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The Problem of Chiral Fermion Theories on the Lattice

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1. Introduction

The electroweak interactions in the Standard Model are described by a *chiral gauge theory* where left- and right-handed components of the fermion fields are transforming differently under the SU(2) ⊗ U(1) gauge symmetry. This implies that left-right symmetry is broken at low energy. Nevertheless, it can be restored at high energy above the scale of spontaneous symmetry breaking. There are two different ways how this can happen:
a.) by enlarging the gauge group to a left-right symmetric one, for instance to $SU(2)_L \otimes SU(2)_R \otimes U(1)$ [1];
b.) by doubling the fermion spectrum with *mirror fermions.*
The mirror fermions are defined in chiral gauge theories by interchanging the transformation properties of the L- and R-handed field components with respect to the gauge group. The idea of mirror fermions is as old as the idea of parity breaking. In fact, the possibility of the existence of "elementary particles exhibiting opposite asymmetry" was discussed already in the classical paper by Lee and Yang [2]. Mirror fermions also occur naturally in connection with many interesting modern theoretical ideas. To mention a few typical examples, mirror fermions were introduced in order to cancel anomalies [3], they occur in grand unified theories with large orthogonal groups [4], in Kaluza-Klein theories [5] and in extended supersymmetry [6].

Since spontaneous symmetry breaking is a non-perturbative phenomenon, for the study of the left-right symmetry restoration a non-perturbative framework (such as lattice regularization) is needed. The first question is, of course, whether one can put chiral gauge theories with mirror fermions on the lattice?

2. Chiral models with mirror fermions on the lattice

For illustration of the lattice formulation of chiral gauge theories with mirror fermions [7,8], let us consider a simple example with SU(2) gauge field $U(x,\mu)$ (x is a lattice point and μ stands for the 8 directions of links starting at x). The fermion doublet Dirac-field ψ_x can be decomposed into a left-handed doublet ψ_{Lx} and two right-handed singlets $(\psi_{Rx1}, \psi_{Rx2}) \equiv \psi_{Rx}$, as in the standard model. The mirror partner of ψ_x is the fermion doublet Dirac-field χ_x, which can be decomposed into a right-handed doublet χ_{Rx} and two left-handed singlets $(\chi_{Lx1}, \chi_{Lx2}) \equiv \chi_{Lx}$. In order to avoid additional lattice fermion species one can take Wilson lattice fermions with an irrelevant parameter r [9].

The fermion mass-term and kinetic term in the locally gauge invariant lattice action [7] is:

$$\sum_x \Big\{ (\mu_{\psi\chi} + 8r) \left[(\bar\chi_{Rx}\psi_{Lx}) + (\bar\chi_{Lx}\psi_{Rx}) + (\bar\psi_{Rx}\chi_{Lx}) + (\bar\psi_{Lx}\chi_{Rx}) \right]$$
$$- K_\psi \sum_\mu \left[(\bar\psi_{Lx+\hat\mu} U(x,\mu)\gamma_\mu \psi_{Lx}) + (\bar\psi_{Rx+\hat\mu}\gamma_\mu\psi_{Rx}) \right]$$
$$- K_\chi \sum_\mu \left[(\bar\chi_{Lx+\hat\mu}\gamma_\mu\chi_{Lx}) + (\bar\chi_{Rx+\hat\mu} U(x,\mu)\gamma_\mu \chi_{Rx}) \right]$$
$$- r \sum_\mu \left[(\bar\chi_{Rx+\hat\mu} U(x,\mu)\psi_{Lx}) + (\bar\chi_{Lx+\hat\mu}\psi_{Rx}) + (\bar\psi_{Rx+\hat\mu}\chi_{Lx}) + (\bar\psi_{Lx+\hat\mu} U(x,\mu)\chi_{Rx}) \right] \Big\} \tag{1}$$

Here $\mu_{\psi\chi}$ is a chiral invariant mass parameter and K_ψ (resp. K_χ) is the hopping parameter for the fermion (resp. mirror fermion). The rest of the euclidean lattice action is standard.

3. Phenomenology of mirror fermions

The fermion spectrum can be characterized by the mass matrix on the (ψ, χ)-basis. The bare mass matrix in the lattice action is:

$$\mu = \begin{pmatrix} 0 & \mu_{\psi\chi} \\ \mu_{\psi\chi} & 0 \end{pmatrix} \tag{2}$$

The corresponding eigenstates are $(\psi \pm \chi)/\sqrt{2}$ representing a degenerate parity doublet. The mass matrix is, of course, renormalized due to the interactions but in the symmetric phase the above form is kept, only the value of $\mu_{\psi\chi}$ is changed. In the phase with spontaneously broken symmetry the Yukawa-couplings to the scalar doublet φ_x, namely

$$G_\psi \left[(\bar\psi_{Rx}\varphi_x^\dagger\psi_{Lx}) + (\bar\psi_{Lx}\varphi_x\psi_{Rx}) \right] + G_\chi \left[(\bar\chi_{Rx}\varphi_x\chi_{Lx}) + (\bar\chi_{Lx}\varphi_x^\dagger\chi_{Rx}) \right] \tag{3}$$

induce diagonal terms in the mass matrix which are proportional to the scalar vacuum expectation value v:

$$\mu = \begin{pmatrix} G_\psi v & \mu_{\psi\chi} \\ \mu_{\psi\chi} & G_\chi v \end{pmatrix} \tag{4}$$

The physical states are mixtures of ψ and χ characterized by some mixing angle α. The corresponding physical fermion masses $\mu_{1,2}$ are, in general, different and are usually of the order of $\max(|\mu_{\psi\chi}|, |v|)$. In the specific case of $\mu_{\psi\chi}^2 = G_\psi G_\chi v^2$, however, one of the masse eigenvalues is zero. The very small fermion masses in the standard model could be due to such a cancellation mechanism. Although this looks at the first sight as an ugly fine tuning of parameters, it is conceivable that there is some dynamical or symmetry principle in a more general theoretical framework determining the parameters of quantum field theory which implies such a relation. In this respect it is worth to emphasize that the smallness of some fermion masses in the usual perturbative setup of the Higgs-sector is due to the fine tuning of the corresponding Yukawa-couplings to values very close to zero. The smallness of the Yukawa-couplings does not necessarily have to do with chiral symmetry.

The most important consequence of the mirror fermions for low energy phenomenology is that the weak currents, instead of being pure $V - A$, have a generic form

$$(V - A)\cos\alpha + (V + A)\sin\alpha \tag{5}$$

Here α is a mixing angle in the $(\psi - \chi)$-basis. The mirror partners of the light fermion families have masses of the order of the scalar vacuum expectation value (i. e. a few hundred GeV) [10]. The phenomenological upper bounds on the mixing angles $\alpha_{e,\mu,u,...}$ were derived in a general framework recently by Langacker and London [11]. In the best cases, namely for the first fermion family and for the muon, the bounds are typically $|\sin\alpha| \leq 0.2$. The lower bounds on mirror fermion masses are similar to the bounds on the masses of a fourth heavy fermion family, typically $m \geq 30\ GeV$ for leptons and $m \geq 50\ GeV$ for quarks. The heavy mirror fermions, if they exist, have rather spectacular decay modes (into 3 leptons or 3 jets etc.) which cannot be missed by the planned detectors at future colliders like SLC, LEP, HERA, LHC, SSC ....

In conclusion, the model with three (or more) mirror pairs of fermion families is an extension of the standard electroweak model consistent with all the presently known phenomenology.

4. Can the mirror fermions be removed?

In a lattice regularized theory with fermions the mirror partners are always present due to the Nielsen-Ninomiya theorem [12], provided some rather plausible assumptions are fulfilled. This is the *fermion doubling* phenomenon on the lattice. Nevertheless, in vectorlike theories as QCD the superfluous additional states can be kept at the cut-off scale and hence are removed from the physical spectrum in the large cut-off ("continuum") limit [9,13]. The question is whether the mirror doublers can also be kept at the scale of the cut-off in chiral gauge theories? Of course, if the mirror partners of the known elementary fermions exist in nature, this question is only of academic interest. However, as long as mirror fermions are not found experimentally it is of an extraordinary theoretical interest.

For the correct formulation of the question it is important to note that the mirror partners *can* be removed from the physical spectrum in scalar-fermion theories without gauge fields. For instance, in the broken phase this can be achieved by an appropriate choice of scalar field vacuum expectation values and Yukawa-couplings. In the above SU(2) example the simplest possibility is to introduce a second scalar doublet which has a vacuum expectation value of order one in lattice units. By tuning the parameters it is possible to arrange in this case that one of the fermion

masses remains at the cut-off scale.

The difficulty comes in the physically interesting case with W- and Z-gauge fields: since there is presumably an upper limit for the renormalized Yukawa-coupling m_{mirror}/v and the scale of the vacuum expectation values are fixed by the W-mass, according to the present knowledge the mirror fermions cannot be removed without removing at the same time also the W- and Z-bosons from the physical spectrum. This is a similar constraint imposed on quantum field theories by the requirement of non-perturbative consistency as the cut-off dependent upper limit for the Higgs boson mass in the SU(2) Higgs model without fermions (for reviews see [14,15]).

An interesting example, where the mirror fermion partners arise dynamically in the hopping parameter expansion, was given in Ref. [16] in the case of the lattice-regularized Gell-Mann-Lévy σ-model [17]. The lattice action with Wilson-fermions and arbitrary field normalizations is:

$$S = \sum_x \Big\{ \mu\phi_{Sx}\phi_{Sx} + \lambda(\phi_{Sx}\phi_{Sx})^2 - \kappa\sum_\mu \phi_{Sx+\hat\mu}\phi_{Sx}$$
$$+ M(\bar\psi_x\psi_x) + G\phi_{Sx}(\bar\psi_x\Gamma_S\psi_x) - K\sum_\mu(\bar\psi_{x+\hat\mu}[r+\gamma_\mu]\psi_x)\Big\} \tag{6}$$

Here the scalar doublet field is represented in an O(4)-notation by ϕ_{Sx}; $S = 0,\ldots,3$, and the 8⊗8 matrices $\Gamma_S = (1, -i\gamma_5\tau_s)$ define the Yukawa-coupling to the fermion doublet field ψ_x. In the limit of infinite bare quartic- and Yukawa-couplings $\lambda \to \infty$, $G \to \infty$ and with convenient field normalizations the action can be written as

$$S = \sum_x \Big\{ \phi_{Sx}(\bar\psi_x\Gamma_S\psi_x) - \kappa\sum_\mu \phi_{Sx+\hat\mu}\phi_{Sx} - K\sum_\mu(\bar\psi_{x+\hat\mu}[r+\gamma_\mu]\psi_x)\Big\} \tag{7}$$

Assuming the triviality of the continuum limit (or which is the same, the relevance of the infrared fixed point at vanishing renormalized couplings $\lambda_r = G_r = 0$), the $\lambda = G = \infty$ model is representative for arbirary values of λ and G. Therefore, the physical content of the model can be studied by using the action in (7).

A useful non-perturbative tool for the study of the lattice σ-model is the double hopping parameter expansion in powers of κ and K. At $\kappa = K = 0$ the path integral can be performed because the fields at different points are decoupled. The partition function at arbitrary κ, K can be expressed as power series in κ, K. The series can be organized according to the topology of graphs similarly to the *linked cluster expansion* in statistical physics [18]. The hopping parameter series can be expected to give a good approximation in those parts of the symmetric phase where the correlation length does not exceed a value of order 1. At correlation lengths of about 1-2 the *scaling region* begins, where the perturbative renormalization group is expected to work in terms of the renormalized couplings. The important rôle of the hopping parameter expansion is to provide the initial values for the integration of the renormalization group equations. Such an analytical approach for the solution of the pure ϕ^4 model was succesfully applied in recent papers by Lüscher and Weisz [19]. In order to obtain a good quantitative description in the region with correlation lengths of order one, a relatively high order hopping parameter expansion is needed. In the σ-model, due to the general structure of graphs, 14th-16th order could be required. In Ref. [16] the qualitative behaviour of the spectrum was studied by a partial resummation of the series ("random walk approximation") along the line $\kappa = 0$. In this limit the action is purely ferminic, the π- and σ-bosons are described by fermion-antifermion bound states. The fermion propagation, due to the $SU(2)_L \otimes SU(2)_R (\equiv O(4))$-symmetry, proceeds by an oscillation to a composite three-fermion state. The corresponding composite fermion operator is

$$\chi_x \equiv \frac{1}{10}\Gamma_S \psi_x (\bar{\psi}_x \Gamma_S \psi_x) \qquad (8)$$

As it can be easily shown, this operator has mirror quantum numbers compared to the elementary fermion ψ_x. In the random walk approximation near the critical point the eigenstates of the fermion mass matrix are $(\psi \pm \chi)/\sqrt{2}$. This is a degenerate parity doublet, which goes over into a split mirror pair in the spontaneously broken phase on the other side of the critical point. The important rôle of the composite state in Eq. (8) is also shown by the effective fermionic action along the $\kappa = 0$ line:

$$S^{\kappa=0}_{eff} = -K \sum_{x,\mu} (\bar{\psi}_{x+\hat{\mu}}[r+\gamma_\mu]\psi_x) - \sum_x \frac{5}{8}\left[(\bar{\chi}_x\psi_x) + (\bar{\psi}_x\chi_x)\right] + \cdots \qquad (9)$$

Here the dots stand for higher powers of the square-bracket.

The lattice-regularized σ-model is a good example to show that the hypercubical lattice has, apparently, many different ways of protection against a chirally asymmetric physical spectrum. Even if the mirror fermion partners are not introduced a priori, they can enter through dynamics. This is a new addition to previous unsuccessful attempts to put chirally asymmetric gauge theories on the lattice (for an incomplete list of references see e. g. [20]). Even if there are some new suggestions in the literature which might perhaps be able to solve this problem [21], nobody really knows whether any of them works. They are also complicated, therefore one can argue for the physical existence of the mirror fermions by simplicity. In principle, it is also possible that the difficulty is due to the use of a (hypercubical) lattice as regulator and there exist other non-perturbative regularization schemes which are well suited also for chirally asymmetric gauge theories. This would, however, be a completely new and rather unexpected situation because it is generally assumed that the physical content of quantum field theories is independent of the regularization scheme.

5. Numerical results in models with Yukawa-coupling

Useful results for understanding the spectrum of scalar-fermion theories can also be obtained by numerical simulations. There are several recent works in this direction [22] (for reviews see also [23]). The first problem is to determine the phase structure. The simplest model with Yukawa-coupling can be defined with a single component scalar field ϕ_x and one naive- or Wilson-fermion ψ_x. The lattice action with general

field normalization is

$$S = \sum_x \Big\{ \mu\phi_x^2 + \lambda\phi_x^4 - \kappa\sum_\mu \phi_{x+\hat{\mu}}\phi_x$$
$$+ M(\bar{\psi}_x\psi_x) + G\phi_x(\bar{\psi}_x\psi_x) - K\sum_\mu(\bar{\psi}_{x+\hat{\mu}}[r+\gamma_\mu]\psi_x) \Big\} \qquad (10)$$

In the limit of infinite bare quartic coupling ($\lambda = \infty$; $\phi_x = \pm 1$) with a massless naive fermion and in a convenient normalization we have:

$$S = \sum_x \Big\{ G\phi_x(\bar{\psi}_x\psi_x) - \kappa\sum_\mu \phi_{x+\hat{\mu}}\phi_x - \frac{1}{2}\sum_\mu(\bar{\psi}_{x+\hat{\mu}}\gamma_\mu\psi_x) \Big\} \qquad (11)$$

I studied the symmetry breaking phase transition in this model in the (κ, G)-plane on small (4^4) lattices [24] by using the bosonization method for dynamical fermions [25]. The numerical work in models with Yukawa-couplings just started recently. For the moment it would be too early to draw any conclusions about them.

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