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STATUS OF LATTICE GLUEBALL MASS CALCULATIONS*

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ABSTRACT

In this talk I review the status of lattice glueball mass calculations. I restrict myself to recent results obtained on large lattices for the gauge group $SU(3)$.

INTRODUCTION

Over the past two years several groups[1] have developed new methods, which, for the first time, allow to calculate glueball masses on large lattices and in the continuum region, i.e. for $\beta = 6/g^2 \leq 6.0$. First results[2-6] of these calculations are available now, and it is the duty of my talk to review them.

As a result of its nonabelian character, QCD possesses a richer spectrum than that of the traditional quark spectroscopy. A whole sector of the spectrum should consist (before mixing) of pure glue states. These glueballs are very much a prediction of QCD, and a firm calculation of their masses would expose QCD to an invaluable quantitative test.

The lattice formulation together with numerical simulations is currently the most promising technique to calculate the glueball mass spectrum. This calculation proceeds in principle from first principles. The results may depend, however, on the lattice spacing a, i.e. the cut-off, and the spatial size of the lattice. For sufficiently large values of β, i.e. for small values of a much smaller than any hadronic scale, the cut-off dependence of the masses computed in lattice units should be given by the two-loop beta function. This

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is referred to as asymptotic scaling, and for a reliable mass calculation one must demonstrate that the masses do obey asymptotic scaling. As far as the size dependence is concerned, there are two kinds of effects. One effect is that the vacuum of small volumes is effectively perturbative (in the sense that the spectrum of states can be computed with perturbation theory[7]), whereas the nonperturbative confinement mechanism - which is an infrared phenomenon - takes over only on larger volumes. There is evidence[8] that confinement originates in the condensation of color magnetic monopoles.[9] This happens for[10] $z = L/\xi \geq 5$, where L is the (linear) size of the lattice and $\xi = (ma)^{-1}$ is the correlation length, with m being the mass of the lightest glueball. In order that the glueball mass calculations are of any relevance to the real world, the foremost requirement is therefore that the lattice volume is large enough so that $z \gg 5$. The other effect concerns larger lattices and is due to interactions of the glueballs with their images arising from the periodicity of the lattice. If one parameterizes these interactions by an effective three-point coupling λ, the following size dependence of the glueball masses has been computed:[11]

$$m(z) = m(\infty)[1 - \frac{\lambda^2}{4\pi} \frac{e^{-\frac{\sqrt{3}}{2}z}}{z}]. \qquad (1)$$

In practice this means that we need to calculate the glueball masses only on a few - though large - lattices and then use equ. (1) to extrapolate them to the infinite volume.

The calculation of the glueball mass spectrum has turned out to be one of the most difficult problems in lattice gauge theory. The problem of earlier mass calculations[12] was that the operators used to project out the glueball states have the property, that their projection onto the lowest-lying states vanishes with the fifth power of the lattice spacing[13] a as a goes to zero. As a result, the signal was rapidly lost in the noise, and the calculations could not be extended beyond $\beta = 5.9$. The so-called continuum region, where we may expect the lattice spacing to be small enough to exhibit asymptotic scaling, begins, however, only at $\beta = 6.0$.

The new methods either take advantage of the fact that operators composed of gauge fields that are averaged over a certain neighborhood[2-4] are much less exposed to short-range fluctuations - which are responsible for the noise - or explicitly construct operators of lower dimensions[5], which lead to a projection that decreases only with the first power of the lattice spacing.[13] This is the optimum one can achieve. Both approaches have led to promising results, which I will review now.

RESULTS

All calculations on large lattices have been done for the pure gauge theory, where one neglects the effects of quarks. So far we have quantitative results only for the 0^{++} and 2^{++} glueballs, which seem to be the lowest-lying states.

It is useful to plot the masses as a function of the variable z, which provides a measure of the physical size of the lattice. This allows us to disentangle finite size effects from violations of asymptotic scaling. In the limit of asymptotic scaling we expect the masses to fall on a universal curve. For larger volumes the z-dependence of this curve can be parameterized by equ. (1).

In this spirit I have compiled the results in fig.1. To convert the lattice units to $\Lambda_{\overline{MS}}$ I have used the two-loop formula

$$a = 28.8\Lambda_{\overline{MS}}^{-1} e^{-\frac{4\pi^2}{33}\beta}\left(\frac{8\pi^2}{33}\beta\right)^{\frac{51}{121}}. \tag{2}$$

The symbols mean:

Symbol	Ref.	β	L
□	2	6.0, 6.05, 6.1	13
×	4	5.9, 6.0, 6.2	12, 16, 20
•	5	6.0, 6.2	14, 16
○	6	6.0	18
△	12	5.9	10
▽	14	≈ 5.83	9

The correlation length is $\xi = (m_{0^{++}}a)^{-1}$. For clarity I have omitted the horizontal error bars on the data points. The last two entries are the only reliable $SU(3)$ mass calculations based on the old methods. All but the bottommost reference use the Wilson action, while ref. 14 uses the fundamental-adjoint action. Reference 4 offers various glueball masses to choose among for each β and L. I have taken the mass from the ratio of the zero-momentum correlation function at time 4 over time 3 except for the 2^{++} at $\beta = 5.9$, where I have taken it from the ratio 3/2. These seem to me their most reliable results.

Let me first discuss the case of the scalar glueball. At present the errors are still too large (at least partly) to draw any *firm* conclusions on scaling and the volume dependence of the 0^{++} mass. (One should be aware that

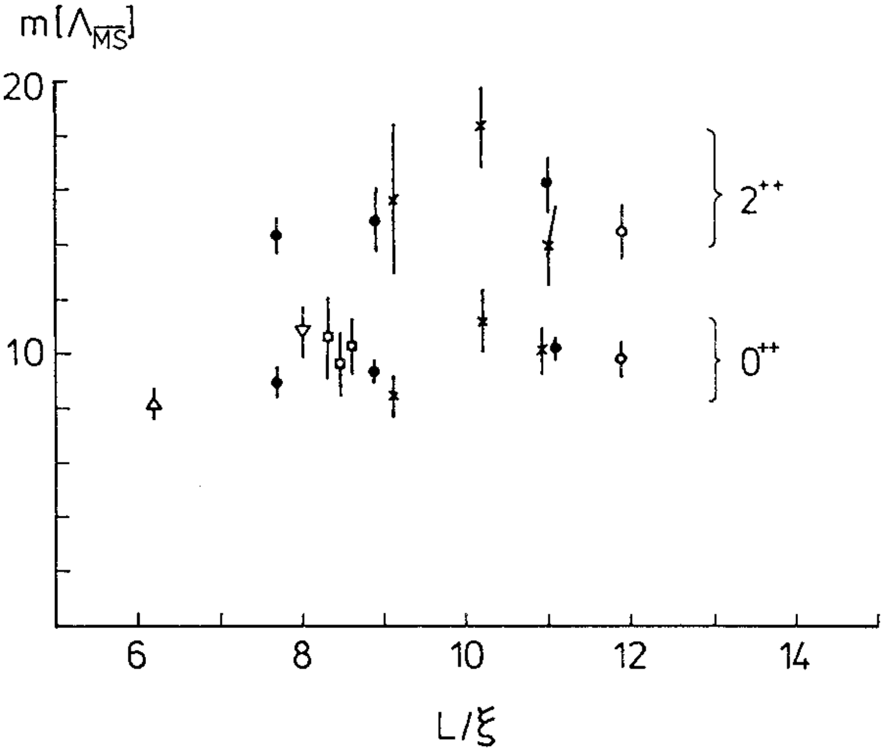

Fig. 1. The masses $m_{0^{++}}$ and $m_{2^{++}}$ in units of $\Lambda_{\overline{MS}}$.

the masses are also afflicted with systematic errors on top of the statistical ones as, e.g., a comparison of the results of refs. 4 and 5 shows where they overlap.) What we can say though is that the results are consistent with asymptotic scaling, and that the variation of the mass with the volume is probably less than 20% over the whole range $6 \leq z \leq 12$. Equation (1) then tells us that the mass will not change noticeably anymore beyond $z = 12$, which leads to the estimate

$$m_{0^{++}} = (10.4 \pm 0.3)\Lambda_{\overline{MS}}. \tag{3}$$

As far as the tensor glueball is concerned, the basic outcome is that its mass is significantly larger than the mass of the scalar glueball. In fig. 2 I have plotted the ratio of the two masses as a function of z. The result is

$$\frac{m_{2^{++}}}{m_{0^{++}}} \approx 1.5 \tag{4}$$

for $8 \leq z \leq 12$. (For comparison: the small volume calculations[7,15] done at $z < 5$ give $m_{2^{++}}/m_{0^{++}} \approx 1$.) It is premature to say anything about the scaling behavior. For that we have to wait until the errors have gone down. I also like to emphasize that in this case the cut-off is only $a^{-1} \lesssim 1.3 m_{2^{++}}$, so that the results may still change if the lattice spacing is decreased further.

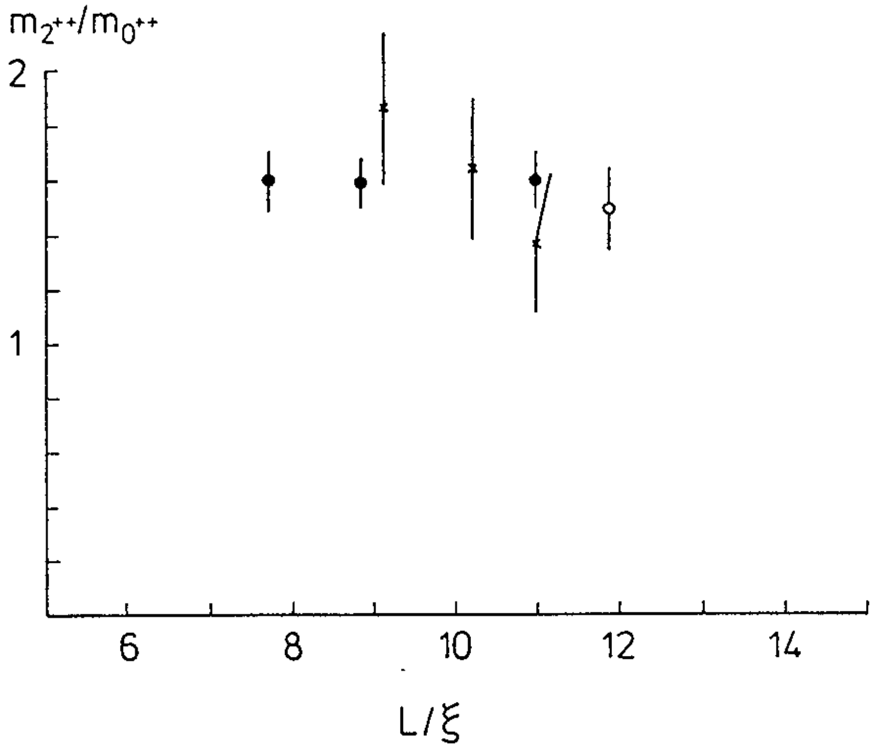

Fig. 2. The mass ratio $m_{2^{++}}/m_{0^{++}}$.

Michael and Teper[4] also quote values for the first excited 0^{++} glueball as well as for the 0^{-+}, 1^{+-} and 2^{+-}. The cut-off here is only $a^{-1} \approx (0.8-1.0)m$, and I am not convinced that these estimates can even serve as upper bounds.

To convert the results to physical units, one may use the string tension K, which is about $\sqrt{K} = 420 MeV$. In fig. 3 I have shown the ratios $m_{0^{++}}/\sqrt{K_\infty}$

and $m_{2^{++}}/\sqrt{K_\infty}$, where K_∞ is the string tension extrapolated to infinite volume using the formula[16] $K(L) = K_\infty - \pi/3L$. This gives $m_{0^{++}} \approx 1.4GeV$ and $m_{2^{++}} \approx 2.1GeV$. It goes without saying that these numbers are likely to change when quarks are included in the calculation, which, however, will not be possible for quite some time. But perhaps one can employ a mixing model[17] to estimate these effects.

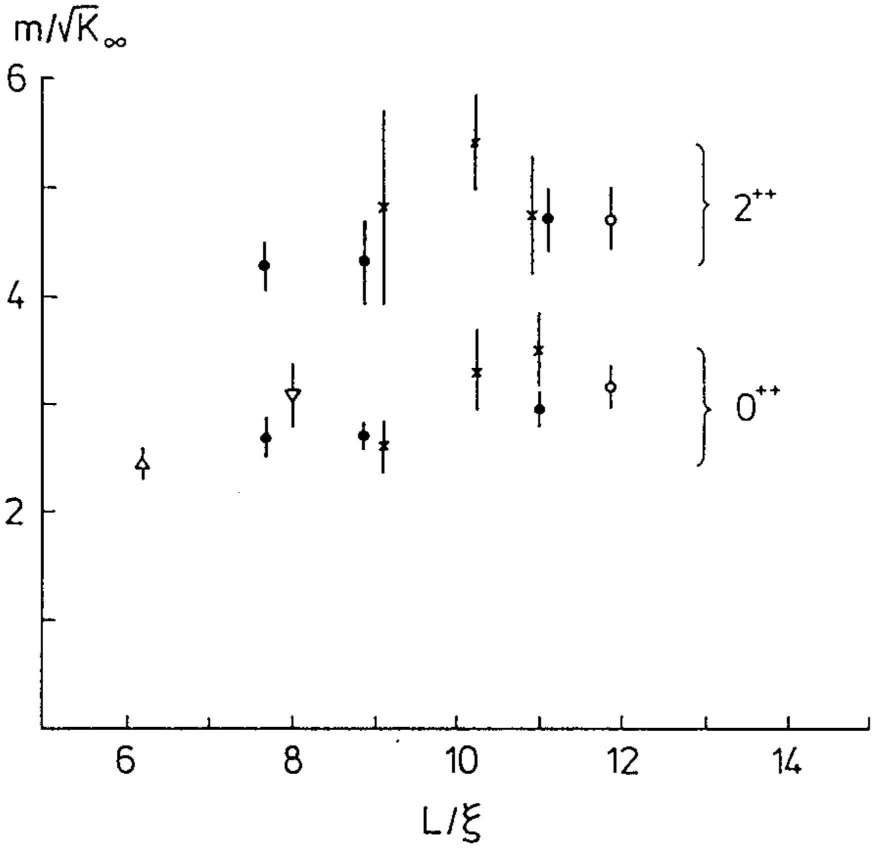

Fig. 3. The ratios $m_{0^{++}}/\sqrt{K_\infty}$ and $m_{2^{++}}/\sqrt{K_\infty}$.

OUTLOOK

The work I have presented in this talk is clearly only the beginning of a new effort to compute the glueball mass spectrum at larger values of β and on large lattices. A lot remains to be done. Most importantly, the errors have to be reduced. It helps that various groups are involved in these calculations, in particular for gaining control over the systematic errors. The next step then is to go to really small values of the lattice spacing, which means to increase β and L further. This should allow us also to reliably estimate the masses of the higher glueball states.

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