YET MORE VERSIONS OF d = 11 SUPERGRAVITY

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Yet More Versions of d=11 Supergravity

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Abstract

Extending previous results, we construct two further versions of $d = 11$ supergravity. The transformation laws are given in a form where the $SO(1,10)$ local Lorentz group is replaced by $SO(1,4) \times USp(8)$ and $SO(1,5) \times [SO(5) \times SO(5)]$, respectively. The spinless fields are assigned to the corresponding representations of $E_{6(+6)}$ and $E_{5(+5)}$.

1. Introduction

It has been known for some time that maximally extended supergravities in D dimensions obtained from $d = 11$ supergravity [1] are manifestly invariant under an internal symmetry group $SL(11 - D)_{global} \times SO(11 - D)_{local}$ [2,3]. This symmetry can be enlarged to the product of a noncompact global group G_{11-D} and a compact local group H_{11-D}. Assuming that the scalar sector in these theories is governed by a nonlinear σ − model based on the coset G_{11-D}/H_{11-D}, one can deduce the relevant groups by equating the number of scalars obtained in the dimensional reduction with the dimension of this coset space. In this way, one gets [2,3] †

D	G_{11-D}	H_{11-D}
8	$E_{3(+3)} = SL(3,\mathbf{R}) \times SL(2,\mathbf{R})$	$SO(3) \times SO(2)$
7	$E_{4(+4)} = SL(5,\mathbf{R})$	$SO(5)$
6	$E_{5(+5)} = SO(5,5)$	$SO(5) \times SO(5)$
5	$E_{6(+6)}$	$USp(8)$
4	$E_{7(+7)}$	$SU(8)$
3	$E_{8(+8)}$	$SO(16)$

(tab. 1)

In [4] it was shown that some of these hidden symmetries are actually present in the $d = 11$ supergravity itself : there exists a reformulation where the tangent space group $SO(1,3) \times SO(7)$ is replaced by $SO(1,3) \times SU(8)$ and the spinless degrees of freedom are assigned to a representation of $E_{7(+7)}$. A crucial step in this construction was the introduction of "composite" gauge connections in the internal dimensions so as to covariantize the theory with respect to $SU(8)$ transformations depending on the internal coordinates. Subsequently, the corresponding version of $d = 11$ supergravity with tangent space group $SO(1,2) \times SO(16)$ was constructed in [5]. These results suggest that, in fact, all of the hidden symmetries displayed in the table can be realized in the full $d = 11$ theory. In this paper we show that this is indeed the case by presenting explicit results for $D = 5$ and $D = 6$ (we have not considered $D \geq 7$ since these cases are less interesting and unlikely to furnish any new insights). This confirms the conclusion that $d = 11$ supergravity is a multi-faceted theory whose ultimate formulation (and possible embedding into an ancestor dual model) remains to be found. It would, of course, be even more interesting if one could extend the results now established for $3 \leq D \leq 6$ to $D \leq 2$ where one expects the emergence of infinite − dimensional groups.‡

We now briefly summarize our notation and conventions; more details can be found in [4,5]. The only difference with [4,5] is that we use a metric η with signature $(+, -, \cdots, -)$. The Lagrangian of $d = 11$ supergravity is given by

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† For $D \geq 9$ the field A_{MNP} does not give any scalars in the reduction. Hence one retains only $GL(2, \mathbf{R})_{global} \times SO(2)_{local}$ after the Weyl rescaling .

‡ For some recent results concerning maximally extended supergravity in two dimensions, see [6].

$$\mathcal{L} = -\frac{1}{2}ER - \frac{i}{2}E\bar\Psi_M\tilde\Gamma^{MNP}D_N\Psi_P - \frac{1}{48}EF_{MNPQ}^2$$
$$+ \frac{1}{\sqrt{2}}\frac{1}{(12)^3}\epsilon^{M\ldots X}F_{MNPQ}F_{RSTU}A_{VWX} +$$
$$+ \frac{\sqrt{2}}{192}EF_{MNPQ}(\bar\Psi_R\tilde\Gamma^{MNPQRS}\Psi_S + 12\bar\Psi^M\tilde\Gamma^{NP}\Psi^Q)$$
(1.1)

where

$$\{\tilde\Gamma^A, \tilde\Gamma^B\} = 2\eta^{AB}, \quad \tilde\Gamma^{A_1\ldots A_{11}} = -i\,\epsilon^{A_1\ldots A_{11}}, \quad \tilde\Gamma^0\tilde\Gamma^A\tilde\Gamma^0 = \tilde\Gamma^{A\,+}.$$
(1.2)

The transformation laws read

$$\delta E_M^A = -\frac{i}{2}\bar\epsilon\tilde\Gamma^A\Psi_M$$

$$\delta A_{MNP} = \frac{\sqrt{2}}{8}\bar\epsilon\tilde\Gamma_{[MN}\Psi_{P]}$$

$$\delta\Psi_M = D_M\epsilon + \frac{i}{288}\sqrt{2}\,(\tilde\Gamma_M{}^{PQRS} - 8\delta_M^P\tilde\Gamma^{QRS})\epsilon\, F_{PQRS}.$$
(1.3)

Higher order fermionic terms are neglected throughout.

2. Field redefinitions and transformation laws

The starting point is a $D + (11-D)$ coordinate split. The curved and flat $d=11$ indices M and A are decomposed into (μ, m) and (α, a), respectively. Fixing the gauge

$$E_M^A = \begin{pmatrix} e_\mu^\alpha & B_\mu^m\, e_m^a \\ 0 & e_m^a \end{pmatrix}$$
(2.1)

reduces the local $SO(1,10)$ Lorentz group to $SO(1, D-1) \times SO(11-D)$. The Weyl rescaling

$$e_\mu'^\alpha = \Delta^s e_\mu^\alpha, \quad \Delta \equiv \det e_m^a, \quad s \equiv \frac{1}{D-2}$$
(2.2)

leads to the standard Einstein action in D dimensions (for $D > 2$). The fermionic fields are redefined as follows

$$\Psi_\mu' = \Delta^{-\frac{1}{2}s}e_\mu'^\alpha(\Psi_\alpha + s\,\tilde\Gamma_\alpha\tilde\Gamma^a\Psi_a) = e_\mu'^\alpha\Psi_\alpha'$$
$$\Psi_\alpha' = \Delta^{-\frac{1}{2}s}\Psi_\alpha$$
$$\epsilon' = \Delta^{\frac{1}{2}s}\epsilon$$
(2.3)

As a general rule, quantities with index μ are obtained from the flat ones via e_μ^a and without the E_μ^a contribution (so that we have a scalar under $11-D$ dimensional general coordinate transformations $\xi^m(x^\mu, y^m)$). In particular we use [4,5]

$$\mathcal{D}_\mu = e_\mu^\alpha\partial_\alpha = \partial_\mu - B_\mu^m\partial_m$$
$$\omega_{\mu AB} = \frac{1}{2}e_\mu^\alpha(\Omega_{AB\alpha} - \Omega_{B\alpha A} - \Omega_{\alpha AB})$$
(2.4)

where

$$\Omega_{AB}{}^C = 2 E_{[A}{}^M E_{B]}{}^N \partial_M E_N{}^C.$$

The supersymmetry transformation of the spin-2 field is given by

$$\delta e_\mu'^\alpha = \delta_{susy}(\Delta^s e_\mu^\alpha) + E_\mu^A\Omega_A^\alpha$$
$$= -\frac{i}{2}\bar\epsilon'\tilde\Gamma^\alpha\Psi_\mu'.$$
(2.5)

The compensating $SO(1,10)$ rotation

$$\Omega_{\alpha a} = \frac{i}{2}\bar\epsilon\tilde\Gamma_a\Psi_\alpha, \quad \Omega_{\alpha\beta} = \frac{i}{2}s\bar\epsilon\tilde\Gamma_{\alpha\beta}\tilde\Gamma^a\Psi_a$$
(2.6)

restores the gauge (2.1). The spin-1 fields transform into

$$\delta B_\mu^m = -\frac{i}{2}\Delta^{-s}e_a{}^m\{\bar\epsilon'\tilde\Gamma^a\Psi_\mu' + \bar\epsilon'^\mu(\eta^{ab} + s\tilde\Gamma^a\tilde\Gamma^b)\Psi_b'\}.$$
(2.7)

Variation of the spin-$\frac{3}{2}$ fields Ψ_μ' (2.3) yields the lengthy expression

$$\delta\Psi_\mu' = \left[\mathcal{D}_\mu - \frac{1}{4}(\hat\omega_{\mu\alpha\beta}\tilde\Gamma^{\alpha\beta} + \omega'_{\mu ab}\tilde\Gamma^{ab}) - \frac{s}{2}\partial_m B_\mu^m\right]\epsilon'$$
$$+ \left(\frac{s}{2}\tilde\Gamma_\mu'\tilde\Gamma^{\bar a\beta} - e_\mu'^{|\alpha}\tilde\Gamma^{\beta]}\right)\left\{-\frac{1}{4}\Delta^s\Omega'_{\alpha\beta a}\tilde\Gamma^a + \frac{\sqrt{2}}{8}\Delta^{-s}F_{\alpha\beta cd}\tilde\Gamma^{cd}\right\}\epsilon'$$
$$- s\Delta^{-s}e_a{}^m\tilde\Gamma^{\bar a}\left\{\partial_m + \frac{1}{4}\Omega'_{m\alpha\beta}\tilde\Gamma^{\alpha\beta} - \frac{1}{4}\omega_{mab}\tilde\Gamma^{ab} - \frac{1+s}{2}(\Delta^{-1}\partial_m\Delta)\right\}\tilde\Gamma_\mu'\epsilon'$$
$$+ \left(\frac{1}{2}-s\right)\Omega'_{a(\alpha\beta)}e_\mu'^\alpha\tilde\Gamma^\beta\tilde\Gamma^{\bar a}\epsilon'$$
$$+ \frac{\sqrt{2}}{96}\Delta^{-s}e_\mu'^\alpha\left\{-s F_{abcd}\tilde\Gamma_\alpha\tilde\Gamma^{\bar abcd} - 4 F_{abcd}\tilde\Gamma^{\bar bcd}\right.$$
$$\left. + (8 s\tilde\Gamma_\alpha\tilde\Gamma^{\beta\gamma\delta} - 12\delta_\alpha^{[\beta}\tilde\Gamma^{\gamma\delta]})F_{\beta\gamma\delta\epsilon}\tilde\Gamma^\epsilon + (3 s\tilde\Gamma_\alpha\tilde\Gamma^{\beta\gamma\delta\epsilon} - 4\delta_\alpha^{[\beta}\tilde\Gamma^{\gamma\delta\epsilon]})F_{\beta\gamma\delta\epsilon}\right\}\epsilon'$$
(2.8)

$$\hat\omega_{\mu\alpha\beta} \equiv \omega_{\mu\alpha\beta} + 2 s\, e'_{\mu[\alpha}e_{\beta]}'^\nu\partial_m B_\nu^m.$$
(2.9)

3. Covariantisation for D=5

To establish local $USp(8)$ covariance we give some notation and definitions for the 5 + 6 coordinate split. The d=11 gamma matrices are represented by

$$\bar{\Gamma}^a = \gamma^\alpha \otimes \Gamma^* \qquad \Gamma^* = i\,\Gamma^{123456} = i\,\Omega = i\,\Gamma^7$$
$$\tilde{\Gamma}^a = \mathbf{1} \otimes \Gamma^a \qquad (3.1)$$

For $\gamma^\alpha(\alpha = 0, \cdots, 4)$ we use the conventions of [3]. Γ^a and Ω are real, antisymmetric 8×8 matrices,

$$\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}\mathbf{1}, \quad \Omega^2 = -\mathbf{1}. \qquad (3.2)$$

Accordingly, the D=11 Dirac indices are split as $(\underline{\alpha}, A)$ where $\underline{\alpha}$ is the D=5 index and A is the "internal" SO(6) (6) spinor index. We will suppress the index $\underline{\alpha}$ in the sequel. The SO(6) part of the local Lorentz group acting on Majorana spinors can be extended to $USp(8)$ with the 36 generators $\Omega, \Gamma^{ab}, iT^{abc}$ satisfying

$$\Lambda^+ = -\Lambda$$
$$(\Omega\Lambda)^T = \Omega\Lambda. \qquad (3.3)$$

We can raise and lower $USp(8)$ indices by means of the symplectic metric Ω. Thus

$$\Psi_A = \Omega_{AB}\Psi^B \qquad (3.4)$$

transforms in the complex conjugate representation of Ψ^B by virtue of (3.3). It follows that $USp(8)$ is compatible with the d=11 Majorana condition which in turn is equivalent to the D=5 pseudo Majorana condition [3]

$$\Psi^A = \gamma_5\,\Omega_{AB}(\Psi^B)^*. \qquad (3.5)$$

The next step is to regroup the 48 spin-$\frac{1}{2}$ fields Ψ_α^A into a representation of $USp(8)$. This is done as follows [2,3]

$$\chi_{ABC} = \frac{3}{2}\sqrt{2}\,i\,\Gamma_{<AB}\Psi'_{C>\alpha}$$
$$= \frac{3}{2}\sqrt{2}\,i\{\Gamma^a_{[AB}\Psi'_{C]\alpha} + \frac{1}{3}\Omega_{[AB}\Gamma^a_{C]D}\Psi'^D_\alpha\}, \qquad (3.6)$$

where $<\cdots>$ denotes antisymmetric tensors which are traceless with respect to Ω. χ_{ABC} satisfies the reality condition

$$\chi^{ABC} = \gamma_5(\chi_{ABC})^*. \qquad (3.7)$$

We note the identity

$$\bar{\chi}^{ABC}\gamma^\mu D_\mu \chi_{ABC} = 6i\,\bar{\Psi}_a(\eta^{ab} + \frac{1}{3}\Gamma^a\Gamma^b)\bar{\Gamma}^\mu D_\mu \Psi_b, \qquad (3.8)$$

which renders the spin-$\frac{1}{2}$ kinetic term diagonal.

From (2.5) and (2.7) we obtain the $USp(8)$ covariant transformation laws

$$\delta e'^\alpha_\mu = \frac{1}{2}\bar{\varepsilon}^A \gamma^\alpha \Psi_{\mu A} \qquad (3.9)$$

$$\delta B_\mu^m = -\frac{1}{2}e_{AB}^m(\bar{\varepsilon}^A\Psi_\mu^B - \frac{\sqrt{2}}{4}\bar{\varepsilon}_C\gamma'_\mu\chi^{ABC}) \qquad (3.10)$$

where the generalized vielbein e_{AB}^m is defined as

$$e_{AB}^m = i\Delta^{-\frac{1}{5}}e_a^m\Gamma_{AB}^a$$

and thereby assigned to the $\underline{27}$ representation of $USp(8)$. As explained in [4] one must introduce a local $USp(8)$ to avoid the emergence of new degrees of freedom; (3.10) then corresponds to a special gauge choice. Raising indices gives $e^{mAB} = (e_{AB}^m)^*$.

Using properties of the d=7 Clifford algebra [2,4] the supersymmetry transformation of e_{AB}^m can be cast into the form

$$\delta_{susy}e_{AB}^m = \frac{\sqrt{2}}{2}\Sigma_{ABCD}e^{mCD} + 2\Lambda_{[A}^C e_{B]C}^m \qquad (3.11)$$

with

$$\Sigma_{ABCD} = \bar{\varepsilon}_{[A}\chi_{BCD]} + \frac{1}{24}\epsilon_{ABCDEFGH}\bar{\varepsilon}^E\chi^{FGH} \qquad (3.12)$$

and the $USp(8)$ parameter

$$\Lambda_B^A = -\frac{i}{8}(\bar{\varepsilon}\Gamma_a\Psi_b)\Gamma^{ab} + \frac{i}{16}(\bar{\varepsilon}\Gamma_{ab}\Psi_c)\Gamma^{abc}. \qquad (3.13)$$

Σ_{ABCD} is antisymmetric, traceless and pseudoreal : $\Sigma^{ABCD} = (\Sigma_{ABCD})^*$ and thus transforms in the $\underline{42}$ representation of $USp(8)$. Discarding the local $USp(8)$ rotation we arrive at a manifestly covariant transformation law for e_{AB}^m. Furthermore the right hand side of (3.11)can be interpreted as a (local) E_6-rotation in the fundamental $\underline{27}$ representation.

We must then rewrite the fermionic variations. Just as in [4,5], (2.8) can be simplified by the introduction of further $USp(8)$ covariant quantities. These are found in exactly the same fashion as in [4,5] and are given by (we omit all primes in the sequel)

$$B_\mu{}^A{}_B = -\frac{1}{2}\omega_{\mu ab}\Gamma^{ab} - \frac{\sqrt{2}}{12}i\Delta^{-\frac{3}{5}}e_\mu^\alpha F_{abcd}\Gamma_{AB}^{bcd}$$
$$-\frac{\sqrt{2}}{48}\Delta^{-\frac{3}{5}}e_\mu^\alpha \varepsilon_{\alpha\beta\gamma\delta\epsilon}F^{\beta\gamma\delta\epsilon}\Omega_{AB} \qquad (3.14)$$

$$A_{\mu ABCD} = \frac{3}{4}\Omega_{\mu(ab)}\Gamma_{<AB}\Gamma_{CD>}^b - \frac{\sqrt{2}}{8}i\Delta^{-\frac{1}{5}}e_\mu^\alpha F_{\alpha abc}\Gamma_{[AB}^a\Gamma_{CD]}^{bc}$$
$$-\frac{\sqrt{2}}{192}\Delta^{-\frac{1}{5}}e_\mu^\alpha \varepsilon_{\alpha\beta\gamma\delta\epsilon}F^{\beta\gamma\delta\epsilon}\Gamma_{a[AB}\Gamma_{CD]}^{a7} \qquad (3.15)$$

$$B_m{}^A{}_B = \frac{1}{2} e_a{}^n \partial_m e_{nb} \Gamma_{AB}^{ab} - \frac{\sqrt{2}}{48} i e_m{}^a F_{abcd} \Gamma_{AB}^{bcd} \tag{3.16}$$

$$\mathcal{A}_{mABCD} = \frac{3}{4} e_a{}^n \partial_m e_{nb} \Gamma^a_{<AB} \Gamma^b_{CD>} - \frac{\sqrt{2}}{32} i e_m{}^a F_{abcd} \Gamma^b_{[AB} \Gamma^{cd}_{CD]}. \tag{3.17}$$

In addition we introduce the spin-1 field strength

$$\mathcal{G}_{\alpha\beta}{}^A{}_B = \frac{i}{4} \Delta^{\frac{1}{4}} \Omega_{\alpha\beta a} \Gamma^a_{AB} - \frac{\sqrt{2}}{8} \Delta^{-\frac{1}{4}} \bar{F}_{\alpha\beta cd} \Gamma^{cd}_{AB} + \frac{\sqrt{2}}{24} i \Delta^{-\frac{1}{4}} \bar{\varepsilon}_{\alpha\beta\gamma\delta\varepsilon} F^{\gamma\delta\varepsilon}{}_a \Gamma^a_{AB}. \tag{3.18}$$

The USp(8) covariant "generalized vielbein postulate" reads

$$\mathcal{D}_\mu e^m_{AB} + \frac{1}{3} \partial_n B^n_\mu e^m_{AB} + \partial_n B^m_\mu e^n_{AB} + B^C_{\mu[A} e_{B]C} + \mathcal{A}_{\mu ABCD} e^{mCD} = 0 \tag{3.19}$$

$$\partial_m e^n_{AB} + B^C_{m[A} e_{B]C} + \mathcal{A}_{mABCD} e^{nCD} = 0. \tag{3.20}$$

With the help of the SO(1,4)× USp(8) covariant derivatives

$$D_\mu \varepsilon^A \equiv \partial_\mu \varepsilon^A - \frac{1}{4} \hat{\omega}_{\mu\alpha\beta} \gamma^{\alpha\beta} \varepsilon^A + \frac{1}{2} B_\mu{}^A{}_B \varepsilon^B$$

$$D_m \varepsilon^A \equiv \partial_m \varepsilon^A + \frac{1}{4} e_\alpha{}^\mu \partial_m e_{\mu\beta} \gamma^{\alpha\beta} \varepsilon^A + \frac{1}{2} B_m{}^A{}_B \varepsilon^B. \tag{3.21}$$

one finally obtains

$$\delta \chi_{ABC} = \frac{3}{2} \sqrt{2} e_{<AB}{}^m D_m \varepsilon_{C>} - \frac{3}{4} \sqrt{2} \gamma^{\alpha\beta} \mathcal{G}_{\alpha\beta <AB} \varepsilon_{C>} - \sqrt{2} \gamma^\mu \mathcal{A}_{\mu ABCD} \varepsilon^D$$
$$- \frac{3}{4} \sqrt{2} e^m{}_{DE} \mathcal{A}_{m DE<AB} \varepsilon_{C>} - \sqrt{2} \mathcal{A}_{m ABCD} e^{mDE} \varepsilon_E \tag{3.22}$$

$$\delta \Psi^A_\mu = [D_\mu(\hat{\omega}, B_\mu) - \frac{1}{6} \partial_m B^m_\mu] \varepsilon^A + \frac{1}{6} (\gamma^{\alpha\beta} \gamma_\mu - 2 e_\mu{}^\alpha \gamma^\beta) \mathcal{G}_{\alpha\beta}{}^A{}_B \varepsilon^B$$
$$+ \frac{1}{3} e^{mAB} [D_m(\gamma_\mu \varepsilon_B) + \frac{1}{4} (\partial_m g_{\mu\nu}) \gamma^\nu \varepsilon_B] - \frac{1}{6} e^m_{CD} \mathcal{A}_m{}^{ABCD} \gamma_\mu \varepsilon_B. \tag{3.23}$$

In the reduction to D=5 where the y-dependence is dropped, these results agree with those of [3].

4. Covariantisation for D=6

Since the procedure should be clear, we just quote the results for D=6 in this section. According to the 6 + 5 coordinate split the d = 11 gamma matrices represented by

$$\tilde{\Gamma}^\alpha = \gamma^\alpha \otimes \mathbf{1}, \quad \gamma^{\alpha\beta\gamma\delta\varepsilon} = \varepsilon^{\alpha\beta\gamma\delta\varepsilon} \Gamma^*$$
$$\tilde{\Gamma}^a = \Gamma^* \otimes \Gamma^a, \quad \Gamma^{abcde} = -i \varepsilon^{abcde} \tag{4.1}$$

γ^α and Γ^a correspond to Γ^α and γ^α of the last chapter with an additional factor of i due to the metric $\eta^{\alpha\beta} = diag(+, \cdots, -), \eta^{ab} = diag(-, \cdots, -)$.
In $D = 6$ there exist Weyl spinors. The projectors are

$$\Pi_\pm = \frac{1}{2}(1 \pm \Gamma^*) \otimes \mathbf{1}. \tag{4.2}$$

We want to extend the internal SO(5) Lorentz group (tab. 1), so we assign the chiral spinor components (which are still (pseudo) Majorana spinors) to representations of SO(5)× SO(5)

$$\varepsilon^A \equiv \Pi_+ \varepsilon', \quad \Psi_{\mu A} \equiv \Pi_+ \Psi'_\mu$$
$$\varepsilon_{\bar{A}} \equiv \Pi_- \varepsilon', \quad \Psi_{\mu \bar{A}} \equiv \Pi_- \Psi'_\mu. \tag{4.3}$$

A and $\bar{A}$ denote the spinor indices of the first and the second SO(5) in SO(5)× SO(5), respectively; a and $\bar{a}$ are the corresponding vector indices. The original SO(5) is, of course, embedded as the diagonal subgroup (just as for D=3 [5]).
We redefine the spin-$\frac{1}{2}$ fields

$$\chi_{a\bar{A}} \equiv \frac{1}{2} \Pi_+ \Gamma^b \Gamma_a \Psi'_b$$
$$\chi_{\bar{a}A} \equiv \frac{1}{2} \Pi_- \Gamma^b \Gamma_a \Psi'_b \tag{4.4}$$

and introduce the "16-bein"

$$e^m_A{}^{\bar{B}} \equiv i \Delta^{-\frac{1}{4}} e_a{}^m \Gamma^a_{AB}. \tag{4.5}$$

This yields SO(5)× SO(5) covariant expressions for the transformation laws (2.5), (2.7).

$$\delta e^\alpha_\mu = -\frac{i}{2} (\bar{\varepsilon}^A \gamma^\alpha \Psi_{\mu A} + \bar{\varepsilon}^{\bar{A}} \gamma^\alpha \Psi_{\mu \bar{A}}) \tag{4.6}$$

$$\delta B_\mu{}^m = -\frac{1}{2} e_A{}^m{}_{\bar{B}} \left(\bar{\varepsilon}^A \Gamma^* \Psi_{\mu B} + \frac{1}{2} (\bar{\varepsilon}^\gamma \gamma_\mu \Gamma^b)^A \chi_{\bar{b}B} \right) + (A \leftrightarrow \bar{A}, a \leftrightarrow \bar{a}) \tag{4.7}$$

Discarding a local SO(5)× SO(5) rotation we obtain for the supersymmetry transformation of $e^m_A{}^{\bar{B}}$ the covariant result

$$\delta e^m_A{}^{\bar{B}} = -\frac{i}{4} \left[\bar{\varepsilon}^A \Gamma_{aA} \chi_{\bar{b}B} - \bar{\varepsilon}^{\bar{A}} \Gamma_{\bar{b}A}{}^{\bar{B}} \chi_{a\bar{B}} \right] (\Gamma^a e^m \Gamma^b)_A{}^{\bar{B}}. \tag{4.8}$$

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One can interpret (4.8) as a (local) $E_{5(+5)} = SO(5,5)$ rotation in the following way. The spinor representation of $SO(5,5)$ has 32 components. In the Weyl basis it is sufficient to consider the 16 upper components. We use the explicit form

$$\hat{\Gamma}^{ab}_{AB,CD} = -\Gamma^{ab}_{AC}\,\delta_{BD}, \quad \hat{\Gamma}^{\hat{a}\hat{b}}_{AB,CD} = -\Gamma^{\hat{a}\hat{b}}_{DB}\,\delta_{AC}$$
$$\hat{\Gamma}^{\hat{a}b}_{AB,CD} = \Gamma^a_{AC}\,\Gamma^b_{DB}.$$ (4.9)

Acting on $e^m_A{}^B$ the generators $-\hat{\Gamma}^{ab}_{IJ}$ and $\hat{\Gamma}^{\hat{a}\hat{b}}_{IJ}$ ($I = A\bar{B}, \ldots$) give rise to the proper $SO(5) \times SO(5)$ transformation.† The introduction of $E_{5(+5)}$ valued fields (B, A) leads to $SO(5) \times SO(5)$ covariant expressions for the fermionic transformation laws. With the abbreviation

$$F_{\alpha ab} = \varepsilon_{abcde}\,F_\alpha{}^{cde}, \quad F_a = \varepsilon_{abcde}\,F^{bcde}$$

one finds

$$B_{\mu\,ab} = -\frac{1}{2}\omega_{\mu\,ab} + \frac{\sqrt{2}}{24}F_{\mu\,ab}$$
$$B_{\mu\,\hat{a}\hat{b}} = \frac{1}{2}\omega_{\mu\,ab} - \frac{\sqrt{2}}{24}F_{\mu ab}$$
$$A_{\mu\,ab} = \frac{1}{2}\Omega_{\mu(ab)} - \frac{1}{4}\eta_{ab}\Omega_{\mu c}{}^c + \frac{\sqrt{2}}{24}F_{\mu ab} = -A_{\mu\,\hat{b}\hat{a}}$$
$$B_{m\,ab} = \frac{1}{2}e_{[a}{}^n \partial_m e_{b]n} - \frac{\sqrt{2}}{192}e_{m[a}F_{b]}$$ (4.10)
$$B_{m\,\hat{a}\hat{b}} = \frac{1}{2}e_{[a}{}^n \partial_m e_{b]n} + \frac{\sqrt{2}}{192}e_{m[a}F_{b]}$$
$$A_{m\,ab} = \frac{1}{2}e_{(a}{}^n \partial_m e_{b)n} - \frac{1}{4}\eta_{ab}(\Delta^{-1}\partial_m\Delta) - \frac{\sqrt{2}}{192}e_{m[a}F_{b]}$$
$$B_{\mu IJ} = -B_{\mu ab}\hat{\Gamma}^{ab}_{IJ} + B_{\mu\,\hat{a}\hat{b}}\hat{\Gamma}^{\hat{a}\hat{b}}_{IJ}, \quad A_{\mu IJ} = A_{\mu ab}\hat{\Gamma}^{ab}_{IJ}$$
$$(B_m, A_m \text{ analogous}).$$

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† This representation is in fact 16 dimensional, because we can impose the $SO(5,5)$ Majorana condition $e^m_I = \hat{C}_{IJ}(e^m_J)^*$, $\hat{C}_{AB,CD} = C_{5\,AC}\,C^{-1}_{5\,DB}$. $e^m_{\bar{A}}{}^{\bar{B}} \equiv (e^m_A{}^B)^*$ transforms in the complex conjugate $SO(5,5)$ representation. In the gauge (4.5): $e^m_{\bar{A}}{}^B = e^m_A{}^{\bar{B}}$.

The "generalized vielbein postulate" reads

$$\mathcal{D}_\mu e^m_I + \frac{1}{4}\partial_n B_\mu{}^n e^m_I + \partial_n B_\mu{}^m e^n_I + \left(\frac{1}{2}B_{\mu IJ} + A_{\mu IJ}\right)e^m_J = 0$$ (4.11)

$$\partial_m e^n_I + \left(\frac{1}{2}B_{m IJ} + A_{m IJ}\right)e^n_J = 0.$$ (4.12)

Furthermore we obtain

$$\mathcal{G}_{\alpha\beta A}{}^B = -\frac{1}{4}\Delta^{\frac{1}{4}}\Omega_{\alpha\beta a}\Gamma^a\Gamma^* + \frac{\sqrt{2}}{8}i\Delta^{-\frac{1}{4}}F_{\alpha\beta ab}\Gamma^{ab} + \frac{\sqrt{2}}{96}i\Delta^{-\frac{1}{4}}\varepsilon_{\alpha\beta\gamma\delta\varepsilon\zeta}F^{\gamma\delta\varepsilon\zeta}\Gamma^*$$ (4.13)

In addition, there appears a three index field strength

$$F^+_{\alpha\beta\gamma a} = \frac{1}{2}\left(F_{\alpha\beta\gamma a} + \frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta\varepsilon\zeta}F^{\delta\varepsilon\zeta}{}_a\right)$$ (4.14)
$$F^-_{\alpha\beta\gamma\hat{a}} = \frac{1}{2}\left(F_{\alpha\beta\gamma a} - \frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta\varepsilon\zeta}F^{\delta\varepsilon\zeta}{}_{\hat{a}}\right).$$

Using $SO(1,5) \times [SO(5) \times SO(5)]$ covariant derivatives

$$D_\mu\varepsilon_A = \left(\mathcal{D}_\mu - \frac{1}{4}\hat{\omega}_{\mu\alpha\beta}\gamma^{\alpha\beta}\right)\varepsilon_A + \frac{1}{2}B_{\mu ab}(\Gamma^{ab}\varepsilon)_A$$ (4.15)
$$D_m\varepsilon_A = \left(\partial_m + \frac{1}{4}(e_\alpha{}^\mu \partial_m e_{\mu\beta})\gamma^{\alpha\beta}\right)\varepsilon_A + \frac{1}{2}B_{m ab}(\Gamma^{ab}\varepsilon)_A$$

(and analogous ones for $A \leftrightarrow \bar{A}$, $a \leftrightarrow \bar{a}$) one arrives at manifestly covariant transformation rules for the fermionic fields

$$\delta\Psi_{\mu A} = \left[D_\mu(\hat{\omega}, B_\mu) - \frac{1}{8}(\partial_m B_\mu{}^m)\right]\varepsilon_A + \left(\frac{1}{8}\gamma_\mu\gamma^{\alpha\beta} - e_\mu{}^\alpha\gamma^\beta\right)\mathcal{G}_{\alpha\beta A}{}^B\varepsilon_B$$
$$- \frac{1}{4}e^m_A{}^B\Gamma^*[D_m(\gamma_\mu\varepsilon_B) + (\partial_m g_{\mu\nu})\gamma^\nu\varepsilon_B]$$
$$+ \frac{1}{8}\gamma_\mu A_{mab}(\Gamma^a e^m\Gamma^b)_A{}^B\varepsilon_B + \frac{\sqrt{2}}{48}i\gamma^{\alpha\beta\gamma}\gamma_\mu F^+_{\alpha\beta\gamma d}(\Gamma^d\Gamma^*\varepsilon)_A$$ (4.16)

$$\delta\chi_{\hat{a}A} = \frac{1}{2}e^m_A{}^B(\Gamma_{\hat{a}}D_m\varepsilon)_B + \frac{1}{4}\gamma^{\alpha\beta}\mathcal{G}_{\alpha\beta A}{}^B(\Gamma_{\hat{a}}\Gamma^*\varepsilon)_B$$
$$- A_{mb\hat{a}}(\Gamma^b e^m)_A{}^B\Gamma^*\varepsilon_B + \frac{1}{4}A_{mb\hat{c}}(\Gamma^b e^m\Gamma^{\hat{c}})_A{}^B(\Gamma_{\hat{a}}\Gamma^*\varepsilon)_B$$
$$- \gamma^\mu A_{\mu b\hat{a}}(\Gamma^b\varepsilon)_A + \frac{\sqrt{2}}{24}i\gamma^{\alpha\beta\gamma}F^-_{\alpha\beta\gamma\hat{a}}\varepsilon_A$$ (4.17)

$$(A \leftrightarrow \bar{A}, \; a \leftrightarrow \bar{a} \text{ for } \delta\Psi_{\mu\bar{A}} \text{ and } \delta\chi_{\bar{a}\bar{A}}).$$

5. Concluding remarks

The H_{11-D} (and E_{11-D}) structure of d=11 supergravity has now been established for $D = 3, 4, 5, 6$. After a coordinate split the various components of the d=11 graviton and the three index "photon" are combined into irreducible representations of the group H_{11-D}. The key ingredient is the generalized vielbein field [4,5].

$$e^m \sim \Delta^{-s} e_a^{\ m} \Gamma^a ,$$ (5.1)

which satisfies a "generalized vielbein postulate" covariant under global E_{11-D} rotations. The fermionic redefinitions coincide with those needed to diagonalize the Rarita-Schwinger terms after dimensional reduction. One obtains H_{11-D} covariant expressions for the supersymmetry transformation laws; the covariance of the equations of motion follows by the usual arguments [4]. There is little doubt that the procedure can also be carried out for the remaining values of D. For $D = 7$ the necessary redefinitions and field assignments can be read off directly from [7].

The general coordinate transformations take a rather unusual form in the covariantized versions of d=11 supergravity. For the elfbein E_M^A compensating $SO(1,10)$ rotations are needed to preserve the gauge (2.1) and a transformation with parameter $\xi^m = -B_\nu^{\ m} \xi^\nu$ is included [4]. For (5.1) one finds

$$\delta_{g.c.} e^m = \xi^\mu \left[\mathcal{D}_\mu e^m + s\, \partial_n B_\mu^{\ n} e^m + \partial_n B_\mu^{\ m} e^n \right] + \xi^n \partial_n e^m - s\, \partial_n \xi^n e^m - \partial_n \xi^m e^n .$$ (5.2)

Comparing with (3.19),(4.11) we see that (B_μ, A_μ) generate a D dimensional general coordinate transformation of e^m. The $11-D$ dimensional part of (5.2) can also be expressed as a E_{11-D} rotation, with parameter $\xi^m(\frac{1}{2} B_m + A_m)$ (cf. (3.20),(4.12)) and a corresponding term in which $\xi^m(e_a^{\ n} \partial_m e_{bn})$ is replaced by $e_a^{\ n} \partial_m \xi^n e_{bn}$.

References

[1] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. 76B (1978) 409

[2] E. Cremmer and B. Julia, Nucl. Phys. B 159 (1979) 141

[3] E. Cremmer and B. Julia, Phys. Lett. 80 B (1978) 48

[3] E. Cremmer in Superspace and supergravity (Cambridge Univ. Press, 1980)

   E. Cremmer, N=8 Supergravity in Erice lectures (1980)

[4] B. de Wit and H. Nicolai, Nucl. Phys. B 274 (1986) 363

   B. de Wit and H. Nicolai, Phys. Lett. 155 B (1985) 47

[5] H. Nicolai, Phys. Lett. 187B (1987) 316

[6] H. Nicolai, Phys. Lett. 194B (1987) 402

[7] H. Nicolai and N.P. Warner, CERN preprint TH 5154 (1988)

[7] E. Sezgin, A. Salam, Phys. Lett. 118 B (1982) 359