J_Z^\mu Z_\mu / (2 \cos \theta_W)$ , where  $Z$  is the massive neutral gauge boson,  $\theta_W \equiv \tan^{-1}(g'/g)$  is the weak angle, and  $g$  and  $g'$  are the  $SU_2$  and  $U_1$  gauge couplings, respectively. The positron electric charge is related by  $e = g \sin \theta$ . The neutral current is given by

$$\begin{aligned} J_Z^\mu &= \sum_i t_{3L}(i) \bar{\psi}_i \gamma^\mu (1 + \gamma^5) \psi_i + t_{3R}(i) \bar{\psi}_i \gamma^\mu (1 - \gamma^5) \psi_i - 2 \sin^2 \theta_W J_{EM}^\mu \\ &= \sum_i \bar{\psi}_i \gamma^\mu (V^i + A^i \gamma^5) \psi_i, \end{aligned} \quad (1)$$

where  $t_{3L}(i)$  is the  $T_3$  eigenvalue of the left-handed component of fermion  $i$  ( $+1/2$  for  $u_i$  and  $\nu_i$ ;  $-1/2$  for  $d_i$  and  $e_i$ ). Similarly,  $t_{3R}(i)$  is the  $T_3$  eigenvalue of the right-handed component of  $\psi_i$ . It is zero in the standard model but could be non-zero

in generalizations with exotic fermions in right-handed doublets. The vector and axial couplings are  $V^i \equiv t_{3L}(i) + t_{3R}(i) - 2\sin^2 \theta_W q_i$  and  $A^i \equiv t_{3L}(i) - t_{3R}(i)$ . At low momenta the neutral current interaction is described by the effective interaction

$$-L_{eff}^{NC} = \rho \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}, \quad (2)$$

where  $\rho \equiv M_W^2/(M_Z^2 \cos^2 \theta_W)$  is unity in the minimal model and in generalizations with extra Higgs doublets, but can differ from one in models which have Higgs triplets, *etc.*, with nonzero vacuum expectation values (VEV's).

The data are now sufficiently precise that one must include higher order (radiative) corrections. For this one must choose a set of renormalized parameters. It is convenient to trade  $g$ ,  $g'$ , and  $M_W$  (which measures the SSB scale), by the fine structure constant (defined conventionally), the Fermi constant (defined in terms of the muon lifetime), and the weak angle. A useful definition (used below) of the latter quantity is the Sirlin mass definition [7], in which one takes the tree level formula  $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$  as the definition of the renormalized  $\sin^2 \theta_W$  to all orders in perturbation theory. An alternative is to use the modified minimal subtraction ( $\overline{MS}$ ) quantity  $\sin^2 \hat{\theta}_W(\mu)$ , where  $\mu$  is conveniently chosen to be  $M_W$  for electroweak processes. The two definitions are related by  $\sin^2 \hat{\theta}_W(M_W) = C(m_t, M_H)\sin^2 \theta_W$ , where  $C = 0.9907$  for top quark and Higgs masses of  $m_t = 45 \text{ GeV}$  and  $M_H = 100 \text{ GeV}$ . Yet another possibility is to take  $M_Z$  rather than  $\sin^2 \theta_W$  as the third fundamental parameter. This will be useful when very precise values of  $M_Z$  are determined at SLC and LEP. In the mass definition scheme one has

$$M_W = \frac{A_0}{\sin \theta_W (1 - \Delta r)^{1/2}}, \quad M_Z = \frac{M_W}{\cos \theta_W} \quad (3)$$

where  $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.281 \text{ GeV}$ . The radiative correction parameter  $\Delta r$  is predicted to be  $0.0713 \pm 0.0013$  for  $m_t = 45 \text{ GeV}$  and  $M_H = 100 \text{ GeV}$ , while  $\Delta r \rightarrow 0$  for  $m_t \sim 245 \text{ GeV}$ .

### Model Independent Analysis

It is convenient to write the terms in  $-L_{eff}^{NC}$  relevant to  $\nu$ -hadron,  $\nu c$ , and  $c$ -hadron processes in a form that is valid in an arbitrary gauge theory (assuming massless left-handed neutrinos). One has

$$-L^{\nu H} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\mu (1 + \gamma^5) \nu \left\{ \sum_i [\epsilon_L(i) \bar{q}_i \gamma_\mu (1 + \gamma^5) q_i + \epsilon_R(i) \bar{q}_i \gamma_\mu (1 - \gamma^5) q_i] \right\}, \quad (4)$$

$$-L^{\nu e} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1 + \gamma^5) \nu_\mu \bar{e} \gamma_\mu (g_V^e + g_A^e \gamma^5) e \quad (5)$$

(for  $\bar{\nu}_e^{(-)}$  the charged current contribution must be included), and

$$-L^{\epsilon H} = \frac{G_F}{\sqrt{2}} \sum_i [C_{1i} \bar{e} \gamma_\mu \gamma^5 e \bar{q}_i \gamma^\mu q_i + C_{2i} \bar{e} \gamma_\mu e \bar{q}_i \gamma^\mu \gamma^5 q_i] \quad (6)$$

The standard model expressions for  $\epsilon_{L,R}(i)$ ,  $g_{V,A}^f$  and  $C_{ij}$  are given in Table 1. The results of model independent fits to the data to determine these parameters are given in Figures 1-3 and in Table 2, along with the predictions of the standard model. It is seen that the standard model expressions for the neutral current couplings of the light fermions are correct to an excellent first approximation.

**$\sin^2 \theta_W$ ,  $m_t$ , and radiative corrections.**

$\sin^2 \theta_W$  and, equivalently,  $M_Z$  have been determined from the  $W$  and  $Z$  masses and in a variety of neutral current processes spanning a very wide  $Q^2$  range. The results [4], shown in Table 3 and Figure 4, are in impressive agreement with each other, indicating the quantitative success of the standard model. The best fit to all data yields  $\sin^2 \theta_W = 0.230 \pm 0.0048$  and  $M_Z = 92.0 \pm 0.7 \text{ GeV}$ , where the errors (as well as those given below for other neutral current parameters) include full statistical, systematic, and theoretical uncertainties. The corresponding value of  $\sin^2 \hat{\theta}_W(M_W)$  (for fixed  $m_t = 45 \text{ GeV}$ ,  $M_H = 100 \text{ GeV}$ ) is  $0.228 \pm 0.0044$ . This is larger by  $\simeq 2.5 \sigma$  than the prediction  $0.214_{-0.004}^{+0.003}$  of minimal  $SU_5$  (for  $\Lambda_{\overline{MS}}^{(4)} = 150_{-75}^{+150} \text{ McV}$ ) and other "great desert" models. It is closer to (but still somewhat below) the prediction of the simplest supersymmetric GUTs. (Typically  $0.237_{-0.004}^{+0.003}$  for  $M_{SUSY} \sim M_W$ , decreasing by  $\sim 0.003$  for  $M_{SUSY} \sim 10 \text{ TeV}$ ). Similar conclusions hold for all values of  $m_t$  and  $M_H$ , as can be seen in Figure 5. The discrepancy with minimal  $SU_5$  can also be seen in Figure 6, in which are shown the couplings  $\alpha_i(Q^2)^{-1}$  determined by the experimental values of  $\alpha$ ,  $\Lambda_{\overline{MS}}^{(4)}$ , and  $\sin^2 \hat{\theta}_W(M_W)$ . It is seen that the  $\alpha_i^{-1}$  do not all meet at a point, although the statistical significance is not compelling.

The radiative corrections are sensitive to the isospin breaking associated with a large  $m_t$ . The  $\sin^2 \theta_W$  values determined from the various reactions are consistent with each other for  $m_t \leq 200 \text{ GeV}$ , but disagree for very large  $m_t$ . A simultaneous fit to  $\sin^2 \theta_W$  and  $m_t$  requires  $m_t < 180 \text{ GeV}$  at 90% *c.l.* for  $M_H \leq 100 \text{ GeV}$ , with a slightly weaker limit for larger  $M_H$ . The allowed region in  $\sin^2 \hat{\theta}_W(M_W)$  and  $m_t$  is shown in Figure 5. Similar limits hold for the mass splittings between fourth generation quarks or leptons. These results assume that there is no compensating

new physics such as non-doublet Higgs representations (discussed below). In the future precise determinations of the  $Z$  mass at SLC and LEP, combined with existing neutral current data (and eventually with  $e^+e^-$  asymmetries), will tightly constrain  $m_t$  or imply new physics [9].

The measured values of  $M_W$  and  $M_Z$  are given in Table 4. They are in excellent agreement with the predictions of the standard model when full radiative corrections (to both the  $W$  and  $Z$  mass formulas and to deep inelastic scattering) are included, but disagree significantly when the corrections are excluded. From the data one obtains  $\Delta r = 0.077 \pm 0.037$ , in excellent agreement with the value  $0.0713 \pm 0.0013$  predicted for  $m_t = 45 \text{ GeV}$  and  $M_H = 100 \text{ GeV}$ . The allowed region for  $\sin^2 \theta_W$  and  $\Delta r$  is shown in Figure 7.

A related parameter [4]  $\delta_W$  is defined by

$$M_W = \frac{A_0}{\sin \theta^0 (1 - \delta_W)^{1/2}} \quad (7)$$

where  $\sin^2 \theta^0$  is the value ( $.242 \pm .006$ ) obtained for the weak angle from deep inelastic scattering if all radiative corrections (to both  $\sigma^{NC}$  and  $\sigma^{CC}$ ) are ignored.  $\delta_W$ , which incorporates all of the radiative corrections relating deep inelastic scattering, the  $W$  and  $Z$  masses, and muon decay, is found to be  $0.112 \pm 0.037$ . This agrees with the prediction  $0.106 \pm 0.004$  (for the same  $m_t$  and  $M_H$  as above) and establishes the existence of radiative corrections at the  $3\sigma$  level.

#### $\rho$ and $\sin^2 \theta_W$

The  $W$  and  $Z$  masses and neutral current data can be used to search for and set limits on deviations from the standard model. For example, the parameter  $\rho \equiv M_W^2 / (M_Z^2 \cos^2 \theta_W)$  can differ from unity if there are Higgs multiplets with weak isospin  $> \frac{1}{2}$  with significant vacuum expectation values. One has the tree level result

$$\rho = \frac{\sum_i (t_i^2 - t_{3i}^2 - t_i) \cdot \langle \phi_i \rangle^2}{\sum_i 2t_{3i}^2 \cdot \langle \phi_i \rangle^2} \quad (8)$$

where  $\langle \phi_i \rangle$  is the VEV of a Higgs field  $\phi_i$  with weak isospin and z-component  $t_i$  and  $t_{3i}$ , respectively.

If the new physics which yields  $\rho \neq 1$  is a small perturbation which does not significantly affect the radiative corrections, one can use the standard model expressions for  $\Delta r$  and the other radiative correction parameters in Table 1. Then  $\rho$  can be regarded as a phenomenological parameter which multiplies  $L_{if}^{NC}$  in (2). (Also, the expression for  $M_Z$  in (3) is divided by  $\sqrt{\rho}$ ; the  $M_W$  formula is unchanged.) The allowed regions in the  $\rho - \sin^2 \theta_W$  plane are shown in Figure 8, and



the values of  $\sin^2 \theta_W$  and  $\rho$  from various reactions are given in Table 5. A global fit to all data yields [4]  $\rho = 0.998 \pm 0.0086$ , remarkably close to unity (justifying the neglect of  $\rho - 1$  in the radiative corrections). This implies 90% *c.l.* upper limits of 0.047 and 0.081 for the VEVs (relative to those of Higgs doublets) for Higgs triplets with  $t_{3i} = 0$  or  $\pm 1$ , respectively.

### Extra $Z$ bosons

Many extensions of the standard model predict the existence of extra  $Z$  bosons. These affect the neutral current because (a) the mass of the ordinary  $Z$  is reduced because of mixing, (b) the couplings of the ordinary  $Z$  are modified by mixing, and (c) the extra  $Z$  boson can be exchanged. For a large class of  $E_6$  models, for example, the extra boson is  $Z(\beta) = \cos \beta Z_\chi + \sin \beta Z_\psi$ , where  $Z_\chi$  and  $Z_\psi$  are the gauge bosons which occur for  $SO_{10} \rightarrow SU_5 \times U_{1\chi}$  and  $E_6 \rightarrow SO_{10} \times U_{1\psi}$ , respectively. The special case  $Z_\eta = \sqrt{\frac{3}{8}} Z_\chi - \sqrt{\frac{5}{8}} Z_\psi = -Z(\beta = \pi - \tan^{-1} \sqrt{\frac{5}{3}})$  occurs in many superstring models. At present the best limits on extra  $Z$ 's are usually from the neutral current data rather than from direct searches at hadron colliders, but the limits are fairly weak (typically 120 - 300 GeV for GUT-inspired bosons) because of the relatively weak coupling to ordinary fermions predicted in most models. The limits for the  $E_6$  bosons are shown in Figure 9 and Table 6 for two cases: (a) in the constrained Higgs case, in which it is assumed that all  $SU_2$  breaking is due to Higgs doublets (analogous to  $\rho = 1$ ), there is a relation between the masses of the two  $Z$ 's and the mixing angle [12]. (b) In the unconstrained case no assumption is made concerning the Higgs sector.

It is seen that the limits on the  $Z_\eta$  are extremely weak. That is because the effects of mixing and of  $Z_\eta$  exchange largely cancel each other for large negative mixing angles. Fortunately, most specific models require positive angles, so the limits are considerably more stringent in those special cases. In particular, in supersymmetric  $E_6$  models (including superstrings) it is generally assumed that the Higgs doublets transform as  $\underline{27}$ -plets. The mixing depends on a parameter  $\lambda = v_1^2/v_2^2$ , where  $v_1$  and  $v_2$  are the expectation values of the two appropriate Higgs doublets. For  $\lambda > 1$ , which is favored by most models (in order to explain why  $m_t \gg m_b$ , for example), one has much more stringent limits, as can be seen in Table 6 and Figure 10.

In the future, indirect limits on heavy  $Z$ 's from LEP/SLC asymmetries should extend up to 500 - 1000 GeV, and direct searches at the SSC would be sensitive to several TeV.

## Other Implications

There are a number of other implications of neutral current data. For example, there are now enough constraints from charged and neutral current experiments to prove directly that all known left-handed fermions transform as  $SU_2$  doublets and that all known right-handed fermions are  $SU_2$  singlets<sup>2</sup> [6]. As one example, the axial couplings of the  $\mu$ ,  $\tau$ ,  $c$ , and  $b$  in Table 2 (given the reasonable assumption that  $A^c$  is canonical) are in agreement with the standard model. This implies that the  $\tau_L$  and  $b_L$  must belong to  $SU_2$  doublets if there are no fermions with exotic charges, and hence that the  $\nu_\tau$  and the  $t$  quark *must exist*. (Many but not all topless models are also excluded by the long  $b$  lifetime and the absence of flavor-changing  $b$  decays.) Similarly, charged and neutral current data can be combined to set fairly stringent limits [14] on the possible mixing between ordinary fermions and the new fermions with unusual weak interactions (*e.g.* right-handed doublets or left-handed singlets) that are predicted by many extensions of the standard model.

Many types of new physics (*e.g.* composite fermions) lead to effective new four fermion operators. For example, Martyn [15] has recently analyzed limits on electron-quark contact terms of the form

$$L^{e q} = \pm \sum_{a,b=L,R} \frac{g^2}{\Lambda_{\pm}^2} \eta_{ab} \bar{e}_a \gamma^\mu e_a \bar{q}_b \gamma_\mu q_b, \quad (9)$$

where each  $\eta_{ab}$  is zero or one, from the  $e^+ e^-$  annihilation cross section. He obtains lower limits on the various  $\Lambda$ 's that are typically in the range 1 - 3  $TeV$  from existing data (with the conventional definition  $g^2 = 4\pi$ .) These should be improved by a factor of two by measurements at HERA [15]. Similarly, the precise experiments on atomic parity violation in cesium from Paris and Boulder [16] are extremely sensitive to the particular electron-quark contact term

$$L^{e q} = \pm \frac{g^2}{\Lambda_{\pm}^2} \bar{e} \gamma^\mu \gamma^5 e (\bar{u} \gamma_\mu u + d \gamma_\mu d). \quad (10)$$

For  $g^2 = 4\pi$  one obtains [17] the extremely impressive limits  $\Lambda_+ > 14.6 TeV$ ,  $\Lambda_- > 10.2 TeV$  at 90% *c.l.* Future improvements in the measurements (and the calculation of the atomic effects) should improve these limits by a factor of 3.

One can also use the atomic parity violation data to set lower limits of order 300  $GeV$  (for couplings of electromagnetic strength) on the masses of lepto-quark bosons [17]. For certain types of couplings these are the only existing constraints.

<sup>2</sup>The only major assumption is that the known particles are not in  $SU_2$  multiplets with particles with exotic electric charges.

## References

- [1] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam in *Elementary Particle Theory*, ed. N. Svartholm (Almquist and Wiksells, Stockholm, 1969) p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. **D2**, 1285 (1970).
- [2] F. J. Hasert *et al.*, Phys. Lett. **46B**, 121, 138 (1973).
- [3] For a review, see J.E. Kim *et al.*, Rev. Mod. Phys. **53**, 211 (1981).
- [4] U. Amaldi, A. Böhm, L. S. Durkin, P. Langacker, A. K. Mann, W. J. Marciano, A. Sirlin, and H. H. Williams, Phys. Rev. **D36**, 1385 (1987).
- [5] G. Costa, J. Ellis, G. L. Fogli, D. V. Nanopoulos, and F. Zwirner, Nucl. Phys. **B297**, 244 (1988).
- [6] P. Langacker, to be published.
- [7] A. Sirlin, Phys. Rev. **D22**, 971 (1980); **D29**, 89 (1984).
- [8] R. Marshall, Rutherford preprint RAL-87-031.
- [9] P. Langacker, W. J. Marciano, and A. Sirlin, Phys. Rev. **D36**, 2191 (1987).
- [10] UA2: R. Ansari *et al.*, Phys. Lett. **186B**, 440 (1987).
- [11] UA1: G. Arnison *et al.*, Phys. Lett. **166B**, 484 (1986).
- [12] P. Langacker and L. S. Durkin, Phys. Lett. **166B**, 436 (1986); P. Langacker Phys. Rev. **D30**, 2008 (1984).
- [13] V. Barger *et al.*, Phys. Rev. **D35**, 2893 (1987); J. Ellis, P. Franzini, and F. Zwirner, Phys. Lett. **202B**, 417 (1988).
- [14] P. Langacker and D. London DESY 83-043, 83-044; J. Maalampi and M. Roos, Helsinki HU-TFT-88-17.
- [15] H.-U. Martyn, Aachen preprint PITHA 87-40.
- [16] Paris: M. A. Bouchiat *et al.*, Phys. Lett. **134B**, 463 (1984); Boulder: S. L. Gilbert *et al.*, Phys. Rev. **A34**, 792 (1986).
- [17] P. Langacker and R. Rückl, to be published.

Table 1: Standard model expressions for the neutral current parameters for  $\nu$ -hadron,  $\nu e$ , and  $e$ -hadron processes. If radiative corrections are ignored,  $\rho = \kappa = 1$ ,  $\lambda = 0$ . The  $O(\alpha)$  values are given in [4].

Quantity	Standard Model Expression
$\epsilon_L(u)$	$\rho_{\nu N}^{NC} [\frac{1}{2} - \frac{2}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda_{uL}]$
$\epsilon_L(d)$	$\rho_{\nu N}^{NC} [-\frac{1}{2} + \frac{1}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda_{dL}]$
$\epsilon_R(u)$	$\rho_{\nu N}^{NC} [-\frac{2}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda_{uR}]$
$\epsilon_R(d)$	$\rho_{\nu N}^{NC} [\frac{1}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda_{dR}]$
$g_V^e$	$\rho_{\nu e} [-\frac{1}{2} + 2\kappa_{\nu e} \sin^2 \theta_W]$
$g_A^e$	$\rho_{\nu e} [-\frac{1}{2}]$
$C_{1u}$	$\rho'_{eq} [-\frac{1}{2} + \frac{4}{3} \kappa'_{eq} \sin^2 \theta_W]$
$C_{1d}$	$\rho'_{eq} [\frac{1}{2} - \frac{2}{3} \kappa'_{eq} \sin^2 \theta_W]$
$C_{2u}$	$\rho_{eq} [-\frac{1}{2} + 2\kappa_{eq} \sin^2 \theta_W]$
$C_{2d}$	$-C_{2u}$

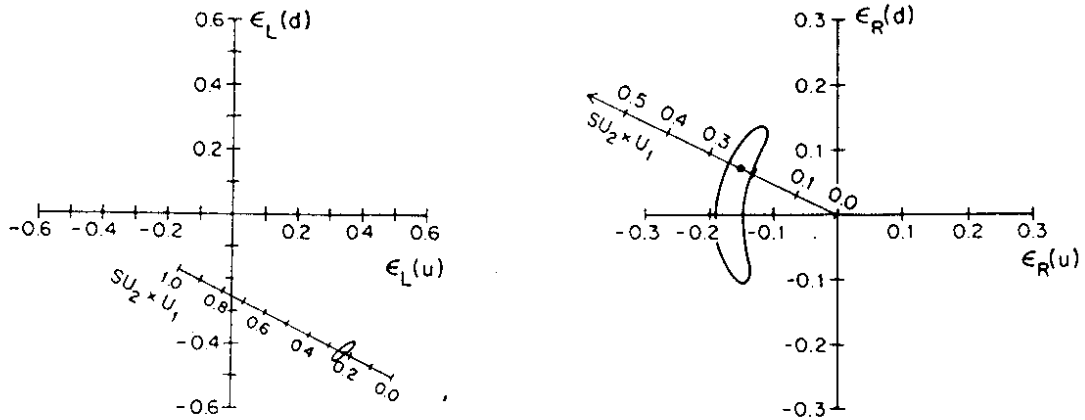


Figure 1: Allowed regions at 90% *c.l.* for the (weak) model independent  $\nu q$  parameters  $\epsilon_i(u)$  and  $\epsilon_i(d)$ ,  $i = L$  or  $R$  and the predictions of the standard model as a function of  $\sin^2 \theta_W$ .

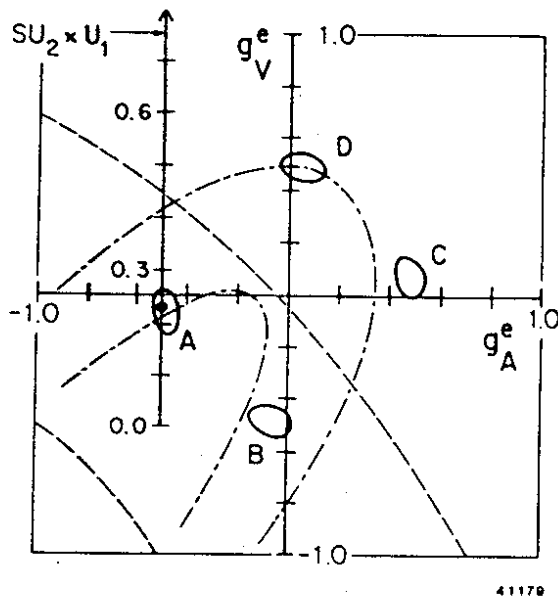


Figure 2: Allowed regions (90% c.l.) for the  $\nu e$  parameters  $g_V^e$  and  $g_A^e$ , for  $\bar{\nu}_\mu e$  (solid lines), reactor  $\bar{\nu}_e e$  (dot-dash), and  $\nu_e e$  (dash). The latter reactions, which involve the interference between charged and neutral current contributions, eliminate two solutions ((C) and (D)) that are allowed by  $\bar{\nu}_\mu e$ . There are still two solutions (axial dominant (A) and vector dominant (B)) that are consistent with all  $\nu e$  data. However, solution (B) is eliminated by the  $e^+e^- \rightarrow \mu^+\mu^-$  forward-backward asymmetry under the (now very reasonable) assumption that the neutral current is dominated by the exchange of a single  $Z$ .

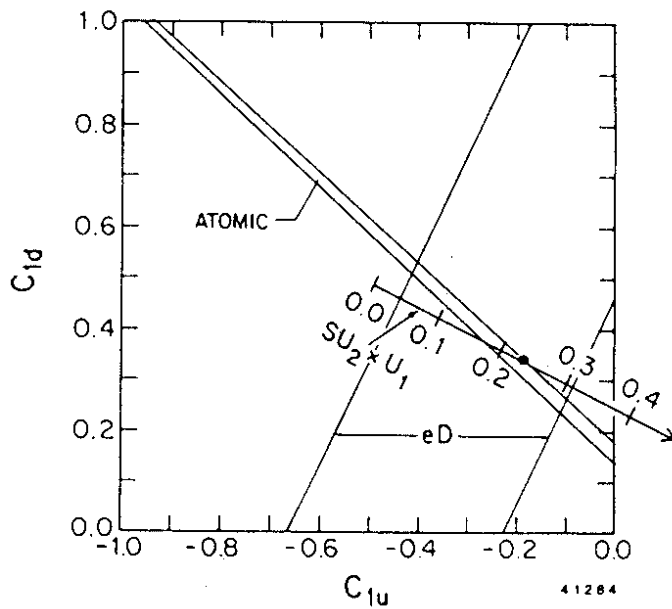


Figure 3: Allowed regions (90% c.l.) for the parity-violating  $eq$  parameters  $C_{1u}$  and  $C_{1d}$  allowed by atomic parity violation and the SLAC polarized  $\epsilon D$  asymmetry (which yield nearly orthogonal constraints), compared with the predictions of the standard model as a function of  $\sin^2 \theta_W$ .

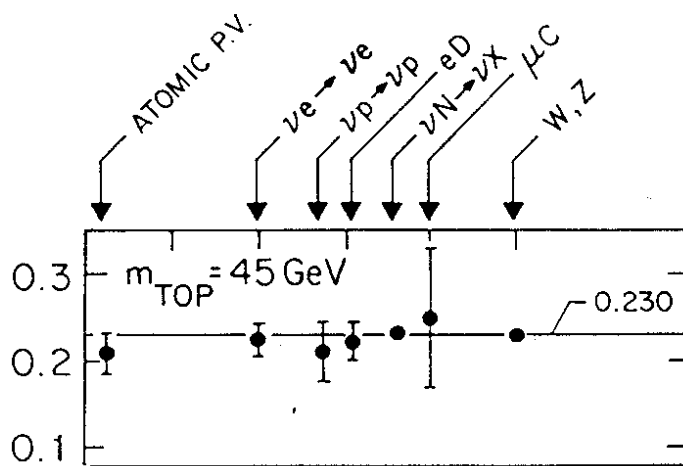


Figure 4:  $\sin^2 \theta_W$  for various reactions as a function of the typical  $Q^2$ , determined for  $m_t = 45 \text{ GeV}$ . The best fit line  $\sin^2 \theta_W = 0.230$  is also shown.

Table 2: Values of the model independent neutral current parameters, compared with the standard model prediction for  $\sin^2 \theta_W = 0.230$ . Correlations are not given for the neutrino-hadron couplings because of the non-Gaussian  $\chi^2$  distributions. However, the neutrino-hadron constraints are accurately represented by the ranges listed for the variables  $g_i^2 \equiv \epsilon_i(u)^2 + \epsilon_i(d)^2$  and  $\theta_i \equiv \tan^{-1}(\epsilon_i(u)/\epsilon_i(d))$ ,  $i = L$  or  $R$ , which are very weakly correlated. Also listed are the products of axial couplings obtained from forward-backward asymmetries in  $e^+e^-$  annihilation, obtained assuming that only a single  $Z$  is relevant [4,8].

Quantity	Experimental Value	Standard Model Prediction	Correlation	
$\epsilon_L(u)$	$0.339 \pm .017$	0.345		
$\epsilon_L(d)$	$-0.429 \pm .014$	-0.427		
$\epsilon_R(u)$	$-0.172 \pm .014$	-0.152		
$\epsilon_R(d)$	$-0.011^{+.081}_{-.057}$	0.076		
$g_L^2$	$0.2996 \pm 0.0044$	0.301		
$g_R^2$	$0.0298 \pm 0.0038$	0.029		
$\theta_L$	$2.47 \pm 0.04$	2.46		
$\theta_R$	$4.65^{+0.48}_{-0.32}$	5.18		
$g_A^e$	$-0.498 \pm .027$	-0.503	-0.08	
$g_V^e$	$-0.044 \pm .036$	-0.045		
$C_{1u}$	$-0.249 \pm 0.071$	-0.191	-0.98	-0.88
$C_{1d}$	$0.381 \pm 0.064$	0.340		0.88
$C_{2u} - \frac{1}{2}C_{2d}$	$0.19 \pm 0.37$	-0.039		
$A^e A^u$	$0.272 \pm 0.015$	1/4		
$A^e A^d$	$0.232 \pm 0.026$	1/4		
$A^e A^e$	$-0.33 \pm 0.08$	-1/4		
$A^e A^b$	$0.27 \pm 0.07$	1/4		

Table 3: Determination of  $\sin^2 \theta_W$  and  $M_Z$  (in  $GeV$ ) from various reactions. The central values of all fits assume  $m_t = 45 GeV$  and  $M_H = 100 GeV$  in the radiative corrections. Where two errors are shown the first is experimental and the second (in square brackets) is theoretical, computed assuming 3 fermion families,  $m_t < 100 GeV$ , and  $M_H < 1 TeV$ . In the other cases the theoretical and experimental uncertainties are combined. When  $m_t$  is allowed to be totally arbitrary the fits to all data yield  $\sin^2 \theta_W = 0.229 \pm 0.007$  and  $M_Z = 91.8 \pm 0.9 GeV$ . The existing  $e^+e^-$  data do not yield a useful determination of  $\sin^2 \theta_W$ : at PEP and PETRA energies the asymmetries are nearly an absolute prediction of the model, and all values of  $\sin^2 \theta_W$  from 0.1 to 0.4 give a good description of the data. (The  $e^+e^-$  asymmetries are nearly independent of  $m_t$  as well.)

Reaction	$\sin^2 \theta_W$	$M_Z$
Deep inelastic (isoscalar)	$0.233 \pm .003 \pm [.005]$	$91.6 \pm 0.4 \pm [0.8]$
$\overset{(-)}{\nu}_{\mu} p \rightarrow \overset{(-)}{\nu}_{\mu} p$	$0.210 \pm .033$	$95.0 \pm 5.2$
$\overset{(-)}{\nu}_{\mu} e \rightarrow \overset{(-)}{\nu}_{\mu} e$	$0.223 \pm .018 \pm [.002]$	$93.0 \pm 2.7$
$W, Z$	$0.228 \pm .007 \pm [.002]$	$92.3 \pm 1.1$
Atomic parity violation	$0.209 \pm .018 \pm [.014]$	$95.1 \pm 3.9$
SLAC eD	$0.221 \pm .015 \pm [.013]$	$93.3 \pm 2.7$
$\mu C$	$0.25 \pm .08$	$89.6 \pm 9.7$
All data	$0.230 \pm 0.0048$	$92.0 \pm 0.7$



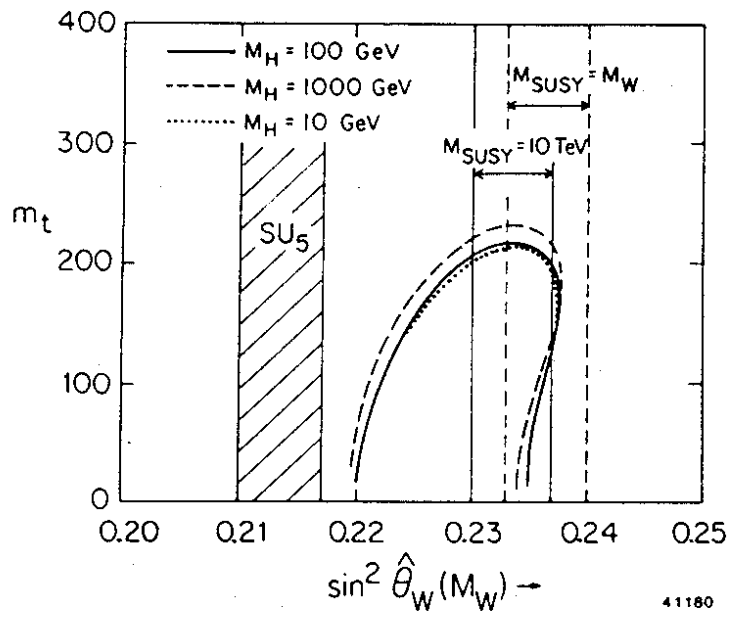


Figure 5: Allowed regions (90% *c.l.*) in  $\sin^2 \hat{\theta}_W(M_W)$  and  $m_t$  for fixed values of  $M_H$ . Also shown are the predictions of ordinary and supersymmetric GUTs, assuming no new thresholds between  $M_W$  or  $M_{SUSY}$  and the unification scale.

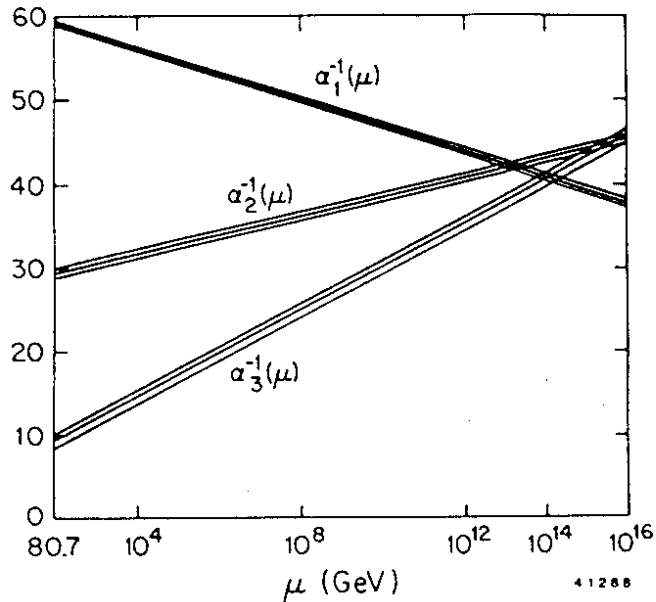


Figure 6: The running couplings  $\alpha_i^{-1}(Q^2)$  (68% *c.l.* bands), from ref [4].

Table 4: The  $W$  and  $Z$  masses (in  $GeV$ ). The first uncertainties are mainly statistical and the second are energy calibration uncertainties that are 100% correlated between  $M_W$  and  $M_Z$  for each group. The last two rows are predictions of the standard model, using  $\sin^2 \theta_W$  determined from deep inelastic scattering, with and without radiative corrections, respectively.

Group	$M_W$	$M_Z$
UA2 [10]	$80.2 \pm 0.8 \pm 1.3$	$91.5 \pm 1.2 \pm 1.7$
UA1 [11]	$83.5^{+1.1}_{-1.0} \pm 2.7$	$93.0 \pm 1.4 \pm 3.0$
UA1 + UA2 combined	$80.9 \pm 1.4$	$91.9 \pm 1.8$
Prediction (with radiative corrections)	$80.2 \pm 1.1$	$91.6 \pm 0.9$
Prediction (without radiative corrections)	$75.9 \pm 1.0$	$87.1 \pm 0.7$

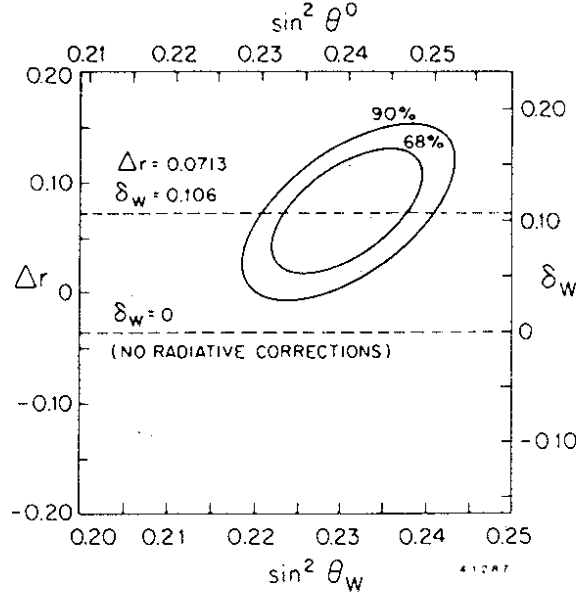


Figure 7: The allowed region in the  $\sin^2 \theta_W - \Delta r$  (or  $\sin^2 \theta^0 - \delta_w$ ) plane determined from deep inelastic (isoscalar) data and the  $W$  and  $Z$  masses.

Table 5: Determination of  $\rho$  and  $\sin^2 \theta_W$  from various reactions. Where two errors are shown the first is experimental and the second (in square brackets) is theoretical.

Reaction	$\sin^2 \theta_W$	$\rho$	Correlation
deep inelastic (isoscalar)	$0.232 \pm 0.014 \pm [0.008]$	$0.999 \pm .013 \pm [0.008]$	.90
$\nu_{\mu} P \rightarrow \nu_{\mu} P$	$0.205 \pm .041$	$0.98 \pm .06 \pm [.05]$	—
$\nu_{\mu} e \rightarrow \nu_{\mu} e$	$0.221 \pm .021 \pm [0.003]$	$0.976 \pm .056 \pm [0.002]$	.12
$W, Z$	$0.228 \pm .008 \pm [0.003]$	$1.015 \pm .026 \pm [0.004]$	.19
all data	$0.229 \pm .0064$	$0.998 \pm .0086$	.63

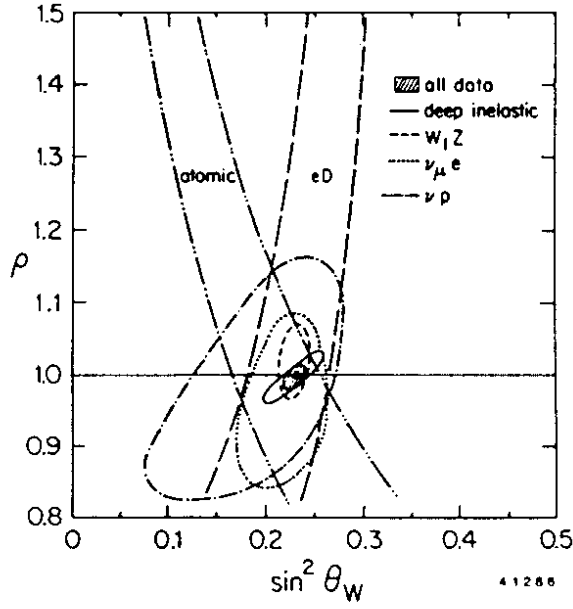


Figure 8: The allowed regions in  $\sin^2 \theta_W - \rho$  at 90% *c.l.* for various reactions.

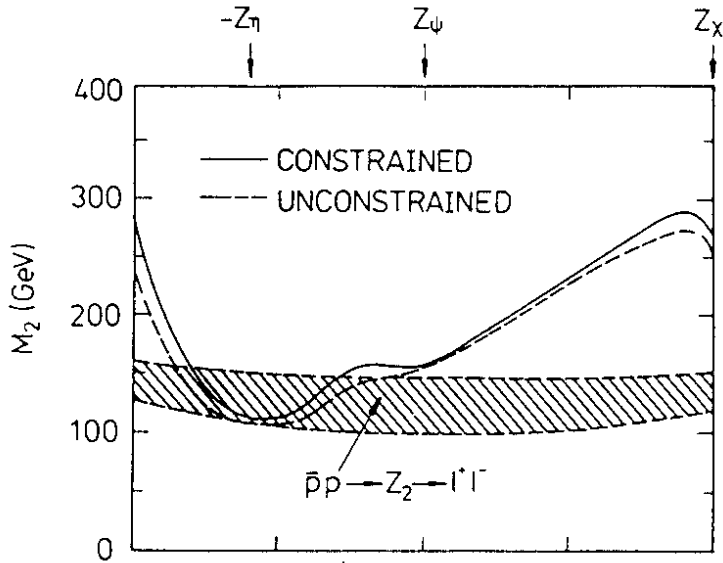


Figure 9: Lower limits on  $M_{Z_2}$  (at 90% *c.l.*) for an  $E_6$  boson  $Z(\beta) = \cos \beta Z_\chi + \sin \beta Z_\psi$  for constrained and unconstrained Higgs. The special cases  $Z_\chi$ ,  $Z_\psi$ , and  $-Z_\eta$  are indicated. Also shown are the direct production limits from searches for  $\bar{p}p \rightarrow Z_2 \rightarrow l^+ l^-$  from Ref. [12,13]. The error band is due to the fact that the limit depends on whether the exotic fermions predicted in the  $E_6$  model are light enough to reduce the  $Z_2 \rightarrow l^+ l^-$  branching ratio.

Table 6: Lower limits (90% c.l.) on the masses in GeV of various extra  $Z$  bosons.  $Z_{LR}$  refers to the extra  $Z$  in the  $SU_{2L} \times SU_{2R} \times U_1$  model. See [4] for the detailed assumptions concerning coupling constants, etc.

	Constrained	Unconstrained
$Z_\lambda$	273	249
$Z_\nu$	154	151
$Z_{LR}$	325	343
$Z_\eta$ ( $\lambda$ free)	111	112
$Z_\eta$ ( $\lambda = 0$ )	116	
$Z_\eta$ ( $\lambda = \frac{1}{2}$ )	222	
$Z_\eta$ ( $\lambda = 1$ )	318	
$Z_\eta$ ( $\lambda = 2$ )	410	
$Z_\eta$ ( $\lambda = 5$ )	496	

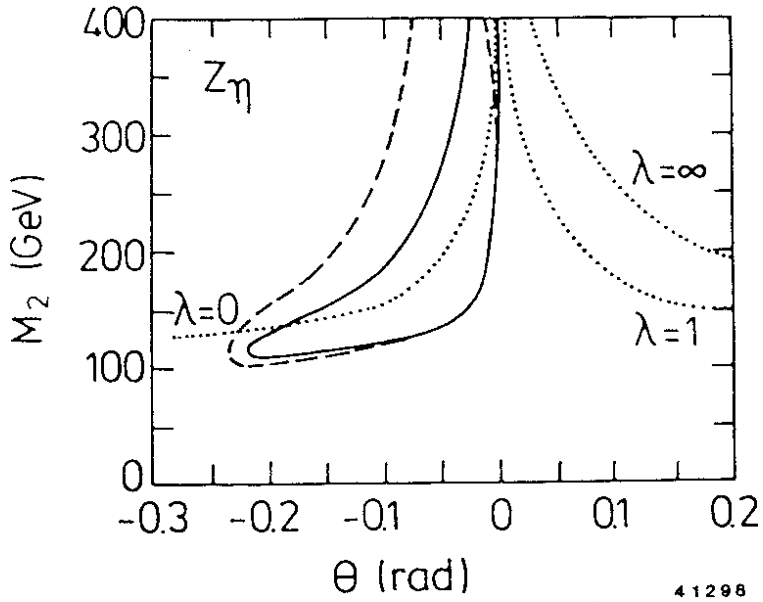


Figure 10: Allowed region for  $M_2$  and the  $Z_1 - Z_2$  mixing angle  $\theta$  (at 90% c.l.) for the superstring-inspired  $Z_\eta$ , along with the additional constraints imposed for fixed  $\lambda \equiv v_1^2/v_2^2$ .