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Abstract

Chiral perturbation theory in the sense of the Weinberg-Gasser- Leutwyler program is presented for weak hadronic processes. It is argued that it represents a meaningful low energy field theory, which in many cases has advantages over the standard approach. Its relation with approaches which go beyond chiral perturbation theory, but rely on it, is discussed. Finally, the role of tadpoles which appear both in chiral perturbation theory and in operator product expansion of weak currents is clarified.

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OPEN (CLOSED) PROBLEMS IN WEAK HADRONIC PROCESSES

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Chiral perturbation theory in the sense of the Weinberg-Gasser- Leutwyler program is presented for weak hadronic processes. It is argued that it represents a meaningful low energy field theory, which in many cases has advantages over the standard approach. Its relation with approaches which go beyond chiral perturbation theory, but rely on it, is discussed. Finally, the role of tadpoles which appear both in chiral perturbation theory and in operator product expansion of weak currents is clarified.

1. Chiral Perturbation Theory - Basics

Chiral perturbation theory (ChPT) represents a viable alternative low energy theory of strong and electroweak interactions. Its importance as an alternative approach to the usual formulation of the Standard Model (SM) is particularly evident in processes where the lack of knowledge of the true QCD confinement is blurring our view of electroweak interactions. Kaon physics is especially sensitive to the above problems- since kaons and pions, being pseudo-Goldstone bosons, are even less reliably described in phenomenological quark models.

Chiral perturbation theory is based on our knowledge of the fundamental symmetries of the QCD lagrangian. The softly broken chiral $SU(3)_L \times SU(3)_R$ symmetry of the QCD lagrangian is an important symmetry of Nature. The formulation of ChPT is based [1,2] on the following *Ansatz*: the most general lagrangian consistent with a given symmetry would result in any given order in perturbation theory, in the most general S-matrix, consistent with incorporated symmetry, analiticity, perturbative unitarity and cluster decomposition. Formulated in this way, ChPT becomes a quantum field theory.

In order to be so, one has to include **all** possible terms in the lagrangian and take account of all graphs in perturbation theory. The classical field theory is, of course, equivalent to tree graphs of quantum field theory. However, to have a consistent field theory, one has to include also loop graphs. Without loops, theory *violates unitarity*. The inclusion of loops

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would generally lead to infinities. To get rid of them, the theory requires counterterms. Using the regularization that preserves chiral symmetry (e.g. dimensional regularization), one finds that counterterms necessary to renormalize one- loop graphs are of order p^4, to be compared with the lowest order tree level which is of order p^2. Evidently, starting with the p^2-order $\mathcal{L}^{(2)}$, one gets counterterms of order p^4 with *the same structure* as the p^4 - order $\mathcal{L}^{(4)}$. Using the terms in $\mathcal{L}^{(4)}$ as counterterms leads to finite results for all Green's functions to one-loop order.

The strong lagrangian at order p^2 (with minimal number of derivatives) is given by the nonlinear σ -model

$$\mathcal{L}^{(2)} = \frac{f^2}{8} tr(\partial_\mu U \partial^\mu U^\dagger) = \frac{f^2}{8} g_{ab}(\phi)\partial_\mu \phi^a \partial^\mu \phi^b \tag{1}$$

for a massless field, with the invariant metric

$$g_{ab}(\phi) = tr(\partial_a U \partial_b U^\dagger). \tag{2}$$

U is the unitary matrix field

$$U(\phi) = \exp i\frac{2}{f}\Phi. \tag{3}$$

$\Phi = \frac{1}{\sqrt{2}}\lambda^a\phi^a$ is given by

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}. \tag{4}$$

The lagrangian $\mathcal{L}^{(2)}$ of the nonlinear σ-model is, of course, nonrenormalizable, and agrees with QCD only at tree level (leading behavior).* To get the full expansion, according to our *Ansatz* one has to add all terms of higher order in p^2 (terms with more derivatives)

$$\mathcal{L}_{strong} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots \tag{5}$$

This represents the full strong lagrangian. Again , the graphs containing n loops are suppressed by $(p^2)^n$ in comparison with the tree graphs. Therefore, the full $\mathcal{L}_{strong}$ in (5) specifies a perturbatively renormalizable scheme. For example, up to order p^6, we would have

p^2- contribution: tree graphs from $\mathcal{L}^{(2)}$

p^4 -contribution: one loop graphs from $\mathcal{L}^{(2)}$ + tree graphs from $\mathcal{L}^{(4)}$

p^6 contribution: two loop graphs from $\mathcal{L}^{(2)}$ + one loop graphs with one vertex from $\mathcal{L}^{(4)}$ + tree graphs with two vertices from $\mathcal{L}^{(4)}$ + tree graphs from $\mathcal{L}^{(6)}$.

* The choice of the effective lagrangian is not unique. At the tree level both linear and nonlinear lagrangian lead to the proper value of two constants f_π and v. At the one loop level , however, they disagree. The linear (renormalizable) σ model leads to relations among the coupling constants [2] which are in disagreement with experiment.

If the quark mass matrix $\mathcal{M}$ is different from zero, all terms in $\mathcal{L}_{strong}$ would pick up additional terms. The strong lagrangian then reads to the lowest order

$$\mathcal{L}^{(2)}_{strong} = \frac{f^2}{8} tr(\partial_\mu U \partial^\mu U^\dagger) + v\ tr(\mathcal{M}U + U^\dagger \mathcal{M}), \tag{6}$$

with

$$\frac{4v}{f^2} = \frac{m^2_{\pi^+}}{m_u + m_d} = \frac{m^2_{K^+}}{m_s + m_u} = \frac{m^2_{K^0}}{m_s + m_d}, \tag{7}$$

and v is proportional to the quark condensate, $v = -1/4 < 0|\bar{q}q|0 >$. The lowest order $\mathcal{L}^{(2)}_{strong}$ at tree level is given basically by two coupling constants, f_π and v, which are expected to be calculable in QCD. Going to the next order, the number of coupling constants increases. Although they should be ultimately calculable in QCD, in ChPT they are arbitrary and have to be determined in physical processes. The complete chiral lagrangian in strong and electromagnetic interactions is known to order p^4 as given in the work of Gasser and Leutwyler [2]. Besides the two couplings, f_π and v in $\mathcal{L}^{(2)}$, the ten coupling constants L_i in $\mathcal{L}^{(4)}$ are uniquely determined. In principle, the same procedure applies to the weak interactions in ChPT.

The chiral realization of quark currents can be obtained by gauging the lagrangian $\mathcal{L}^{(2)}_{strong}$ locally*. This leads to the unique currents $(V-A)_\mu$ and $(V+A)_\mu$:

$$\begin{aligned} (V-A)_\mu &= i\tfrac{f^2}{2} U \partial_\mu U^\dagger \\ (V+A)_\mu &= i\tfrac{f^2}{2} U^\dagger \partial_\mu U. \end{aligned} \tag{8}$$

The extension of ChPT to weak processes is in principle straightforward; in practice, however, the predictive power is spoiled by a large number of unknown coupling constants. The rich field of applications are rare K-decays because of many decay channels, which enables one to fix counterterms uniquely. This program has been put forward recently by Ecker *at al.*[3], showing the powerfulness of ChPT in processes where a complicated interplay between short- and long-distance effects and difficulties in calculating matrix elements cause the standard approaches to fail.

In weak $\Delta S = 1$ kaon decays, CP-violation *etc.* one has unfortunately only a few channels and this fact makes it very difficult to go beyond the tree level chiral lagrangian. Nevertheless, the chiral realization of weak lagrangians[4] has been found to be very useful as it "supports" other approaches like the QCD lattice Monte Carlo simulation[5]. Some recent programs that aim to go beyond ChPT, such as QCD Duality Sum Rules[6-9] or the $1/N_c$ expansion[10-13] rely heavily on ChPT.

In the following chapters we present the weak interaction sector of ChPT and comment on recent attempts to go beyond ChPT.

* One applies a local linear transformation to the U field, $\delta U = -iT^A\theta^A(x)$, where $\theta^A(x)$ is an arbitrary gauge function and finds the conserved current by using $\delta\mathcal{L}(\phi) = -j^A_\mu \partial^\mu \theta^A(x)$.

2. $\Delta I = 1/2$ Rule in ChPT

...me tanquam umbra sequitur.

A typical weak process, like $K \to \pi\pi$, is given in the Standard Model by lowest possible order in weak interactions and *all* orders in strong interactions. The basic identification is given as follows:

$$< \pi\pi|\mathcal{L}_{eff} \sim \int d^4x D_F(xM_W) T(j^\mu(x) j^\dagger_\mu(0))|K >^{dressed} \equiv < \pi\pi|\mathcal{L}_{chiral}|K >, \qquad (9)$$

where $\mathcal{L}_{chiral}$ is the effective lagrangian in ChPT. Applying the Wilson operator product expansion (OPE) to the l.h. side of (9) leads to the set of quark operators with definite symmetry properties [14]. It is then possible to find their respective chiral realization.

Instead of presenting the complete weak sector of ChPT sistematically, we choose to select some particular examples (see Fig. 1) where the advantages and drawbacks of weak ChPT become illustrative.

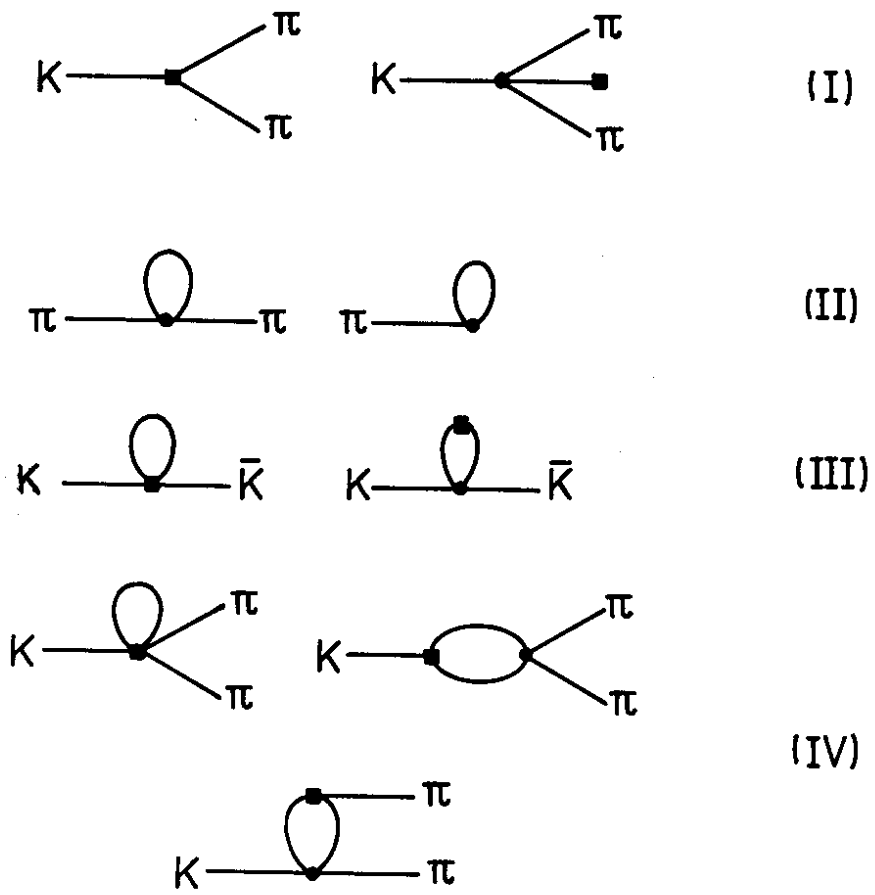

Figure 1. Processes in ChPT. (I) tree level K^0 decay, (II) f_π and wave function renormalization, (III) loop corrections to $K^0 - \bar{K}^0$ mixing, (IV) loop corrections to K^+ decay. A square is an insertion of the weak lagrangian and a circle a strong interaction vertex.

For example, to order p^2, the octet part of the $\Delta S = 1$ weak lagrangian is given by [4]

$$\mathcal{L}^{(2)}_{\Delta S=1} = g_8 \tilde{\mathcal{L}}_8 + h_8 \Theta, \tag{10}$$

where g_8 and h_8 are *unknown coupling constants*, not fixed by ChPT alone. The operators $\tilde{\mathcal{L}}_8$ and Θ have the following realization to the lowest order in derivatives and masses:

$$\begin{aligned} \tilde{\mathcal{L}}_8 &= tr(\Lambda \partial_\mu U \partial^\mu U^\dagger) \\ \Theta &= \tfrac{8v}{f^2} tr(\Lambda U \mathcal{M} + \Lambda (U\mathcal{M})^\dagger). \end{aligned} \tag{11}$$

The operator Θ is the tadpole operator which contributes in general to the off-shell Green's functions, but does not contribute to the S-matrix (see Fig. 1-I).

At tree level there is basically only one coupling constant, g_8, for the $\Delta I = 1/2$ part of $\mathcal{A}(K \to 2\pi)$. It is not fixed by chiral symmetry alone and to determine it, one has to go beyond ChPT. The value of g_8 is known only in the large N_c limit, in which $\mathcal{L}^{(2)}_{\Delta S=1}$ is reduced to the product of bare currents.

With g_8 determined in the large N_c limit, the amplitude $\mathcal{A}(K \to 2\pi)$ coincides with the large N_c vacuum saturation approximation for the matrix elements of local 4-quark operators[10].

Going to the p^4-level lagrangian $\mathcal{L}^{(4)}_{weak}$ introduces many unknown coupling constants. Typically, the amplitude receives contributions from one loop graphs calculated with $\mathcal{L}^{(2)}_{weak}$ and from tree graphs stemming from $\mathcal{L}^{(4)}_{weak}$. A typical structure obtained by using dimensional regularization is (up to irrelevant factors)

$$\mathcal{A}(p^4) \rightsquigarrow \Gamma \frac{\mu^{2\epsilon}}{\epsilon} + m^2 \ln \frac{m^2}{\mu^2} + w^{bare}. \tag{12}$$

Here μ is a renormalization scale, Γ is a calculable number, m is the meson mass and w^{bare} is a bare coupling constant (counterterm) in $\mathcal{L}^{(4)}_{weak}$.

Defining renormalized (finite) w^{ren}gives

$$\begin{aligned} w^{ren} &= w^{bare} + \Gamma \mu^{2\epsilon} \tfrac{1}{\epsilon}, \\ \mathcal{A}(p^4) &\rightsquigarrow m^2 \ln \tfrac{m^2}{\mu^2} + w^{ren}(\mu), \end{aligned} \tag{13}$$

where the coupling constants $w^{ren}(\mu)$ obviously depend on the renormalization point μ. The whole expansion in (9) is, of course, μ- independent. Clearly, in calculating loops one is also able to renormalize divergences via (13), but w^{ren} is *unknown*. The same is true in the cutoff regularization, provided *all counterterms* are kept*.

* Using a cut-off and naive Feynman rules the results are not chirally invariant [15]. Adding a term $-i\delta^4(0) \ln \det g_{ab}(\phi)$ to the lagrangian, where g_{ab} is defined in the Lagrangian (1), makes the results chirally invariant. This term takes care of the difference between the naive measure and a chirally invariant one in the functional integral [16]. However, this term does not contribute to any of the processes discussed in this talk.

The role of meson loops and/or counterterms is very important, as can be seen by studying rare K decays in ChPT [3]. Some of these processes, like radiative K decays, proceed in the next to leading order, since the lowest order contribution vanishes. So, the physical amplitudes get contributions *only* from loops and/or counterterms. * In the next we discuss the large N_c expansion in ChPT and beyond. It turns out to be a very useful tool in treating many problems in strong interactions. Its application to weak interactions should, however, be taken *cum grano salis*. Point is that in the original formulation of the large N_c expansion[17] the *punctum saliens* is the expectation that the true expansion parameter could be not exactly $1/N_c = 1/3$, but rather something like $1/4\pi N_c$ or even $1/4\pi^2 N_c$, as it happens in QED where an expansion in the coupling constant e becomes in reality an expansion in $\alpha = e^2/4\pi$. Unfortunately, at least in the weak interaction sector, one often finds large subleading corrections** that exceed 30%, which makes the whole expansion doubtful, at least in the sense of Ref. 17.

Going to the next order in $1/N_c$ introduces two types of contributions: **i)** tree level graphs from $\mathcal{L}$ with more than one flavor trace, i. e. from $\mathcal{L}^{(4)}$, **ii)** loop contributions with one meson loop from $\mathcal{L}^{(2)}$. The loop graphs are suppressed by factors $p^2/(16\pi^2 f^2)$ and hence suppressed by $1/N_c$.

The low energy constant f_π gets renormalized[2] (see Fig. 1-II). The same is true for f_K. The cutoff dependence comes in in different ways[11]: **i)** A possible Λ^4 contribution is absent because of chiral symmetry; **ii)** Λ^2-terms are of the form $\Lambda^2\times$ tree-level $\mathcal{L}^{(2)}$ (by power counting and chiral symmetry); **iii)** The $\ln\Lambda^2$-terms are of the form $\ln\Lambda^2\times$ tree level $\mathcal{L}^{(4)}$. It is then possible to absorb the cutoff dependence defining

$$\begin{aligned} f^{ren} &= f - \tfrac{3\Lambda^2}{16\pi^2 f} \\ L_5^{ren} &= L_5 + \tfrac{1}{16\pi^2}\tfrac{3}{16}\ln\tfrac{\Lambda^2}{\mu^2} \\ L_4^{ren} &= L_4 + \tfrac{1}{16\pi^2}\tfrac{1}{16}\ln\tfrac{\Lambda^2}{\mu^2}. \end{aligned} \tag{14}$$

This leads to the finite values for f_π and f_K. The results obtained coincide with the ones obtained using dimensional regularization[2,11]. The cutoff has disappeared and the logarithmic μ-dependence is left over. It gets cancelled with the μ-dependence of renormalized constants $L_i^{ren}(\mu)$.This also remains true for the physical amplitudes in weak decays.

The weak 4-quark lagrangian transforms as $(8_L, 1_R)$ and $(27_L, 1_R)$ under $SU(3)_L\times SU(3)_R$.

* The processes can be classified as follows[3] **i)** Vanishing p^4-couplings. In this case, the underlying chiral symmetry forces loops to be finite. **ii)** The loop contribution is finite, but there is also a (scale independent) finite counterterm contribution. **iii)**The loops diverge. In this case, ChPT must allow for a renormalization scale dependent counterterms.

**The leading $1/N_c$ calculation gives the ratio of amplitudes $\mathcal{A}(K^0\to\pi^+\pi^-)/\mathcal{A}(K^+\to\pi^+\pi^0)$ of the order 1 instead of the order 10. The point is that the Wilson coefficients are of order 1 (virtual gluons are subleading in $1/N_c$) and matrix elements are simply given by vacuum insertion (without color suppressed terms). Then, one has to gain a factor of 10 by doing $1/N_c$ corrections.

All terms in the weak lagrangian have the structure

$$\mathcal{L}_{ijkl} = c(\mu_{QCD})\, \bar{q}_j \gamma_\mu \frac{1}{2}(1-\gamma_5) q_i \; \bar{q}_l \gamma^\mu \frac{1}{2}(1-\gamma_5) q_k, \tag{15}$$

where $c(\mu_{QCD})$ is a Wilson coefficient that depends on the renormalization point μ_{QCD} (which is introduced by QCD renormalization of local operators).

The leading $1/N_c$ behavior of (15) is rather simple[10,11]. Since virtual gluon loops are suppressed by $1/N_c$, there is no strong interaction connection between two currents in (15). Therefore (15) can be written as

$$\mathcal{L}_{ijkl} = (L_\mu)_{ij}\, (L^\mu)_{kl}. \tag{16}$$

Including next to leading $1/N_c$ corrections *spoils* the simple factorization exhibited in (16). One has

$$\mathcal{L}_{ijkl} = (L_\mu)_{ij}\, (L^\mu)_{kl} + \text{ tree level from other operators } + \text{ loops in(16)}. \tag{17}$$

Each of the new operators also includes subleading $1/N_c$ corrections but with a different coefficient. Unfortunately, as we have already emphasized, they introduce new free couplings at the next to leading order in $1/N_c$. This reflects the composite structure of the four-quark operator, which is not simply a product of bare currents, and whose overall scale is unknown. As an example the operator relevant to $K^0 - \overline{K^0}$ mixing (see Fig. 1-III) reads

$$\begin{aligned}\mathcal{L}_{s\bar{d}s\bar{d}} = \; & -[\tfrac{f^4}{16} + g_1](U\partial_\mu U^\dagger)_{s\bar{d}}(U\partial^\mu U^\dagger)_{s\bar{d}} \\ & -[4vL_5 + g_2](U\partial_\mu U^\dagger)_{s\bar{d}}(U\partial^\mu U^\dagger \mathcal{M}^\dagger U^\dagger - \partial^\mu U \mathcal{M} + \mathcal{M}^\dagger \partial^\mu U^\dagger - U\mathcal{M}\partial^\mu U U^\dagger)_{s\bar{d}} \\ & -(8vL_4 + g_3)(U\partial_\mu U^\dagger)_{s\bar{d}}(U\partial^\mu U^\dagger)_{s\bar{d}} tr(\mathcal{M}U + \mathcal{M}^\dagger U^\dagger) - g_4(\mathcal{M}^\dagger U^\dagger)_{s\bar{d}}(\mathcal{M}^\dagger U^\dagger)_{s\bar{d}} + \cdots\end{aligned} \tag{18}$$

The leading term is of order N_c^2 (leading factorizable contribution). The unknown constants g_i are of order N_c. The nonleading factorizable contributions are contained in the terms proportional to L_i. The dots stand for the rather long list of other operators with the same transformation properties under $SU(3)_L \times SU(3)_R$.

The renormalization procedure for weak amplitudes is rather simple. After renormalization of f_K and f_π, the rest of loop divergences can be renormalized by redefinition of bare g_i. In particular, the g_1 may be used as a counterterm to cancel the quadratic divergence. Let us take as an example the $K^0 - \bar{K}^0$ mixing. The complete expression is rather long. If we collect only quadratically divergent terms and the terms of the second order in momenta, we get*

$$< K^0|\mathcal{L}_{s\bar{d}s\bar{d}}|\overline{K^0} >= (\frac{f^2}{2} + \frac{8g_1}{f^2})m_K^2 - \frac{\Lambda^2}{16\pi^2}5m_K^2 + \cdots \tag{19}$$

Using now (14) and

$$g_1^{ren} = g_1 - \frac{\Lambda^2 f^2}{16\pi^2}\frac{1}{4}, \tag{20}$$

*Λ^4 divergences cancel as they should, since they would otherwise break chiral symmetry. For the same reason no $m_\pi^2\Lambda^2$ is present.

one gets the final result with no cutoff dependence:

$$< K^0|\mathcal{L}_{s\bar{d}s\bar{d}}|\overline{K^0} >= (\frac{f^{ren2}}{2} + \frac{8g_1^{ren}}{f^{ren2}})m_K^2 + \cdots \qquad (21)$$

The subtraction* in (21) is consistent with respect to the large N_c behavior, since g_1 and the quadratic divergence are of the same order in N_c. In the same way one handles the logarithmic divergences, using all possible counterterms in (18).

In conclusion, one can renormalize away all quadratic and logarithmic divergences. The logarithmic μ dependence is cancelled by the corresponding μ dependence of the counterterms. The results obtained in cutoff regularization agree with the results[18] obtained in dimensional regularization.**

3. Beyond ChPT

As it became obvious from the preceding chapter, the $1/N_c$ analysis of the next to leading order of weak amplitudes in ChPT does not bring more predictive power, since the loop corrections are of the same order in $1/N_c$ as are the unknown coupling constants (counterterms).

To have complete subleading $1/N_c$ contributions, one has to include the finite parts of g_i because they are contributions of the same order in $1/N_c$ as the divergences. Since they are totally unknown in ChPT alone, the predictive power of the large N_c expansion beyond the trivial leading order (factorization) is rather poor. In order to get more information on g_i, one has to go beyond ChPT.

An intriguing attempt to go beyond ChPT represents the large N_c approach of Bardeen *et al.*[13] Based on an intuitive physical insight rather than a formal field theory formulation of Chapters 1 and 2, it differs from the *Ansatz* (9) at the very beginning. Instead of using the chiral realization for the lagrangian as in (10), Bardeen *et al.* find chiral realization for the local 4-quark operator in the leading $1/N_c$ expansion. The weak lagrangian then becomes of a hybrid type, typically of the form

$$\mathcal{L} \sim c(\mu)_{QCD}\mathcal{O}_{chiral}, \qquad (23)$$

where $c(\mu)_{QCD}$ is the Wilson coefficient calculated in QCD and $\mathcal{O}_{chiral}$ is the chiral realization of the local 4-quark operator. Clearly, the hybrid expression (23), in order to be consistent, needs the additional scale dependence of the matrix element of the operator $\mathcal{O}_{chiral}$, which

*The same counterterm also removes the quadratic divergences in other weak decays. As an example, the $K^+ \to \pi^+\pi^0$ analogon of (19) (see Fig. 1-IV) reads as follows[11]

$$< \pi^+\pi^0|\mathcal{L}_{u\bar{s}d\bar{u}}|K^+ >= \frac{i}{2\sqrt{2}}[(\frac{f}{2} + \frac{8g_1}{f^3})(m_K^2 - m_\pi^2) - \frac{\Lambda^2}{16\pi^2 f}\frac{7}{2}(m_K^2 - m_\pi^2)] + \cdots \qquad (22)$$

By inspection one can easily convince oneself that the same renormalization (20) removes the quadratic divergences in (22).

**Dimensional regularization implicitly uses f^{ren} and g_1^{ren} since the quadratic divergence is subtracted in the evaluation of integrals.

has to cancel the μ-dependence of the Wilson coefficient $c(\mu)_{QCD}$. In order to be so, one has to solve the problem of matching the two "technically" different theories, QCD and ChPT. In other words, the renormalization properties of ChPT should match the renormalization properties of QCD.

The proposal made in ref. 13 is to introduce a scale in the matrix element of $\mathcal{O}_{chiral}$ via loop corrections to the leading tree diagram. The *punctum saliens* in their approach is the introduction of the physical cutoff Λ of the order of renormalization scale μ_{QCD}, instead of removing it in the process of renormalization, as we did in Chapter 2. In this way a scale is introduced which preasumably has to match the scale in the Wilson coefficient. This philosophy has a physical background in the fact that short distance effects ($x < \mu^{-1}$) are integrated out in the form of Wilson coefficients and long distance effects ($x > \mu^{-1}$) are implicitly included in the calculation of matrix elements. The last ones are proposed [13] to be controlled by the physical cutoff Λ. Since the cutoff dependence is not removed, in order to be consistent, the quadratic divergence has to be kept.

The second difference in respect to the ChPT formulation in Chapters 1 and 2 is that no counterterms are allowed and consequently, the effects of, e. g. vector mesons and higher resonances, have to be added separately. As we discussed in preceding chapters, in ChPT these effects are contained in coupling constants (counterterms).

It is instructive to compare the expression for a typical weak amplitude in ChPT and in the large N_c approach. Since the approximate identification $\mu_{QCD} \sim \Lambda$ has been used in Ref. 13, one sees that the logarithmic scale dependence is the same. There is, however, one essential difference. In ref. 13 one has instead of the counterterms g_i, the quadratic Λ dependence. Naively, one would guess that the Λ^2-dependence mimics the counterterms. However, this is not true for the following reasons: i) The counterterms g_i contain besides the long distance effects also the short distance ones. In the approach of ref. 13 the latter are factorized in the form of Wilson coefficients. ii) The counterterms in ChPT also contain the effects of higher resonances (vector mesons etc.). In the approach of ref. 13 they have, according to authors, to be added separately.

From the above points it is clear that it is very difficult to achieve one-to-one correspondence between ChPT and the large N_c approach of Bardeen *et al.*

One question raised by Bijnens and myself in Ref. 11 is whether the approach of Bardeen *et al.* is missing the $1/N_c$ subleading counterterms g_i. Again, it is not trivial to answer this question. Bardeen *et al.* claim explicitly that e. g. vector resonances have to be added later as an improvement in their present calculation. So, at least that presumably dominant part of counterterms is not present in their calculation.* In my opinion, it is almost

*Low energy ChPT is perfectly consistent with resonances. It can be shown[2] that the renormalized low energy couplings at $\mu \sim 0.5 - 1GeV$ are almost exclusively reproduced by ρ-exchange, a ρ-field being introduced via an asymetric tensor field $\rho^i_{\mu\nu}(x)$ which transforms as the nonlinear realization $D^{(1)}$ of $SU(2)_L \times SU(2)_R$ (couplings of the type $\epsilon_{ijk}\rho^i_\mu \phi^j \partial^\mu \phi^k$ would break chiral symmetry).

impossible without calculation to guess how important that contribution is. ChPT would implicitly require the importance (largness) of counterterms to keep the whole correction in weak amplitudes moderate. Otherwise the perturbative expansion would break, since the loop contributions are in itself huge. This would imply the importance of missing corrections due to vector mesons etc. in Bardeen *et al.*, at least in the range $\Lambda \sim 0.8GeV$.

The question of matching the QCD scale μ_{QCD} with the cutoff scale of meson loops is even more difficult to answer. First of all, one has (at the present level of calculation in ref. 13 to match the logarithmic μ_{QCD} dependence of Wilson coefficient with the quadratic Λ dependence of meson loops, i. e. matching should be studied in the chiral limit (Wilson coefficients are calculated in this limit). Then, the chiral logarithms are absent and matching is, *horribile dictu*, disappointing. However, it would be premature to claim mismatch, since higher resonance contributions could improve matching, smoothing the quadratic behavior of the scale Λ and forcing the approximate logarithmic behavior. This actually has to be the case if the matching of two different scales has any sense. Unfortunately, I do not see any *a priori* argument that it should be the case and therefore, in my opinion, the matching, although in principle possible, needs the proof.*

An interesting attempt to go beyond ChPT are *QCD Duality Sum Rules*, proposed by Pich and de Rafael[6]. They rely on the tree level ChPT in the sense that e. g. *renormalized* g_8 is determined by using the proposed duality between QCD and ChPT [6–8]. To my knowledge this is the only approach (appart from lattice QCD) where the μ-dependence of the matrix element is exactly controlled. It turns out that it comes from *perturbative radiative corrections* to the *matrix element*. The μ-dependence obtained in such a way exactly cancels the μ-dependence of the Wilson coefficient.**

The QCD duality approach works very well for "normal" transitions, like $K^0 - \bar{K}$ mixing and $K^+ \to \pi^+\pi^-$ decay[6,7]. It failed, however, at least in the first attempt[8], to explain the $\Delta I = 1/2$ rule. Intriguing progress has been made recently[9], indicating the reason for failure, as well as the possible dynamical explanation of the $\Delta I = 1/2$ rule. The huge finite α_s corrections found show that the $\Delta I = 1/2$ processes are, contrary to the $\Delta I = 3/2$ ones or $K - \bar{K}$ mixing, **totally nonperturbative.** In this case OPE and LLA become untrustable. This may have serious consequences for all present approaches.

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*In this context it is interesting to note that in some cases (e. g. penguin operator) one can get[12] *exact* μ-cancelation in the leading $1/N_c$ expansion in ChPT (since the penguin coefficient scales as the square of the running quark mass), *without* introducing a physical cutoff *à la* Bardeen *et al.*

**It is of interest to note that Goity has done his $1/N_c$ analysis[19] using the identification (9). In order to match the μ-dependence he wrote a new Wilson expansion (at lower (chiral) scale) whose inverse Wilson coefficients match the μ-dependence of the original Wilson coefficients. This seems to me to be *the* correct way of doing it - much in the same way as it happens in QCD sum rules.

IV. *Ex Nihilo Nihil* - RULING OUT SELF -PENGUIN TADPOLES

The role of the $d-s$ self-energy diagrams in nonleptonic weak interactions has been studied for a long time by a number of authors[20]. The interest in this problem arises because the self- energy diagram could in principle contribute, in the form of tadpoles, to the $\Delta I = 1/2$ processes. If it were large enough, it would provide an elegant explanation of the $\Delta I = 1/2$ rule. However, Chia has recently shown[21] that, when renormalization is properly done, the tadpole contribution turns out to be prohibitively small, mainly because of the GIM mechanism.

Recently, Shabalin[22] has suggested an interesting and intriguing mechanism that turns a quadratic GIM suppression into a logarithmic one. The effect basically comes from leading logarithmic one-gluon corrections to the bare diagram. Unfortunately, it has been shown by Peccei, Picek and myself[23] that the full QCD correction in LLA, reduces the tadpole contribution to a negligible amount (zero in the chiral limit). As a number of papers has appeared recently[24], still trying to survival tadpoles, I shall present arguments against it, both in the framework of OPE in QCD and in ChPT*.

In the framework of OPE of two weak currents there appear a set of two-quark operators with canonical dimension ≤ 6. The first two, $\theta_1 = \bar{s}d$ and $\theta_2 = \bar{s}\gamma \cdot \partial d$ may be transformed away by renormalization. The next two operators contain two and three derivatives, respectively; $\theta_3 = \bar{s}\Box d$ and $\theta_4 = \bar{s}\Box\gamma \cdot \partial d$. θ_4 is an operator that would lead to a tadpole-type contribution (Wilson coefficient of order g^0), but is suppressed by GIM, i. e. by $(m_c^2 - m_u^2)/M_W^2$. An appropriate *normalization condition* imposed on self- energy fixes the counterterms to be added to the original lagrangian in order to remove divergences. This suffices to make the self-energy contribution numerically *unimportant.*

The situation changes radically by inclusion of gluon corrections, as emphasized by Shabalin[22]. The GIM cancellation changes from the powerlike to the logarithmic one, $\sim \alpha_s \ln(m_c^2/\mu^2)$. According to Shabalin this would be enough, to explain the $\Delta I = 1/2$ rule.

In the following we shall make the full OPE analysis in LLA using the renormalization group. In general, one encounters three classes of operators, *class I*: gauge invariant, which do not vanish by the QCD equation of motion, *class II*: gauge invariant, which *vanish* by the equation of motion and *class III*, gauge variant operators, which vanish by the equations of motion. It is clear that only class I contributes to physical matrix elements (S-matrix). Class III is clearly unphysical. Class II should give no contribution to the S-matrix, provided it does not mix under renormalization with class I.** The argument is based on the fact that

*The self-energy tadpole contribution should not be misidentified with the so called "eye" graph that appears as a tentative solution to the $\Delta I = 1/2$ puzzle in the lattice calculations[5]. The "eye" graph is the symbolically written *matrix element* of the *4-quark operator* and the self-energy tadpole is a *2-quark operator* which does not lead to an "eye" graph. (The "eye" graph appears as a result of contraction of two quark fields of a 4-quark operator in a soft-gluonic background).

**Classical QCD equations of motion are changed by quantum corrections and renormalization and may not be used in the Green's functions. These changes, however, have no influence in the self-energy operator Σ_{ds} is a part of an operator that *vanishes* by QCD equations of motion (i. e. belongs to class *II*). It remains to show that such operators, eventually generated during renormalization, do not mix with normal 4-quark operators from class *I*.

Let us define local 4-quark operators as

$$\begin{aligned} \mathcal{O}_{(A,B)} &= (\bar{\psi}\gamma_\mu\gamma_5\mathbf{A}\psi)(\bar{\psi}\gamma^\mu\mathbf{B}\psi) \\ \mathcal{O}_{(A\lambda,B\lambda)} &= (\bar{\psi}\gamma_\mu\gamma_5\mathbf{A}\lambda^a\psi)(\bar{\psi}\gamma^\mu\mathbf{B}\lambda^a\psi) \end{aligned} \tag{24}$$

For simplicity we have restricted ourselves to the parity-violating part of the operators. **A** and **B** are matrices in the flavor space and λ^a are color matrices. Any 4-quark operator can be expressed in terms of the operators (24).

In order to solve the RGE for Wilson coefficients, one has to find the anomalous dimension matrix γ_{ij} of the operators, which is defined via*

$$\mu\frac{d}{d\mu}\mathcal{O}_i^{ren} = -\gamma_{ij}\mathcal{O}_j^{ren}. \tag{25}$$

The set of operators (24) has to be inserted[25] in all possible 1-particle irreducible (1PI) truncated Green's functions.**

Inserting the operators (24) in set I and set II of Fig.2, one gets besides the original operators also the new one***

$$\mathcal{O}_C = -\frac{2}{g}D^\mu F^a_{\mu\nu}(\bar{\psi}\gamma^\nu\gamma_5\mathbf{C}\lambda^a\psi). \tag{28}$$

This is the first new operator generated in the process of renormalization. It is the famous *penguin* operator, as can be seen by using QCD equations of motion

$$\mathcal{O}_C = (\bar{\psi}\gamma^\mu\gamma_5\mathbf{C}\lambda^a\psi)(\bar{\psi}\gamma_\mu\mathbf{1}\lambda^a\psi) \equiv \mathcal{O}_{(C\lambda,\lambda)}, \tag{29}$$

i. e. one gets the 4-quark operator.

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calculation of physical amplitudes, i. e. the S-matrix and may be used there.

*The relation between bare and renormalized operators, $\mathcal{O}_i^{bare} = Z_{ij}\mathcal{O}_j^{ren}$, defines the renormalization matrix Z_{ij} from which γ_{ij} may be calculated, $\gamma_{ij} = Z_{ik}^{-1}\mu(d/d\mu)Z_{kj}$.

**The relation between bare and renormalized Green's function (with n quarks and m gluons) with operator insertion is given by

$$\Gamma^{bare}_{\mathcal{O}^{bare}} = Z_{2F}^{-n/2}Z_{3A}^{-m/2}Z_{\mathcal{O}}\Gamma^{ren}_{\mathcal{O}^{ren}} \tag{26}$$

to be compared with the analogous relation for the Green's function without operator insertion

$$\Gamma^{bare} = Z_{2F}^{-n/2}Z_{3A}^{-m/2}\Gamma^{ren}. \tag{27}$$

$Z_{\mathcal{O}}$ iz the renormalization matrix of the operator $\mathcal{O}$ and Z_{2F} and Z_{3A} are renormalization constants for the quark and gluon field, respectively.

***To the results one has to add a factor Z_{2F}^2, cf. eq.(26).

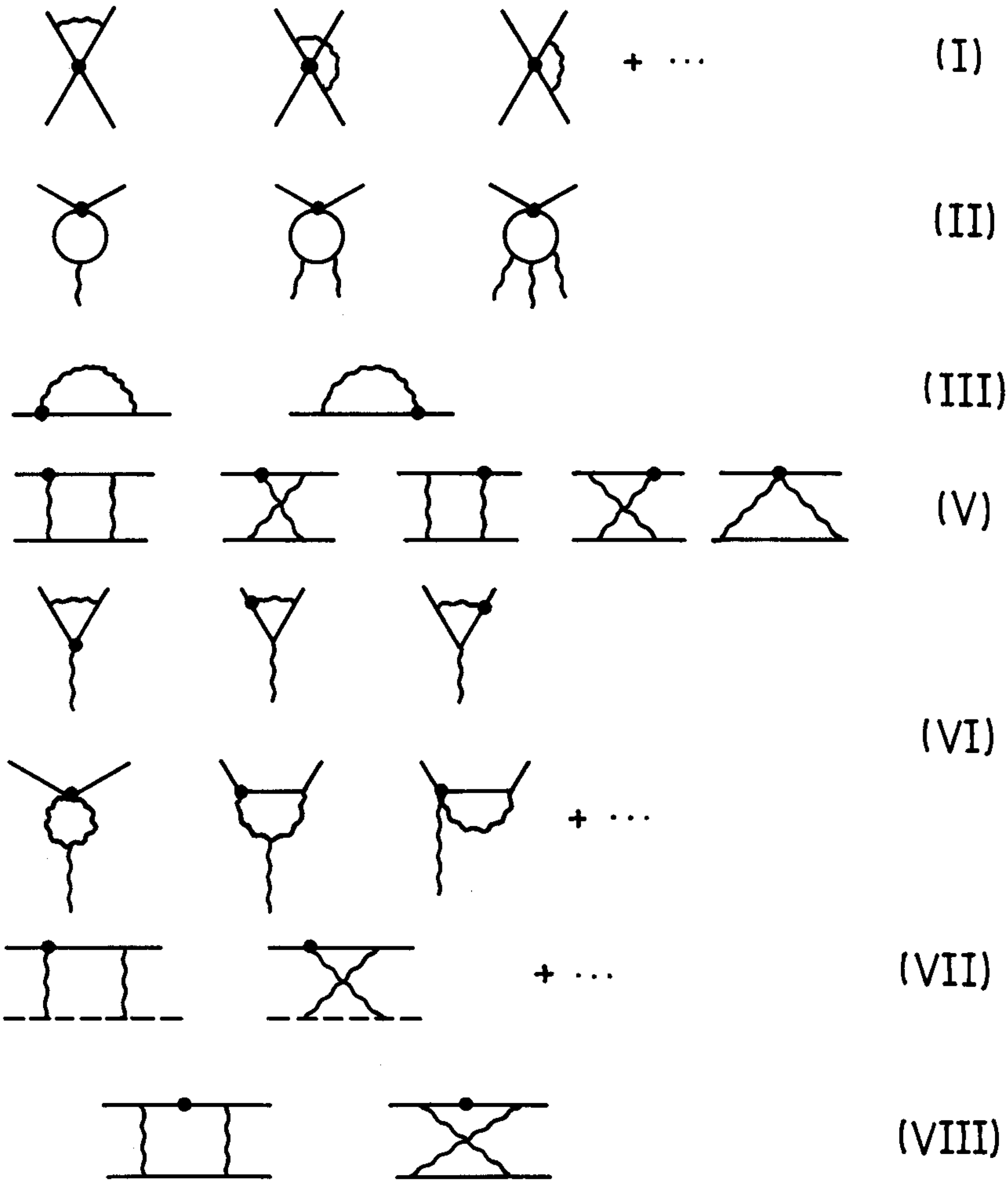

Figure 2. 1*PI* Green's functions with operator insertion. The symbol • indicates the insertion of the operator. Full lines are quarks, wavy lines are gluons and dashed lines are ghosts.

In principle, one is not allowed to use equations of motion in the Green's functions and therefore we should proceed with renormalization of the operator $\mathcal{O}_C$ in (29). Insertion of $\mathcal{O}_C$ in set III of Fig.2 induces new operators

$$\begin{aligned}\mathcal{D}_C &= \tfrac{4i}{g^2}\bar{\psi}\hat{D}\hat{D}\hat{D}\gamma_5\mathbf{C}\psi \\ \mathcal{G}_C &= -\tfrac{2}{g}\partial^\mu A^a_\mu D^{ab}_\nu(\bar{\psi}\gamma^\nu\gamma_5\mathbf{C}\lambda^b\psi).\end{aligned} \tag{30}$$

The first one is the leading operator of Shabalin's leading self-penguin and the second one is the gauge variant operator. Adding up all contributions coming from further insertions of $\mathcal{O}_C$ (sets III - VII) one sees that the operators $\mathcal{D}_C$ and $\mathcal{G}_C$ survive.* There is, however, no operator with ghosts, i. e. graphs of set VII give zero. To proceed, one has to insert the self-penguin operator $\mathcal{D}_C$ in all possible $1PI$ Green's functions and show that $\mathcal{D}_C$ does not mix with other operators, i. e. that

$$\mathcal{D}_C^{bare} = Z_\mathcal{D}\mathcal{D}_C^{ren}. \tag{31}$$

For example, to show that $\mathcal{D}_C$ does not induce any 4-quark operator, one has to insert it in 4-quark Green's functions, i. e. sets V and VIII (Fig. 2). The result is zero[23]. The same procedure applies to the gauge variant operators.

In ChPT, a situation with the tadpole contribution is even more transparent. The operator Θ contributes to the amplitudes $\mathcal{A}(K^0 \to \pi^+\pi^-)$, $\mathcal{A}(K^0 \to \pi^0)$ and $\mathcal{A}(K^0 \to 0)$. For example, the first graph in Fig. 1-I gives $\sim \frac{4i}{3}\frac{m_s - m_d}{f^3}g_\Theta$. This is, however, exactly cancelled by the contribution of the second graph in Fig. 1-I. On the other side, if one writes the amplitude $\mathcal{A}(K \to \pi\pi)$ in terms of the amplitudes $\mathcal{A}(K \to \pi)$ and $\mathcal{A}(K \to 0)$ (PCAC relation), one finds again that the tadpole contributions in the last two amplitudes cancel each other and $\mathcal{A}(K \to \pi\pi)$ is tadpole-free.

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*Again one has to add a factor $Z_{2F}Z_{3A}^{1/2}Z_g^{-1}$, $Z_g = Z_{1F}Z_{3A}^{-1/2}Z_{2F}^{-1}$.

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