DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 88-147
October 1988

Particle Physics and Cosmology

A.D. Linde

Lebedev Physical Institute, Moscow, USSR

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 88-147
October 1988

ISSN 0418-9833

Particle Physics and Cosmology

A.D. Linde
Lebedev Physical Institute, Leninsky prospect 53,
Moscow 11724, USSR

1 Introduction

With the invention of unified theories of strong, weak, electromagnetic and gravitational interactions, elementary particle physics has entered a very interesting and unusual stage of its development. It appears that in the context of these theories we can predict much more than we can actually verify by the standard experimental methods of high energy physics.

The energy scale at which the unified nature of all four fundamental interactions is expected to become manifest is not very different from the Planck mass M_P, about 10^{-5} grams, where quantum gravity effects become important. (The Planck mass is that mass for which the Compton wavelength l_P, about 10^{-33} cm, equals the Schwarzschild radius.) Its rest energy M_Pc^2, about 10^{19} GeV, corresponds to the kinetic energy of a small airplane. By contrast, the 80-km-circumference Superconducting Super Collider the Americans hope to build in the near future will accelerate particles up to 10^4 GeV. The largest accelerator ring one could build on Earth, with a circumference of 40.000 km, could not accelerate particles beyond about 10^8 GeV, a center-of-mass energy occasionally to be seen in cosmic-ray collisions. But this still leaves us 12 orders of magnitude short of the energy necessary for a direct test of the unified theories. Of course, there are some indirect tests, such as the searches for proton decay and for supersymmetric partners of ordinary particles. But trying to get a correct theory of all fundamental interactions with only such low-energy experiments is like looking for the correct unified electroweak theory by using nothing but radiotelescopes. (Note that $M_P/E_W \sim E_W/E_\gamma$, where $E_W \sim 10^2$ GeV is the unification energy scale in the theory of weak and electromagnetic interactions, $E_\gamma \sim 10^{-5}$eV is the typical energy of photons in a radiowave).

The only accelerator that could ever produce particles energetic enough for a direct testing of the unified theories of all fundamental interactions is our universe itself. The Big Bang scenario as it stood ten years ago, which I will call the hot-universe theory, asserts that the universe was born at some moment $t = 0$ about 15 billion years ago, in a state of infinitely large density ρ. Of course, one cannot really speak of classical space-time for the earliest moments, when kT/c^2 was greater than the Planck mass, and ρ exceeded M_P/l_P^3, making quantum fluctuations of the metric predominant. (We will use a convenient unit system that sets k, c and $\hbar$ all equal to

1, so that $l_P = 1/M_P$ and the Planck density is M_P^4, roughly $10^{94}g/cm^3$.) It is just at such times, when the average particle energy exceeded M_P, that the unity of all four fundamental interactions would have been manifest.

With the rapid expansion of the universe, the average energy of particles, given by the temperature, decreases rapidly, and the universe becomes cold. The temperature falls as the reciprocal of $a(t)$, the scale factor, or "radius", of the universe. This means that particle interactions at extremely large energies can have occurred only at the very early stages of the evolution of the universe. One might think it very difficult to extract useful and reliable information from the unique experiment carried out about 10^{10} years ago. Thus, it came as a great surprise to those who study elementary particles that the investigation of physical processes at the very early stages of the universe can rule out most of the existing unified theories.

For example, all the grand unified theories predict the existence of superheavy stable particles carrying magnetic charge: magnetic monopoles. These objects have a typical mass 10^{16} times that of the proton. According to the standard hot-universe theory, monopoles should appear at the very early stages of the universe, and they should now be as abundant as protons. In that case the mean density of matter in the universe would be about 15 orders of magnitude higher than its present value of about $10^{-29}g/cm^3$ [1].

It was shown that all theories with spontaneous breaking of a discrete symmetry (including the simplest $SU(5)$ model, the models of spontaneous CP violation, most of the theories with axions, etc.) lead to the existence of superheavy domain walls, which would be in a drastic contradiction with observational cosmology [2]. It was shown that in most of the theories based on $N = 1$ supergravity the primordial abundance of gravitinos contradicts cosmological data by about 10 orders of magnitude [3], whereas the energy density stored in the so-called Polonyi fields contradicts cosmological data by 15 orders of magnitude for the simplest models [4], and by 6 orders of magnitude for the no-scale models [5]. Several models based on superstring theory lead to a cosmologically unacceptable type of axions [6]. Most of the Kaluza-Klein theories based on $N = 1$ $d = 11$ supergravity predict the present vacuum energy to be $O(-M_P^4)$, which would contradict the observational data by 125 orders of magnitude. The situation with superstring theories seemed, at first glance, to be somewhat better. To be fair, however, one must say that no sufficiently good cosmological model based on superstrings has been suggested so far, though some interesting ideas have been already proposed, see e.g. [7].

Cosmological considerations make it possible to obtain strong constraints on the number of light neutrino species, $N_\nu \leq 4$ [8,9], on the Higgs meson mass in the Glashow-Weinberg-Salam model, $m_H \geq 10GeV$ [10] and on the axion mass, $10^{-5}eV \leq m_A \leq 10^{-3} - 10^{-4}eV$ [11,12]. This list can be continued, but the above-mentioned examples are quite sufficient to explain why so many experts in elementary particle physics at present study cosmology. A more general reason is that no true unification of gravity and all other interactions is possible without a detailed investigation of one of the most important manifestations of this unification, which is our universe. This point became especially clear after the development of Kaluza-Klein theories and the theory of superstrings. In these theories the properties of the four-dimensional space-time are deeply connected with the properties of compactification, and, consequently, with elementary particle physics.

Despite many efforts, some of the problems listed above still remain unsolved, which probably means that the corresponding theories are actually unrealistic. Fortunately, however, the problems of primordial monopoles, gravitinos, domain walls and some other important cosmological problems such as the flatness problem, the homogeneity and isotropy problem, etc. can be solved simultaneously in the context of a relatively simple scenario of the universe evolution - the inflationary universe scenario [13]-[17]. This scenario makes it possible also to remove some constraints on the parameters of the elementary particle theory, for example, the constraint on the axion mass $m_A \geq 10^{-5} eV (f_A \leq 10^{12} GeV)$ [18]. The invention of this scenario has modified considerably the standard cosmological paradigm. One of the most important modifications was performed within the last two years, i.e. after the High Energy Physics Conference in Berkeley. Therefore, we will start with the discussion of the "standard" inflationary universe scenario (inflation before the Berkeley Conference). Then we will discuss some recent developments in the inflationary cosmology [19,20] and some new trends in the theory of baryogenesis and in the theory of galaxy formation. We will finish with the discussion of exciting possibilities related to baby-universe formation and the cosmological constant problem [21]-[31]. However, even though we will go as far as to consider the possibility of existence of life inside black holes [32], we will be unable to describe all recent results related to particle physics and cosmology, and we refer the readers to the interesting talks given in the section of Astrophysics and Cosmology at this Conference.

2 The "Standard" Inflationary Scenario

2.1 The stage of inflation

Historically, there were several different versions of the inflationary universe scenario. At present, it seems that the simplest and, simultaneously, the most general one is the chaotic inflation scenario [17]. To describe it, let us consider the simplest model based on the theory of a massive non-interacting scalar field ϕ with the Lagrangian

$$L = +\frac{M_P^2}{16\pi}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 \qquad (1)$$

Here $M_P^{-2} = G$ is the gravitational constant, $M_P \sim 10^{19} GeV$ is the Planck mass, R is the curvature scalar, m is the mass of the scalar field $\phi, m \ll M_P$. If the classical field ϕ is sufficiently homogeneous in some domain of the universe (see below), then its behaviour inside this domain is governed by the equations

$$\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi, \qquad (2)$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_P^2}(\frac{1}{2}\dot{\phi}^2 + V(\phi)). \qquad (3)$$

Here $V(\phi)$ is the effective potential of the field ϕ (in our case $V(\phi) = \frac{1}{2}m^2\phi^2$), $H = \dot{a}/a, a(t)$ is the scale factor of the locally Friedmannian universe (inside the domain under consideration), $k = +1, -1$, or 0 for a closed, open or flat universe, respectively. If the field ϕ initially is sufficiently large ($\phi \geq M_P$), then the functions $\phi(t)$ and $a(t)$ rapidly approach the asymptotic regime

$$\phi(t) = \phi_0 - \frac{mM_P}{2(3\pi)^{1/2}}t, \qquad (4)$$

$$a(t) = a_0 exp(\frac{2\pi}{M_P^2}(\phi_0^2 - \phi^2(t))). \qquad (5)$$

According to (2.4) and (2.5), during a time $\tau \sim \phi/mM_P$ the value of the field ϕ remains almost unchanged and the universe expands quasi-exponentially:

$$a(t + \Delta t) \sim a(t)exp(H\Delta t) \qquad (6)$$

for $\Delta t \leq \tau = \phi/mM_P$. Here

$$H = \frac{2\pi^{1/2}}{\sqrt{3}}\frac{m\phi}{M_P} \qquad (7)$$

Note that $H \gg \tau^{-1}$ for $\phi \gg M_P$.

The regime of quasi-exponential expansion (inflation) occurs for $\phi \geq \frac{1}{5}M_P$. For $\phi \leq \frac{1}{5}M_P$ the field ϕ oscillates rapidly, and if this field interacts with other matter fields (which are not written explicitly in eq. (1)), its potential energy $V(\phi) \sim \frac{m^2\phi^2}{2} \sim m^2M_P^2$ is transformed into heat. The reheating temperature T_R may be of the order $(mM_P)^{1/2}$ or somewhat smaller, depending on the strength of the interaction of the field ϕ with other fields. It is important that T_R does not depend on the initial value ϕ_0 of the field ϕ. The only parameter which depends on ϕ_0 is the scale factor $a(t)$, which grows $\exp((2\pi/M_P^2)\phi_0^2)$ times during inflation.

If, as is usually assumed, a classical description of the universe becomes possible only when the energy-momentum tensor of matter becomes smaller than M_P^4, then at this moment $\partial_\mu\phi\partial^\mu\phi \leq M_P^4$ and $V(\phi) \leq M_P^4$.

Therefore, the only constraint on the initial amplitude of the field ϕ is given by $\frac{1}{2}m^2/\phi^2 \leq M_P^4$. This gives a typical initial value of the field ϕ:

$$\phi_0 \sim \frac{M_P^2}{m} \qquad (8)$$

Let us consider for definiteness a closed universe of a typical initial size $O(M_P^{-1})$. It can be shown that if initially $\partial_\mu\phi\partial^\mu\phi$ becomes much smaller than $V(\phi)$, the evolution of the universe becomes describable by (2)-(7), and after inflation the total size of the universe becomes larger than

$$l \sim M_P^{-1} exp(\frac{2\pi}{M_P^2}\phi_0^2) \sim M_P^{-1} exp(\frac{2\pi M_P^2}{m^2}) \qquad (9)$$

For $m \sim 10^{-4} M_P$ (which is necessary to produce density perturbations $\frac{\delta\rho}{\rho} \sim 10^{-3}$, see below)

$$l \sim M_P^{-1} \exp(2\pi 10^8) \geq 10^{10^8} cm, \tag{10}$$

which is much greater than the size of the observable part of the universe $\sim 10^{28} cm$.

After such a large inflation the term k/a^2 in (3) becomes negligibly small compared with H^2, which means that the universe becomes flat and its geometry locally Euclidean. This implies that the total density of the universe ρ becomes almost exactly equal to the critical density $\rho_c = \frac{3M_P^2}{8\pi} H^2$, i.e.

$$\Omega = \frac{\rho}{\rho_c} \sim 1. \tag{11}$$

For similar reasons the universe becomes locally homogeneous and isotropic. The density of all 'undesirable' objects (monopoles, domain walls, gravitinos) created before or during inflation becomes exponentially small, and they never appear again if the reheating temperature, T_R, is not too large.

We should like to emphasize that for a realisation of this scenario it is sufficient that initially $\partial_\mu \phi \partial^\mu \phi \leq V(\phi) \sim M_P^4$ in a domain of a smallest possible size $l \sim M_P^{-1}$. Since $\partial_\mu \phi \partial^\mu \phi \leq M_P^4, V(\phi) \leq M_P^4$ in any classical spacetime, the above-mentioned initial conditions are quite natural. For a more detailed discussion of initial conditions which are necessary for inflation see [33].

2.2 Reheating

Reheating of the universe after inflation was studied in a number of different papers. However, the investigation was somewhat incomplete, and the results were never presented in a systematic way. Here we would like to mention several results which might be of some interest [34]. Let us assume that the field ϕ in (1) interacts with another scalar field χ and with a fermion field ψ, with the masses $m_\chi, m_\psi \ll m$. We will take the interaction Lagrangian, in the simplest form, to be

$$\mathcal{L}_{int} = \nu \phi \chi^2 - h \bar{\psi} \psi \phi, \tag{12}$$

where ν and h are some coupling constants. It can be shown that the reheating temperature, after the decay of the classical scalar field ϕ into particles χ and ψ is given by [34]

$$T_R \sim 10^{-1} \sqrt{\Gamma_{tot} m_P} \tag{13}$$

where Γ_{tot} is the total decay rate of a ϕ-particle, $\Gamma_{tot} = \Gamma(\phi \to \chi\chi) + \Gamma(\phi \to \bar{\psi}\psi)$,

$$\Gamma(\phi \to \chi\chi) = \frac{\nu^2}{16\pi m}, \tag{14}$$

$$\Gamma(\phi \to \bar{\psi}\psi) = \frac{h^2 m}{8\pi}, \tag{15}$$

In order to make a numerical estimate one should note that the constants ν and h must be taken sufficiently small, since otherwise quantum corrections will change considerably the shape of $V(\phi)$ at $\phi \sim M_P$. It can be shown that this leads to the constraints $h^2 \leq \frac{8m}{M_P} \sim 10^{-3}, \nu \leq 5m \sim 10^{16} GeV$. Consequently

$$\Gamma(\phi \to \chi\chi) \leq m \sim 10^{-4} M_P, \tag{16}$$

$$\Gamma(\phi \to \bar{\psi}\psi) \leq \frac{m^2}{M_P} \sim 10^{-8} M_P \tag{17}$$

For completeness we must note that if the particles ϕ decay only due to gravitational effects, their typical decay rate is given by

$$\Gamma_g \sim \frac{m_\phi^3}{M_P^2} \sim 10^{-12} M_P \tag{18}$$

Note that we have obtained a hierarchy of different decay rates. From (16) - (18) it follows that if the field ϕ can decay into more light scalar particles χ, the reheating temperature can be very large

$$T_R \leq 10^{-1} \sqrt{m M_P} \sim 10^{16} GeV. \tag{19}$$

If the field ϕ can decay only to fermions, the reheating temperature becomes much smaller,

$$T_R \leq 10^{-1} m \sim 10^{14} GeV, \tag{20}$$

and in the cases where only gravitational effects are possible

$$T_R \leq 10^{-1} m \sqrt{\frac{m}{M_P}} \sim 10^{12} GeV. \tag{21}$$

The existence of the hierarchy of possible values of T_R (19) - (21) implies that by excluding (or adding) new channels of decay one can easily make T_R either very large or extremely small. In particular, it is possible to have T_R as large as $10^{15} - 10^{16} GeV$, which is necessary for the standard mechanism of baryogenesis and, if one wishes, for the production of heavy cosmic strings. On the other hand, one can propose models in which $T_R << 10^{12} GeV$ and even models in which the reheating process never completes.

As a particular semi-realistic example, one can consider $SU(5)$ theory with the effective potential

$$\begin{aligned} V = &\frac{a}{4}(Tr\Phi^2)^2 + \frac{b}{2}(Tr\Phi^4) - \frac{M_\Phi^2}{2}(Tr\Phi^2) \\ &-\alpha H_5^\dagger H_5 (Tr\Phi^2) + \frac{\lambda_1}{4}(H_5^\dagger H_5)^2 - \beta H_5^\dagger \Phi^2 H_5 \\ &+m_5^2 H_5^\dagger H_5 - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\lambda_2}{2}\phi^2 H_5^\dagger H_5 \end{aligned} \tag{22}$$

with $a \sim b \sim \alpha \sim g^2, \lambda \sim 10^{-10} \gg \lambda_2^2$, $m \sim 10^{14} GeV$. In this model T_R can be as large as $10^{14} - 10^{15} GeV$, the standard mechanism of baryogenesis based on the decay of the Higgs boson H_3 may work, and the density perturbations after inflation have a desirable amplitude $\frac{\delta\rho}{\rho} \sim 10^{-3} - 10^{-4}$ at a galaxy scale.

2.3 Baryosynthesis

The discovery of a possible mechanism of generation of the baryon asymmetry of the universe [35] was one of the most important events in modern cosmology. The existence of such a mechanism is very important also for the realization of the inflationary universe scenario. Indeed, in the pre-inflationary cosmology one could just assume that the universe had the excess of particles over antiparticles *ab initio*. However, after inflation, the pre-existing baryon density becomes exponentially small, and it should be created anew.

As was emphasized by Zeldovich, cosmological observations, which show that the universe is flat, homogeneous and isotropic, and which imply therefore that it should have been inflationary, serve at present as the sole experimental evidence of the nonconservation of baryon number. Indeed, without the baryon nonconserving processes, the universe after inflation would contain an almost exactly equal number of baryons and antibaryons. The first mechanism of baryon production suggested in [35] was based on the theory of nonequilibrium and CP-violating processes of superheavy particle decay in an expanding universe. Implementation of this mechanism in the context of the inflationary cosmology proved to be not quite trivial. The universe soon after inflation is very far from thermodynamic equilibrium, which is good for baryosynthesis [36]. However, if the reheating temperature T_R is much smaller than $10^{14} - 10^{15} GeV$, then the standard mechanism of baryosynthesis does not work, or at least becomes very inefficient. Therefore, it would be good to have an efficient mechanism of low-temperature baryon production. The search for such a mechanism was also stimulated by a very important paper by Kuzmin, Rubakov and Shaposhnikov [37], in which it was shown that non-perturbative effects near the critical temperature of the phase transition in the Glashow-Weinberg-Salam model (i.e. at $T \sim 200 GeV$) can wash out any excess of baryons produced earlier by B-L conserving processes.

A detailed discussion of this effect, of the possible ways to circumvent the above-mentioned difficulty and to generate baryon asymmetry of the universe during the epoch of the electroweak phase transition is contained in the talk by Shaposhnikov at this conference.

Within the last three years many other mechanisms of low-temperature baryon production were proposed. One of the most interesting mechanisms was suggested by Affleck and Dine [38]. It is based on the observation that in supersymmetric theories baryon and lepton charges can be carried not only by quarks and leptons, but by classical squark-slepton (scalar) fields as well. This mechanism can be naturally implemented [39] in the context of the chaotic inflation scenario, since in this scenario large classical scalar fields either can exist from the very beginning of the universe evolution, or they can be generated by quantum effects to be discussed below. An

important feature of this scenario is that the final stage of the baryon production process occurs at a temperature $T \sim 100 GeV$ [39], and, therefore, the baryons produced by this mechanism do not disappear due to the nonperturbative effects discussed above. Realization of this mechanism in some supergravity- and superstring-inspired models was considered in a series of papers [40]. Other interesting models of the low-temperature baryosynthesis were considered in [41].

2.4 Scalar field fluctuations, perturbations of density and galaxy formation

According to quantum field theory, empty space is not entirely empty. It is filled with quantum fluctuations of all types of physical fields. These fluctuations can be regarded as waves of physical fields with all possible wave-lengths, moving in all possible directions. If the values of these fields, averaged over some macroscopically large time, vanish, then the space filled with these fields seems to us empty and can be called the vacuum.

In the exponentially expanding universe the vacuum structure is much more complicated. The wave-lengths of all vacuum fluctuations of the scalar field ϕ grow exponentially in the expanding universe. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops propagating, and its amplitude freezes at some nonzero value $\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more new perturbations of the classical field with wavelengths greater than H^{-1}. The average amplitude of such perturbations generated during a time interval H^{-1} (in which the universe expands by a factor of e) is given by

$$|\delta\phi(x)| \approx \frac{H}{2\pi}. \tag{23}$$

Perturbations of the field lead to adiabatic perturbations of density $\delta\rho \sim V'(\phi)\delta\phi$, which after inflation grow, and acquire the amplitude [42,43,34]

$$\frac{\delta\rho}{\rho} = \frac{48}{5}\sqrt{\frac{2\pi}{3}}\frac{V_\phi^{3/2}}{M_P^3 V'(\phi)}, \tag{24}$$

where ϕ is the value of the classical field $\phi(t)$ (4), at which the fluctuation we consider has the wavelength $l \sim k^{-1} \sim H^{-1}(\phi)$ and becomes frozen in amplitude. In the theory of the massive scalar field with $V(\phi) = \frac{m^2}{2}\phi^2$

$$\frac{\delta\rho}{\rho} = \frac{48}{5}\sqrt{\frac{\pi}{3}}\frac{m}{M_P}(\frac{\phi}{M_P})^2. \tag{25}$$

Taking into account of (4), (5) and also of the expansion of the universe by about 10^{30} times after the end of inflation, one can obtain the following result for the density perturbations with the wavelength $l(cm)$ at the moment when these perturbations begin growing and the process of the galaxy formation starts:

$$\frac{\delta\rho}{\rho} \sim 1.5\frac{m}{M_P}\ln\, l(cm). \tag{26}$$

At a galaxy scale ($l \sim 10^{21} - 10^{22}cm$)

$$\frac{\delta\rho}{\rho} \sim 70\frac{m}{M_P}, \tag{27}$$

which gives a desirable amplitude $\frac{\delta\rho}{\rho} \sim 10^{-3}$ for $m \sim 10^{-4}M_P$. [1]

An important feature of the density perturbation spectrum (26) is its very slow (logarithmic) dependence on the scale l. One can say that after inflation the universe becomes flat (i.e. $\Omega = \frac{\rho}{\rho_c} \sim 1$), and the spectrum of density perturbations is also flat (i.e. almost scale-independent). However, whereas the universe actually becomes almost exactly flat in all known models of inflation, the spectrum of density perturbations in some models can differ from the flat one, and perturbations can be even non-Gaussian [44]-[47]. Phase transitions during inflation may lead also to important nonperturbative effects [45,48,49]. Another interesting possibility is related to the cosmic string formation after inflation [50]. All such models *a priori* do not seem as natural as the simplest models where adiabatic perturbations with a flat spectrum are produced. However, when it appears that $m_e \sim 0.5 \cdot 10^{-3}m_p$, where m_e and m_p are the electron and proton mass, respectively, we do not say that it is unnatural and, consequently, wrong. In a similar way one should consider cosmological observation data. If it will prove impossible to explain the origin of the large-scale structure of the universe with the help of adiabatic perturbations with a flat spectrum, then it will become necessary to investigate the alternative possibilities mentioned above, even though they can be realized in a less general class of theories.

One should note that the investigation of the large-scale structure of the universe, of the anisotropy of the primordial microwave background radiation and of the dark matter in the universe provide very important tools for verification of the new cosmological theory. For example, if it will be shown that the mass of the electron neutrino m_{ν_e} is much bigger than 30eV (which, of course, does not seem likely), then one will be able to deduce almost unambiguously that the present density of the universe is much bigger than the critical density ρ_c, and it will be difficult to reconcile this result with the inflationary universe scenario.

A detailed discussion of various cosmological and astrophysical observational data, as well as of the proposed methods for detection of dark matter in the laboratory was

[1] Note that there is some discrepancy in the literature concerning the necessary value of $\frac{\delta\rho}{\rho}$, since some authors include into the definition of $\frac{\delta\rho}{\rho}$ the factor $(4\pi)^{-1/2}$, whereas some other obtain a slightly different coefficient in (24). In our estimates we follow ref. [43], which seems to contain the most accurate calculation of $\frac{\delta\rho}{\rho}$.

given in a series of very informative talks at this Conference, which are contained in the Proceedings.

3 Self-Reproducing Universe

In our previous investigation we only considered local properties of the inflationary universe, which is quite sufficient for a description of the observable part of the universe of the present size $l \sim 10^{28}$ cm. Indeed, in accordance to (26) our universe remains relatively homogeneous on a scale

$$l \leq l^* \sim exp(C\frac{M_P}{m})cm \sim 10^{3000}cm, \tag{28}$$

where $C = 0(1)$. The corresponding density perturbations were formed at the time, when the scalar field $\phi(t)$ was bigger than ϕ^*, where

$$\phi^* \sim \frac{1}{3}M_P\sqrt{\frac{M_P}{m}} \sim 30M_P, \tag{29}$$

see (25). (Note that $V(\phi^*) \sim \frac{m^2}{2}(\phi^*)^2 \sim mM_P^3 << M_P^4$). On a scale $l > l^*$ the universe becomes extremely inhomogenous due to quantum fluctuations produced during inflation. We are coming to a paradoxical conclusion, that the global properties of the inflationary universe are determined not by classical but by quantum effects.

Let us try to understand the origin of such a behaviour of the inflationary universe.

A very unusual feature of the inflationary universe is that processes separated by distances l greater than H^{-1} proceed independently of one another. This is so because during exponential expansion any two objects separated by more than H^{-1} are moving away from each other with a velocity ν exceeding the speed of light. (This does not contradict special relativity because ν is not the speed of any signal; it is just the rate at which the general expansion of the universe separates two distant points.) As a result, any observer in the inflationary universe can see only those processes occurring nearer than H^{-1}.

An important consequence of this general result is that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it. Any two inflationary domains displaced by more than H^{-1} cannot collide or eat one another, or do each other any damage. Their expansion is due not to the annexation of the territory of their neighbors, but rather to the peaceful (and very rapid) growth in their own volume, as allowed by general relativity. In this sense any inflationary domain of initial size exceeding $2H^{-1}$ can be considered as a separate mini-universe, expanding independently of what occurs outside it.

To investigate the behavior of such a mini-universe, with an account taken of quantum fluctuations, let us consider an inflationary domain of initial size roughly H^{-1} containing a sufficiently homogeneous field whose initial value ϕ greatly exceeds

M_P. Eq.(4) tells us that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by

$$\Delta\phi = \frac{M_P^2}{4\pi\phi}. \tag{30}$$

By comparison of (23) and (30) one can easily see that if ϕ is much less than $\phi^* \sim \frac{1}{3}M_P\sqrt{\frac{M_P}{m}}$, the decrease of the field ϕ due to its classical motion is much larger than the amplitude of the quantum fluctuations $\delta\phi$ generated during the same time. But for large ϕ (up to the classical limit of $10^4 M_P$), $\delta\phi(x)$ will exceed $\Delta\phi$, i.e. the Brownian motion of the field ϕ becomes more rapid than its classical motion. Because the typical wavelength of the fluctuation field $\delta\phi(x)$ generated during this time is H^{-1}, the whole domain volume after Δt will effectively have become divided into e^3 separate domains (mini-universes) of diameter H^{-1}. In almost half of these domains the field ϕ grows by $|\delta\phi(x)| - \Delta\phi$, which is not very different from $|\delta\phi(x)|$ or $H/2\pi$, rather than decreases. During the next time interval $\Delta t = H^{-1}$ the field grows again in half of these mini-universes. It can be shown that the total physical volume occupied by a permanently growing field ϕ increases with time like $exp(3 - \ln 2)Ht$, and the total volume occupied by a field that does not decrease grows almost as fast as $\frac{1}{2}e^{3Ht}$.

Because the value of the Hubble constant $H(\phi)$ is proportional to ϕ, the main part of the physical volume of the universe is the result of the expansion of domains with nearly the maximal possible field value, M_P^2/m, for which $V(\phi)$ is close to M_P^4. There are also exponentially many domains with smaller values of ϕ. Those domains in which ϕ eventually becomes smaller than about $30M_P$ give rise to the mini-universes of *our* type. In such domains, ϕ eventually rolls down to the minimum of $V(\phi)$, and these mini-universes are subsequently describable by the usual Big Bang theory. However a considerable part of the physical volume of the entire universe remains forever in the inflationary phase [18]-[20].

Similar results are also valid for the old [51] and the new inflationary scenarios [52], in which some part of the volume of the universe can always remain in a state corresponding to a local extremum of $V(\phi)$ at $\phi = 0$. In our chaotic-inflation case, the results are even more surprising. Not only can the universe stay permanently on the top of a hill as in the old and new inflationary scenario; it can also climb perpetually up the wall toward the largest possible values of its potential energy density [18]-[20]. How can it be that the universe unceasingly produces inflationary mini-universes in energetically unfavorable states with large $V(\phi)$? What energy source supports such a process?

The answer is that the probability for a successful climb up the wall is very small indeed, but those domains in which ϕ jumps high enough are immediately rewarded by a huge growth of their volumes.

The energy source that supports inflation is the gravitational energy associated with $a(t)$, the scale factor of the universe. This gravitational energy is negative, so that the total energy of a closed universe, being the sum of the positive energy of matter and the negative gravitational energy, is zero. Just this unbounded reservoir of

gravitational energy makes possible the exponentially rapid growth of the total energy of matter during inflation. The negative gravitational energy seeks any opportunity to become more negative, that is, to make inflation a nonstop process. Just this possibility is realized in any inflationary domain with $\phi \geq \phi^*$.

Thus in our scenario the universe, in which there was initially at least one domain of a size on the order of H^{-1} filled with a sufficiently large and homogeneous field ϕ, unceasingly reproduces itself and becomes immortal. One mini-universe produces many others, and this process goes on without end, even if some of the mini-universes eventually collapse.

But this means that it is vanishingly improbable that our mini-universe would have been the first in the sequence of mini-universes. Moreover, it no longer seems necessary to assume that there actually was some first mini-universe appearing from nothing or from an initial singularity at some moment $t = 0$ before which there was no space-time at all.

From general topological theorems about singularities in cosmology it does not actually follow that our universe was created *as a whole* at some moment before which the universe did not exist. The usual supposition that the whole universe appears from the unique Big Bang singularity at $t = 0$ is based on the implicit assumption that the universe *as a whole* is sufficiently homogeneous. Indeed, the observable part of our universe is very homogeneous. Observed density fluctuations are less than a part in a thousand and there has been no reason to expect that the universe is inhomogeneous on a larger scale beyond the 10^{10}-light-year horizon. In a homogeneous universe one can use the density $\rho(t)$ as a measure of time. In that case it can be shown that the universe appears *as a whole* from a singularity at $t = 0$. The initial density $\rho(0)$ is infinite, and it becomes possible to describe the whole universe in terms of classical space-time after the Planck time M_P^{-1} (about 10^{-43} seconds), when the energy density everywhere simultaneously becomes smaller than the Planck density M_P^4.

With the invention of the inflationary scenario the situation changes drastically. At present only inflation can explain why the observable part of the universe is so homogeneous, but from inflation it also follows that on a much larger scale the universe is extremely *in*homogeneous. In some parts of the universe the energy density ρ is now of the order of M_P^4, 125 orders of magnitude higher than the 10^{-29} or $10^{-30} g/cm^3$ we can see nearby. In such a scenario there is no reason to assume that the universe was initially homogeneous and that all its causally disconnected parts started their expansions simultaneously.

If the universe is infinitely large (like the Friedmann open or flat universe), then it cannot have had a single beginning; a simultaneous creation of infinitely many causally disconnected regions is totally improbable. Therefore the universe cannot be infinite *ab initio*, or it must exist eternally as a huge self-reproducing entity. Some of its parts appear at different times from singularities, or may die in a singular state. New parts are constantly being created from the space-time foam when $V(\phi)$ exceeds the Planck density, or they may revert to the foamlike state again as a result of large fluctuations in ϕ. But the evolution of the universe as a whole has no end, and it may have had no beginning.

On the other hand, the process of the universe self-reproduction occurs only in the inflationary domains. One may wonder, therefore, what will be the fate of the domain of the universe in which we live now. An investigation of this problem shows [32] that we probably live in a place where $\delta\rho(l) \sim \rho, \delta\rho(l) > 0$ on some scale $l \sim l^*$ (28). Such a domain after a time $t \sim M_P^{-2}\rho_c(l^*)^3$ will form a black hole, which may collapse within the time of the same order. Here ρ_c is the present energy density of the universe.

Does this mean that mankind is doomed to die inside a black hole? It seems that we have some small chance to survive, or at least to send some information about ourselves to those who will live after us.

Black holes of the type discussed above are formed in domains of a size $l \geq l^*$. These domains are formed from small inflationary domains, which contained initially some field $\phi \geq \phi^*$. But such domains permanently produce new and new inflationary domains with $\phi \geq \phi^*$. Therefore the black holes of a size $0(l^*)$ may contain inflationary domains, whose internal size unlimitedly grows. Thus, though we will be unable to escape from the black hole after it is formed, we may try to survive moving toward the self-reproducing domains inside it [32]. It is not quite clear whether it will be possible to use this trick for a long time, since finally we may find ourselves in a black hole, in which occasionally there will be no inflationary domains, and this will be the end of the game. Fortunately, the typical time of the black hole formation is extremely large ($t \sim 10^{10000}$ years in the simplest models), so we still have some time to think how to overcome this difficulty.

The model we have studied above was, of course, oversimplified. In realistic theories of elementary particles there exist many different types of scalar fields ϕ_i. The potential energy $V(\phi_i)$ often has many different local minima, in which the universe may live for an extremely long time, much greater than the 10^{10} years of our observable domain. For example, in the supersymmetric $SU(5)$ theory, $V(\phi_i)$ has several different minima of almost equal depth. Because the laws governing the interactions of elementary particles at the low energies at which we do experiments depend on the values of the classical fields ϕ_i, each of these minima corresponds to a different low-energy physics. In one of them the $SU(5)$ symmetry between all types of interactions remains unbroken - that is, the scalar fields ϕ_i remain equal to zero. In other minima various symmetry breaking patterns are realized, and in only one of these minima is the broken symmetry of the weak, strong and electromagnetic interactions that which we in fact observe.

During inflation there are large-scale fluctuations of all the fields ϕ_i. As a result, the inflationary universe becomes divided into an exponentially large number of inflationary mini-universes, with the scalar fields taking all possible values. As inflation ends in some mini-universes, these scalar fields roll down to all possible minima of $V(\phi_i)$. The universe becomes divided into many different exponentially large domains, realizing all possible types of symmetry breaking between the fundamental

interactions. In some of these mini-universes the low-energy physics is quite different from our own. We cannot now see them because the size of our own domains is much greater than the size of its 10^{10}-light-year observable portion. We could not live in these domains because our kind of life requires our kind of low-energy physics.

It is very important that in the inflationay universe there is lots of room for all possible types of symmetry breaking and for all possible types of life. There is, therefore, no longer any need to require that in the true theory the minimum of $V(\phi_i)$ corresponding to our type of symmetry breaking be the only one or the deepest one. This new cosmopolitan viewpoint may greatly simplify the task of building realistic models of the elementary particles.

The change of the values of the scalar fields ϕ_i – that is to say, the change of the vacuum state – is the simplest kind of "mutation" that may occur during inflation. Much more interesting possibilities appear if one considers chaotic inflation in the higher-dimensional Kaluza-Klein theories. In those domains in which the energy density of the field ϕ grows to the Planck density, quantum fluctuations of the metric at a length scale of M_P^{-1} become of order unity. In such domains an inflationary d-dimensional universe can squeeze locally into a tube of smaller dimensionality $d - n$ (or vice versa). If this tube is also inflationary (in $d - n$ dimensions) and the initial length of the tube is greater than M_P^{-1} (which is quite probable near the Planck density), then its further expansion proceeds independently of its prehistory and of the fate of its mother universe. In an eternally existing universe such processes should occur even if their probability is very small. In fact the probability of such processes is small only if $V(\phi)$ is far below M_P^4.

Thus the inflationary universe becomes divided into different mini-universes in which all possible types of compactification produce all sorts of dimensionalities [53]. By this argument we find ourselves inside a four-dimensional domain with our kind of low-energy physics not because other kinds of mini-universes are impossible or improbable, but simply because our kind of life cannot exist in other domains.

This may have important implication for the building of realistic Kaluza-Klein and superstring theories. For example, it is extremely complicated, if not impossible, to construct a theory in which only one type of compactification can occur, leading precisely to a four-dimensional inflationary universe with the low-energy particle physics of our experience. But from the point of view discussed here, there is no need to require that the results compactification and inflation have wrought in our realm be the only possible results, or the best. It is enough to find a theory in which such a compactification is possible. This problem is still difficult, but it is much easier than the one we have been trying to solve.

4 Inflation and the Wave Function of the Universe

One of the most ambitious approaches to cosmology is based on the investigation of the Wheeler-DeWitt equation for the wave function Ψ of the universe [54]. However, this equation has many different solutions, and *a priori* it is not quite clear which of these solutions describes our universe.

A very interesting idea was suggested by Hartle and Hawking [55]. According to their work, the wave function of the ground state of the universe with a scale factor a filled with a scalar field ϕ in the semi-classical approximation is given by

$$\psi_0(a,\phi) \sim \; exp\,(-S_E(a,\phi)). \tag{31}$$

Here $S_E(a,\phi)$ is the Euclidean action corresponding to the Euclidean solutions of the Lagrange equation for $a(\tau)$ and $\phi(\tau)$ with the boundary conditions $a(0) = a, \phi(0) = \phi$. The reason for choosing this particular solution of the Wheeler-DeWitt equation was explained as follows. Let us consider the Green's function of a particle which moves from the point $(0,t')$ to the point $\mathbf{x}, t$:

$$< \mathbf{x}, 0|0, t' >= \sum_n \psi_n(\mathbf{x})\psi_n(0)\; exp\,(iE_n(t-t'))$$
$$= \int d\mathbf{x}(t)\; exp\,(iS(\mathbf{x}(t))), \tag{32}$$

where Ψ_n is a complete set of energy eigenstates corresponding to the energies $E_n \geq 0$.

To obtain an expression for the ground-state wave function $\Psi_0(\mathbf{x})$, one should make a rotation $t \to -i\tau$ and take the limit as $\tau \to -\infty$. In the summation (32) only the term $n = 0$ with $E_0 = 0$ survives, and the integral transforms into $\int dx(\tau)\exp(-S_E(\tau)))$. Hartle and Hawking have argued that the generalisation of this result to the case of interest in the semiclassical approximation would yield (31).

The gravitational action corresponding to the Euclidean section S_4 of de Sitter space dS_4 with $a(\tau) = H^{-1}(\phi)\cos H\tau$ is negative,

$$S_E(a,\phi) = -\frac{1}{2}\int d\eta[(\frac{da}{d\eta})^2 - a^2 + \frac{\Lambda}{3}a^4]\frac{3\pi M_P^2}{2} = -\frac{3M_P^4}{16V(\phi)} \tag{33}$$

Here η is the conformal time, $\eta = \int dt/a(t), \Lambda = 8\pi V/M_p^2$. Therefore, according to [55]

$$\psi_0(a,\phi) \sim \exp(-S_E(a,\phi)) \sim \exp(\frac{3M_P^4}{16V(\phi)}). \tag{34}$$

This means that the probability P of finding the universe in the state with $\phi = const, a = H^{-1}(\phi) = (3M_P^2/8\pi V(\phi))^{1/2}$ is given by

$$P(\phi) \sim |\psi_0|^2 \sim \exp(\frac{3M_P^4}{8V(\phi)}). \tag{35}$$

This expression has a very sharp maximum as $V(\phi) \to 0$. Therefore the probability of finding the universe in a state with a large field ϕ and having a long stage of inflation becomes strongly diminished.

There exists an alternative choice of the wave function of the universe. It can be argued that the analogy between the standard theory (32) and the gravitational theory (33) is incomplete. Indeed, there is an overall minus sign in the expression for $S_E(a,\phi)$ (33) which indicates that the gravitational energy associated with the scale

factor a is negative. (This is related to the well-known fact that the total energy of a closed universe is zero, being a sum of the positive energy of matter and the negative energy of the scale factor a.) In such a case, to obtain ψ_0 from (33) one should rotate t not to $-i\tau$, but to $+i\tau$ which leads to [56]

$$P \sim |\psi_0(a,\phi)|^2 \sim \exp(-2|S_E(a,\phi)|) \sim \exp(\frac{-3M_P^4}{8V(\phi)}) \tag{36}$$

Actually, this result is valid only if the evolution of the field ϕ is very slow, so that this field acts only as a cosmological constant $\Lambda(\phi) = 8\pi V(\phi)/M_P^2$ in (33). Fortunately, this is indeed the case during inflation. Later the same result was obtained by another method, devised by Zeldovich and Starobinsky [57], Rubakov [58], and Vilenkin [59]. This result can be interpreted as the probability of quantum tunnelling of the universe from $a = 0$ (from 'nothing') to $a = H^{-1}(\phi)$. In complete agreement with our previous argument (36), one predicts that a typical initial value of the field ϕ is given by $V(\phi) \sim M_P^4$ (if one does not speculate about the possibility that $V(\phi) >> M_P^4$), which leads to a very long state of inflation.

It must be said that there is no rigorous proof of either (31) or (36), and the physical meaning of creation of everything from 'nothing' is far from clear. Therefore a deeper understanding of the physical processes in the inflationaly universe is necessary in order to investigate the wave function of the universe $\Psi_0(a,\phi)$ and to suggest a correct interpretation of this wave function. With this purpose we shall try to investigate the global structure of the inflationary universe, and go beyond the minisuperspace approach used in the derivation of (31) and (36). This can be done with the help of a stochastic approach to inflation [60,20], which is a more formal way to investigate the Brownian motion of the scalar field ϕ studied in the previous section.

The evolution of the fluctuating field ϕ in any given domain can be described with the help of its distribution function $P(\phi)$, or in terms of its average value $\bar{\phi}$ in this domain and its dispersion $\Delta = (\langle\delta\phi^2\rangle)^{1/2}$. However, one will obtain different results depending on the method of averaging: one can consider the distribution $P_c(\phi)$ over the non-growing coordinate volume of the domain (i.e. over its physical volume at some initial moment of inflation), or the distribution $P_P(\phi)$ over its *physical* (proper) volume, which grows exponentially at a different rate in different parts of the domain. It can be shown that the dispersion of the field ϕ in the coordinate volume Δ_c is always much smaller than $\bar{\phi}_c$ for $V(\bar{\phi}_c) << M_P^4$. Therefore the evolution of the averaged field $\bar{\phi}_c$, can be described approximately by (2)-(5). However, if one wishes to know the resulting spacetime structure and the distribution of the field ϕ after (or during) inflation, it is more appropriate to take an average $\bar{\phi}_P$ over the physical volume, and in some cases the behaviour of $\bar{\phi}_P$ and Δ_P differs considerably from the behaviour of $\bar{\phi}_c$ and Δ_c.

The Brownian motion of the field ϕ can be described by the diffusion equation

$$\frac{\partial P_c}{\partial t} = \frac{\partial}{\partial\phi}[\frac{\partial(\mathcal{D}P_c)}{\partial\phi} + \frac{P_c}{3H}\frac{\partial V}{\partial\phi}], \tag{37}$$

where the coefficient of diffusion $\mathcal{D} = H^3/8\pi^2$. This equation for the case $H(\phi) =$ const. was first derived by Starobinsky [60]; for a more detailed derivation see [61]. For the special case $\frac{\partial V}{\partial \phi} = 0$ this equation was obtained by Vilenkin [62].

The stationary solution $(\partial P_c/\partial t = 0)$ would be [60]

$$P_c \sim exp(3M_P^4/8V(\phi)) \tag{38}$$

which is equal to the square of the Hartle-Hawking wave function of the universe (35). At the first glance, this result is a direct confirmation of the Hartle-Hawking prescription for the wave function of the universe. (In fact, this is the *only* more or less rigorous confirmation of the Hartle-Hawking prescription which is known to me at present).

However, in all realistic cosmological theories, in which $V(\phi) = 0$ in its minimum the distribution (38) is not normalizable. The source of this difficulty can be easily understood: any stationary distribution may exist only due to a compensation of a classical flow of the field ψ downwards to the minimum of $V(\phi)$ by the diffusion motion upwards. However, diffusion of the field ϕ discussed above exists only during inflation, i.e. only for $\phi \geq M_P, V(\phi) \geq V(M_P) \sim m^2M_P^2 \sim 10^{-8}M_P^4$. Therefore (38) would correctly describe the stationary distribution $P_c(\phi)$ in the inflationary universe only if $V(\phi) \geq 10^{-8}M_P^4$ in the absolute minimum of $V(\phi)$, which, of course, is unrealistic [20]. Therefore, at present, we do not know how one can use the Hartle-Hawking result (34), (35) for the description of inflation. Now let us study a general non-stationary solution of (37) describing the time-evolution of the initial distribution $P_c(\phi, t = 0) = \delta(\phi - \phi_0)$. This solution for the theory $\frac{m^2}{2}\phi^2$ is [20].

$$P_c(\phi, t) = exp[-\frac{3(\phi - \phi(t))^2M_P^4}{2m^2(\phi_0^4 - \phi^4(t))}], \tag{39}$$

which shows that at the first stage of the process during the time $\tau \leq \frac{\phi_0 2(3\pi)^{1/2}}{mM_P}$ (see (4)), the maximum of the distribution $P_c(\phi, t)$ almost does not move, whereas the dispersion linearly grows. Then, at $t >> \tau$, the maximum of $P_c(\phi, t)$ moves to $\phi = 0$, just as the classical field $\phi(t)$ (4). However, during the first period of time the volume of domains filled by the field ϕ increases approximately by $exp(3H(\phi)\tau) \sim exp(c_1\frac{\phi_0\phi}{M_P^2})$ where $c_1 = 0(1)$. After this time the distribution $P_p(\phi, \tau)$ looks (approximately) as follows

$$P_p(\phi, \tau) \sim P_c(\phi, \tau)exp(3H(\phi)\tau) \sim exp[-\frac{\phi^2M_P^4}{c_2m^2\phi_0^4} + c_1\frac{\phi_0\phi}{M_P^2}], \tag{40}$$

where $c_2 = 0(1)$. One can easily verify that, if $\phi_0 > \phi^* \sim M_P\sqrt{\frac{M_P}{m}}$, then the maximum of $P_p(\phi, t)$ during the time τ becomes shifted to some field ϕ which is bigger than ϕ_0. This just corresponds to the process of eternal self-reproduction of the inflationary universe studied in the previous section. For completeness we shall mention here another solution of (37). If the initial value of the field ϕ is very large, $\phi_0 \geq M_P^2/m$, i.e. if one starts with the spacetime foam with $V(\phi_0) \geq M_P^4$, then the evolution of the field ϕ in the first stage (rapid diffusion) becomes more complicated (the naively

estimated dispersion $\Delta_c^2 \sim H^3\Delta t$ soon becomes greater than ϕ_0^2). In this case the distribution of the field ϕ is not Gaussian. The solution of eq. (37) at the stage of diffusion from ϕ_0 to some field ϕ with $V(\phi) << M_P^4$ is given by

$$P_c(\phi) \sim exp(-\frac{3\sqrt{3}\pi M_P^3}{m^3\phi t}). \tag{41}$$

This solution describes quantum creation of domains of a size $l \geq H^{-1}(\phi)$, which occurs due to the diffusion of the field ϕ from $\phi_0 \geq M_P^2/m$ to $\phi << \phi_0$. Direct diffusion with formation of a domain filled with the field ϕ is possible only during the time $= c(2\sqrt{3\pi}\phi/mM_P), c = 0(1)$. At larger times a more rapid process is a diffusion to some field $\tilde{\phi} > \phi$ and a subsequent classical rolling down from $\tilde{\phi}$ to ϕ. Therefore one may interpret a distribution $P_c(\phi)$ formed after a time $t = c(2\sqrt{3\pi}\phi/mM_P)$ as a probability of a quantum creation of a mini-universe filled with a field ϕ [20],

$$P_c(\phi) \sim exp(-c\frac{3M_P^4}{2m^2\phi^2}) \sim exp(-2c\frac{3M_P^4}{8V(\phi)}), \tag{42}$$

which is in agreement with the previous estimate for the probability of quantum creation of the universe eq. (36) [56]-[59].

5 Universe Multiplication and the Cosmological Constant Problem

The cosmological constant problem is considered now as one of the most difficult problems of elementary particle theory. This problem was first recognized more than ten years ago, when it was understood that the effective potential $V(\phi)$ of a classical scalar field ϕ, being multiplied by $\frac{8\pi}{M_P^2}$, plays the role of the cosmological constant Λ in the Einstein equations [63]. From the comparison of the cosmological observational data with the solutions of the Einstein equations with $\Lambda \neq 0$ it follows that the observed value of the curvature scalar R at present corresponds to an extremely small value of the vacuum energy density, $V(\phi) \leq 10^{-29}g\ cm^{-3}$. The main problem here is to understand the reason why after a large chain of phase transitions with breaking of various symmetries in unified theories of elementary particles the vacuum energy in our asymmetric vacuum state exactly (or almost exactly) vanishes. Indeed, a most natural value of the vacuum energy density which could be expected in Kaluza-Klein theories is of the order of the Planck energy density, $\rho_P \sim M_p^4 \sim 10^{94}g\ cm^{-3}$, which is at least 125 orders of magnitude greater than the present vacuum energy density $V(\phi) \leq 10^{-29}g\ cm^{-3}$. In grand unified theories a typical value of the vacuum energy density is of the order $10^{80} - 10^{85}g\ cm^{-3}$, in the electroweak theory it is $O(10^{25}g\ cm^{-3})$. It is very difficult to understand, how all these contributions to the total value of the vacuum energy, being summed up, give us something as small as $V(\phi) \leq 10^{-29}g\ cm^{-3}$.

In the last year there were several interesting suggestions how to solve this problem. Here we would like to discuss some different proposals related to the suggestion of [64], that it is possible to solve the cosmological constant problem by considering many universes interacting with each other only globally.

In order to explain what does this mean, we first consider the original model proposed in [64]. This model describes two universes, X and Y, with coordinates x_μ and y_α, respectively $(\mu, \alpha = 0, 1, \ldots, 3)$ and with metrics $g_{\mu\nu}(x)$ and $\bar{g}_{\alpha\beta}(y)$, containing fields $\phi(x)$ and $\bar{\phi}(y)$ with the action of the following unusual type:

$$S = N \int d^4x d^4y \sqrt{g(x)} \sqrt{\bar{g}(y)}$$
$$\times [\frac{M_P^2}{16\pi} R(x) + L(\phi(x)) - \frac{M_P^2}{16\pi} R(y) - L(\bar{\phi}(y))] \quad (43)$$

Here N is some normalization constant. This action is invariant under general coordinate transformations in each of the universes separately. A novel symmetry of the action is the symmetry under the transformation $\phi(x) \to \bar{\phi}(x), g_{\mu\nu}(x) \to \bar{g}_{\alpha\beta}(x)$ and under the subsequent change of the overall sign, $S \to -S$. We call this the antipodal symmetry, since it relates to each other the states with positive and negative energies.

An immediate consequence of this symmetry is the invariance under the change of the values of the effective potentials $V(\phi) \to V(\phi) + c, V(\bar{\phi}) = V(\bar{\phi}) + c$, where c is some constant. Consequently, nothing in this theory depends on the value of the effective potentials $V(\phi)$ and $V(\bar{\phi})$ in their absolute minima ϕ_0 and $\bar{\phi}_0$. (Note, that $\phi_0 = \bar{\phi}_0$ and $V(\phi_0) = V(\bar{\phi}_0)$ due to the antipodal symmetry.) This is the basic reason why it proves possible to solve the cosmological constant problem in our model.

However, our main reason to invoke this new symmetry was not just to solve the cosmological constant problem. Just as the theory of mirror particles originally was proposed in order to make the theory CP-symmetric while maintaining CP-asymmetry in its observable sector, the theory (43) is proposed in order to make the theory symmetric with respect to the choice of the sign of energy. This removes the old prejudice that, even though the overall change of sign of the Lagrangian (i.e. both of its kinetic and potential terms) does not change the solutions of the theory, one *must say* that the energy of all particles is positive. This prejudice was so strong, that several years ago people prefered to quantize *particles* with *negative energy* as *antiparticles* with *positive energy*, which caused the appearance of such meaningless concepts as negative probability. We wish to emphasize that there is no problem to perform a consistent quantization of theories which describe particles with negative energy. All difficulties appear only when there exist interacting species with both signs of energy. (Actually, this is one of the main problems of quantum cosmology, where one should quantize fields with positive energy, as well as the scalar factor a with negative energy, see the previous section.) In our case no such problem exists, just as there is no problem of antipodes falling down from the opposite side of the earth. The reason is that the fields $\bar{\phi}(y)$ do not interact with the fields $\phi(x)$, and the equations of motion for the fields $\bar{\phi}(y)$ are the same as for the fields $\phi(x)$ (the overall minus sign in front of $L(\bar{\phi}(y))$ does not change the Lagrange equations). Similarly, gravitons from different universes do not interact with each other. However, some interaction between the two universes does exist. Indeed, the Einstein equations in

our case are:

$$R_{\mu\nu}(x) - \frac{1}{2} g_{\mu\nu} R(x) = -8\pi G T_{\mu\nu}(x) - g_{\mu\nu} \langle \frac{1}{2} R(y) + 8\pi G L(\bar{\phi}(y)) \rangle, \quad (44)$$

$$R_{\alpha\beta}(y) - \frac{1}{2} \bar{g}_{\alpha\beta} R(y) = -8\pi G T_{\alpha\beta}(y) - \bar{g}_{\alpha\beta} \langle \frac{1}{2} R(x) + 8\pi G L(\phi(x)) \rangle \quad (45)$$

Here $T_{\mu\nu}$ is the energy-momentum tensor of the fields $\phi(x), T_{\alpha\beta}$ is the energy-momentum tensor of the fields $\bar{\phi}(y)$, the sign of averaging means

$$\langle R(x) \rangle = \frac{\int d^4x \sqrt{g(x)} R(x)}{\int d^4x \sqrt{g(x)}}, \quad (46)$$

$$\langle R(y) \rangle = \frac{\int d^4y \sqrt{\bar{g}(y)} R(y)}{\int d^4y \sqrt{\bar{g}(y)}}, \quad (47)$$

and similarly for $\langle L(x) \rangle$ and $\langle L(y) \rangle$. Thus, the novel feature of the theory (43) is the existence of a *global* interaction between the universes X and Y: The integral *over the whole history* of the Y-universe changes the vacuum energy density of the X-universe.

In general, the computation of the averages of the type (46), (47) may be a rather sophisticated problem. Fortunately, however, in the inflationary universe scenario (at least, if the universe is not self-reproducing, see below), this task is rather trivial. Namely, the universe after inflation becomes almost flat and its lifetime becomes exponentially large. In such a case, the dominating contribution to the average values $\langle R \rangle$ and $\langle L \rangle$ comes from the late stages of the universe evolution at which the fields $\phi(x)$ and $\phi(\bar{a})$ relax near the absolute minima of their effective potentials. As a result, the average value of $-L(\phi(x))$ almost exactly coincides with the value of the effective potential $V(\phi)$ in its absolute minimum at $\phi = \phi_0$, and the averaged value of the curvature scalar $R(x)$ coincides with its value at the late stages of the universe evolution, when the universe transforms to the state corresponding to the absolute minimum of $V(\phi)$. Similar results are valid for the average values of $-L(\bar{\phi}(y))$ and of $R(y)$ as well. In such a case one can easily show [64] that at the late stages of the universe evolution, when the fields $\phi(x)$ and $\bar{\phi}(y)$ relax near the absolute minima of their effective potentials, the *effective* cosmological constant automatically vanishes,

$$R(x) = -R(y) = \frac{32}{3} \pi G [V(\phi_0) - V(\bar{\phi}_0)] = 0 \quad (48)$$

If the universe is self-reproducing, then one may encounter some difficulties when computing the averages (46), (47), since they may become infrared-divergent and the result of computation may depend on the cut-off. This question is not completely investigated yet due to the very complicated large-scale structure of the self-reproducing universe. However, one can easily avoid such questions in the theories in which $V(\phi)$ grows rapidly enough at $\phi \geq \phi^*$, since there will be no universe self-reproduction in such theories. Another problem is that the integral over d^4y in (43) renormalizes the effective Planck constant, and one should take a very small normalization factor $[N \sim exp(-\lambda^{\frac{1}{2}})$ in the theory $\lambda\phi^4]$ in order to compensate this renormalization. Another possibility is that in constructing the quantum theory in a doubled universe, one just should do it in each of the noninteracting universes separately, without an account taken of the above mentioned renormalization of N. Note also that the mechanism of the cosmological term cancellation suggested above works independently of the value of N.

The model considered above can be easily extended. For example, one can consider not only a doubled universe, but a multiuniverse as well, with the Lagrangian being a sum of different Lagrangians of different theories of different fields living in different coupled universes. Within such a theory it may be possible to justify the anthropic principle even in its most radical form.

Recently an extremely interesting generalization of this construction was proposed by Coleman [24], Giddings and Strominger [26] and Banks [25], which is based on previous works by Hawking [21], Lavrelashvili, Rubakov and Tinyakov [22] and Giddings and Strominger [23] on the wormholes and coherence loss in quantum gravity, and also on Hawking's proposal concerning the cancellation of the cosmological term [66]. The main idea is that our universe can be split into disconnected pieces by quantum gravity effects. Baby universes created from the parent universe can carry from it an electron-positron pair, or some other combinations of particles and fields, unless this is forbidden by conservation laws. Such a process can occur in any place in our universe. Many ways were suggested to describe such a situation. The simplest one is to say that the existence of baby universes leads to a modification of the effective Hamiltonian density [24]-[26].

$$\mathcal{H}(x) = \mathcal{H}_0(\phi(x)) + \sum \mathcal{H}_i[\phi(x)]A_i \qquad (49)$$

The Hamiltonian (49) describes the fields $\phi(x)$ on the parent universe at distances much greater than the Planck scale. $\mathcal{H}_0$ is the part of the Hamiltonian which does not involve topological fluctuations. $\mathcal{H}_i(\phi)$ are some local functions of the fields ϕ, and A_i are combinations of creation and annihilation operators for the baby universes. These operators do not depend on x since the baby universes cannot carry away momentum. Coleman argues [24] that the demand of locality, on the parent universe,

$$[\mathcal{H}(x), \mathcal{H}(y)] = 0 \qquad (50)$$

for spacelike separated x and y, implies that the operators A_i must all commute. Therefore, they can be simultaneously diagonalized by the "$\alpha - states$":

$$A_i|\alpha_i> = \alpha_i|\alpha_i> . \qquad (51)$$

If the state of the baby universe is an eigenstate of the A_i, then the net effect of the baby universes is to introduce infinite number of undetermined parameters (the α_i) into the effective Hamiltonian (49): one can just replace the operators A_i by their

eigenvalues. If the universe initially is not in the A_i eigenstate, then, nevertheless, after a series of measurements the wave junction soon collapses to one of the A_i eigenstates [24]-[26].

This gives rise to an extremely interesting possibility related to the basic principles of physics. We were accustomed to believe that the main purpose of physics is to discover the Lagrangian (or Hamiltonian) of the theory which correctly describes our world. However, the question arises: if our universe did not exist sometimes in a distant past, in which sense could one speak about the existence of the laws of Nature which govern the universe? We know, for example, that the laws of our biological evolution are written in our genetic code. But where were the laws of physics written at the time when there was no universe (if there was such time)? The possible answer now is that the final structure of the (effective) Hamiltonian becomes fixed only after measurements are performed, which determine the values of coupling constants in the state in which we live with better and better precision. Different effective Hamiltonians describe different laws of physics in different (quantum) states of the universe, and by making measurements we reduce the variety of all possible laws of physics to those laws which are valid in the (classical) universe where we live.

We will not discuss this problem here any more, since it would require a thorough discussion of the difference between the orthodox (Copenhagen) and the many-world interpretation of quantum mechanics. We would like to mention only that this theory, just as the universe multiplication model considered above, opens new interesting possibilities to justify the anthropic principle.

Another important extension of the ideas of refs. [24]-[26] is connected with the possibility that the wave function of the universe depends on the values of the coupling constants in each quantum state, and it can be peaked near some particular values of these constants. A most interesting application is the possible explanation of the vanishing of the cosmological constant suggested by Coleman [28]. The main idea is closely related to the previous suggestion by Hawking [66] based upon the Hartle-Hawking wave function of the universe (34), (35). According to Hawking [66], if the cosmological constant can take any given value in our universe, then the probability to find ourselves in the universe with the cosmological constant $\Lambda = \frac{8\pi}{M_P^2}V(\phi)$ would be given by (35)

$$P(\Lambda) \sim exp(-2S_E(\Lambda)) = exp\frac{3\pi M_P^2}{\Lambda}. \qquad (52)$$

In the theory discussed above the cosmological constant, like other constants, actually can take different values. However, in this theory one should not only take into account one-universe Euclidean configurations. Rather one should sum over all configurations of babies and parents (connected by Euclidean wormholes), which finally gives [28]

$$P(\Lambda) \sim exp(exp\frac{3\pi M_P^2}{\Lambda})) \qquad (53)$$

From (53) it follows that it is most probable to live in a universe with $\Lambda = 0$.

Is this conclusion reliable? It is very difficult to give an answer to this question. As we have discussed in sect. 4, it was possible to justify the use of the wave function

Rev. Lett. **60** (1988) 1793;
1988) 1797;
. Olive, D. Schramm and G. Steigman, Phys.

22 (1965) 378; Dokl. Akad. Nauk SSSR **192**

Pis. Astron. Zh. **1** (1975) 7.
Sci. **38** (1975) 67.

) (1979) 682; Phys. Lett. **91B** (1980) 99.

1) 347.

982); **114B** (1982) 431; **116B** (1982) 335, 340.
Phys. Rev. Lett. **48** (1982) 1220.

983) 177.

988) 437.

(1986) 395; Physica Scripta **T15** (1987) 169;

V.F. Mukhanov, Int. J. Mod. Phys. **A2** (1987)

(1987) 277; Phys. Rev. **D37** (1988) 904.

v and P.G. Tinyakov, JETP Lett. **46** (1987)
7.

, Nucl. Phys. **B306** (1988) 890.

A008 (1988).

SCIPP 88/09 (1988), to be published in Nucl.

preprint HUTP-88/A006 (1988).

Phys. Lett. **209B** (1988) 39.

A022 (1988).

Santa Barbara preprint (1988).

v and P.G. Tinyakov, INR preprint (1988);
v, INR preprint (1988).

[31] I. Klebanov, L. Susskind and T. Banks, SLAC preprint (1988).

[32] A.D. Linde, Life after inflation, to be published in Phys. Lett. B.

[33] A.D. Linde, Phys. Lett. **162** (1985) 281.

[34] A.D. Linde, **Particle Physics and Inflationary Cosmology** (Gordon and Breach, New York, in preparation).

[35] A.D. Sakharov, JETP Lett. 5 (1967) 24;
V.A. Kuzmin, JETP Lett. **12** (19709 335;
M. Yoshimura, Phys. Rev. Lett. **41** (1978) 281;
A.Yu. Ignatiev, N.V. Krasnikov, V.A. Kuzmin and A.N. Tavkhelidze, Phys. Lett. **76B** (1978) 436;
S. Weinberg, Phys. Rev. Lett. **42** (1979) 850.

[36] A.D. Dolgov and A.D. Linde, Phys. Lett. **116B** (1982) 329.

[37] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. **155B** (1985) 83.

[38] I. Affleck and M. Dine, Nucl. Phys. B **249** (1985) 361.

[39] A.D. Linde, Phys. Lett. **160B** (1985) 243.

[40] B. Campbell, J. Ellis, D.V. Nanopoulos and K.A. Olive, Mod. Phys. Lett. **A1** (1986) 389;
J. Ellis, K. Enquist, D.V. Nanopoulos and K.A. Olive, Phys. Lett. **191B** (1987) 343.

[41] M. Fukugita and T. Yanagida, Phys. Lett. **174B** (1986) 45;
S. Dimopoulos and L.J. Hall, Phys. Lett. **196B** (1987) 135;
A. Dannenberg, L.J. Hall, LBL preprint 23728 (1987);
A.G. Cohen and D.B. Kaplan, Phys. Lett. **199B** (1987) 251.

[42] V.F. Mukhanov and G.V. Chibisov, JETP Lett. **33** (1981) 523.
S.W. Hawking, Phys. Lett. **115B** (1982) 339.
A.A. Starobinsky, Phys. Lett. **117B** (1982) 175.
A.H. Guth and S.-Y. Pi, Phys. Lett. **49** (1982) 1110.
J. Bardeen, P.J. Steinhardt and M. Turner, Phys. Rev. **D28** (1983) 679.
R. Brandenberger, Rev. Mod. Phys. **57** (1985) 1.

[43] V.F. Mukhanov, JETP Lett. **41** (1985) 493;
V.F. Mukhanov, L.A. Kofman and D.Yu. Pogosyan, Phys. Lett. **193B** (1987) 427.

[44] A.D. Linde, JETP Lett. **40** (1984) 1333; Phys. Lett. **158B** (19859 375;
L.A. Kofman, Phys. Lett. **173B** (1986) 400.

eously in the whole universe, so it cannot make
nultaneously.
. is the following [69]. If the mechanisms of refs.
unction of the universe probably must have not
alues of Λ. One of these peaks corresponds to
nother corresponds to $V(\phi_2) = 0$, $V(\phi_i) \neq 0$
ments, an observer finds himself in a universe
This means that he lives in a quantum state
... $\neq 0$. Any observer of our type can live only
$\leq V(\phi) \leq 10^{-27} g \cdot cm^{-3}$ [70]. Since a typical
ic theories is many orders of magnitude bigger
ur type can find himself only in the domain of
ϕ_2, independently of the value of its volume, as
by domains corresponding to other minima of

constraints on the value of the present vacuum
) $\leq 10^{-27} g \cdot cm^{-3}$ [70] differs only by two orders
m the observational bounds $-10^{-29} g \cdot cm^{-3} \leq$
possible to strengthen the anthropic constraints
galaxy formation at $\Lambda \neq 0$, then one could be
ant problem in the context of the approach of
nputation of the wave function of the universe.
possibility to strengthen anthropic bounds on
val of mankind inside huge black holes which
3. Indeed, it can be shown, along the lines of
ankind and proliferation of life in the universe
$(-cM_p/m)$, where $c = 0(1)$, m is the inflaton
ed in a separate publication, this leads to the
vacuum energy density $V(\phi) < 10^{-10^4} g \cdot cm^{-3}$,

nfortable by avoiding any use of the anthropic
but I would like to emphasize that after the
gy and of the baby-universe theory the weak
es a well-definite scientific meaning, and, being
l.

lations between particle physics and cosmology
n before. The main hopes in particle physics
of superstrings, and the authors of this theory
standing of the basic principles of this theory
cosmological) implications may take 10 or even

[45] L.A. Kofman and A.D. Linde, Nucl. Phys. **B282** (1987) 555.

[46] T.J. Allen, B. Grinstein and M. Wise, Phys. Lett. **197B** (1987) 66.

[47] L.A. Kofman and D.Yu. Pogosyan, Tartu preprint A-8 (1988).

[48] J. Silk and M. Turner, Phys. Rev. **D35**(1986) 419.

[49] L.A. Kofman, A.D. Linde and J. Einasto, Nature **326** (1987) 48.

[50] Ya.B. Zeldovich, Mon. Nat. R. Astr. Soc. **192** (1980) 663;
A. Vilenkin, Phys. Rep. **121** (1985) 263.

[51] J.R. Gott, Nature **295** (1982) 304,
K. Sato, H. Kodama, M. Sasaki and K. Maeda, Phys. Lett. **B108** (1982) 35.

[52] P.J. Steinhardt, in **The Very Early Universe**, G.W. Gibbons, S.W. Hawking, S. Siklos, eds., Cambridge U.P. Cambridge, England (1982), p. 251;
A.D. Linde, **Nonsingular Regenerating Inflationary Universe**, Cambridge University preprint (1982).
A. Vilenkin, Phys. Rev. **D27** (1983) 2848.

[53] A.D. Linde and M.I. Zelnikov, Max Planck Inst. preprint MPI-PAE/PTh 43/88 (1988), submitted to Phys. Lett. B.

[54] J.A. Wheeler, in: **Relativity, Groups and Topology** eds. C.M. DeWitt and J.A. Wheeler (Benjamin, New York, 1968);
B.S. DeWitt, Phys. Rev. **160** (1967) 1113.

[55] J.B. Hartle and S.W. Hawking, Phys. Rev. **D28** (1983) 2960.

[56] A.D. Linde, JETP **60** (1984) 211; Lett. Nuovo Cim. **39** (1984) 401.

[57] Ya.B. Zeldovich and A.A. Starobinsky, Sov. Astron. Lett. **10** (1984) 135.

[58] V.A. Rubakov, Phys. Lett. **148B** (1984) 280.

[59] A. Vilenkin, Phys. Rev. **D30** (1984) 549.

[60] A.A. Starobinsky, in: **Fundamental Interactions** (MGPI Press, Moscow, 1984), p. 55; in: **Current Topics in Field Theory, Quantum Gravity and Strings**, Lecture Notes in Physics, eds. H.J. de Vega and N. Sanchez (Springer, Heidelberg 1986) **206**, p. 107.

[61] A.S. Goncharov and A.D. Linde, Sov. I. Part. Nucl. **17** (1986) 369;
S.-J. Rey, Nucl. Phys. **B284** (1987) 706.

[62] A. Vilenkin, Phys. Rev. **D27** (1983) 2848.

[63] A.D. Linde, JETP Lett. **19** (1974) 183;
M. Veltman, Rockefeller University preprint (1974); Phys. Rev. Lett. **34** (1975) 77;
J. Dreilein, Phys. Rev. Lett. **33** (1974) 1243.

[64] A.D. Linde, Phys. Lett. **200B** (1988) 272.

[65] J.D. Barrow and F.J. Tipler, **The Anthropic Cosmological Principle** (Oxford University Press, Oxford, 1986).

[66] S. Hawking, Phys. Lett. **134B** (1984) 403.

[67] N.V. Dragon, Phys. Lett. **113B** (1982) 288;
P.H. Frampton and T.W. Kephart, Phys. Rev. Lett. **48** (1982) 1237;
F. Buccella, J.P. Deredinger, S. Ferrara and C.A. Savoy, Phys. Lett. **115B** (1982) 375.

[68] S. Weinberg, Phys. Rev. Lett. **48** (1982) 1776.

[69] V.A. Rubakov, private communication. A similar but somewhat different point of view is expressed in ref. [31].

[70] S. Weinberg, Phys. Rev. Lett. **59** (1987) 2607.

[71] J. Schwarz, Int. J. Mod. Phys. **A2** (1987) 592.