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Estimates of the gluon-fusion contribution to the proton structure function at exceedingly small x

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Abstract

Updated estimates are given for the contribution of gluon fusion with virtual photons to the proton structure function at very small x. The sensitivity of these estimates to the assumed input of parton distributions and gluon structure functions are examined in detail.

1 Introduction

The importance of the fusion of gluons with virtual photons (andZ^0) has been recognized by many [1,2,3,4,5] as the major process contributing to e-p deep inelastic scattering at very small x. The Q^2 evolution of the target nucleon structure functions are described by the Lipatov-Altarelli-Parisi equations [6,7] ,where the incoming energy does not appear explicitly.However,we note that the kinematic lower bound of x depends on the incoming energy as $x_{min} = \frac{Q^2}{2ME_L}$. Consequently, our ability to explore the very small x domain is dependent on the available energy in a given deep inelastic lepton-nucleon experiment.It is not suprising, that with the e-p collider HERA nearing completion,there has been a renewed interest in the process of fusion of gluons with electro-weak gauge bosons. However, detailed calculations have been confined to estimates of heavy quark production [8,9,10] and exotic processes,such as Higgs production[11,12,13] .The purpose of this communication is to provide a detailed and up to date analysis of gluon-photon fusion and its influence on the nucleon structure function behaviour at small x.

For our calculation we utilize the Weizacker -Williams Approximation(WWA)[14].This enables us to write the cross sections of interest as a factorizable product of the

$\gamma(Z^0)g \to q\bar{q}$ cross section (or equivalently the gluon structure function F_i^g) and the flux factor $f_{g/p}$ corresponding to the gluon density within the target proton. The original WWA was based on the assumption that both intermediate vector bosons are essentially real and thus only the transverse polarizations are retained . This assumption can be relaxed without modifying the factorization property,and we have:

$$\frac{d^2\sigma}{dQ^2dx_g} = \frac{4\pi\alpha^2}{Q^4x_p}\{(1-y)F_2^g(x_g,Q^2) + x_gy^2F_1^g(x_g,Q^2)\}.f_{g/p}(z)dz \quad (1)$$

using the kinematic notation illustrated in Fig.1.

$$s = (k+p_1)^2;\ \hat{s} = W_{\gamma g}^2 = (q_1+q_2)^2;\ q_1^2 = -Q^2; \quad (2)$$

with the standard kinematic variables

$$x_g = \frac{Q^2}{2q_1.q_2};\ y = \frac{q_1.q_2}{k.q_2};\ z = \frac{q_1.q_2}{q_1.p_1} \quad (3)$$

The kinematic variable x_g differs from x_p the standard scaling variable pertinent to deep e-p inelastic scattering

$$x_p = \frac{Q^2}{2q_1.p_1} = zx_g \quad (4)$$

with

$$x_g \geq x_p.$$

Thus, the contribution of the photon-gluon fusion to the deep e-p inelastic scattering cross section is given by:

$$\frac{d^2\sigma}{dQ^2dx_p} = \int_{ax_p}^1\int_{z_{min}}^1 \frac{d^3\sigma}{dQ^2dx_gdz}dx_g\delta(z-\frac{x_p}{x_g})dz \quad (5)$$

where $a = 1 + \frac{4m_q^2}{Q^2} \sim 1$. Equation (5) relates to the nucleon structure functions:

$$\frac{d^2\sigma}{dQ^2dx_p} = \frac{4\,\pi\,\alpha^2}{x_pQ^2}\{(1-y)F_2^{ep}(x_p,Q^2) + x_pyF_1^{ep}(x_p,Q^2)\} \quad (6)$$

Throughout this study we assume that the Callan-Gross relation $F_2 = 2x_pF_1$ holds.

The use of WWA simplifies our calculations and enables us to investigate explicitly how different parameterizations of the the gluon structure function F_i^g, and $f_{g/p}$ the gluon distribution in the proton, effect our final results. It has been established by previous investigations that the difference between the WWA and an explicit calculation is rather small [8,9,10,13,15]. Since our analysis is confined to $Q^2 < 5000\,GeV^2$ we have conveniently omitted the Z^0 contribution, the inclusion of which is straight forward.

The formalism we have just presented is analogous to the well known two-photon EPA approximation [16,17],however, this analogy should not be carried too far. The two-photon process deals with virtual photons which are coherently emitted from charged incoming particles.Thus we may use a single tag pole approximation for

$f_{\gamma/p}$.In the case of photon-gluon fusion, $f_{g/p}$ is very different being a convolution of the gluon density within an emitting quark, and the quark's distribution within the target proton.This convolution has not been calculated reliably thus far,and we follow previous calculations [18,21] and utilize phenomenological parameterizations for $f_{g/p}$.

The photon-gluon fusion process has an important role, as input in the LAP equations [6,7] to determine the Q^2 evolution of the structure functions. Following GHR [18] we write

$$\partial F_2^{ep}(x_p,Q^2)/(\partial \ln Q^2) = (\alpha_s(Q^2)x_p)/(2\,\pi)\{\int_{x_p}^1 \frac{dz}{z}\frac{1}{z}F_2^{ep}(z,Q^2)P_{qq}(\frac{x_B}{z}) + \sum_{q\bar{q}} e_q^2 \int_{x_p}^1 \frac{dz}{z} G(z,Q^2)P_{qg}(\frac{x_p}{z},Q^2) + \sum_H C_H \int_{ax_p}^1 \frac{dz}{z} G(z,Q^2)P_{Hg}(\frac{x_p}{z},Q^2)\} \quad (7)$$

where the first term on the r.h.s. contains the quark-quark splitting functions, while the second and third terms,involve the quark-gluon splitting function.The third term represents the contribution of the heavy quarks via photon-gluon fusion to F_2^{ep}. We will only concern ourselves in the following with the second term in equation (7) i.e. the contribution of the u,d,s and c quarks via photon-gluon fusion to F_2^{ep}.

The first order leading log approximation(LLA) for

$$P_{qg}(w,Q^2) = 0.5(w^2 + (1-w)^2)\log\frac{Q^2}{\Lambda^2} \quad (8)$$

To simplify the notation we write

$$F_2^{ep}(x_p,Q^2)/x_p = \int_{x_p}^1 \frac{dz}{z}\{zG(z,M_g^2)F_2(\frac{x_p}{z},Q^2)\} \quad (9)$$

which is the contribution of the light quarks to the proton structure function. zG(z) denotes the gluon distribution within the target proton and $f_2(y)$ the structure function for the subprocess $\gamma^* + g \to q + \bar{q}$. We discuss the two terms seperately:

2 Parameterization of the parton distributions

Older parameterizations such as that suggested by GHR [18] are only applicable for $x > 0.01$, GHR has $\Lambda_{QCD} = 0.4\ GeV$. With the promise in the near future of small angle tagging devices at HERA, one will be able to investigate events at $x_p = 10^{-4}$, for which the GHR parameterizations are not adequate. Although there is not much experimental data available below $x \leq 0.01$, new fits based on different plausible extrapolation as $x \to 0$ have been made. In this investigation we will use a set of distributions suggested by Eichten et al.[20] . The structure functions labelled EHLQ1 have $\Lambda_{QCD} = 0.2\ GeV$, while EHLQ2 has $\Lambda_{QCD} = 0.29\ GeV$. Eichten et al.[20] is a

leading log calculation , and valid for $5\ GeV^2 \leq Q^2 \leq 10^8\ GeV^2$ and $10^{-4} \leq x \leq 1$. Next to leading order(NLO) evolution to the splitting functions have been used by Martin,Roberts and Stirling (MRS) [19] , however this only gives small corrections and the resulting proton structure functions are very similar to those obtained with the LLA. We will also use two parameterizations suggested by MRS [19], MRS1 has $\Lambda_{QCD} = 107\ MeV$. and gives results which do not differ much from those of Duke and Owen DO1 [21], which is a LLA calculation with $\Lambda_{QCD} = 200\ MeV$. MRS3 has an initial gluon distribution of the form

$$xG(x,Q_0^2) = \frac{C}{\sqrt{x}}(1-x)^4(1+9x) \quad (10)$$

and $\Lambda_{QCD} = 178$ MeV.

In figure 2 we compare the $F_2^{ep}(x_p)$ resulting from the different quark distributions for $Q^2 = 10, 10^2, 10^3 GeV^2$ and for $x_{min} \leq x_p \leq 0.3$,assuming a nominal value of $E_L = 50,000GeV$ at HERA. We define

$$F_2^{ep}(x,Q^2) = \sum_f e_f^2\{xq_f(x,Q^2) + x\bar{q}_f(x,Q^2)\} \quad (11)$$

e_f denotes the electric charge of the quark, q_f and $\bar{q}_f$ are the quark and the antiquark densities given in the different schemes. We only consider photon exchange, as the Z^0 exchange contribution is neglegible at these Q^2 values.

We note that for $Q^2 = 10^3\ GeV^2$, $x_{min} \sim 0.011$ and here there is very little difference between the proton structure functions predicted in the different schemes.GHR is somewhat low,but it might be stretching things to extrapolate to such low x in this scheme. See fig.2 (c). For $Q^2 = 10^2\ GeV^2$,$x_{min} \sim 0.0011$(see fig. 2 (b)) differences become appreciable.and at x_{min} we note that MRS3 parameterization yields a F_2^{ep} which is twice that found with EHLQ1.For $Q^2 = 10\ GeV^2$, $x_{min} \sim 10^{-4}$ (see fig 2(a)) and there is a factor of more than three between the proton structure functions calculated in the two schemes. We will therefore use these two parameterizations,MRS3 (largest) and EHLQ1 (smallest) to illustrate our results, and assume that they represent a plausible upper and lower bound.

3 The process $\gamma^* + g \to q + \bar{q}$.

As our parameterization (eqn. (9)) only includes terms to leading order in QCD,two arbitrary scales are present,namely the renormalization scale M_s^2 which appears in the running coupling constant $\alpha_s(M_s^2)$, and the factorization scale M_g^2 which is present in the gluon distribution function $G(x,M_g^2)$. To simplify matters we unite both scales and represent them by a single parameter μ^2, as do [24,10,9]. The question remains,what is a suitable physical scale ? .As Ingelman and Schuler [24] have noted,the deep inelastic scale Q^2 is not appropriate,since the structure function receives important contributions from Q^2 close to zero. .For want of anything better we follow [9,24] , and choose $\hat{s}$,the invariant energy of the subprocess $\gamma^* + g \to q + \bar{q}$ as the scale for both the gluon density, and α_s . We note that $\hat{s} = Q^2(\frac{1-x_g}{x_g})$,also that the smallest values of x_g give the dominant contribution to the integral in equation (9).Thus for values of most interest $\hat{s} > Q^2$, and hence using $\alpha_s(Q^2)$ instead of $\alpha_s(\hat{s})$ would increase our results accordingly. We have chosen the following three representative models for the gluon structure function $f_2^{\gamma+g\to q+\bar{q}}$.

3.1 Leading Log Approximation (LLA)

For which

$$f_2^{\gamma+g\to q+\bar{q}}(y,Q^2) = \frac{\alpha_s(\hat{s})}{2\,\pi}\sum_q e_q^2\{\frac{y^2+(1-y)^2}{2}\}\ln(\frac{Q^2}{\Lambda^2}) \qquad (12)$$

The QCD running coupling constant is given by

$$\alpha_s(\hat{s}) = \frac{12\,\pi}{(33-2N_f)\ln(\frac{\hat{s}}{\Lambda^2})} \qquad (13)$$

with N_f ,the number of flavours = 3. We take Λ = 200 MeV.

3.2 Quark Parton Model (QPM)

Budnev et al [22] have written down the cross section for the interaction $\gamma^* + \gamma* \to e^- + e^+$,replacing m_e by m_q and inserting the appropriate charges and colour factors gives us the cross section for $\gamma^* + g \to q + \bar{q}$ i.e. for the production of a pair of massive free quarks by photon-gluon fusion . We use the constituent quark masses $m_u = m_d$ = 300 MeV , m_s=500 MeV and m_c = 1.5 GeV.

3.3 QCD motivated parameterization (HR)

Hill and Ross [23] use operator product expansion and renormalization group methods to calculate the structure function for $\gamma^* + \gamma^* \to q + \bar{q}$,their treatment includes higher order QCD effects and retains terms of $O(\frac{m_q^2}{Q^2})$. Here

$$f_2^{\gamma+g\to q+\bar{q}}(y,Q^2) = \sum_q \frac{e_q^2\alpha_s}{2\,\pi}\{(-y+6y^2-6y^3)v)$$
$$+\ln(\frac{P_-}{P_+})(y-2y^2+2y^3+\frac{4m_q^2}{q^2}y^2(1-3y)-\frac{8m_q^4}{Q^4}y^3)$$
$$-y^2(-2m_Q^2+\frac{4m_q^4}{Q^2})(\frac{1}{P_-}-\frac{1}{P_+})\}$$

where

$$P_+ = -(\hat{s}+Q^2)(1-v)/2 : P_- = -(\hat{s}+Q^2)(1+v)/2 \qquad (14)$$

and $v^2 = (1-\frac{4m_q^2}{\hat{s}})$.

4 Results and Conclusion

Our numerical results are summarized in Figs 3-5. As we have seen the quark's contribution by gluon-fusion to $F_2^{ep}(x,Q^2)$ depends on two input gluon properties, the gluon distribution within the proton target, and the gluon structure function.Our calculations suggest an approximate factorization of the final result, which enables us to discuss the properties of the two inputs seperately.
a) Our results are not very sensitive to the different parameterizations suggested for the gluon distribution in the proton $xG(x,Q^2)$.This is most clearly seen at Q^2 = 1000 GeV^2. Even with Q^2 = 100 GeV^2 the differences between the various distributions are rather small, except with MRS3 which yields significantly higher results for $x < 0.01$ (see Fig. 3).
b) Our results are sensitive to the assumed gluon structure function $F_2^g(x_g,Q^2)$. We observe that for the parameterizations labelled QPM,HR and LLA, our final results are approximately in the ratio 4:2:1 ,and are almost independent of Q^2 and x variation. (See Figs. 4 and 5).
Clearly the intriguing question remains, to how small a value of x does one have to measure, in order to explore the gluon distribution within the target,bearing in mind that we are in a domain where the background from other parton distributions is expected to be small. Our analysis suggests that the answer to this question depends mostly on our knowledge of the gluon structure function, and to a lesser extent on the assumed parton distributions. These are rather well known for x > 0.05 ,and provide rather severe constraints on the small x distributions. A detailed study of the gluon structure function , is thus crucial for any further analysis of the proton structure function.

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Figure captions

Figure 1 First order QCD diagrams for photon-gluon fusion

Figure 2 Sum of quark contributions to F_2^{ep} (as defined in eqn.(11) for the different parton distributions discussed in the text.

Figure 3 The structure function $f_2^{\gamma+g\to q+\bar{q}}$ using the Hill-Ross result eqn. (14),for the different gluon distribution parameterizations discussed in text. (a) for $Q^2 = 10\ GeV^2$; (b) $Q^2 = 100\ GeV^2$ (c) $Q^2 = 1000\ GeV^2$.

Figure 4 Using the parton distributions MRS3. Quark contribution via photon-gluon fusion to F_2^{ep} for different parameterizations of the structure function $f_2^{\gamma+g\to q+\bar{q}}$. The full line denotes the sum of the quark contributions to F_2^{ep} as defined in eqn (11). (a) for $Q^2 = 10GeV^2$,(b) $Q^2 = 100GeV^2$, (c) $Q^2 = 1000GeV^2$.

Figure 5 As in Fig. 4 ,with the parton distributions EHLQ1.

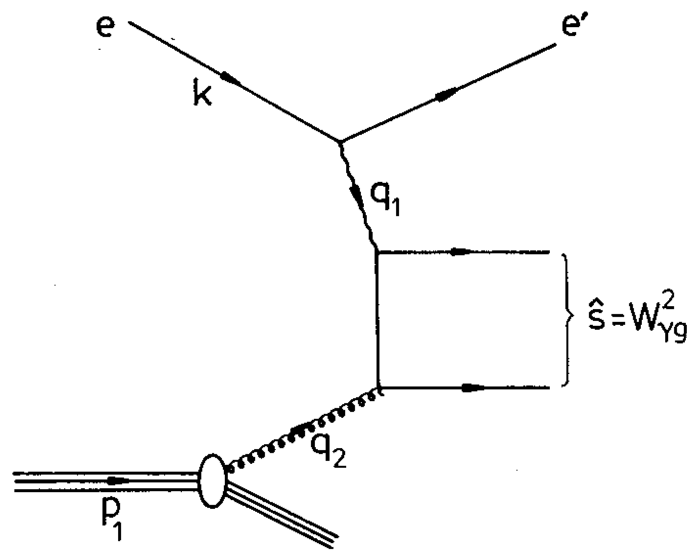


Fig.1

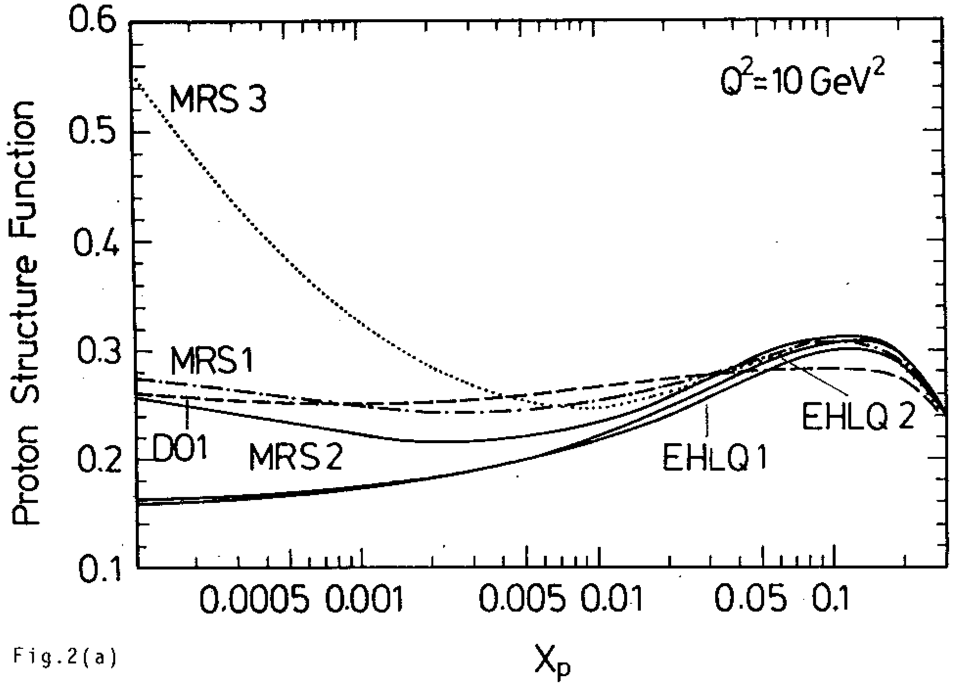
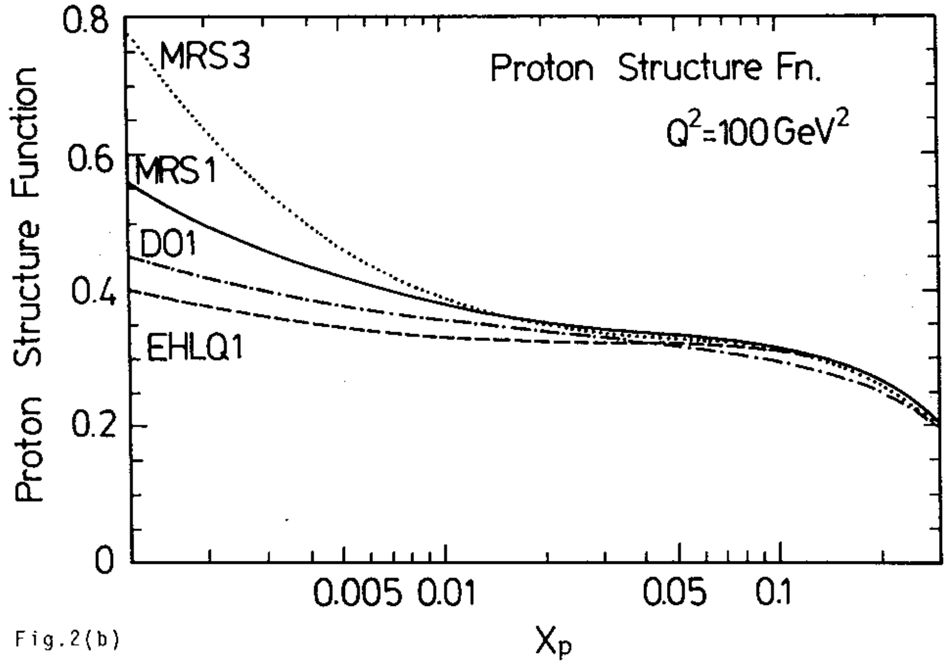


Fig.2(a)



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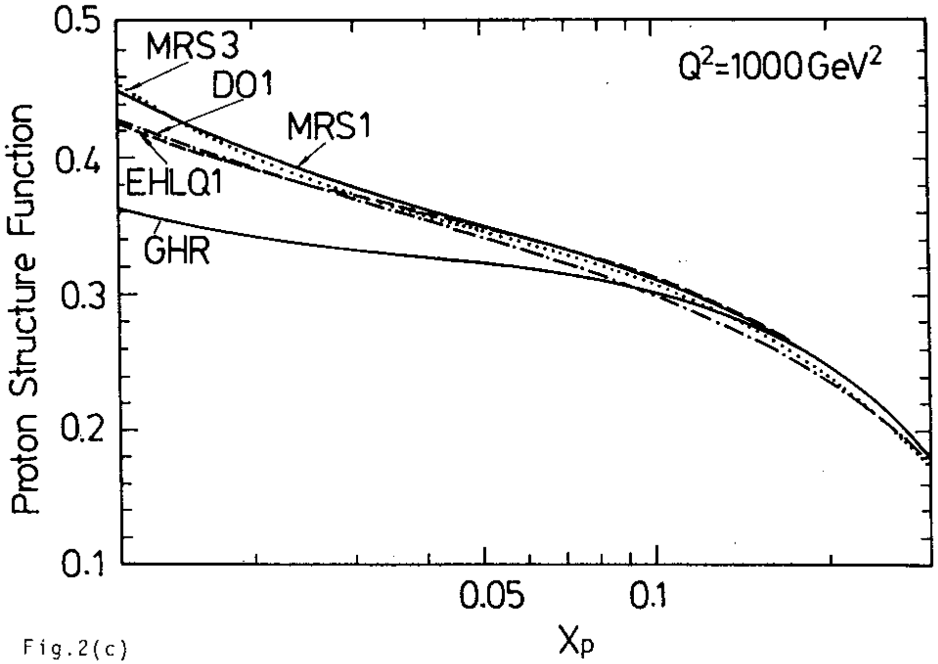

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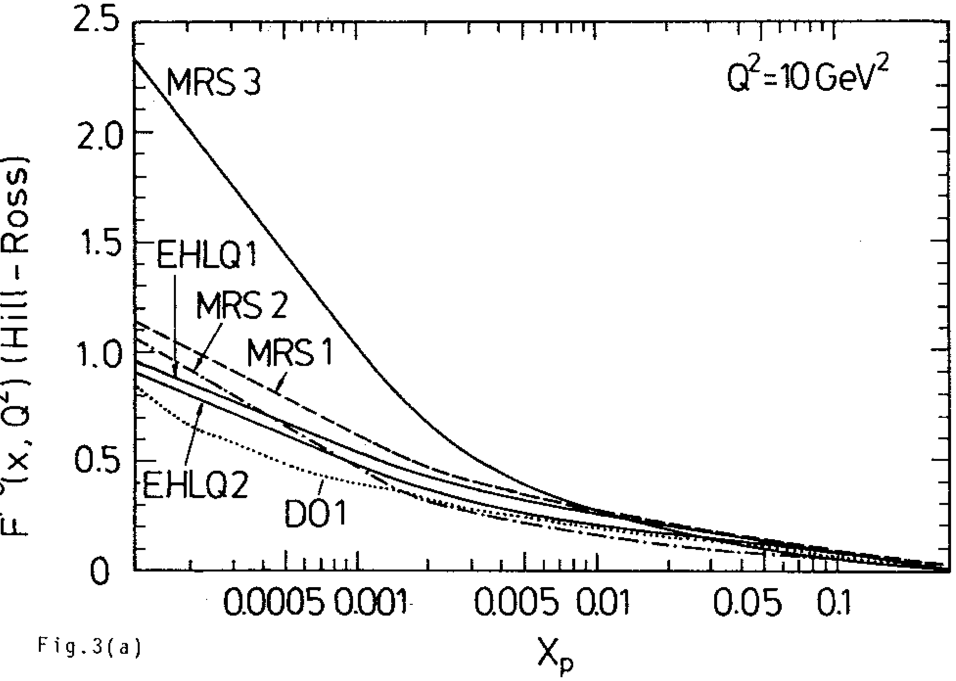

Fig.3(a)

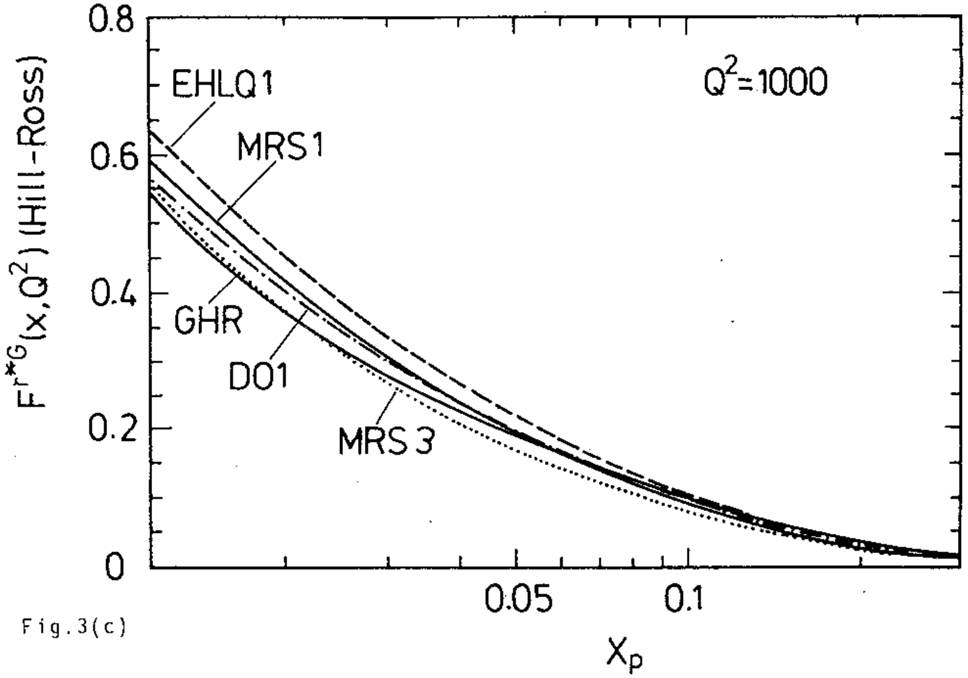

Fig.3(c)

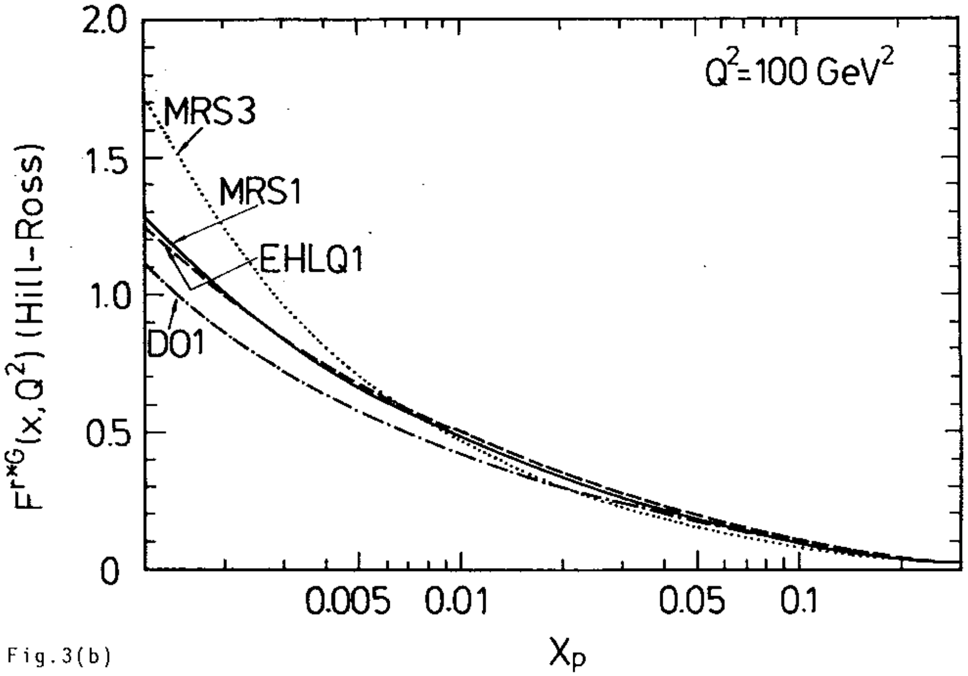

Fig.3(b)

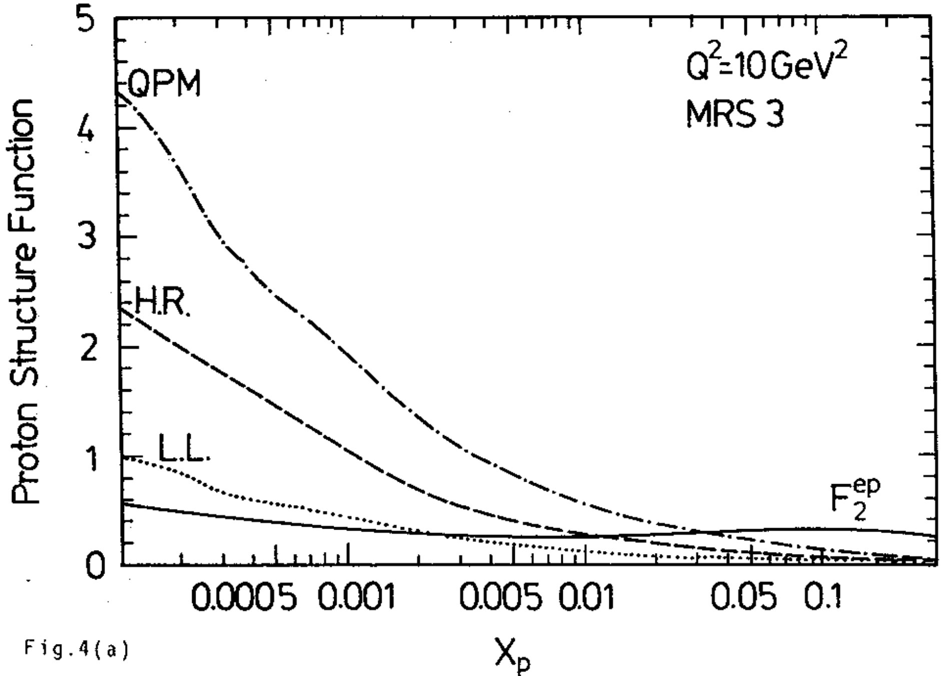

Fig.4(a)

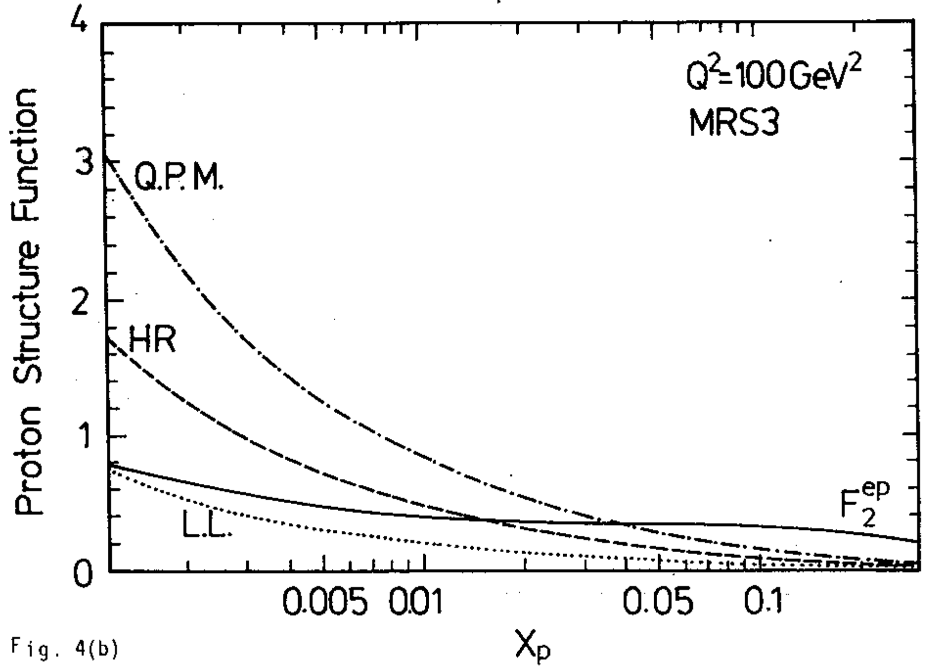

Fig. 4(b)

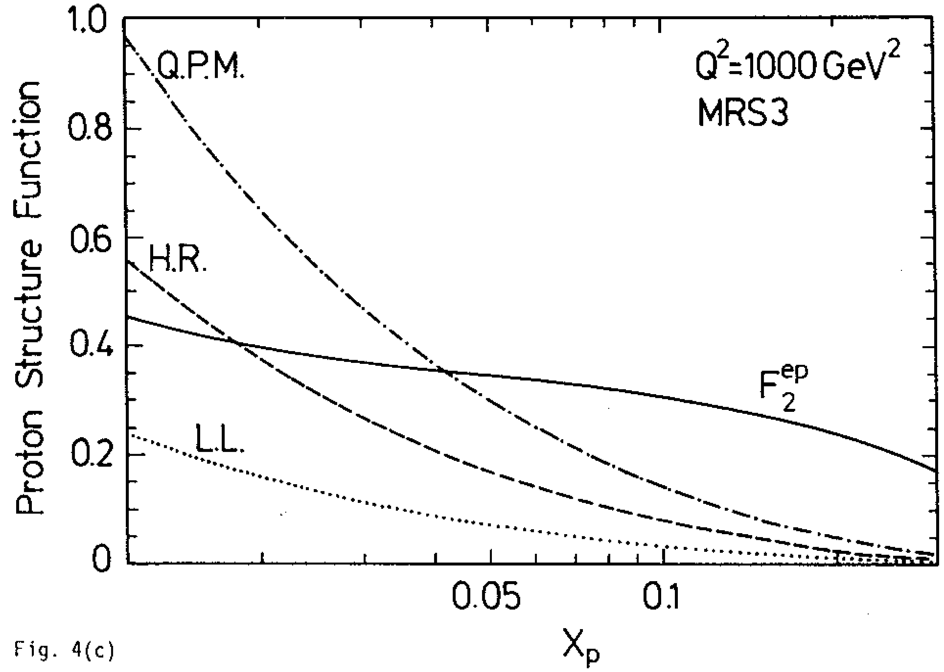

Fig. 4(c)

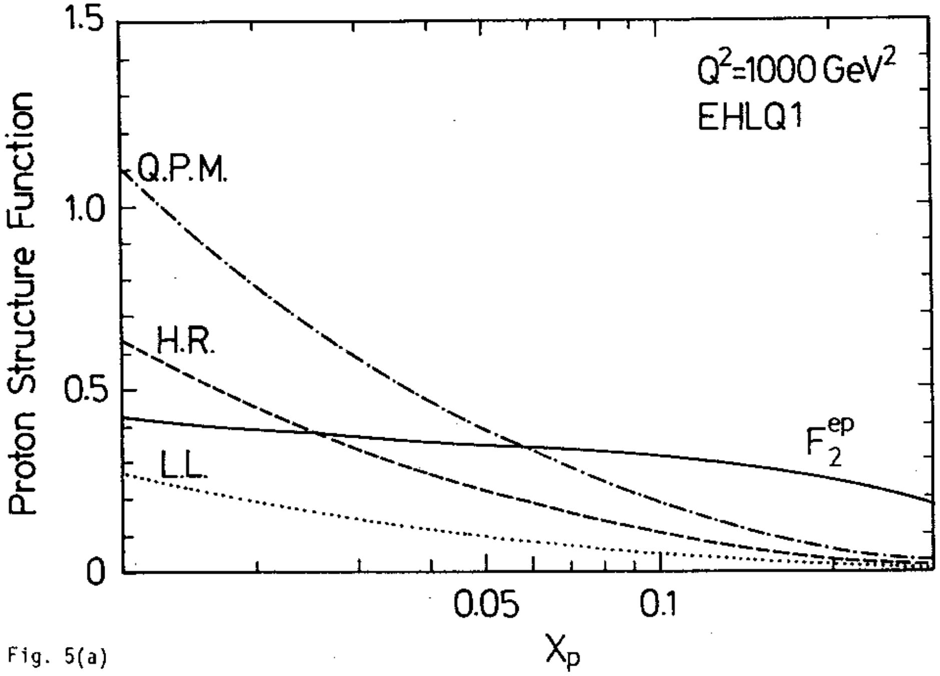

Fig. 5(a)

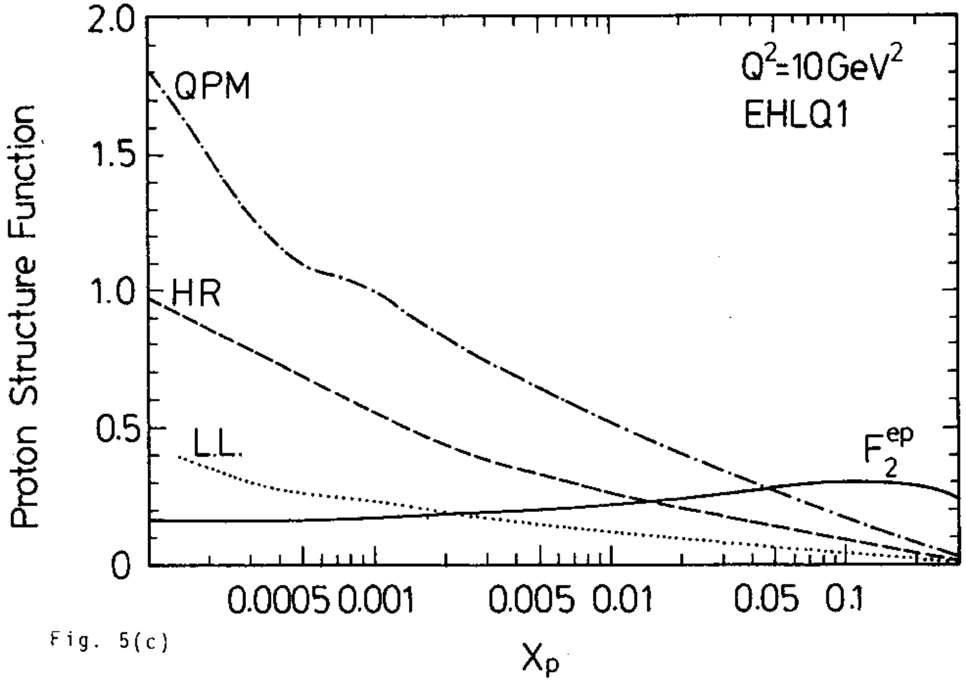

Fig. 5(c)

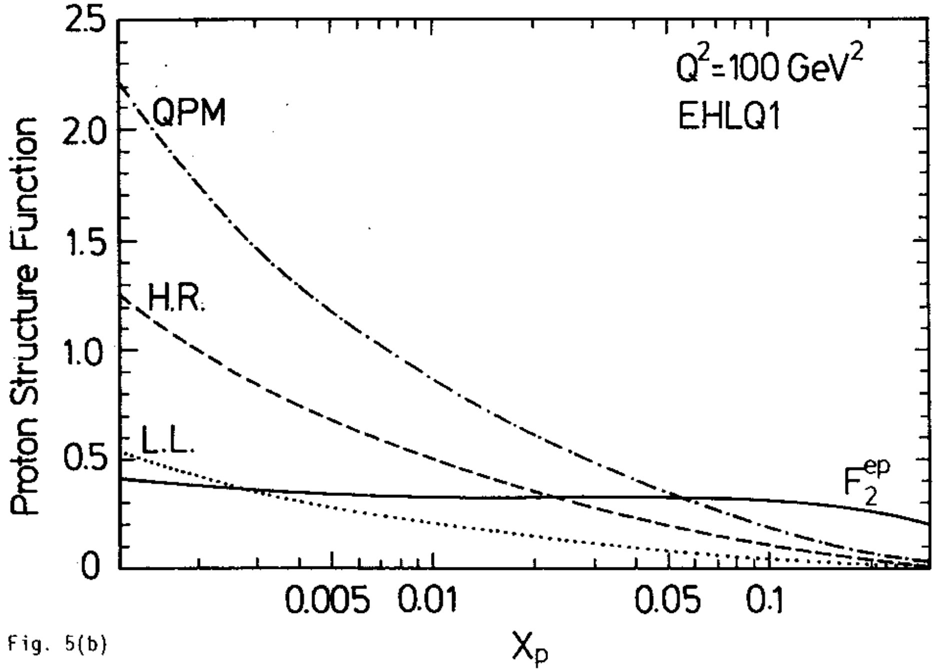

Fig. 5(b)