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THE STRUCTURE OF THE FERMION MASS MATRICES FROM SYMMETRIES AND HIGHER DIMENSIONS

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ABSTRACT

All small quark and charged lepton masses and the small generation mixing angles can be understood in terms of a generation symmetry. This scenario introduces only one small parameter λ, characterizing the magnitude of spontaneous generation symmetry breaking. In the fermion mass matrices λ appears with various powers, which can be computed by group theory. These concepts emerge from higher dimensional unification, although a realistic compactification remains to be found.

1. STRUCTURE FROM SYMMETRY

The masses of quarks and charged leptons and the generation mixing angles show a very pronounced structure. The quark mixing angles are small and allow the identification of quark generations. Furthermore, one can associate to every d-type quark a charged lepton with similar mass. The mass eigenvalues exhibit a hierarchical pattern. They fall roughly into four groups: The so far unobserved top quark must be much heavier than all other known fermions. The second group, with masses in the GeV region, consists of b and c quark and the τ lepton. Next one finds m_s and m_μ somewhat above 100 MeV. Finally, the first generation masses m_d, m_u, m_e are in the MeV range.

1) Talk given at "Quark '88", Tbilisi 1988

Note that these four groups do not reflect exactly the generation pattern. Another hierarchy appears in the magnitude of the quark mixing angles: $\vartheta_{23} \approx \vartheta_{12}^2$, $\vartheta_{13} \approx \vartheta_{12}^3$.

In the standard model[1)] this structure is not explained. It is parametrized by dimensionless Yukawa couplings which differ by several orders of magnitude. They are related to the mass eigenvalues and the hierarchical mixings[2)] in the Kobayashi-Maskawa matrix. There have been many attempts[3)4)5)] to understand at least part of this structure from symmetries, special properties of mass matrices, radiative effects etc. In general the phenomenologically successful approaches still need small Yukawa couplings of a very different size at some step. I explore here the possibility that <u>all</u> the structure can be understood in terms of symmetry.[6)] I want to study models where the size of all dimensionless couplings is related to the gauge couplings g. In particular all Yukawa couplings have similar magnitude and are of order g. In these models the physics responsible for the fermion mass structure is associated with a mass scale M much above the Fermi scale. There will be many scalar doublets with mass of the order M. At low energies, however, the theory will look like the standard model with one Higgs doublet and associated small Yukawa couplings.

The central idea is that in the limit of an unbroken generation symmetry only the top quark acquires a mass, whereas all other chiral fermions remain massless. The smaller fermion masses and the generation mixing angles arise only as generation symmetry breaking effects. They will be suppressed by various powers of a small parameter. This parameter λ is the ratio between the scale φ_s, at which the generation group is spontaneously broken, and M. It is a measure for the size of generation symmetry breaking. Typical quark and lepton masses are of the order $m_i \sim \lambda^{P_i} \times M_W$ (M_W is the W boson mass) and mixings obtain as

$\vartheta_{ij} \sim \lambda^{P_{ij}}$. The power P of the suppression factor λ can be calculated from group theory.

2. THE m_b/m_t RATIO IN TERMS OF SYMMETRY

Let me start to illustrate these ideas in a simple one generation model with an abelian generation symmetry $U(1)_G$ (in addition to the gauge symmetries of the standard model). I want to demonstrate how the small mass ratios m_b/m_t and m_τ/m_t can be understood in terms of this symmetry. Assume that (t,b), (ν_τ,τ) and t^c have generation charge q=0 whereas b^c and τ^c have q=1. (I use a notation where all fermions are left handed, b^c being the anti-bottom quark etc.) Consider a model with two weak doublet scalars with charges q=0,1. The generation symmetry allows the following Yukawa couplings of the electrically neutral scalars

$$\mathcal{L}_Y = h_t t^c t \varphi_0 + h_b b^c b \varphi_1^* + h_\tau \tau^c \tau \varphi_1^* + h.c. \qquad (1)$$

At the unification scale M all Yukawa couplings should be of the order of the gauge coupling g. The difference in magnitude of m_t and m_b must therefore be related to a different size of the expectation values of φ_0 and φ_1.

The generation symmetry will be spontaneously broken by a singlet scalar χ with generation charge q=1. The most general renormalizable potential for the electrically neutral scalars φ_0, φ_1 and χ is

$$\begin{aligned} V = {} & \mu_\chi^2 \chi^*\chi + \mu_0^2 \varphi_0^*\varphi_0 + \mu_1^2 \varphi_1^*\varphi_1 \\ & + \tfrac{1}{2}\lambda_\chi (\chi^*\chi)^2 + \tfrac{1}{2}\lambda_0 (\varphi_0^*\varphi_0)^2 + \tfrac{1}{2}\lambda_1 (\varphi_1^*\varphi_1)^2 + \lambda_{01}\varphi_0^*\varphi_0\varphi_1^*\varphi_1 \\ & + \tilde{\alpha}\varphi_0^*\varphi_1\chi^* + \tilde{\alpha}^*\varphi_1^*\varphi_0\chi \\ & + \beta_0 \varphi_0^*\varphi_0\chi^*\chi + \beta_1\varphi_1^*\varphi_1\chi^*\chi \end{aligned} \qquad (2)$$

I assume that the couplings with dimension of mass reflect the unification scale M, $\mu_i^2 = \nu_i M^2$, $\tilde{\alpha} = \alpha M$ and

all dimensionless parameters in the potential($\lambda, \beta, \nu, \alpha$) are typically of the order g^2. Depending on the parameters, the potential (2) has a minimum for which both $U(1)_G$ and $SU(2)_L x U(1)_Y$ are spontaneously broken by vacuum expectation values $|\langle\chi\rangle| = \varphi_S$ and $\{|\langle\varphi_0\rangle|^2 + |\langle\varphi_1\rangle|^2\}^{1/2} = \varphi_L$. I am interested in a situation where the generation symmetry is broken somewhat below the unification scale, whereas the Fermi scale φ_L is much smaller than M

$$\frac{\varphi_S}{M} = \lambda \ll 1 \qquad (3)$$

$$\varphi_L \ll \varphi_S \qquad (4)$$

The ratios φ_L/M and λ will be the only small quantities[1] introduced in this theory (except g).

For nonvanishing χ the mass matrix for the doublets φ_0, φ_1 is given by

$$M_\varphi^2 = \begin{pmatrix} \nu_0 M^2 + \beta_0\chi^2 , & \alpha M\chi \\ \alpha M\chi & , \nu_1 M^2 + \beta_1 \chi^2 \end{pmatrix} \qquad (5)$$

(For simplicity I omit phases from now on.) In the limit of vanishing χ the generation symmetry forbids any mixing between φ_0 and φ_1. For $\chi \neq 0$, however, the mass eigenstates $\tilde{\varphi}_0$, $\tilde{\varphi}_1$ are linear combinations of the charge eigenstates φ_0, φ_1:

$$\begin{aligned} \varphi_0 &= \cos\gamma\, \tilde{\varphi}_0 - \sin\gamma\, \tilde{\varphi}_1 \\ \varphi_1 &= \cos\gamma\, \tilde{\varphi}_1 + \sin\gamma\, \tilde{\varphi}_0 \end{aligned} \qquad (6)$$

---

1) These small ratios may obtain as a result of tuning parameters or ultimately be explained by the short distance dynamics of unification. As an alternative they could follow from the complexity of long distance (QCD) physics.

For a small scale of generation symmetry breaking $\langle\chi\rangle = \varphi_S$ $\ll M$ the mixing between charge eigenstates is of the order

$$\gamma \approx \frac{\alpha M \varphi_S}{(\nu_0-\nu_1)M^2+(\beta_0-\beta_1)\varphi_S^2} = O(\lambda) \qquad (7)$$

Let me assume that the mass eigenvalue for $\tilde{\varphi}_1$ is positive ($M_1^2 \sim g^2M^2$) whereas the one for $\tilde{\varphi}_0$ is negative ($\nu_0 \lesssim (\alpha^2/\nu_1-\beta_0)\lambda^2$). Electroweak symmetry breaking is then related to the dynamics of $\tilde{\varphi}_0$ ($\langle\tilde{\varphi}_0\rangle=\varphi_L$, $\langle\tilde{\varphi}_1\rangle=0$). For $\varphi_L \ll \varphi_S$ the low energy Higgs doublet corresponds to the excitation $\tilde{\varphi}_0$, whereas $\tilde{\varphi}_1$ can be completely neglected at long distances.

Nevertheless, the fermion mass matrix reflects the fact that $\tilde{\varphi}_0$ is a linear combination of the generation charge eigenstates φ_0 and φ_1. In leading order only the top quark acquires a mass from electroweak symmetry breaking.

$$m_t \approx h_t \varphi_L \approx M_W \qquad (8)$$

However, due to the mixing (6), there is a nonzero expectation value $\langle\varphi_1\rangle$ of the order $\lambda\varphi_L$. Therefore b and τ acquire small masses induced by generation symmetry breaking.

$$m_{b,\tau} = h_{b,\tau}\,\gamma\,\varphi_L \approx \lambda g \varphi_L \approx \lambda M_W \qquad (9)$$

The low energy Yukawa couplings of $\tilde{\varphi}_0$ to b and τ are of the order λg and therefore much smaller than g. One may call φ_0 the "leading doublet" since the low energy Higgs doublet consists dominantly of a q=0 state. The effects of generation symmetry breaking can be represented graphically (fig. 1). Since the fermion bilinear operator appearing in the mass term for b and τ has generation charge q=1, the masses m_b, m_τ must be proportional to $\langle\chi\rangle$. One concludes by simple dimensional arguments

$$m_{b,\tau} \sim g\varphi_L\varphi_S/M\,;\quad m_{b,\tau} = c_{b,\tau}\,\lambda M_W \qquad (10)$$

The coefficients $c_{b,\tau}$ are given by ratios of dimensionless coupling constants and depend on the detailed dynamics of the model. According to my assumptions they are of order one. Realistic masses require $\lambda \approx 1/40$.

The idea that spontaneously broken symmetries generate hierarchies among expectation values of scalars and therefore also between fermion masses was introduced earlier in a different context.[4] It can be easily generalized to an arbitrary number of doublets in arbitrary representations of an arbitrary generation symmetry. (Higher dimensional unification typically leads to infinitely many doublets with various generation charges.) As long as all mass eigenvalues are positive except one, the low energy scalar corresponds to a leading doublet with small admixtures of scalars in other representations. (In addition, the spontaneous generation symmetry breaking can manifest itself through the effects of heavy fermions[5)6)]. I omit this possibility here for simplicity). Let me turn back to the abelian generation symmetry $U(1)_G$ with doublets of arbitrary charge q. If the leading doublet has generation charge q_0, the mixing angle γ_q for a charge eigenstate φ_q is $\sim\lambda^{|q-q_0|}$. In particular, for $q_0=0$, the expectation value of a doublet with charge q is of the order

$$\langle\varphi_q\rangle \sim \lambda^{|q|}\varphi_L \qquad (11)$$

For arbitrary fermions the Yukawa couplings of φ_q are all assumed to be of the order g (unless they are forbidden by the generation symmetry). The effective Yukawa coupling of the low energy doublet to a fermion bilinear operator $\psi_i\psi_j$ is then of the order

$$h_{ij} \sim g\lambda^{|q_i+q_j|} \qquad (12)$$

with $q_{i,j}$ the generation charges of the fermions $\psi_{i,j}$. In the above example, one may take the generation charge of b^c and τ^c to be two instead of one. This implies $m_{b,\tau} \sim \lambda^2 M_W$

and requires $\lambda \approx 1/6$. A top quark mass substantially smaller than M_W could be obtained for $q(t^c)=1$.

3. A REALISTIC MASS STRUCTURE FOR THREE GENERATIONS

The generalisation to three (or more) generations and to arbitrary generation groups is now straightforward.[6] I will assume that all Higgs doublets, which are allowed to interact with the chiral fermions, are indeed present in the theory. For simplicity I continue with the abelian generation group $U(1)_G$ of the last section, supposing that the leading doublet has vanishing generation charge. Consider the following fermion charges

$$\begin{aligned} & q(t,b) = q(t^c) = q(\nu_\tau,\tau) = 0 \\ & q(c^c) = 1 \\ & q(c,s) = q(b^c) = q(s^c) = q(\nu_\mu,\mu) = q(\tau^c) = q(\mu^c) = 2 \\ & q(u,d) = q(u^c) = q(d^c) = q(\nu_e,e) = 3 \\ & q(e^c) = 4 \end{aligned} \tag{13}$$

Using the low energy Yukawa couplings (12) (or, equivalently, the expectation values of charge eigenstates (11) or an evaluation of graphs similar to fig. 1) one finds the following fermion mass matrices

$$\frac{M_U}{M_W} = \begin{pmatrix} c_{tt} , & c_{tc}\lambda^2 , & c_{tu}\lambda^3 \\ c_{ct}\lambda , & c_{cc}\lambda^3 , & c_{cu}\lambda^4 \\ c_{ut}\lambda^3 , & c_{uc}\lambda^5 , & c_{uu}\lambda^6 \end{pmatrix} \tag{14}$$

$$\frac{M_D}{M_W} = \begin{pmatrix} c_{bb}\lambda^2 , & c_{bs}\lambda^4 , & c_{bd}\lambda^5 \\ c_{sb}\lambda^2 , & c_{ss}\lambda^4 , & c_{sd}\lambda^5 \\ c_{db}\lambda^3 , & c_{ds}\lambda^5 , & c_{dd}\lambda^6 \end{pmatrix} \tag{15}$$

$$\frac{M_L}{M_W} = \begin{pmatrix} c_{\tau\tau}\lambda^2 , & c_{\tau\mu}\lambda^4 , & c_{\tau e}\lambda^5 \\ c_{\mu\tau}\lambda^2 , & c_{\mu\mu}\lambda^4 , & c_{\mu e}\lambda^5 \\ c_{e\tau}\lambda^4 , & c_{e\mu}\lambda^6 , & c_{ee}\lambda^7 \end{pmatrix} \tag{16}$$

A diagonalization of these matrices leads to the following eigenvalues and mixings

$$\begin{aligned} m_i &= c_i \lambda^{P_i} M_W \\ \vartheta_{ij} &= c_{ij} \lambda^{P_{ij}} \end{aligned} \tag{17}$$

where the powers P_i, P_{ij} are calculated[3] from (14)-(16). The dimensionless coefficients c reflect the details of the model and are assumed to be of order one. For an illustration I take all c_i and c_{ij} in (17) equal to one. The relations (17) hold at the unification scale M and renormalization corrections should be included for a scaling down to the Fermi scale. For $gM = 10^{17}$ GeV and $\lambda = 1/6$ one obtains a set of interesting relations[6]

$$\begin{aligned} \vartheta_{12} &= \lambda = 1/6 \\ \vartheta_{23} &= \vartheta_{12}^2 = 0.028 \\ \vartheta_{13} &= \vartheta_{12}^3 = 0.005 \\ m_\tau &= \vartheta_{23} M_W = 2.3 \text{ GeV} \\ m_b &= 2.3\, m_\tau = 5.3 \text{ GeV} \\ m_c &= \vartheta_{12} m_b = 880 \text{ MeV} \\ m_s &= m_c^2/m_b = 145 \text{ MeV} \\ m_\mu &= \vartheta_{23} m_\tau = 63 \text{ MeV} \\ m_{u,d} &= \vartheta_{12}^2 m_s = 4 \text{ MeV} \\ m_e &= \vartheta_{12}^3 m_\mu = 0.3 \text{ MeV} \end{aligned} \tag{18}$$

The top quark mass would be around 150 GeV. Of course, the coefficients c_i, c_{ij} are expected somewhat different from one, but it is impressive how well this simple model works. The charges (13) are not the only possibility for obtaining realistic mass patterns. A systematic computerized scan for viable generation charges has been performed[6] and many solutions were found. Nevertheless, it is not at all trivial that the scheme works and reproduces all small masses and mixings in terms of a single small parameter λ. I consider the existence of successful examples, as the ones discussed above, as a strong hint in favour of the explanation of the fermion mass structure by symmetry. One should emphasize that in this approach (for given λ) the order of magnitude of all masses and mixings is calculable from group theory. (This extends to generation groups different from $U(1)_G$, as for example non abelian continuous groups or discrete symmetries.) The identification of the "correct" generation group can constitute an important link to fundamental unification.

4. FERMION MASSES AND MIXINGS FROM HIGHER DIMENSIONS

So far I have used a couple of assumptions, like Yukawa couplings of order g, dimensionless scalar couplings of order g^2, the existence of a generation group, and many scalars in various representations of this group. I will argue in this section that these assumptions are typically realized in higher dimensional unification. Other assumptions, like the existence of a small scale ratio λ, electroweak symmetry breaking by a leading doublet with all other doublet masses of the order M, and the gauge hierarchy between the Fermi scale and M, will, in contrast, depend on more detailed dynamics of "compactification" in higher dimensional theories.

Higher dimensional theories, after compactification, lead to four dimensional theories with infinitely many particles. (The general remarks of this section apply to string theories as well as higher dimensional field theories.) Typical masses of these particles are of the order of the inverse length scale L_0 characteristic for the "internal space". One may identify the unification

scale gM discussed above with the compactification scale L_0^{-1}. (L_0^{-1} is near the Planck mass M_p.) In these theories there should be some reason why massless or very light particles occur. One has to explain why the quark masses are much smaller than M_p. The vanishing or light masses of photons, gluons, W- and Z- bosons are related to gauge symmetries. Similarly, the light masses of quarks and leptons should be explained by their chiral behaviour with respect to the standard gauge symmetry SU(3)xSU(2)xU(1). Chiral fermions can only acquire a mass through spontaneous electroweak symmetry breaking, $m_i \sim \varphi_L$, in analogy to the W- and Z-bosons. The number and charges of chiral fermions depend on topological properties of internal space. More precisely, the number of generations is related to an index- the chirality index.[8]

The mass problem for chiral fermions can be divided into two relatively independent parts: first one should understand the scale of electroweak breaking φ_L. This sets the overall scale for the masses of quarks and charged leptons. Second one needs the Yukawa couplings of the associated low energy scalar doublet. (The expectation value of this doublet determines the Fermi scale φ_L.) These Yukawa couplings will determine the ratios between the fermion masses and the W-boson mass M_W. They describe the <u>structure</u> of the fermion mass matrices. The second part involves two subproblems: One has to compute the Yukawa couplings between the chiral fermions and the relevant scalar doublets and one must identify the low energy doublet in terms of the (infinitely) many doublets of the effective four dimensional theory. Unless there are some topological restrictions[9] all scalar doublets which can have Yukawa couplings to the chiral fermions will be present.

In higher dimensional theories the Yukawa couplings are in principle calculable. They can be expressed in terms of integrals over three wave functions on internal space[10]. Explicit calculations as well as more general arguments show that typical Yukawa couplings are of the same order as the gauge coupling g or they vanish due to some symmetry. This is easily understood intuitively, since often the weak scalar doublets and the gauge bosons belong to the same higher dimensional bosonic multiplet. (For example, the scalars may correspond to internal components of the higher dimensional gauge fields.) Similarly, one finds that the quartic scalar couplings are typically of the order g^2, whereas the quadratic mass terms are proportional $g^2M^2 \approx L_o^{-2}$.

This leads to an apparent problem for the fermion masses in higher dimensional unification - the structure problem: Why do the top quark mass and the electron mass differ by more than five orders of magnitude, with a rich structure in between? If the top quark and the electron couple to the same scalar field one would expect their masses to have comparable size. Although some unknown dynamical effect or cancellation could perhaps explain some small mass ratio, it seems highly unlikely to me that all the small masses and the very pronounced structure of these masses are accidental and should come out right just by luck if one computes the relevant Yukawa couplings. One needs a symmetry that prevents the electron to interact with the same scalar as the top quark. Such symmetries appear indeed frequently in the effective four dimensional theory after compactification. They are remnants of the much wider higher dimensional symmetries and may be called generation symmetries. Compactification of higher dimensions can lead to different sorts of generation symmetries, like local[11] or global[12] continuous generation groups (as the group $U(1)_G$ discussed in the preceeding sections), or discrete symmetries[12][13]. (In addition there may be grand unified symmetries[14], or subgroups of them commuting with SU(3)xSU(2)xU(1). They may be helpful for an understanding of mass ratios within

a generation, but they are insufficient for a solution of the structure problem since the respective quantum numbers do not differentiate between generations.)

If the fermion bilinear e^ce has generation quantum numbers different from t^ct, the electron and the top quark must couple to different scalar doublets with different quantum numbers. The scalar which couples to t^ct has then a vanishing Yukawa coupling to the electron. However, the structure problem is not yet solved. So far the generation symmetry only explains why the electron could remain massless although the top quark acquires a mass from electroweak symmetry breaking. But the electron has a small nonzero mass and one has to explain it. Generation symmetries are helpful only if the low energy doublet is not in a simple representation of the generation group. It must rather be a mixture of different representations, with mixing angles controlled by the magnitude of generation symmetry breaking. For this reason, I consider the approach of identifying the low energy doublet with some zero mass mode on a Calabi Yau space (given by the corresponding Betti number) not as very promising for an understanding of the structure problem for fermion masses. These scalars are typically in a given representation of the (discrete) generation group. (Besides, Calabi Yau spaces are not exact solutions and Betti numbers are in general not conserved by small deformations.) Scalars with various different quantum numbers must play a role in generating the rich fermion mass structure.

One is then left with two alternatives: Either one identifies the various scalars with the towers of massive excitations in the harmonic expansion around some state with unbroken generation symmetry. In this case, the unification scale M is near the compactification scale and the generation symmetry must be broken at very high energies (near L_o^{-1}). The large off diagonal terms in the scalar mass matrix (5) imply that the diagonal term for the leading doublet cannot vanish. This is still consistent with a vanishing scalar mass on the symmetric space ($\nu_0 = 0$) provided the symmetry breaking effects only shift the zero mode in representation space ($\beta_0 \simeq \alpha^2/\nu_1$). In this sense the search for nonzero Betti numbers on a symmetric space can be considered as a first step. A small lowest order scalar mass ($\nu_0 \neq 0$), however, is conceavable as well. Or, as a second alternative, there may be many low energy scalar doublets with generation symmetry breaking at a low scale. (This could be identified with the Fermi scale φ_L.) This approach seems preferred by some believers of low energy supersymmetry. Since in this case the scale M is very low, a lot of additional problems must be handled (strangeness violating neutral currents, conservation of B, L_e, L_μ, electron - muon universality etc.). For this reason, I personally prefer the first alternative.

In conclusion, the prospects for an understanding of the structure of the fermion mass matrices in terms of a generation symmetry and its spontaneous breaking look promising. Realistic four dimensional models can explain all small masses and mixings in terms of the quantum numbers and one small parameter λ. The unsatisfactory point in these models concerns the selection of the "correct" generation symmetry. Higher dimensional unification may be a step towards this selection. For compactification on a given "ground state" the generation group is well determined, but the problem is now shifted to the selection of the "correct" ground state. No higher dimensional model leading to a realistic fermion mass pattern is known so far (although some simple models get very close[6]). In my opinion, the next crucial step toward a solution of the fermion mass problem concerns criteria for the selection of the generation symmetry, and the mechanism and order of magnitude of its breaking.

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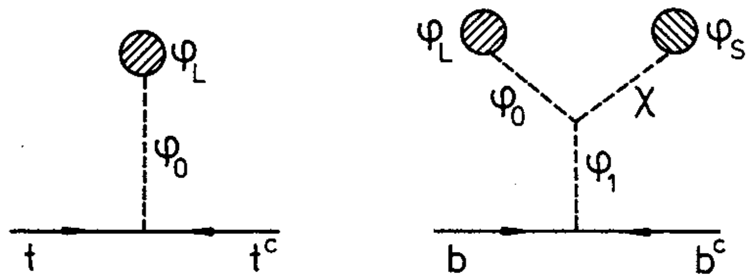

Fig.1