DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 88-158
November 1988

Non-Perturbative Aspects of the Higgs Sector in the Standard Electroweak Theory

I. Montvay
Deutsches Elektronen-Synchrotron DESY, Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 88-158
November 1988

ISSN 0418-9833

Non-perturbative aspects of the Higgs sector in the standard elektroweak theory*

I. Montvay
Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg

November 10, 1988

Abstract

Non-perturbative aspects of quantum field theories with elementary scalars are reviewed. Such theories play an important rôle in connection with the Higgs mechanism of mass generation in the standard model. Recent non-perturbative results in pure ϕ^4 models imply an upper limit $m_H \leq 630\ GeV$ on the mass of the Higgs boson. The effect of the SU(2) gauge coupling on the pure ϕ^4 sector is discussed. It is pointed out that the influence of the Yukawa-couplings of the Higgs scalar to heavy fermions may be important, because a non-trivial infrared fixed point structure can arise. The problem of chiral fermion gauge theories is summarized. In these theories the chiral fermions always appear in mirror pairs if the mirror Yukawa-couplings are attracted by the trivial infrared fixed point. This provides a strong motivation of the experimental search for mirror fermions and for their indirect effects in low energy phenomenology. In case if mirror fermions do exist in nature, future high energy colliders have a very important rôle in the exploration of their properties.

1 Introduction

The $SU(2) \otimes U(1)$ electroweak interaction of elementary particles is weak, therefore the question naturally arises, why is it necessary to study the electroweak theory non-perturbatively? The answer to this question has several parts:

- A mathematical aspect: quantum field theories cannot be defined by perturbation theory, unless the perturbative expansion is convergent. The perturbation series, however, can only be expected to be asymptotic.

*Lecture given at the 7th INFN Eloisatron Project Workshop, Erice, June 1988

- A general physical aspect: spontaneous symmetry breaking providing the vacuum expectation value of the Higgs scalar field is a genuinely non-perturbative phenomenon. There is a lot of accumulated knowledge about such phenomena in statistical physics which may be very useful.

- A special physical aspect: even if the $SU(2) \otimes U(1)$ gauge couplings are weak the quartic scalar self-coupling can in principle be strong, implying a non-perturbative Higgs sector.

Before going into the details of recent non-perturbative investigations of the Higgs sector let us summarize some general concepts defining the theoretical framework. It is natural to assume that quantum field theory defined on a flat space-time is valid only up to the Planck-scale ($1.2\ 10^{19}\ GeV$) where the effects of quantum gravity become important. Beyond this natural physical cut-off some new framework is necessary describing the physics at still higher energies. This more general framework may be the "theory of everything", it may involve "superstrings", "wormholes" or "chaos with random dynamics" etc. From the point of view of quantum field theory the most important aspect is that it has to specify the free parameters at the natural cut-off scale.

Within quantum field theory the simplest possibility is that the minimal $SU(3) \otimes SU(2) \otimes U(1)$ model is valid up to the Planck scale. This might, however, be impossible either because this assumption is mathematically inconsistent or because it is in conflict with some presently unknown experimental result. Another possibility is that the $SU(3) \otimes SU(2) \otimes U(1)$ model is embedded at some scale intermediate between $100\ GeV$ and $10^{19}\ GeV$ in some larger quantum field theory involving e.g. supersymmetry, grand unification etc. In a renormalizable quantum field theory the cut-off dependence is weak if the cut-off scale (Λ) is much higher that the scale of physical masses (m). In most cases the dependence behaves as some positive integer power of the ratio m/Λ. Since the electroweak scale is so much smaller than the Planck-scale ($m/\Lambda \simeq 10^{-17}$) the cut-off dependence can be neglected for every practical purpose and a simple hypercubical lattice can be used for the cut-off. One has to keep in mind, however, that the replacement of the continuous space-time by a discrete lattice is only a mathematical tool and is only valid as long as the details of the discretization (lattice type, form of lattice action etc.) do not matter.

Every quantum field theory has a number of free parameters which are primarily defined by the independent bare parameters in the lattice action. Since these are free parameters, the determination of there values lies beyond the scope of quantum field theory. Every point in the space of bare parameters is equally possible. There is no inherent concept of "naturalness" for the choice of bare parameters. In this sense there is *parameter democracy*. In other words, there is nothing wrong in fine tuning the bare parameters of a given quantum field theory. Just the contrary, some bare (mass) parameters have to be tuned always to a high accuracy, because $m/\Lambda \simeq 10^{-17} << 1$. The consequence of the smallness of physical masses in lattice units (= cut-off units) is that the physically interesting points are very close to the *critical hypersurface* in bare parameter space, where the masses in lattice units are zero. Therefore, parameter democracy holds only in the immediate vicinity of this critical hypersurface. Aspects like "naturalness" or "simplicity" may play a rôle in the choice between different possible quantum field theories. For instance, theories with a smaller number of free parameters can generally be considered simpler and more natural. Since symmetries usually reduce the number of free parameters considerably, they are the most common source of "naturalness". From this point of view the standard SU(3) ⊗ SU(2) ⊗ U(1) model is only moderately natural because of the large number (about 20) of its free parameters.

2 Non-perturbative results in pure ϕ^4 models

The simplest working laboratories for renormalizable quantum field theories with a possibility of spontaneous symmetry breaking are the ϕ^4 models with a quartic self-interaction. For an N-component scalar field $\phi_n(x)$, $n = 1, \ldots, N$ the interaction term in the action can be written as

$$\sum_{n_1 n_2 n_3 n_4} \lambda_{n_1 n_2 n_3 n_4} \phi_{n_1}(x) \phi_{n_2}(x) \phi_{n_3}(x) \phi_{n_4}(x) \tag{1}$$

The simplest case is the single component model $N = 1$. The complex scalar doublet in the standard model corresponds to $N = 4$. Another simple limit is $N \to \infty$, where the $1/N$-expansion offers a possible non-perturbative framework.

A basic property of pure ϕ^4 models is the *triviality of the continuum limit* [1]. This means that in the infinite cut-off limit $\Lambda/m \to \infty$ mathematical consistency requires a vanishing renormalized quartic self-coupling: $\lambda_r \to 0$. This is at the first sight a rather surprising result, which shows in a dramatic way the importance of the mathematical aspect of non-perturbative investigations mentioned in the introduction. The point is that in a perturbation theory framework the renormalized coupling is *assumed* to be a free parameter. Questions about the mathematical consistency of this assumption are usually difficult to formulate. Even if an apparent inconsistency appears it can be interpreted as a limit of applicability of perturbation theory.

The simplest possibility for a non-perturbative definition of quantum field theories is *lattice regularization.* The euclidean space-time continuum (with imaginary time) is approximated by a finite hypercubical lattice with sites x. The N-component scalar field ϕ_{nx} lives on the sites of the lattice. In the case of O(N)-symmetry the lattice action S contains two independent relevant parameters. The field normalization is a further irrelevant parameter which is, however, sometimes useful to keep, therefore S can be written as

$$S = \sum_x \left\{ \mu \phi_{nx} \phi_{nx} + \lambda (\phi_{nx} \phi_{nx})^2 - \kappa \sum_\nu \phi_{nx+\hat{\nu}} \phi_{nx} \right\} \tag{2}$$

Here an automatic summation over the O(N)-index n is understood, but not over the lattice sites x. One out of the three parameters can be fixed by an appropriate choice of the field normalization. For instance, in lattice bare perturbation theory $\kappa = \frac{1}{2}$ is convenient, whereas in numerical studies (especially for large λ) $\mu = 1 - 2\lambda$ is a correct choice. In this case the two free parameters are: the quartic self-coupling λ and the *hopping parameter* κ which stands in front of the quadratic coupling of nearest neighbours ϕ_{nx} with $\phi_{nx+\hat{\nu}}$, $\nu = \pm1, \pm2, \pm3, \pm4$. With this choice of parameters the lattice action (2) goes over into

$$S = \sum_x \left\{ \phi_{nx} \phi_{nx} + \lambda (\phi_{nx} \phi_{nx} - 1)^2 - \kappa \sum_\nu \phi_{nx+\hat{\nu}} \phi_{nx} \right\} \tag{3}$$

The limit of infinitely strong bare self-coupling $\lambda \to \infty$ is particularly interesting. In this case the length of the field is frozen to

$$\phi_{nx} \phi_{nx} = 1 \tag{4}$$

This looks like losing one of the degrees of freedom. For instance, in the standard Higgs sector with $N = 4$, where the field components are usually denoted by σ_x and π_{rx}, $r = 1,2,3$, the above constraint implies

$$\sigma_x^2 = 1 - \pi_{rx}\pi_{rx} \tag{5}$$

In this case the $\lambda = \infty$ limit is usually called the *non-linear σ-model* , in contrast to the linear σ-model for $\lambda < \infty$ [2]. According to Eq. (5) at $\lambda = \infty$ σ_x is a function of the π-field and therefore one is tempted to assume that the σ-field, which is the physical Higgs-field in the standard Higgs sector, is removed from the physical spectrum. This is, however, not correct. The physical spectrum at $\lambda = \infty$ is not essentially different from the physical spectrum at finite λ. A state with the same quantum numbers as the σ-field remains in the physical spectrum, as one can see in a numerical simulation [3]. In other words, the σ-field cannot be removed from the spectrum by taking the infinite bare coupling limit. The state with the quantum numbers of σ can be considered to be a bound state of two (or more) π's. Therefore, the non-linear σ-model with the action

$$S = -\kappa \sum_{x,\nu} \phi_{nx+\hat{\nu}}\phi_{nx} \tag{6}$$

is equivalent to the linear σ-model defined by Eq. (3). Here the restriction to the simplest minimal actions (3,6) is important. For instance, the non-linear sigma model as a low-energy approximation of QCD contains many higher dymensional couplings, which do influence the physical content in an essential way.

This simple example shows, how difficult the correct identification of the spectrum of a quantum field theory can be, due to the appearance of all sorts of non-perturbative bound states. Another important point is, that perturbative renormalizability is not necessary. In perturbation theory the non-linear σ-model is non-renormalizable, whereas the $\lambda < \infty$ case is renormalizable. If one defines these models non-perturbatively, the $\lambda \to \infty$ limit is smooth and finite. The $\lambda = \infty$ model is *in the same universality class* as $\lambda < \infty$. Instead of perturbative renormalizability one is interested in the existence of critical points with zero mass in lattice units ($m/\Lambda = 0$). If such critical points do exist, one can perform a continuum limit by going to one of them. In the vicinity of critical points there is a quasi-continuum situation where the cut-off dependence is negligible and the model can be used for the description of a physical theory.

The critical line for the O(N)-symmetric ϕ^4 model (3) is qualitatively shown in Fig. 1 (see [4,5,6,7] and references therein). Below the critical line there is the symmetric phase with vanishing vacuum expectation value. Above the critical line the symmetry is spontaneously broken by the non-zero vacuum expectation value of the scalar field. In both phases near the critical line there is the *scaling region* where the mass in lattice units m/Λ is small and the cut-off effects are negligible. An important information about the model is contained in the *curves of constant physics.* These are the curves in the (κ, λ)-plane where the renormalized coupling λ_r is constant. Their qualitative behaviour is shown in Fig. 2 (for more details see the papers in Ref. [4,5]). As it is also shown by the figure, the renormalized coupling is zero on the critical line in accordance with the triviality of the continuum limit. The consequence of the triviality of the continuum limit is that for increasing ratio of the cut-off to the renormalized mass Λ/m_r there is an upper limit for λ_r. In Fig. 3 this is represented by the excluded area. The mapping from the bare parameters to the renormalized parameters is such that no points in the excluded area can be reached by any choice of the bare parameters.

The numerical determination of the upper limit for the renormalized coupling in the phase with spontaneously broken symmetry involves some interesting technical problems. Without going into details here let us mention the problem of infrared singularities due to the presence of Goldstone-bosons in the O(N)-symmetric ϕ^4 models with $N \geq 2$. As it is well known, the zero mass Goldstone-bosons are the consequence of spontaneous symmetry breaking. The long range correlations due to the zero mass Goldstone-bosons (and the corresponding infrared singularities) imply strong finite volume effects in the numerical simulations. These can, however, be used to extract the required infinite volume information from the study of finite volume systems [8,9]. In a similar way, the study of finite volume effects in the single component ϕ^4 models ($N = 1$) can also be used to obtain interesting physical information, for instance about low energy scattering and vacuum tunneling [10,11].

The cut-off dependent upper limit on the renormalized coupling in the broken phase of the O(4)-symmetric ϕ^4 model implies an upper limit on the ratio of the Higgs-boson mass to the W-mass in the standard model. The renormalized quartic coupling is usually defined as

$$\lambda_r = \frac{3m_r^2}{v_r^2} \tag{7}$$

where v_r denotes the renormalized vacuum expectation value. For small SU(2) gauge coupling (g) the physical Higgs-boson mass (m_H) is equal to a good approximation to the renormalized mass in the pure ϕ^4 model ($m_H = m_r$), whereas the W-mass is given by the Dashen-Neuberger formula [12]

$$m_W = \frac{1}{2} g v_r \tag{8}$$

This relation is a consequence of the O(4) Ward-identities and together with Eq. (7) implies

$$\frac{m_H^2}{m_W^2} = \frac{4\lambda_r}{3g^2} \tag{9}$$

From this relation and the upper limit on λ_r one obtains [5,6,7]

$$m_H < 630 \ GeV \tag{10}$$

This is at the lowest cut-off where the lattice regularization is applicable, actually at $\Lambda > 2m_r$. Taking the natural value of the cut-off $\Lambda = 1.2 \cdot 10^{19} \ GeV$ the result is

$$m_H < 145 \ GeV \tag{11}$$

These are surprisingly low limits which imply the absence of a strongly interacting Higgs sector, because the tree-level unitarity limit signalizing strong interaction is at $m_H \simeq 1 \ TeV$ [13]. In the case of the very low cut-off corresponding to Eq. (10) the upper limit depends a little on the way how the discretization was done (lattice structure, lattice action) [14], but the corresponding change in Eq. (11) is rather small. For $\Lambda/m_r \to \infty$ the asymptotic behaviour of the upper limit is given by the perturbative renormalization group equations (see Sec. 4):

$$\lambda_r < \frac{4\pi^2}{\ln(\Lambda/m_r)} \tag{12}$$

This is why the value of the upper limit in Eq. (11) is so similar to the limits obtained by the requirement that the renormalized coupling be small up to a scale close to the Planck-scale ("perturbative grand unification") [15].

3 The inclusion of the gauge couplings

The weak $SU(2) \otimes U(1)$ gauge couplings are usually assumed to be small perturbations on the pure ϕ^4 model. This assumption is plausible, but an exact proof is not easy in the case if the bare quartic scalar coupling is large. For a not too large number of fermions the SU(2)- and U(1)-couplings behave differently, because SU(2) is asymptotically free and U(1) is not. Most of the non-perturbative work was up to now done on the SU(2)-coupling, therefore let us concentrate here on it (for studies of the phase structure of the full $SU(2) \otimes U(1)$ Higgs model see [16]).

The phase structure of the standard SU(2) Higgs model was extensively studied in a large number of papers (for references see the reviews [17]). The bare parameter space and the established phase structure is shown in Fig. 4. On the surface shown in the figure there is a phase transition which is most probably of first order everywhere except for some parts of the boundary. Above the phase transition surface there is the *Higgs phase* where the W-boson gets a mass due to the Higgs mechanism. In the *confining phase* below this surface the model describes a QCD-like theory with scalar "quarks". The usual assumption is that the standard electroweak model is in the Higgs phase. The confining phase is relevant in the formulation of the *strongly interacting electroweak model* [18]. In this phase the broken symmetry is "restored", therefore the phase transition is usually called *symmetry restoring phase transition* [19].

The effect of the small SU(2) gauge coupling on the ϕ^4 model can be described in the framework of the weak gauge coupling expansion [20], where the Green's functions of the Higgs model are expressed by power series in the gauge coupling. The coefficients of the series depend on the Green's functions of the pure ϕ^4 model. With reasonable assumptions one can show that the higher terms of the expansion give only small corrections, but at the same time one can also see why the smallness of the corrections is not completely trivial. Namely, in the higher loop contributions an integration over all momenta has to be performed, and at the momenta near the cut-off scale the scalar self-interaction is roughly equal to the bare quartic coupling which can be large. A direct control over the effect of the small gauge coupling can be achieved in numerical simulations which are possible at small gauge couplings in the Higgs phase [21]. The results are consistent with the weak gauge coupling expansion. The direct simulation with a weak gauge coupling has the advantage that the finite mass of the gauge W-boson acts as an infrared regulator and therefore there is no problem with infrared divergencies. The upper limit on the Higgs-boson mass can also be obtained in this way and the results [22,23] are consistent with the upper limit obtained in the ϕ^4 model (see previous section).

Besides the gauge couplings in the standard model there are also the Yukawa-couplings of the fermions which can influence the Higgs sector. In particular, heavy fermions imply strong Yukawa-couplings which can have an important effect because they can change the renormalization group behaviour qualitatively. These questions will be discussed in the next section.

4 Infrared and ultraviolet fixed points

4.1 Pure scalar ϕ^4 model

The absence of strong interactions in the ϕ^4 model can be qualitatively understood from the infrared behaviour of the renormalization group equations (RGE's). For this purpose the convenient form of the RGE's involves the renormalized coupling (λ_r) as a function of the cut-off (actually, the ratio of the cut-off to the physical mass Λ/m) [24]. Using the natural variable

$$\tau \equiv \ln \frac{\Lambda}{m} \tag{13}$$

the RGE can be written as

$$\frac{d\lambda_r(\tau)}{d\tau} = -\beta_r(\lambda_r, \tau) = -\frac{1}{16\pi^2}\, 4\lambda_r^2 + \cdots \tag{14}$$

Here β_r is an appropriate Callan-Symanzik β-function, which in the scaling region depends only on λ_r. (The τ-dependence implies scale breaking.) The last equality in (14) shows the universal one-loop contribution, which dominates for small λ_r. This equation determines the change of the renormalized coupling for fixed bare coupling λ, that is along vertical lines in Fig. 1. A numerical check of the RGE behaviour in the single component ϕ^4 model at infinite bare quartic coupling (Ising limit) was performed in Ref. [25], and a good agreement with the three-loop β-function was found. Near the critical point λ_r is small, therefore the one-loop term dominates and drives the solution for $\tau \to \infty$ (i.e. for $m/\Lambda \to 0$) to the fixed point of the renormalization group equation at $\lambda_r = 0$. The asymptotic behaviour near $\lambda_r = 0$ is, in accordance with Eq. (12),

$$\lambda_r \simeq \frac{4\pi^2}{\tau} \tag{15}$$

Since $\tau \to \infty$ is the limit when the physical mass in cut-off units tends to zero, $\lambda_r = 0$ is called an *infrared fixed point* (IRFP) of the renormalization group equation. The qualitative consequence of the IRFP at $\lambda_r = 0$ is that once the cut-off is large compared to the physical mass, the renormalized coupling is small because the solution of Eq. (14) is attracted to the IRFP.

The situation in a U(1) gauge theory like QED can be similar to ϕ^4, because the leading term of the Callan-Symanzik β-function is similar to Eq. (14). This would imply the triviality of the continuum limit of QED and, consequently, a cut-off dependent upper limit on the fine structure constant. However, if the cut-off is at the Planck-scale this upper limit is much higher than the physical value 1/137. In the continuum formulation of perturbation theory the triviality of the continuum limit is signalized by inconsistencies which were discovered long ago by Landau ("Landau-ghosts") [26].

The triviality of the continuum limit follows from Eq. (14) only if the leading term in Eq. (14) is at least qualitatively correct also for large renormalized couplings (see Fig. 5). In the ϕ^4 model this is most probably true but in QED there are arguments that the behaviour of the β-function is different at large couplings [27,28]. In order to illustrate how a non-trivial continuum limit can arise let us consider a more complicated β-function depicted in Fig. 6. It starts at $\lambda_r = 0$ as a parabola (see Eq. (14)), but for larger λ_r it has two other zeros. The second zero at $\lambda_r = \lambda_i$ is another IRFP which is attractive for increasing τ. The intermediate zero at $\lambda_r = \lambda_u$ is a repulsive fixed point of the RGE (14). One can consider another RGE which describes the change of the bare coupling as a function of the cut-off for fixed renormalized coupling:

$$\frac{d\lambda(\tau)}{d\tau} = \beta(\lambda, \tau) = \frac{1}{16\pi^2}\, 4\lambda^2 + \cdots \tag{16}$$

The functional form of the β-function in this equation is similar but not exactly equal to β_r in Eq. (14). The leading term for small coupling is the same and the qualitative behaviour for large couplings is also given by Fig. 6. Let us assume that the zeros of $\beta(\lambda)$ are at the same place as those of $\beta_r(\lambda_r)$, for instance $\beta(\lambda = \lambda_u) = 0$. (This can always be achieved by an appropriate redefinition of the renormalized coupling.) At $\lambda = \lambda_u$ the differential equation (16) has an attractive fixed point, which is called *ultraviolet fixed point* (UVFP) because for $\tau \to \infty$ the cut-off is infinitely large compared to the physical mass. (Note that an equation similar to (16) also describes the change of the renormalized coupling in the continuum theory as a function of μ/m, where μ is the renormalization scale.)

Combining the two equations (14) and (16) one can construct the curves of constant renormalized coupling (λ_r) in the bare parameter space. Staying for simplicity in the symmetric phase, one obtains Fig. 7. (In the broken phase a qualitatively similar picture is repeated, only upside down.) Since the UVFP corresponds to a singular point of the theory, there may be all sorts of singularities near to it. For instance, the critical line may have a cusp or some other type of singularity at $\lambda = \lambda_u$.

The consequence of Fig. 7 is that at the UVFP it is possible to define a non-trivial continuum limit. Near this point the renormalized coupling can have any value between the two neighbouring IRFP's, namely

$$0 \le \lambda_r \le \lambda_i \tag{17}$$

In QCD the existence of the continuum limit is guaranteed by an UVFP, which is at zero coupling corresponding to asymptotic freedom. The triviality of the continuum limit in the ϕ^4 model is equivalent to the absence of an UVFP in the β-function. As mentioned above, in QED there might be a non-trivial UVFP, therefore Fig. 6 can be qualitatively correct (perhaps without the second IRFP).

The importance of the IRFP's can also be inferred from Fig. 7. First, they imply limits for the renormalized coupling in the continuum limit (see Eq. (17)). Second, as it is shown by the figure, on the critical line outside the UVFP the value of the renormalized coupling is given by the IRFP's. This means that defining a continuum limit at some critical point different from the UVFP, the value of the renormalized coupling always tends to an IRFP. Therefore, continuum theories with IRFP couplings are special points in the space of all theories (in this context see Ref. [29], where the notion of IRFP's was approached more generally).

4.2 Inclusion of gauge couplings

After this general discussion of infrared and ultraviolet fixed points let us return to the renormalization group behaviour in the standard SU(2) Higgs model. In the following let us always consider the version of RGE's corresponding to Eq. (14), which gives the change of the renormalized couplings for fixed bare couplings. If the SU(2) gauge coupling is added to the ϕ^4 model there are two renormalized couplings: the quartic coupling λ_r and the gauge coupling g_r. Along the lines $\lambda = const.$; $\beta \equiv 4/g^2 = const.$ in Fig. 4 the change of λ_r and g_r^2

as a function of $\tau \equiv \ln(\Lambda/m)$ is determined by

$$\frac{d\lambda_r(\tau)}{d\tau} = -\beta_\lambda(\lambda_r, g_r^2, \tau) = -\frac{1}{16\pi^2}(4\lambda_r^2 - 9\lambda_r g_r^2 + \frac{27}{4}g_r^4) + \cdots$$

$$\frac{dg_r^2(\tau)}{d\tau} = -\beta_g(\lambda_r, g_r^2, \tau) = \frac{1}{16\pi^2}\frac{43}{3}g_r^4 + \cdots \qquad (18)$$

Here the one-loop terms are explicitly given. The SU(2) gauge coupling has an UVFP at $g_r^2 = 0$, corresponding to asymptotic freedom. The general behaviour of β_λ for fixed g_r is shown by Fig. 8. Since this function has no zeros, nothing can stop the decrease of λ_r which, therefore, goes to negative values and makes the theory unstable. The consequence is the first order Weinberg-Linde phase transition [19] at the surface separating the two phases (see Fig. 4).

4.3 Inclusion of Yukawa-couplings

The inclusion of light fermions with small Yukawa-couplings to the scalar field does not influence the Higgs sector in an essential way. The only noticeable difference is due to the change in the β-functions of the gauge couplings. Heavy fermions and the corresponding strong Yukawa-couplings are, however, important. In particular, heavy quarks establish a strong coupling between QCD and the electroweak sector and induce a qualitatively new renormalization group behaviour. In order to illustrate this let us consider here the RGE's for a colour triplet weak doublet quark. The SU(2) gauge coupling is not essential here, therefore let us only consider the SU(3) colour coupling (g_r), the Yukawa-coupling (G_r) and the scalar quartic coupling:

$$\frac{dg_r^2(\tau)}{d\tau} = -\beta_g(g_r^2, G_r^2, \lambda_r, \tau) = \frac{1}{16\pi^2}\frac{58}{3}g_r^4 + \cdots$$

$$\frac{dG_r^2(\tau)}{d\tau} = -\beta_G(g_r^2, G_r^2, \lambda_r, \tau) = -\frac{1}{16\pi^2}(24G_r^4 - 16G_r^2 g_r^2) + \cdots$$

$$\frac{d\lambda_r(\tau)}{d\tau} = -\beta_\lambda(g_r^2, G_r^2, \lambda_r, \tau) = -\frac{1}{16\pi^2}(4\lambda_r^2 + 48\lambda_r G_r^2 - 288G_r^4) + \cdots \qquad (19)$$

For $\tau \to \infty$ the colour gauge coupling grows, the higher corrections to the one-loop β-function become important and at some point perturbation theory breaks down. In the perturbative region, where one-loop gives a good approximation, the right hand sides of the last two equations have zeros (see Fig. 9). Therefore, as long as the colour gauge coupling is small and slowly varying, the other two couplings are "dragged" with it, close to the values where β_G and β_λ vanish. This situation can be called *quasi-IRFP*. Since in this case the quartic and Yukawa-couplings can be predicted, there is a large number of papers in the literature exploiting this possibility (see for instance [30]). Of course, once the colour coupling gets large, the perturbative β-functions are not applicable, the quasi-IRFP becomes irrelevant and the question of the continuum limit at $\tau \to \infty$ remains open. In an imaginary world with only heavy quarks, where the colour coupling remains perturbative at the quark mass scale and therefore all the couplings can stay perturbative, one can show that the quasi-IRFP does not imply a non-trivial continuum limit of the quartic and Yukawa-couplings [31]. This means that for infinite cut-off the model tends to pure QCD with heavy quarks and a non-interacting

Higgs-boson. The additional couplings go to zero for $\tau \to \infty$, although very slowly. For fixed g_r we have [31]:

$$G_r(\tau) \to \tau^{-\frac{5}{58}} \qquad\qquad \lambda_r(\tau) \to \tau^{-\frac{10}{29}} \tag{20}$$

Such a slow change has practically no consequences. For instance, for the Yukawa-coupling the asymptotic change between 100 and 10^{19} GeV is only a decrease by a factor of about 1.4. In other words, in the case of a quasi-IRFP the upper limits on the quartic and Yukawa-couplings are slowly changing with the cut-off. The value of the upper limit for the Higgs-boson mass is in this case somewhat higher than in Eq. (11). For the natural cut-off at the Planck-scale both this limit and the upper limits for heavy quark masses are in the range of $200 - 300$ GeV, depending on the details of the model [30].

4.4 Is there a non-trivial infrared fixed point?

Since models with a true IRFP are special, it is conceivable that for some yet unknown reason they play a important rôle in the understanding of the standard model. The above example of a quasi-IRFP fulfils almost all requirements. The only missing ingredient is the infrared attractive zero of the β-function for the gauge coupling. In the one-loop approximation this is impossible, therefore one has to go to higher loops or to a non-perturbative approach. In fact, as it was pointed out by Banks and Zaks [32], in the 2-loop approximation some non-abelian gauge theories with an intermediate number of flavours have an IRFP. The 2-loop β-function of the gauge coupling can be written in models with fermions as

$$\beta_r(g_r^2) = -2\beta_0 \frac{g_r^4}{16\pi^2} - 2\beta_1 \frac{g_r^6}{(16\pi^2)^2} \tag{21}$$

The constants $\beta_{0,1}$ depend on the group and on the number of fermions. In the case of SU(2) and a number N_f of fermions in the fundamental representation we have

$$\beta_0 = \frac{22}{3} - \frac{2}{3}N_f \qquad\qquad \beta_1 = \frac{136}{3} - \frac{49}{6}N_f \tag{22}$$

The same for SU(3) is:

$$\beta_0 = 11 - \frac{2}{3}N_f \qquad\qquad \beta_1 = 102 - \frac{38}{3}N_f \tag{23}$$

This shows that in the case of SU(2) for N_f between 6 and 10, and in the case of SU(3) for N_f between 9 and 16 the 2-loop β-function has an IRFP zero, as indicated by Fig. 10. Taking as an example 6 standard families, or which is from this point of view the same, 3 mirror pairs of standard families (see next section), we have $N_f = 12$ both for SU(3) and SU(2) and the 2-loop β-function predicts a non-trivial IRFP for the SU(3) colour coupling at

$$\frac{g_r^2}{16\pi^2} = \frac{3}{50} \tag{24}$$

This implies a non-trivial IRFP also for the quartic and Yukawa-couplings (see above), whereas the IRFP for the SU(2) and U(1) couplings is at zero. (The absolute value of the 2-loop β-function for the SU(2) coupling is, however, very small.)

Therefore, the 2-loop beta functions predict the possibility of non-trivial IRFP's. The question is, of course, whether the IRFP remains there after taking into account the higher loop corrections and the mass- (threshold-) effects. Also the question of the phase structure of such models has to be clarified, for instance, whether there is a second order phase transition allowing a non-trivial continuum limit or not.

5 Chiral gauge theories and mirror fermions

5.1 Chiral gauge theories with mirror fermions on the lattice

The electroweak interactions in the standard model are described by a *chiral gauge theory* where left- and right-handed components of the fermion fields are transforming differently under the $SU(2) \otimes U(1)$ gauge symmetry. This implies that left-right symmetry is broken at low energy. Nevertheless, it can be restored at high energy above the scale of spontaneous symmetry breaking. There are two different ways how this can happen:

- by enlarging the gauge group to a left-right symmetric one, for instance to $SU(2)_L \otimes SU(2)_R \otimes U(1)$ [33];
- by doubling the fermion spectrum with *mirror fermions*.

The mirror fermions are defined in chiral gauge theories by interchanging the transformation properties of the L- and R-handed field components with respect to the gauge group. The idea of mirror fermions is as old as the idea of parity breaking. In fact, the possibility of the existence of "elementary particles exhibiting opposite asymmetry" was discussed already in the classical paper by Lee and Yang [34]. Mirror fermions also occur naturally in connection with many interesting modern theoretical ideas. To mention a few typical examples, mirror fermions were introduced in order to cancel anomalies [35], they occur in grand unified theories with large orthogonal groups [36], in Kaluza-Klein theories [37], in extended supersymmetry [38] and also in superstring inspired models [39].

Since spontaneous symmetry breaking is a non-perturbative phenomenon, for the study of the left-right symmetry restoration a non-perturbative framework (such as lattice regularization) is needed. Chiral gauge theories with mirror fermions were intorduced in lattice regularization in Ref. [40,31]. Let us denote the "normal" fermions with $V - A$ coupling to the W-boson by ψ and the mirror partners with $V + A$ coupling by χ. The generic form of the fermion mass matrix in the broken symmetry phase on the (ψ, χ)-basis is

$$\mu = \begin{pmatrix} G_\psi v & \mu_{\psi\chi} \\ \mu_{\psi\chi} & G_\chi v \end{pmatrix} \tag{25}$$

Here v is the vacuum expectation value of the scalar doublet field, G_ψ, respectively G_χ, are the Yukawa-couplings of the ψ- and χ-fermions and $\mu_{\psi\chi}$ is a chiral invariant mass parameter. The physical states are mixtures of ψ and χ characterized by some mixing angle α. The corresponding physical fermion masses $\mu_{1,2}$ are, in general, different and are usually of the order of $\max(|\mu_{\psi\chi}|, |v|)$. In the specific case of $\mu^2_{\psi\chi} = G_\psi G_\chi v^2$, however, one of the mass eigenvalues is zero. The very small fermion masses in the standard model (for neutrinos, electron, u- and d-quarks etc.) could be due to such a cancellation mechanism. Although this looks at the first sight as an ugly fine tuning of parameters, it is conceivable that there is some dynamical or symmetry principle in a more general theoretical framework determining the parameters of quantum field theory which implies such a relation. In this respect it is worth to emphasize that in the usual perturbative setup of the Higgs sector the smallness of the fermion masses compared to the vacuum expectation value is due to the fine tuning of the corresponding Yukawa-couplings to values very close to zero. (The smallness of the Yukawa-couplings does not necessarily have to do with local chiral symmetry.)

As it will be discussed in some detail below, models with three mirror pairs of fermion families can be constructed which are consistent with all the presently known phenomenology (see for example Ref. [41]). It is a very interesting experimental question whether this way of parity symmetry restoration is realized in nature. Mirror fermions should be searched for directly in high energy production experiments or indirectly by looking for their effects in low energy phenomenology. Of course, as long as mirror fermions are not found experimentally, we have to ask the exciting theoretical question, whether the mirror partners can be removed from the physical spectrum of a quantum field theory?

5.2 Can the mirror fermions be removed?

In a lattice regularized theory with fermions the mirror partners are always present due to the Nielsen-Ninomiya theorem [42], provided some rather plausible assumptions are fulfilled. This is the *fermion doubling* phenomenon on the lattice. Nevertheless, in vectorlike theories as QCD the superfluous additional states can be kept at the cut-off scale and hence are removed from the physical spectrum in the large cut-off ("continuum") limit [43]. The question is whether the mirror doublers can also be kept at the scale of the cut-off in chiral gauge theories?

For the correct formulation of the question it is important to note that the mirror partners *can* be removed from the physical spectrum in scalar-fermion theories without gauge fields. For instance, in the broken phase this can be achieved by an appropriate choice of scalar field vacuum expectation values and Yukawa-couplings. In a model with SU(2) gauge symmetry the simples possibility is to introduce a second scalar doublet which has a vacuum expectation value of order one in lattice units. It can be shown that by tuning the parameters it is possible to arrange in this case that one of the masses of a mirror fermion pair remains at the cut-off scale.

The difficulty comes in the physically interesting case with W- and Z-gauge fields, because then the scale of the scalar doublet vacuum expectation values is fixed by the W- and Z-masses. Let us first assume that in the large cut-off limit the Yukawa-couplings $G_{\psi,\chi}$ are attracted by the trivial IRFP at vanishing (renormalized) couplings, as it is suggested by the perturbative β-functions. In this case, similarly to the triviality upper limit for the Higgs-boson mass, there are cut-off dependent upper limits for the masses of the mirror fermion partners which follow from the requirement of mathematical consistency of the quantum field theory. As a consequence, the mirror partners cannot have much higher masses than the vacuum expectation value.

The only viable alternative to the physical existence of mirror fermions seems to be that there is a non-trivial UVFP in the Yukawa-couplings of fermions $G_{\psi,\chi}$. In this case, as it was generally discussed in the previous section, the upper limit on the renormalized Yukawa-couplings of the mirror fermion partners can be very large (in principle also infinite, see Eq. (17)), and the mirror families can be moved in the continuum limit to very high (maybe also infinite) masses. The existence of a non-trivial UVFP may also be connected to the existence of chirally asymmetric phases in quantum gauge field theories with mirror fermions [44].

At present it is not known whether a non-trivial UVFP in the Yukawa-couplings of mirror fermion pairs does exist or not. Nevertheless, its existence or non-existence is a genuine property of the quantum field theory, because it is generally assumed that the physical content of a quantum field theory is independent of the regularization scheme. Therefore, the possibility of removing the mirror fermion partners from the physical spectrum is a general quantum field theory problem and not only a question in lattice regularization.

5.3 Phenomenology of mirror fermions

As it was stated before, it is possible to construct extensions of the minimal standard model which contain mirror fermions at the 100 GeV scale and at the same time are consistent with all presently known phenomenology. Needless to say that if such mirror fermions do indeed exist in the 100 GeV range, then future high energy colliders are very important for the exploration of their properties.

Let us first consider the constraints on mirror fermion models imposed by phenomenology. The most important consequence of the mirror fermions at low energies is that the weak currents, instead of being pure $V - A$, have a generic form

$$(V - A)\cos\alpha + (V + A)\sin\alpha \tag{26}$$

Here α is a mixing angle in the (ψ, χ)-basis. In a model with three mirror pairs of standard fermion families an important constraint is the absence of lepton number violations and the absence of flavour changing neutral currents. These constraints can be satisfied, for instance, with fermion mixing schemes having a one-to-one correspondence between fermions and mirror fermions (such mixing schemes can be called *monogamous*) [41]. In such a scheme the mixing in the heavy mirror quark sector is given by the same Kobayashi-Maskawa matrix as in the light quark sector (including the top quark).

The phenomenological upper bounds on the mixing angles $\alpha_{e,\mu,u,...}$ were derived in a more general framework recently by Langacker and London [45]. (For earlier works see also the references in [46].) In the best cases, namely for the first fermion family and for the muon, the bounds are typically $|\sin\alpha| \leq 0.2$. The lower bounds on mirror fermion masses are similar to the bounds on the masses of a fourth heavy fermion family, typically $m \geq 30\ GeV$ for leptons and $m \geq 50\ GeV$ for quarks.

The important couplings for the production and decay of mirror fermions are the off-diagonal couplings to real or virtual W- and Z-bosons (see Fig. 11). In the monogamous mixing scheme these couplings are always between corresponding pairs, that is between electrons and mirror electrons, u-quarks and mirror u-quarks etc. They are always proportional to the corresponding $\sin\alpha$. Depending on the values of the masses and mixing angles, the dominant decay pattern of the mirror fermions can be either a direct decay to the light partner, or first a decay to some lighter mirror fermion. In the monogamous scheme these latter are suppressed by the fact that the mass splittings between the mirror families are relatively small, in fact similar to the mass splittings among the light families. Therefore, the decay signature of a heavy mirror fermion is quite spectacular: the mirror leptons can decay to 3 leptons or to a lepton plus 2 jets, the mirror quarks to 3 jets or to a jet plus a lepton pair. At very high energy hadron colliders, like the Eloisatron, the mirror fermions can probably be pair-produced. This would allow a detailed study of their spectrum and of many of their decay modes.

References

[1] K. G. Wilson, Phys. Rev. **B4** (1971) 3184;
K. G. Wilson, J. Kogut, Phys. Rep. **12C** (1974) 75;
R. Schrader, Phys. Rev. **B14** (1976) 172;
M. Aizenmann, Phys. Rev. Lett. **47** (1981) 1; Commun. Math. Phys. **86** (1982) 1;
J. Fröhlich, Nucl. Phys. **B200** [**FS4**] (1982) 281;
C. Aragao de Carvalho, C. S. Caracciolo, J. Fröhlich, Nucl. Phys. **B215** [**FS7**] (1983) 209;
J. Fröhlich, in *Progress in gauge field theory*, Cargese lecture 1983, ed. G. 't Hooft et al., Plenum Press 1984; for further references see this review.

[2] M. Gell-Mann, M. Lévy, Nuovo Cimento, **16** (1960) 705;
B. W. Lee, *Chiral dynamics*, Gordon and Breach 1972, New York

[3] I. Montvay, Phys. Lett. **150B** (1985) 441

[4] M. Lüscher, P. Weisz, Nucl. Phys. **B290** [**FS20**] (1987) 25; Nucl. Phys. **B295** [**FS21**] (1988) 65

[5] M. Lüscher, P. Weisz, Phys. Lett. **212B** (1988) 472; DESY preprint 88-146 (1988)

[6] A. Hasenfratz, K. Jansen, C. B. Lang, T. Neuhaus, H. Yoneyama, Phys. Lett. **199B** (1987) 531;
A. Hasenfratz, K. Jansen, J. Jersák, C. B. Lang, T. Neuhaus, H. Yoneyama, Jülich preprint HLRZ-88-02 (1988)

[7] J. Kuti, L. Lin, Y. Shen, Phys. Rev. Lett. **61** (1988) 678; San Diego preprint UCSD/PTH 88-05 (1988)

[8] H. Neuberger, Phys. Rev. Lett. **60** (1988) 889; Nucl. Phys. **B300** [**FS22**] (1988) 180

[9] H. Leutwyler, Nucl. Phys. Proc. Suppl. **B4** (1988) 248;
J. Gasser, H. Leutwyler, Nucl. Phys. **B307** (1988) 763

[10] M. Lüscher, Comm. Math. Phys. **105** (1986) 153

[11] I. Montvay, P. Weisz, Nucl. Phys. **B290** [**FS20**] (1987) 327;
K. Jansen, J. Jersák, I. Montvay, G. Münster, T. Trappenberg, U. Wolff, Phys. Lett. **213B** (1988) 203;
K. Jansen, I. Montvay, G. Münster, T. Trappenberg, U. Wolff, DESY preprint (1988)

[12] R. Dashen, H. Neuberger, Phys. Rev. Lett. **50** (1983) 1897

[13] B. W. Lee, C. Quigg, H. B. Thacker, Phys. Rev. **D16** (1977) 1519

[14] G. Bhanot, K. Bitar, Phys. Rev. Lett. **61** (1988) 798

[15] L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. **B136** (1978) 115;
N. Cabibbo, L. Maiani, G. Parisi, R. Petronzio, Nucl. Phys. **B158** (1979) 295

[16] R. E. Shrock, Phys. Lett. **162B** (1985) 165; Nucl. Phys. **B267** (1986) 301; Phys. Rev. Lett. **56** (1986) 2124; Nucl. Phys. **B278** (1986) 380

[17] J. Jersák, in *Lattice gauge theory - a challenge to large scale computing* (Wuppertal, 1985), edited by B. Bunk, K. H. Mütter, K. Schilling, Plenum 1986, p. 133;
I. Montvay, in *Proceedings of the International Europhysics Conference on High Energy Physics* (Uppsala, 1987), edited by O. Botner, Vol. I. p. 298;
R. E. Shrock, Nucl. Phys. Proc. Suppl. **B4** (1988) 373;
H. G. Evertz, M. Marcu, DESY preprint 88-133 (1988), to appear in *Proceedings of the 12th Johns Hopkins Workshop on Current Problems in Particle Theory* (Baltimore, 1988)

[18] M. Claudson, E. Farhi, R. Jaffe, Phys. Rev. **D34** (1986) 873

[19] D. A. Kirzhnitz, A. D. Linde, Phys. Lett. **42B** (1972) 471;
S. Weinberg, Phys. Rev. **D9** (1974) 3357;
S. Weinberg, Phys. Rev. Lett. **36** (1976) 294;
A. D. Linde, JETP **23** (1976) 64

[20] I. Montvay, Phys. Lett. **172B** (1986) 71; Nucl. Phys. **B293** (1987) 479

[21] W. Langguth, I. Montvay, P. Weisz, Nucl. Phys. **B277** (1986) 11

[22] W. Langguth, I. Montvay, Z. Phys. **C36** (1987) 725

[23] A. Hasenfratz, T. Neuhaus, Nucl. Phys. **B297** (1988) 205

[24] E. Brezin, J. C. Le Guillou, J. Zinn-Justin, in *Phase transitions and critical phenomena,* eds. C. Domb, M. S. Green (Academic Press, London, 1976) vol. 6, p. 125

[25] I. Montvay, G. Münster, U. Wolff, Nucl. Phys. **B305** [**FS23**] (1988) 143

[26] L. D. Landau, in *Niels Bohr and the Development of Physics,* edited by W. Pauli (Mc Graw Hill, New York, 1955)

[27] K. Johnson, M. Baker, R. Willey, Phys. Rev. **136B** (1964) 111;
S. Adler, Phys. Rev. **D5** (1972) 3021;
P. Fomin, V. Gusinyn, V. Miransky, Yu. Sitenko, Riv. Nuovo Cimento, **6** (1983) 1;
V. Miransky, Nuovo Cimento, **90A** (1985) 149;
C. N. Leung, S. T. Love, W. A. Bardeen, Nucl. Phys. **B273** (1986) 649

[28] J. Kogut, E. Dagotto, A. Kocić, Phys. Rev. Lett. **60** (1988) 772

[29] W. Zimmermann, Commun. Math. Phys. **97** (1985) 211

[30] B. Pendleton, G. Ross, Phys. Lett. **98B** (1981) 291;
C. T. Hill, Phys. Rev. **D24** (1981) 691;
S. Dimopoulos, S. Theodorakis, Phys. Lett. **154B** (1985) 153

[31] I. Montvay, Nucl. Phys. Proc. Suppl. **B4** (1988) 443

[32] T. Banks, A. Zaks, Nucl. Phys. **B196** (1982) 189

[33] S. Weinberg, Phys. Rev. Lett. **29** (1972) 388;
J. C. Pati, A. Salam, Phys. Rev. **D10** (1974) 275;
G. Senjanovic, R. N. Mohapatra, Phys. Rev. **D12** (1975) 1502

[34] T. D. Lee, C. N. Yang, Phys. Rev. **104** (1956) 254

[35] D. J. Gross, R. Jackiw, Phys. Rev. **D6** (1972) 477;
P. Fayet, in *Proceedings of the 17th Rencontre de Moriond on elementary particles*, ed. J. Tran Thanh Van, Gif-sur-Yvette (Editions Frontiers, 1982), p. 483

[36] G. Senjanovic, F. Wilczek, A. Zee, Phys. Lett. **141B** (1984) 389

[37] E. Witten, Nucl. Phys. **B186** (1981) 412

[38] F. de Aguila, M. Dugan, B. Grinstein, L. Hall, G. G. Ross, P. West, Nucl. Phys. **B250** (1985) 225

[39] A. L. Kagan, C. H. Albright, Phys. Rev. **D38** (1988) 917

[40] I. Montvay, Phys. Lett. **199B** (1987) 89

[41] I. Montvay, Phys. Lett. **205** (1988) 315

[42] H. B. Nielsen, M. Ninomiya, Nucl. Phys. **B185** (1981) 20; errata: Nucl. Phys. **B195** (1982) 541

[43] M. Bochicchio, L. Maiani, G. Martinelli, G. Rossi, M. Testa, Nucl. Phys. **B262** (1985) 331

[44] E. Eichten, J. Preskill, Nucl. Phys. **B268** (1986) 179

[45] P. Langacker, D. London, Phys. Rev. **D38** (1988) 886

[46] J. Maalampi, M. Roos, Helsinki preprint HU-TFT-88-17 (1988)

Figure captions

Fig. 1. The qualitative behaviour of the critical line(C) in the (λ, κ)-plane of the O(N)-symmetric ϕ^4 model. The hatched area on both sides of the critical line is the scaling region where the cut-off is much higher than the physical scale.

Fig. 2. The curves of constant physics (i. e. constant renormalized coupling) in the (κ, λ)-plane for the O(N)-symmetric ϕ^4 model. The dashed-dotted line is the critical line where the mass in lattice units is zero. The arrows point in the direction of increasing cut-off (decreasing lattice spacing).

Fig. 3. The cut-off dependent upper limit on the renormalized coupling in the $(\Lambda/m_r, \lambda_r)$-plane (m_r is the renormalized mass, λ_r the renormalized coupling). The hatched area is excluded.

Fig. 4. The bare parameter space of the standard SU(2) Higgs model. λ and κ are the bare parameters of the scalar field and $\beta \equiv 4/g^2$ stands for the SU(2) gauge coupling. Above the phase transition surface (T) there is the Higgs phase, below it the confining phase.

Fig. 5. The qualitative behaviour of the Callan-Symanzik β-function in the ϕ^4 model. The β-function has no other zeros besides the IRFP at $\lambda_r = 0$. The curves of constant physics look in this case like Fig. 2.

Fig. 6. An illustrative Callan-Symanzik β-function. It has two infrared fixed points (IRFP) at $\lambda_r = 0$ and at $\lambda_r = \lambda_i$ and an ultraviolet fixed point (UVFP) at $\lambda_r = \lambda_u$.

Fig. 7. The qualitative behaviour of the curves of constant physics for the β-function shown in Fig. 6. C is the critical line, and here only the symmetric phase is shown for simplicity.

Fig. 8. The qualitative behaviour of the Callan-Symanzik β-function for λ_r ($g_r = fixed$) in the standard SU(2) Higgs model.

Fig. 9. The qualitative behaviour of the Callan-Symanzik β-functions for the Yukawa-coupling (β_G) and for the quartic coupling (β_λ) at small couplings according to Eq. (19).

Fig. 10. The 2-loop β-function of the gauge coupling for intermediate number of fermion flavours.

Fig. 11. The off-diagonal couplings of the mirror fermions (F) to the corresponding light fermions (f).

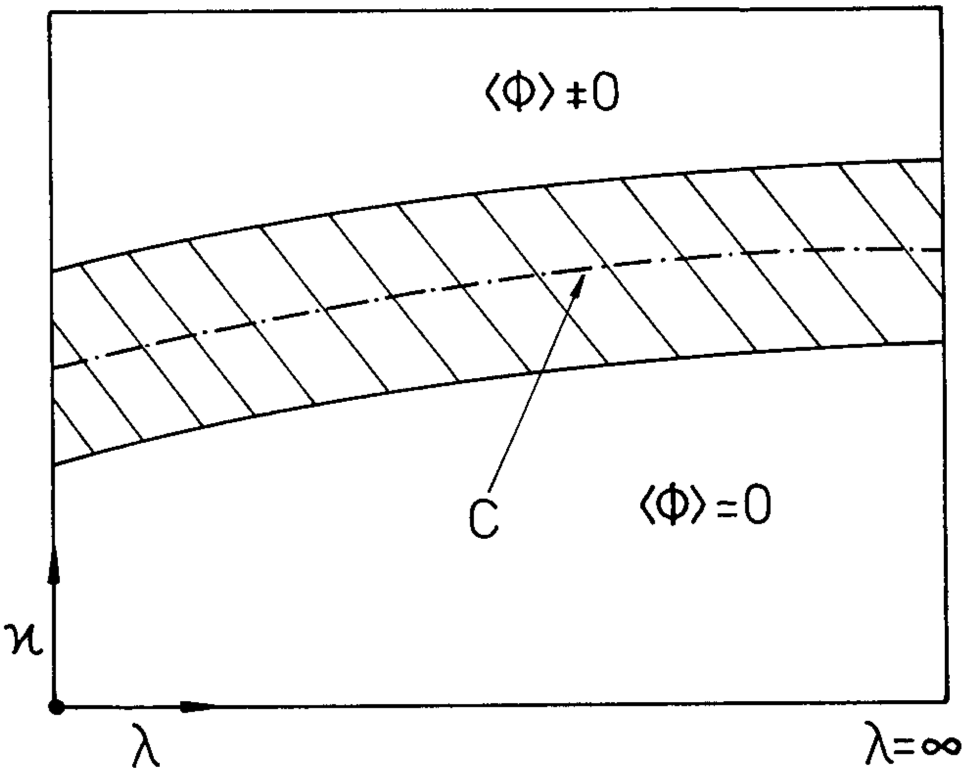

Fig. 1

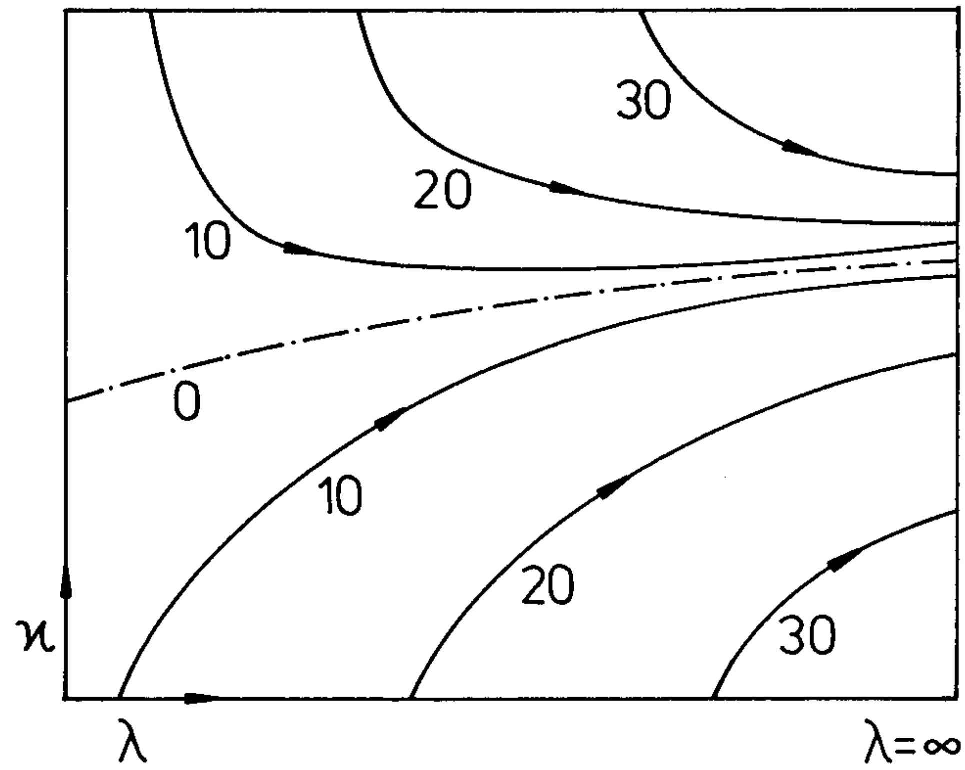

Fig.2

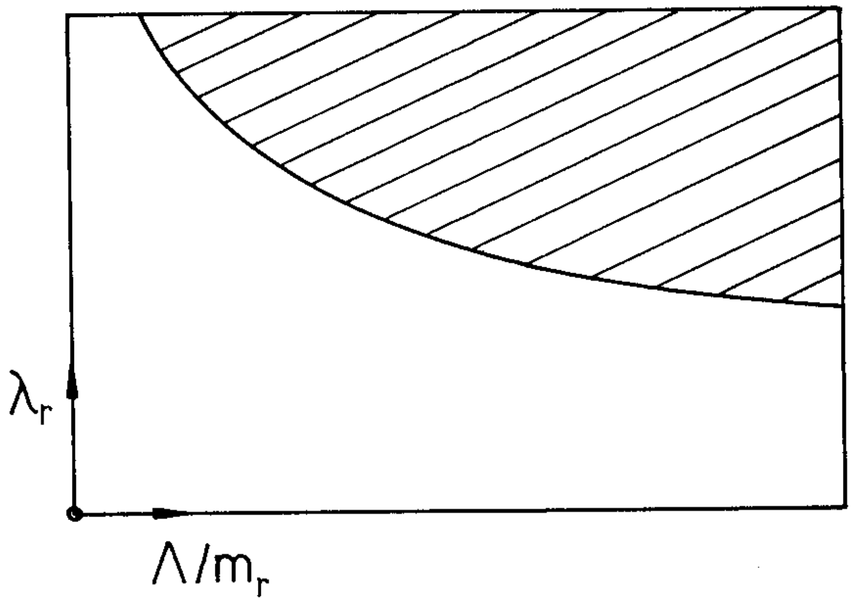

Fig.3

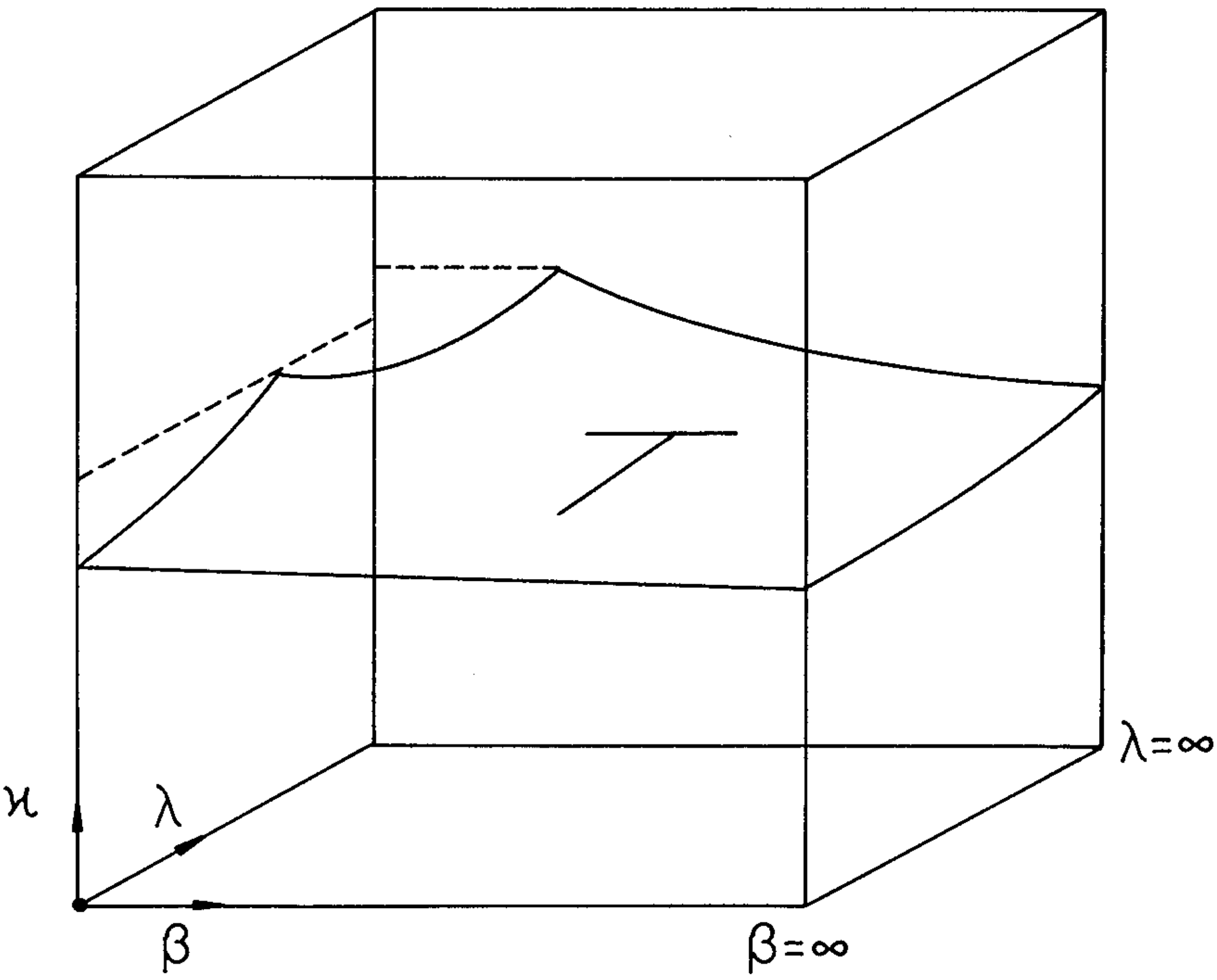

Fig. 4

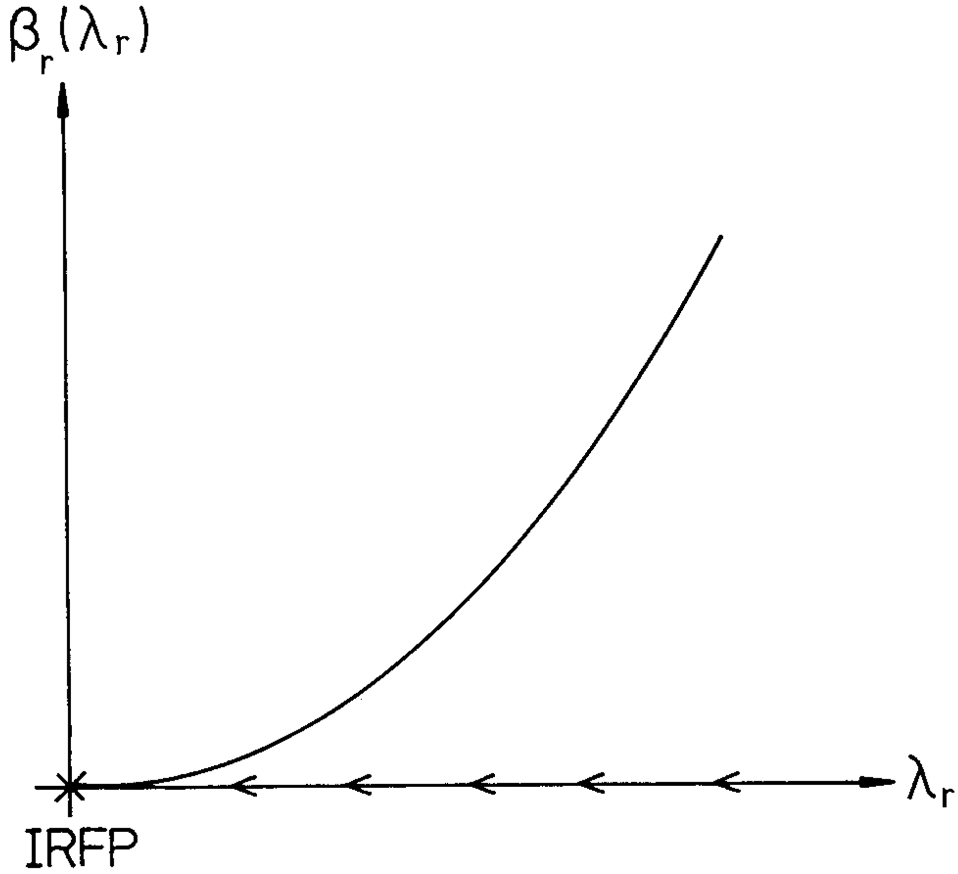

Fig.5

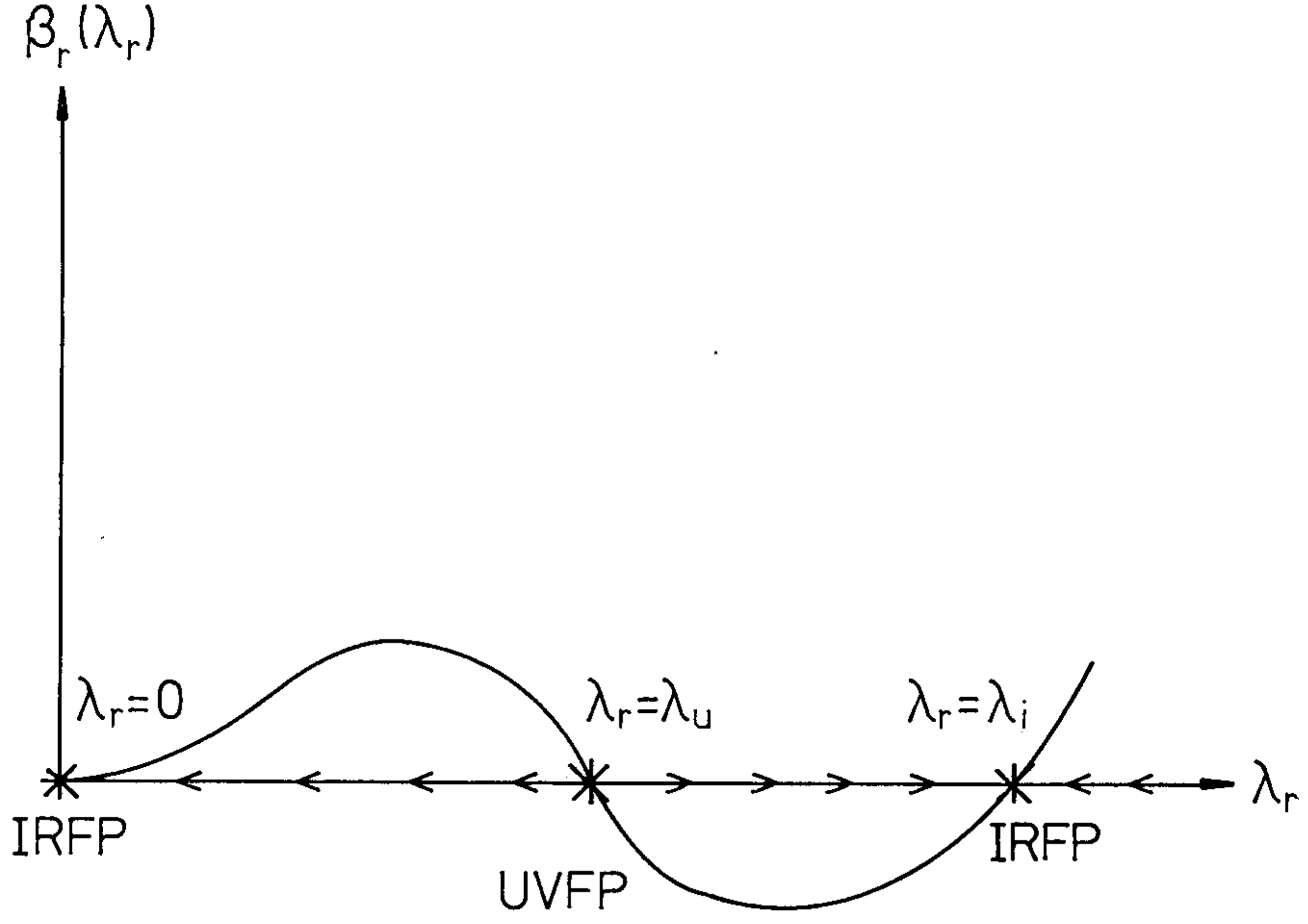
$\beta_r(\lambda_r)$
$\lambda_r=0$
$\lambda_r=\lambda_u$
$\lambda_r=\lambda_i$
λ_r
IRFP
UVFP
IRFP

Fig.6

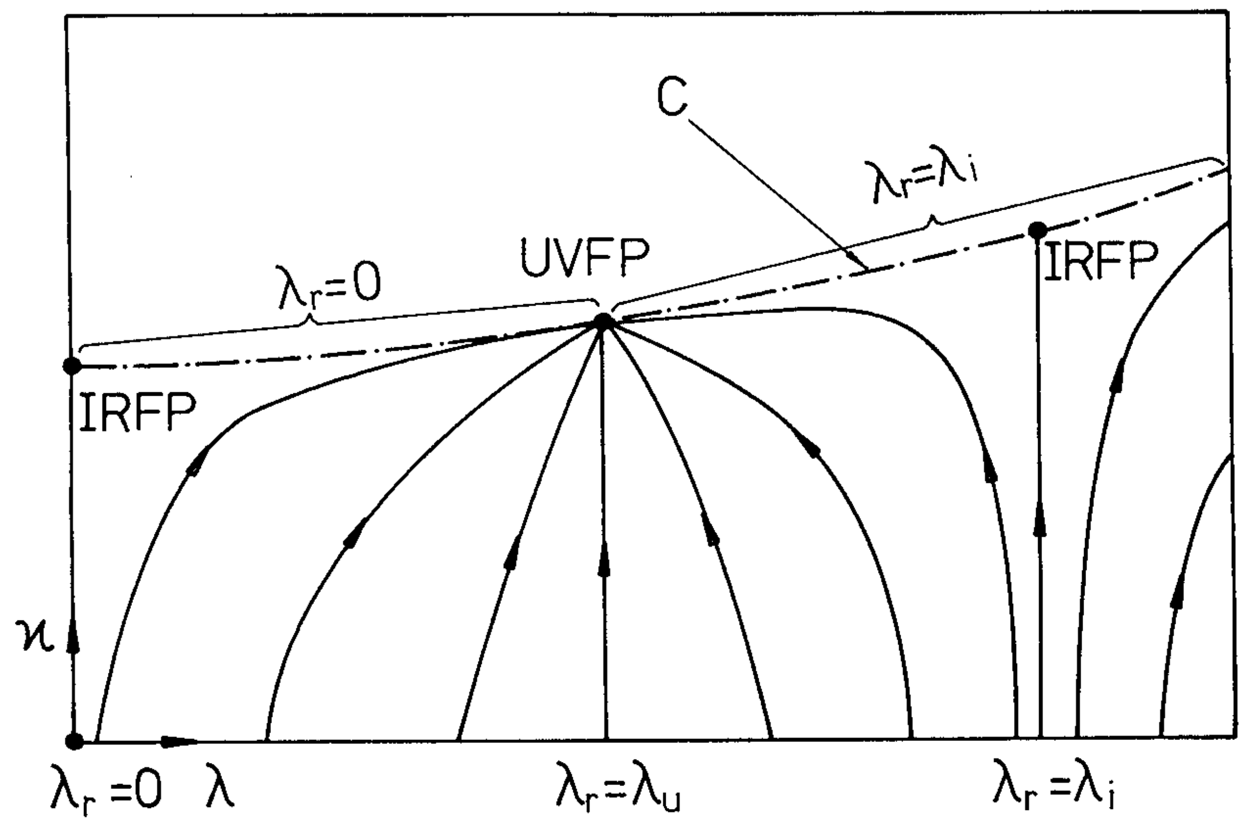

Fig.7

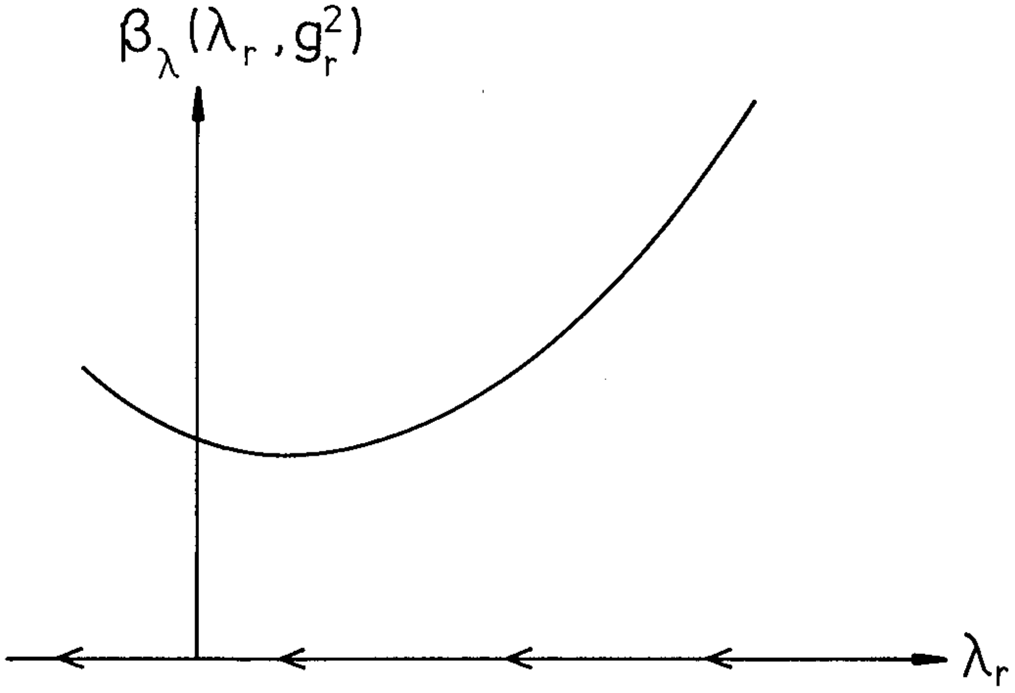

Fig. 8

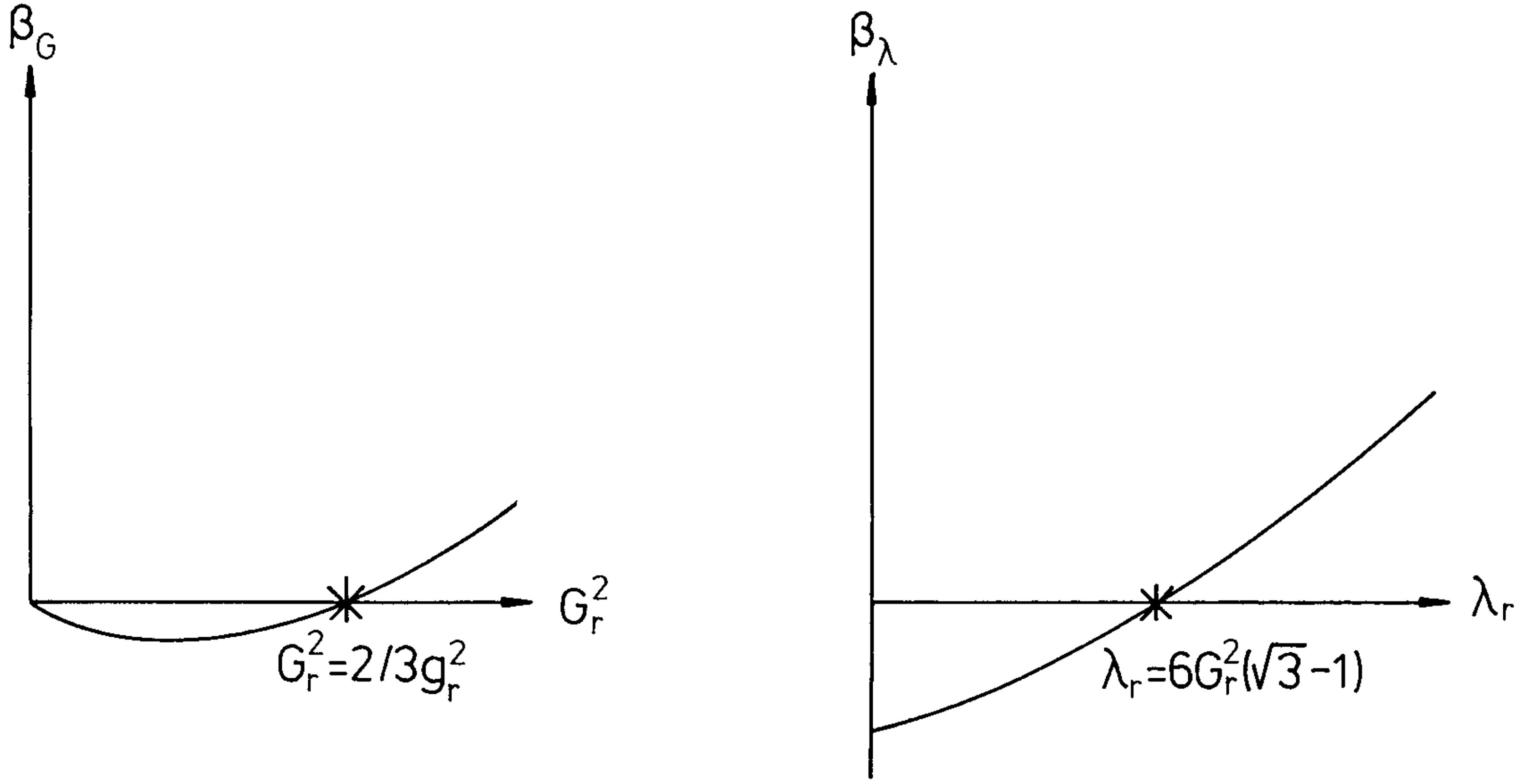

Fig.9

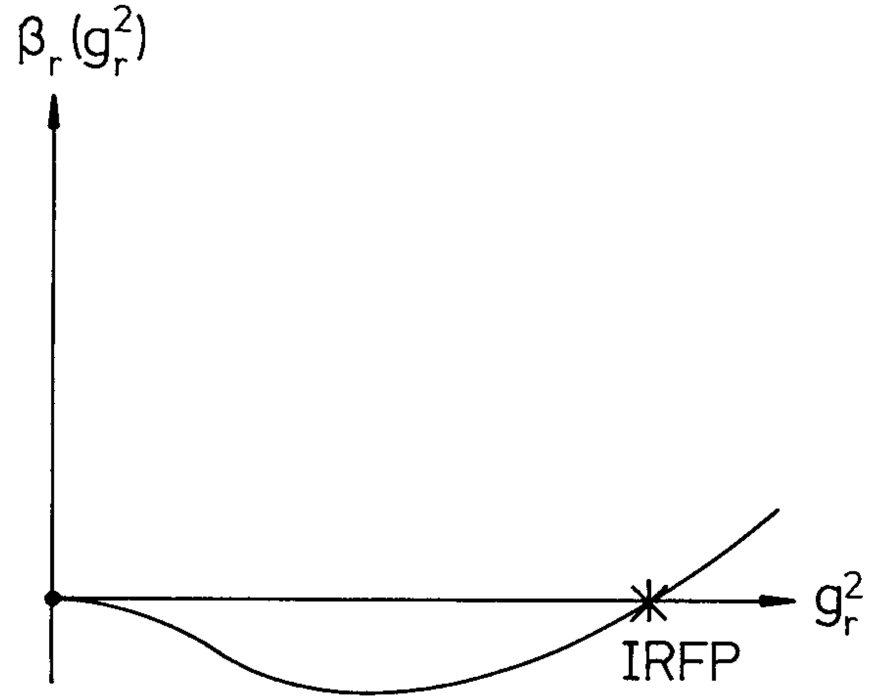

Fig.10

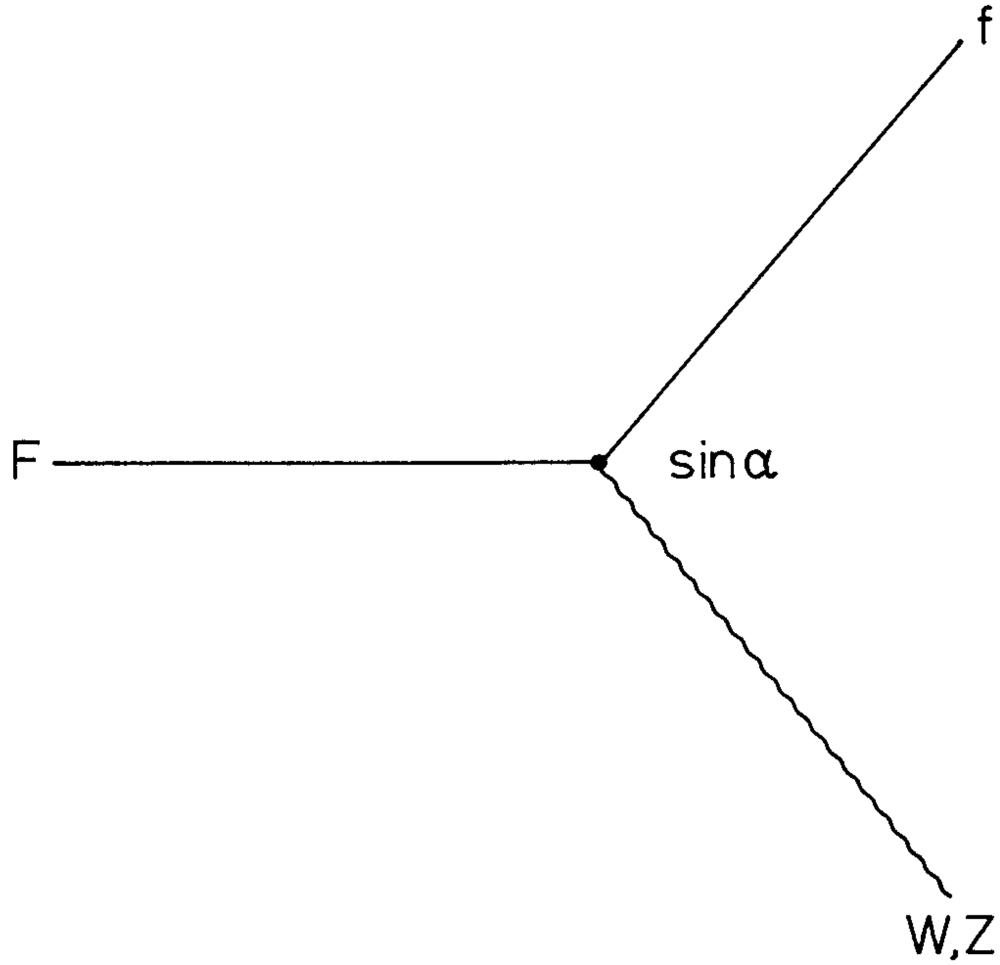

Fig.11