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Weak Decays: Theoretical Summary

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Abstract

The present status of the Cabibbo Kobayashi Maskawa matrix is reviewed and the prospects for improving our knowledge of the various elements by K and B decay experiments in the 1990's are critically examined. Windows which these investigations may provide for physics beyond the standard model are also discussed. Particular attention is devoted to the expected patterns of CP violation in the B system.

1 The Cabibbo Kobayashi Maskawa Paradigm

The $SU(2) \times U(1)$ gauge theory of Glashow Salam and Weinberg [1] is the present paradigm for the electroweak interactions. Within the standard model, hadronic flavor changing weak transitions have a somewhat less secure role, since they require some knowledge of the mass generation mechanism in the quark sector. The simplest possibility to give mass to the fermions in the theory makes use of Yukawa interactions involving the doublet Higgs field. These interactions give rise to, what might be called, the Cabibbo Kobayashi Maskawa (CKM) paradigm: Quarks of different flavor are mixed in the charged weak currents by means of an unitary matrix V. However, both the electromagnetic current and the weak neutral current remain flavor diagonal. Departures from this pattern such as, for example, finding evidence for a flavor changing neutral current transition like $K \to \mu e$, would belie this simple hypothesis and be a sign for new physics.

Second order weak processes are on even less secure ground, since they can be affected by both beyond the standard model virtual contributions, as well as new physics direct contributions. Thus they provide very stringent tests of the CKM paradigm. For example, both the $\Delta S = 2$ transition responsible for K-$\overline{K}$ mixing, and the CP violating parameter ϵ in neutral Kaon decays are governed by the size of the CKM matrix elements entering in Fig. 1. However, if low mass superpartners existed, or if not too massive diquarks existed, additional contributions, like those shown in Fig. 2, would vitiate these predictions. Hence, accurate comparisons between experiment and the predictions of the CKM paradigm can give

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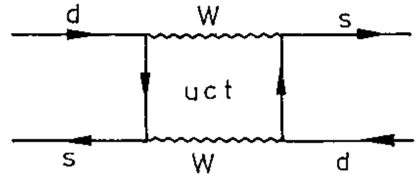

Figure 1: W exchange graph for the $\Delta S = 2$ process, which depends on V_{is} and V_{id} $(i = u, c, t)$.

indications either for, or against, new physics. It is clear that one of the main tasks in the 1990's will be to either establish – at a higher level of accuracy – the CKM paradigm, or to disprove it.

The task of testing the CKM paradigm is much simpler if there exist **only** 3 generations. In this case, as is well known, the CKM matrix depends on three real angles and only one phase, since the other phases entering in the 3×3 unitary matrix V can be absorbed by a redefinition of the quark fields. For 4 generations, on the other hand, one needs 6 angles and 3 phases to determine V. In this study, as a benchmark, we have focused on the 3 generation case. There are a number of very good practical reasons to do so:

1. We shall soon know from SLC and/or LEP whether the number of neutrino species – and thus, presumably, the number of generations – is 3.
2. Unitarity $[V^\dagger V = 1]$ strongly restricts V_{ij}, even without any explicit evidence for a t quark.
3. In the case of 3 generations, experiment indicates a nice hierarchical pattern in V, as a function of the Cabibbo angle: $\sin\theta_c = \lambda$

$$V \sim \begin{vmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{vmatrix} \qquad (1)$$

A convenient form of the CKM matrix, which displays this hierarchy in a simple way, is[1]

$$V = \begin{vmatrix} c_1c_3 & s_1c_3 & s_3e^{-i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{vmatrix} \qquad (2)$$

[1]This is now the form of the CKM matrix "officially" adopted by the Particle Data Group [2].

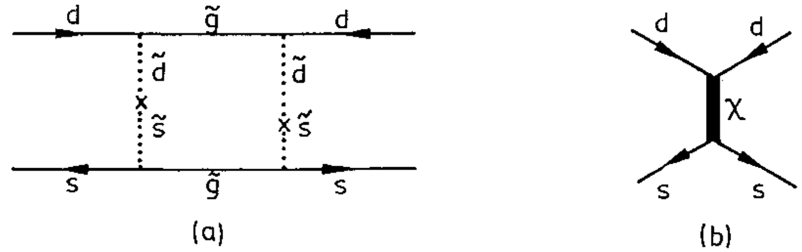

Figure 2: Possible supersymmetric contribution (a) and diquark contribution (b) to the $\Delta S = 2$ process.

In the above, s_i, c_i are shorthand for $\sin\theta_i$ and $\cos\theta_i$. The experimental hierarchy of Eq. (1) is realized by writing

$$s_1 = \lambda; \quad s_2 = A\lambda^2; \quad s_3 = A\rho\lambda^3 \; , \tag{3}$$

where A and ρ are real numbers. Since $\lambda \simeq 0.22$ one can approximate [3]

$$V = \begin{vmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\rho\lambda^3 e^{-i\delta} \\ -\lambda(1+A^2\rho\lambda^4 e^{i\delta}) & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\delta}) & -A\lambda^2 & 1 \end{vmatrix} \tag{4}$$

Note that the only elements which have a potentially large phase in the CKM matrix V, in this parametrization. are the ub entry, $|V_{ub}|e^{-i\delta}$, and the td entry, $|V_{td}|e^{-i\phi}$, where the phase ϕ is related to δ in an obvious way. The unitarity of the CKM matrix, along with the hierarchy, implies a simple interrelation among the matrix elements

$$V_{ub}^* + V_{td} = \lambda V_{cb} \; . \tag{5}$$

That is, the elements V_{ub}^*, V_{td} and λV_{cb} form a triangle in the complex plane [4]. The phases δ and ϕ are two of the angles of this triangle, as shown in Fig. 3. As we shall see, CP violating asymmetries in B decays can directly measure the various angles of the unitarity triangle.

1.1 Status of the CKM Matrix

Apart from the Cabibbo angle, which is well determined, the remaining parameters of the CKM matrix today are still quite uncertain. Roughly speaking, λ is determined to 1%, A to 10%, while the uncertainty in ρ and in the phase δ is of the order of 100%! The value of λ, obtained by Leutwyler and Roos [5] from an analysis of K_{l3} decays and Hyperon β-decay, is

$$V_{us} = \lambda = 0.221 \pm 0.002 \; . \tag{6}$$

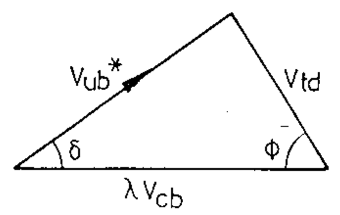

Figure 3: The unitarity triangle

This value, plus unitarity, implies a value for V_{ud} which is in excellent agreement (to the per mil level) with the direct determination of this matrix element, from a comparison of μ-decay with 0^+-0^+ nuclear transitions [6].

The parameter A in the CKM matrix, being related to V_{cb}, is fixed by the B lifetime and by the semileptonic B decay spectrum. In principle, since $|V_{ub}|^2 \ll |V_{cb}|^2$, the B lifetime measurement gives directly A. However, since in a simple spectator model $\tau_B \sim m_b^{-5}$, the uncertainty in the value of the b quark mass is far too large to allow for a good determination of A. To do better, one must eliminate m_b in favor of m_B – the B meson mass. This requires constructing some (simple) bound state model, whose parameters can be fitted from the lepton spectrum in B semileptonic decays [7]. A recent analysis along these lines, by Altarelli and Franzini [8], gives

$$A = 1.05 \pm 0.17 \quad \leftrightarrow \quad V_{cb} = 0.051 \pm 0.008 \; . \tag{7}$$

One of us (CSK) has reanalyzed this question independently, by using CRYSTAL BALL data [9] and the bound state model of Ref. [7], obtaining similar results to those given in Eq. (7). In particular, using the new value for the B semileptonic branching ratio [10,11] $B(B \to l\nu_l X) = 0.10 \pm 0.01$, this reanalysis gives $A = 1.03 \pm 0.14 \;\leftrightarrow\; V_{cb} = 0.050 \pm 0.007$, in excellent agreement with the results of [8]. Unfortunately, at present, it appears very difficult to reduce the error on A below the 10% level.

The parameter ρ is related to V_{ub}. Since no evidence exists for charmless semileptonic B decays, one can obtain an upper bound on ρ from the study of the lepton spectra in these processes. This is a, somewhat, model dependent bound since the detailed predictions for the lepton spectrum shape depend on the bound state model one assumes for the B meson. Adopting again the simple model of [7] and analyzing the CRYSTAL BALL data [9] – using the parameter set that provides the best fit – yields[2]

$$\left|\frac{V_{ub}}{V_{cb}}\right| < 0.18 \; . \tag{8}$$

We note that the recent analyses of the ARGUS and CLEO collaborations [10,11] each give a very similar limit $|\frac{V_{ub}}{V_{cb}}| < 0.16$. In terms of ρ, Eq. (8) implies

$$\rho < 0.8 \; . \tag{9}$$

In principle, the observation by the ARGUS collaboration [12] of the charmless B decay $B \to p\bar{p}\pi(\pi)$ provides a lower bound for ρ. However, the ARGUS observation has not been confirmed by CLEO [13]. Furthermore, extracting a reliable lower bound for ρ from the ARGUS value is difficult theoretically. Nevertheless, if one had to hazard a guess from present day B decay evidence, one would put $\rho \geq 0.2$-0.3.

The phase δ and ρ are constrained by the observation of CP violation in the Kaon system and by B-$\overline{B}$ oscillations. Since there would be no CP violation if δ or ρ vanished[3], the observed value of ϵ in Kaon decays provides bounds for these parameters. The allowed region for δ and ρ is correlated. The standard analysis of Buras et al. [14] gives for ϵ a formula which is essentially proportional to the imaginary part of the box graph in Fig. 1. The final answer depends on the value assumed for m_t, even though the t quark graph is Cabibbo suppressed by a factor of λ^4 with respect to the other contributions, since it is proportional to m_t^2. There is, furthermore, another uncertainty connected with the value of the K-$\overline{K}$ matrix element of the effective Hamiltonian, which is not precisely known theoretically[4], with the allowed region in the ρ-δ plane shrinking or expanding depending on what one assumes for this matrix element.

The B_d-$\overline{B_d}$ mixing parameter $x_d = \frac{(\Delta m)_{B_d}}{\Gamma_{B_d}}$ is given by the real part of a box graph similar to that of Fig. 1, with $s \leftrightarrow b$. For x_d, the t quark contribution in the box graph really dominates and one has that

$$x_d \sim |V_{td}|^2 \sim (1 + \rho^2 - 2\rho\cos\delta) \; . \tag{10}$$

Thus B_d-$\overline{B_d}$ mixing can be used to restrict the range for ρ and δ. Unfortunately, again there are corresponding uncertainties; x_d also depends on m_t (essentially quadratically) and on the value of the hadronic matrix element[5].

We show in Fig. 4, as an illustration, the allowed region in the ρ-δ plane from the combined fit of ϵ and x_d. In this Figure, taken from [15], we let m_t range over 40 GeV $< m_t <$ 180 GeV, fix $B_K = \frac{2}{3}$ and assume 100 MeV $< f_{B_d} B_{B_d}^{\frac{1}{2}} <$ 200 MeV. In this analysis, furthermore, ρ was restricted to be $\rho < 0.9$ [compare with Eq. (9)]. A knowledge of m_t would considerably restrict the allowed region in the ρ-δ plane. Fig. 5 illustrates this, even in the case when one assumes, besides the previously considered uncertainty for the hadronic matrix element in x_d, also a rather large uncertainty in the estimate of the Kaon matrix element $[1/3 < B_K < 1]$.

[2] Conventionally one quotes a bound for $R = \frac{\Gamma(b \to u)}{\Gamma(b \to c)}$, which is related to the Kobayashi Maskawa matrix element ratio (8) up to a phase space factor. For the best fit parameter set, this phase space factor is 2.10 ± 0.15.

[3] $\delta = \pi$ is also a CP conserving point.

[4] Conventionally, one normalizes one's ignorance by quoting a value for the, so called, B_K parameter – the ratio of the matrix element $\langle K^0 | [\bar{d}\gamma_\mu(1-\gamma_5)s]^2 | \overline{K^0} \rangle$ to that obtained by vacuum saturation.

[5] The conventional normalization of one's ignorance here is done via the parameter combination $f_{B_d}^2 B_{B_d}$, because in this case the B_d decay constant is also unknown.

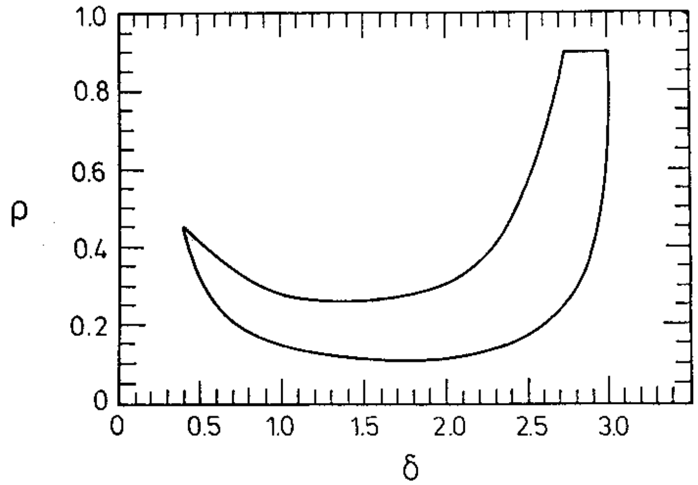

Figure 4: Allowed region in the ρ-δ plane, from [15], for the range of parameters given in the text.

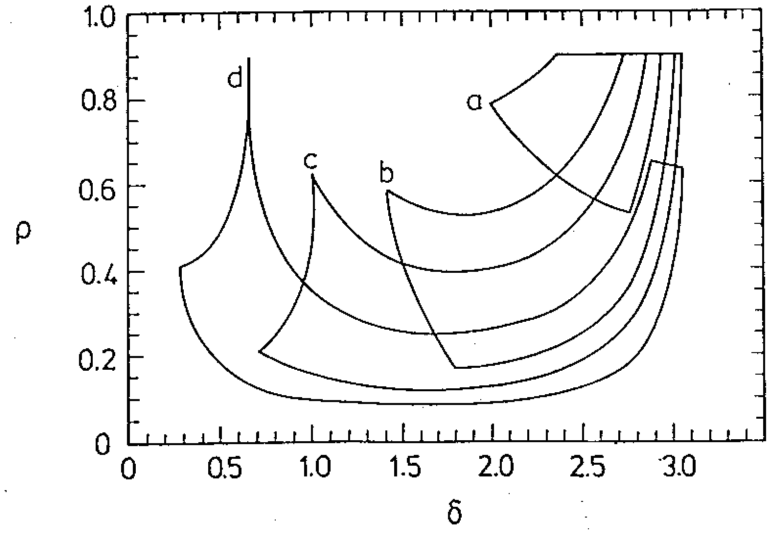

Figure 5: Allowed regions in the ρ-δ plane for various values of m_t: (a) $m_t = 60$ GeV; (b) $m_t = 90$ GeV; (c) $m_t = 120$ GeV; (d) $m_t = 180$ GeV. From [15].

To really test the CKM paradigm it will be necessary, in the future, to considerably restrict the rather large region now allowed in the ρ-δ plane. In principle, this can be achieved in a number of ways:

- More accurate theoretical calculations of the various hadronic matrix elements would be very helpful. It is possible – although by no means certain – that the large effort now underway in lattice calculations of weak matrix elements [16] may bear fruit in the 1990's and provide us with predictions with an accuracy of 10%. Without this kind of precision, even beautiful new data, like the NA31 result on ϵ' [17], cannot provide further constraints on the CKM model [18].
- Confirmation of the ARGUS direct signal for charmless B decay may provide a lower bound on ρ. However, again, here one is faced with difficulties in trying to reliably estimate some two-body hadronic matrix element. A measurement of the process $B \to \rho l \nu_l$ may, in this respect, be more helpful to pin down $|V_{ub}|$.
- Measurement of the B_s-$\overline{B_s}$ mixing parameter x_s will provide, in conjunction with x_d and an assumption about (or a calculation of) the ratio $f_{B_s} B_{B_s}^{\frac{1}{2}} / f_{B_d} B_{B_d}^{\frac{1}{2}}$, a further constraint on ρ and δ.
- The decay $K^- \to \pi^- \nu\bar{\nu}$ measures a differently weighted combination of CKM matrix elements than those appearing in ϵ and x_d. Thus, this decay can provide additional constraints, particularly since the estimate of the hadronic matrix elements here is much more under control. A measurement of the very rare process $K_L^0 \to \pi^0 \nu\bar{\nu}$ would also provide new information.
- The measurement of different CP violating hadronic asymmetries in B decays directly tests the phase structure of the CKM matrix. Hence, these experiments could provide the most direct test of the CKM paradigm.

In the subsequent sections, we shall address some of the theoretical issues connected with the points raised above.

2 A Few Crucial K Decays

The NA31 [17] positive evidence for **direct** $\Delta S = 1$ CP violation is a very important result, since it kills the still extant superweak explanation [19] for CP violation in the Kaon system. Nevertheless, it is not possible to obtain an accurate theoretical estimate, within the CKM paradigm, to compare with the NA31 result [17]:

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} = (3.3 \pm 1.1) \times 10^{-3} \,. \tag{11}$$

In the CKM model ϵ' arises through the Penguin graph of Fig. 6. Unfortunately, the matrix element of the associated Penguin operator $O_p = (\bar{s}\gamma^\mu(1-\gamma_5)\lambda_a d)(\bar{q}\gamma_\mu\lambda_a q)$ between a Kaon and two pions is theoretically uncertain. The theoretical explanation for the ratio (11) is, furthermore, also affected by our inability to calculate ϵ precisely. This notwithstanding, the central value measured by NA31 is in excellent agreement with the central value of the

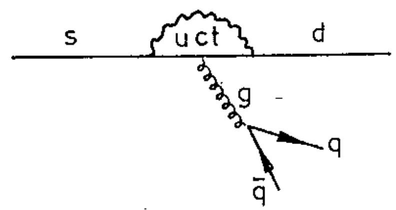

Figure 6: Penguin graphs contributing to ϵ'.

theoretical expectation in the CKM model [18], but a theoretical excursion of a factor 2-3 is permitted.

The above status of affairs suggest the following three recommendations:

1. It is important to pursue an independent measurement of $\frac{\epsilon'}{\epsilon}$ to check that $\epsilon' \neq 0$. This is presently being done at Fermilab and will soon be attempted at LEAR[6]. However, given the theoretical uncertainties in precisely predicting a value for ϵ', the knowledge of a very accurate experimental value for ϵ' seems superfluous.
2. One should explore other K decays which are sensitive to direct CP violation. Among these, the decay channel $K_L^0 \to \pi^0 e^+ e^-$ may provide a particularly interesting example, as we shall discuss in more detail below.
3. With the benefit of hindsight, at least in the CKM model, the Kaon system appears to be the **wrong place** to look for CP violation. The dominant, 2^{nd} order weak transitions between an s and a d quark (shown in Fig. 7) are approximately real. For the corresponding transitions from b to d, the real and imaginary parts are of the same order in the Cabibbo angle. Thus, if one believes in the CKM paradigm, the place to look for CP violation is in the B system.

2.1 The decay $K_L^0 \to \pi^0 e^+ e^-$

The weak decay $K_L^0 \to \pi^0 e^+ e^-$ is particularly interesting dynamically since there are numerous physical mechanisms at play simultaneously. The decay amplitude $K_L^0 \to \pi^0 e^+ e^-$ can be related to that of the process $K_L^0 \to \pi^0 V^*$, where V^* is an effective $J = 1$ CP even state. This latter amplitude violates CP, since $(\pi^0 V^*)_{J=0}$ has CP $= +1$. Through this channel, therefore,

[6]The CP LEAR experiment does not quite measure $\frac{\epsilon'}{\epsilon}$, but a hadronic asymmetry in the Kaon system – quite analogous to those we will discuss in the B system – which is proportional to ϵ'.

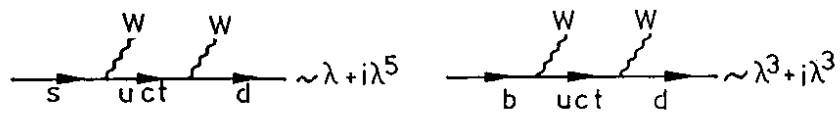

Figure 7: (a) 2nd order weak transition from s to d; (b) 2nd order weak transition from b to d.

$K_L^0 \to \pi^0 V^*$ proceeds either by **indirect** CP violation, because K_L^0 has a small CP even admixture $[K_L^0 = \epsilon K_1]$, or through the **direct** CP violating decay of the CP odd component of K_L^0: $K_2 \to \pi^0 V^*$. In addition, however, there is an effective two photon contribution to the process, as shown schematically in Fig. 8. This contribution is CP conserving, but is of higher order in α.

One can estimate (CD,FG) [20] the associated branching ratio due to the indirect CP violating contribution as follows. If one assumes a pure $\Delta I = \frac{1}{2}$ transition, then

$$\begin{aligned} B(K_L^0 \to \pi^0 e^+ e^-)_{\text{ind CP viol}} &= B(K^+ \to \pi^+ e^+ e^-) \cdot \frac{\Gamma(K^+ \to \text{all})}{\Gamma(K_L^0 \to \text{all})} \cdot \\ & \quad \frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} \cdot \frac{\Gamma(K_L^0 \to \pi^0 e^+ e^-)_{\text{ind CP viol}}}{\Gamma(K_1 \to \pi^0 e^+ e^-)} \\ &= 2.7 \times 10^{-7} \cdot 4.2 \cdot 1 \cdot |\epsilon|^2 \\ &\simeq 6 \times 10^{-12} \, . \end{aligned} \tag{12}$$

This number is in agreement with the estimates of [21,22] and is typically of the same order or less than that obtained from a calculation of the relevant graphs, shown in Fig. 9, causing the direct CP violating transition. The direct CP violating amplitude, being mostly a short distance electroweak contribution, is calculable [23]. However, the actual answer one obtains depends on the unknowns of the CKM paradigm: the values of m_t and the range in the ρ-δ plane. A recent analysis done by two of us (CD,FG) with I. Dunietz [20], including QCD corrections, indicates that

$$B(K_L^0 \to \pi^0 e^+ e^-)_{\text{dir CP viol}} \simeq (2-20) \times 10^{-12} \, , \tag{13}$$

where the larger branching ratios correspond to bigger values for m_t.

The CP conserving 2γ contribution to the decay $K_L^0 \to \pi^0 e^+ e^-$ was estimated earlier [22] to be negligible. If this were the case, this decay could provide a useful test of the CKM paradigm, since the direct CP violating contribution is certainly non-negligible and

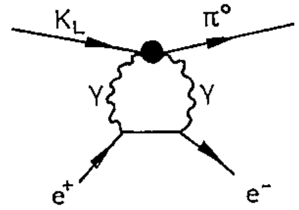

Figure 8: CP conserving contribution to $K_L^0 \to \pi^0 e^+ e^-$, via a two photon intermediate state.

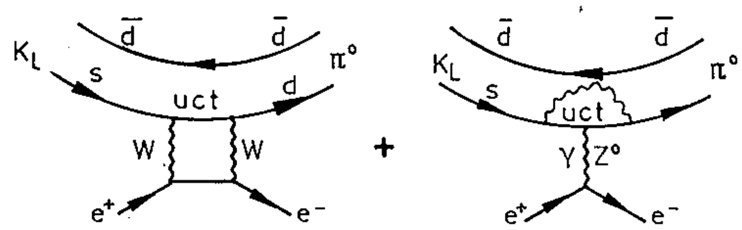

Figure 9: Graphs contributing to the direct CP violating decay of $K_L^0 \to \pi^0 e^+ e^-$.

may in fact dominate[7]. However, some recent papers [24,25] have questioned this point. In particular, using a vector dominance model for the subprocess $K_L^0 \to \pi^0\gamma\gamma$, these authors arrive at a CP conserving branching ratio[8]

$$B(K_L^0 \to \pi^0 e^+ e^-)_{\rm CP\ conserving} \simeq (8 - 16) \times 10^{-12}\ , \tag{14}$$

Several comments are in order:

1. The vector dominance estimate for the CP conserving amplitude is not in disagreement with the formal results of chiral perturbation theory [26]. The tiny estimate for this amplitude obtained by Donoghue et al [22] concerns terms which are of $O(p^4)$ (where p is the pion 4-momentum), but which are strongly suppressed by their helicity structure, giving a rate proportional to m_e^2. The results of [24,25] rely on terms which are of $O(p^6)$ in chiral perturbation theory, but which are not chirally suppressed because an additional tensor structure is now allowed. Nevertheless, the coefficient of the p^6 term which leads to Eq. (14) appears to be numerically much larger than what one would expect simply by counting factors of f_π [21].

2. One may be able to resolve the issue of how large the CP conserving contribution for the $K_L^0 \to \pi^0 e^+ e^-$ decay is by measuring directly the process $K_L^0 \to \pi^0\gamma\gamma$. A sizable $O(p^6)$ contribution would distort the spectrum of the two photons, predicted by chiral perturbation theory [26], and lend support to the vector dominance calculation.

3. Sehgal [24] has made the amusing observation that, in principle, there is a way to disentangle the CP conserving from the CP violating contribution in the decay $K_L^0 \to \pi^0 e^+ e^-$, by looking at the energy distribution of the electrons and positrons. The presence of the CP violating amplitudes leads to an asymmetry in this distribution, between e^+ and e^-. However, although the asymmetry can be large [24], the number of events one is likely to collect in the 1990's would be far too few for any meaningful test.

2.2 The decays $K^\pm \to \pi^\pm \nu\bar{\nu}$ and $K_L^0 \to \pi^0\nu\bar{\nu}$

The weak decay $K^\pm \to \pi^\pm \nu\bar{\nu}$ does not directly test CP violation. However, in the CKM paradigm, for the case of three generations, it is sensitive to V_{td} and, therefore, indirectly to the phase δ. The relevant graphs, at the quark level, for this decay are the analogues to those in Fig. 9, but with $e \leftrightarrow \nu_e$ and $\bar{d} \leftrightarrow \bar{u}$. These graphs give rise to an effective Lagrangian at short distances for this process, of the form

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_l A_l \bar{s}\gamma^\mu(1-\gamma_5)d\bar{\nu_l}\gamma_\mu(1-\gamma_5)\nu_l\ , \tag{15}$$

with A_l a calculable function of the quark and lepton masses and the CKM angles [23]. Using (15), the matrix element for $K^- \to \pi^-\nu\bar{\nu}$ is directly related to that for $K^- \to \pi^0 e^- \bar{\nu}_e$ and

[7] In the superweak model, where only the $\Delta S = 2$ parameter ϵ is nonvanishing, there would be no direct CP violating contribution and one would expect a branching ratio like that given in Eq. (12).

[8] The value obtained by [25] is larger than this, but we believe this is due to a trivial error in their phase space estimate.

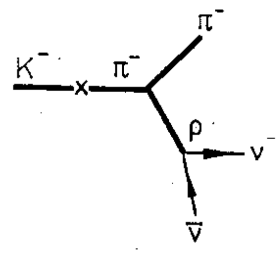

Figure 10: Possible long distance contribution to the decay $K^- \to \pi^-\nu\bar{\nu}$.

the number of neutrino species:

$$B(K^- \to \pi^-\nu\bar{\nu}) = \frac{2\sum_l |A_l|^2}{|V_{us}|^2} B(K^- \to \pi^0 e^- \bar{\nu}_e)\ . \tag{16}$$

This result ignores possible long distance contributions to this decay, of the type shown in Fig. 10. However, a simple estimate [27] shows these contributions to be negligibly small, leading to a branching ratio about 2 to 3 orders of magnitude below that arising from Eq. (16).

Using the results of Inami and Lim [23] for the functions A_l (and ignoring a numerically irrelevant contribution which arises from keeping $m_\tau \neq 0$) and the experimental value for the K_{e3} branching ratio, one finds

$$B(K^- \to \pi^-\nu\bar{\nu}) = 6 \times 10^{-7} N_\nu |F(y_c) + A^2\lambda^4(1-\rho e^{i\delta})F(y_t)|^2 \eta\ . \tag{17}$$

Here N_ν is the number of neutrino species [$N_\nu = 3$ for the three generation case we are considering], while the function $F(y_i)$, with $y_i = m_i^2/M_W^2$, calulated in [23], is given by

$$F(y) = \frac{y}{4}\left[\frac{3(y-2)}{(y-1)^2}\ln y + \frac{y+2}{y-1}\right]. \tag{18}$$

The parameter η in Eq. (17) is a QCD correction to the short distance calculation of Eq. (15), which has been estimated to be close to unity [28] and so will be ignored in what follows[9], given the other uncertainties.

The rate (17) has an irreducible uncertainty connected to the value of the A parameter and that of the charm mass to be used. For instance, for $m_c = 1.5 \pm 0.2$ GeV, and A as given in Eq. (7), typically the branching ratio for $B(K^- \to \pi^-\nu\bar{\nu})$ is only determined to a 50% accuracy. The overall rate is quite sensitive to m_t, ρ, and δ, since the c quark and t quark contributions in (17) are quite comparable. Given the present constraints on these

[9] In effect, it appears worthwhile to repeat the calculation of [28], since this was done only in certain kinematical limits concerning m_t.

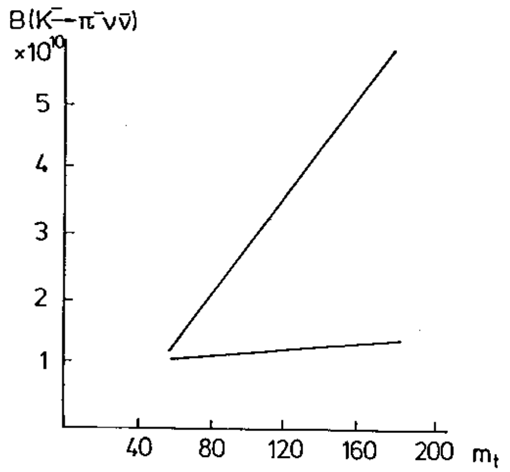

Figure 11: Expected value of $B(K^- \to \pi^- \nu\bar{\nu})$ as a function of m_t.

parameters – and using the central values for m_c and A – a straightforward analysis (DL, RDP) gives the wedge shaped prediction for the branching ratio $B(K^- \to \pi^- \nu\bar{\nu})$ shown in Fig. 11, plotted as a function of m_t. The upper values for this branching ratio are obtained for large values of ρ and δ, while the lower line in Fig. 11 corresponds to small values for ρ and δ. Given a value for m_t, a measurement of this branching ratio can provide useful information on the CKM parameters ρ and δ. This requires, however, that one should measure a rate which is, typically, a few times 10^{-10} that of the main K^- decay mode – a tough experimental challenge!

Although, as Fig. 11 shows, the predictions of the three generation model for $B(K \to \pi\nu\bar{\nu})$ lie around 10^{-10}, these expectations can be considerably changed if there are more generations, or if there are additional interactions – like those caused by a charged Higgs boson.

The existence of a fourth generation can dramatically alter the three generation standard model predictions for the branching ratios of rare decays. Relaxed unitarity constraints on the 3×3 CKM matrix yield wider ranges of allowed values for some of the 4×4 CKM matrix elements, V_{ij}, which need no longer scale according to the "natural" hierarchy of Eq. (1). This, combined with the large span of possible fourth family fermion masses (as well as the uncertainty in the t-quark mass, m_t), is responsible for the extension of the SM branching ratios.

One of us (JH) has examined the rare process $K^{\pm} \to \pi^{\pm}\nu\bar{\nu}$ (along with $b \to s\gamma$ and $b \to se^+e^-$) in the four generation model. Apart from Penguin-type diagrams, the exchange of the fourth family up-type quark, t', in the process $K \to \pi\nu\bar{\nu}$ also receives a contribution from the fourth generation charged lepton, L, in the box diagrams. In the calculation, the unknown fermion masses were taken in the ranges

$$\begin{aligned} 45 < \; & m_t \; < 180 \text{ GeV} \\ 100 < \; & m_{t'} \; < 400 \text{ GeV} \\ 50 < \; & m_L \; < 300 \text{ GeV}, \end{aligned} \tag{19}$$

subject to $m_{t'} > m_t$. In the spirit of Ref. [29], an extensive numerical search was performed

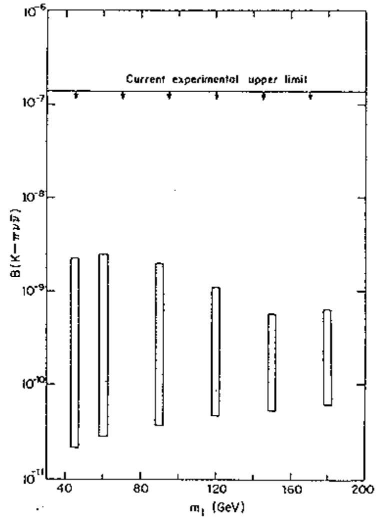

Figure 12: $B(K^- \to \pi^- \nu\bar{\nu})$ predictions with four generations (hollow boxes) as a function of m_t.

for given pairs of t and t' masses over the six mixing angles and three phases in the 4×4 CKM matrix and constrained to fit the existing experimental data discussed in Sec. 1. Using the set of angles which pass these constraints, the ranges of the branching ratios for each process, including QCD corrections, were calculated.

The branching ratio $B(K^- \to \pi^- \nu\bar{\nu})$ within the four generation model is displayed in Fig. 12. The hollow rectangles correspond to the allowed range of branching ratios in the four family case. It is clear from the figure that the addition of a fourth family can greatly extend the branching ratio range, and can either enhance or suppress the branching ratio compared to the three generation standard model value.

This same comment applies if one extends the Higgs sector of the standard model to two doublets, so that one has also physical charged Higgs bosons in the theory. For **fixed** CKM matrix elements V_{ij}, the presence of charged Higgses always **adds** to the standard model rate for $K^- \to \pi^- \nu\bar{\nu}$. However, having charged Higgses in the theory changes the constraints on the V_{ij}. In particular, one now has a larger allowed range for V_{td} and this can change the $K^- \to \pi^- \nu\bar{\nu}$ branching ratio, and may in fact allow values below those predicted by the standard model. An example is provided by Fig. 13, obtained by two of us (CG, JNN), which shows this branching ratio for various values of m_t, as a function of the charged Higgs mass, for fixed[10] $V_{td} = .01$. Obviously, going beyond the standard model renders any analysis much more uncertain, since it involves many extra unknown parameters!

The last Kaon decay which is interesting to consider, even though it is very difficult to attempt to measure, is the process $K_L^0 \to \pi^0 \nu\bar{\nu}$. This decay rate is closely related to that for $K_L^0 \to \pi^0 e^+ e^-$. However, in contrast to this case – as has been realized recently by Littenberg [30] – the direct CP violating amplitude really dominates over the indirect amplitude. Furthermore, there is no two photon CP conserving contribution. Hence, this process provides a very clean (albeit rare!) test for the CKM paradigm.

[10]This matrix element can remain fixed at this value, even as m_t varies, since x_d has an additional charged Higgs contribution.

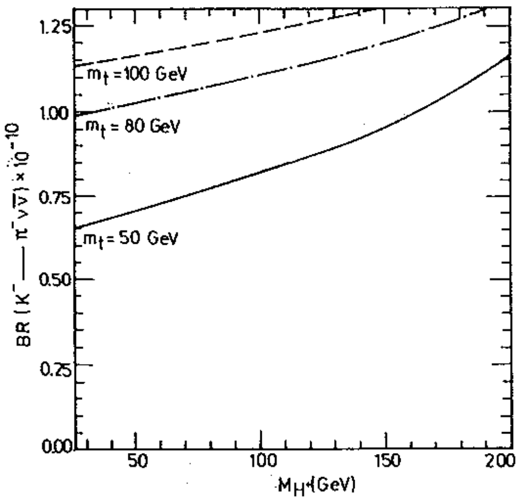

Figure 13: $B(K^- \to \pi^- \nu\bar{\nu})$ predictions in a model with charged Higgses as a function of m_{H^+}, and various values of m_t, with $V_{td} = .01$.

That the indirect CP violating amplitude is very small can be gathered by an estimate completely analogous to that in Eq. (12). Instead of having the branching ratio $B(K^- \to \pi^- e^+ e^-) \sim 3 \times 10^{-7}$, the estimate of the indirect CP violating amplitude here involves $B(K^- \to \pi^- \nu\bar{\nu}) \sim 3 \times 10^{-10}$, which is three orders of magnitude smaller. The direct CP violating amplitude, on the other hand, involves the imaginary part of the amplitude entering in Eq. (17). Typically this imaginary part is not very much smaller than the real part. Thus in this case, indeed, the direct CP violating amplitude dominates.

The branching ratio for the decay $K_L^0 \to \pi^0 \nu\bar{\nu}$, assuming three generations and neglecting QCD corrections, is given by

$$\begin{aligned} B(K_L^0 \to \pi^0 \nu\bar{\nu}) &= 1.8 \times 10^{-6} F^2(y_t) A^4 \lambda^8 \rho^2 \sin^2\delta \\ &\simeq 1.2 \times 10^{-11} F^2(y_t) \rho^2 \sin^2\delta \ , \end{aligned} \tag{20}$$

where the second line uses the central value for A. Since $F(y_t) \sim O(1)$ and the allowed region for ρ and $\sin\delta$ permits these parameters also to be of order unity, this branching ratio could be as large as 10^{-11}. However, taking the whole range of allowed values for ρ and δ, along with the correlated values for m_t, considerably smaller branching ratios can also ensue. We find (DL,RDP)

$$4 \times 10^{-13} < B(K_L^0 \to \pi^0 \nu\bar{\nu}) < 2 \times 10^{-11} \ , \tag{21}$$

where the lower limit is essentially m_t independent.

3 Some (non CP violating) B Physics

If one wants to look for deviations from the CKM paradigm in **rate**, one should select processes in which, if at all possible:

1. The standard model prediction for the decay rate is large,

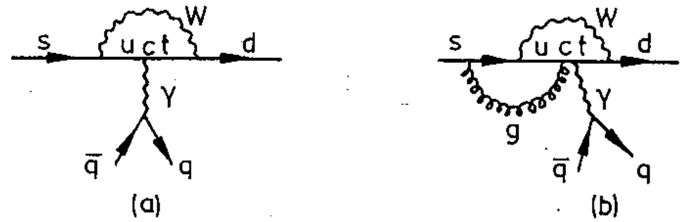

Figure 14: (a) Lowest order electromagnetic Penguin contribution to $b \to s\gamma$; (b) Gluonic correction to the $b \to s\gamma$ process.

2. Exotic (new physics) contributions can give sizable effects,

3. The standard model process, per se, is already interesting.

Clearly, it is helpful experimentally to be able to study channels in which the decay rate is not hopelessly small. Furthermore, it is imperative to select processes in which new physics effects could have a substantial impact. However, if these effects turn out not to be there – a normal occurrence! – it is nice if the process itself is interesting, even within the standard model. The decay $b \to s\gamma$, discussed below, fulfills all three of these criteria very nicely.

3.1 $b \to s\gamma$

The radiative decay of a b quark to an s quark proceeds through the electromagnetic penguin contributions shown in Fig. 14. In the three generation CKM model, there is not much uncertainty in the rate prediction connected with the electroweak sector, except for the value of m_t[11]. However, QCD corrections – of which a typical diagram is shown in Fig. 14b – substantially modify the rate [31]. Basically what happens is that, including gluonic effects, the GIM suppression factor of $(m_t^2 - m_c^2)/M_W^2$, coming from the lowest order diagram, becomes only logarithmic: $\alpha_s \ln(m_t^2/m_c^2)$. This produces quite a sizable difference in the rate, as can be appreciated from Fig. 15. So the decay $b \to s\gamma$ is theoretically interesting as a QCD laboratory.

The predicted branching fraction for this decay is quite large $[B(b \to s\gamma) \sim (2\text{-}4) \times 10^{-4}$, depending on $m_t]$. Unfortunately, one cannot directly access this inclusive process experimentally, but must look for some exclusive signal. This obviously reduces the rate and, more damagingly, introduces considerable theoretical uncertainty. Typically, one looks for

[11] The c and t quark loops dominate in Fig. 14 and V_{cb} and V_{ts} are reasonably well known, since they depend only on A and not ρ and δ.

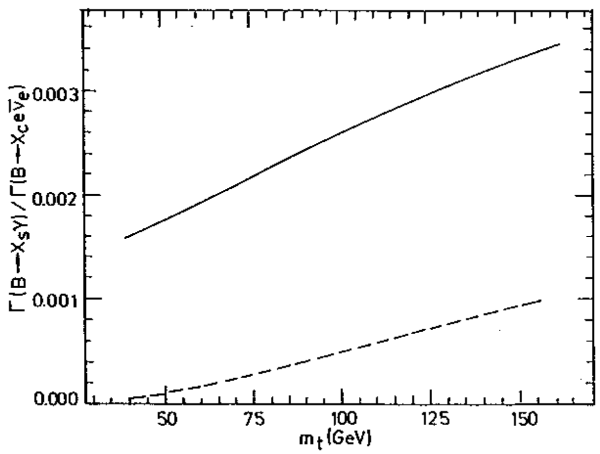

Figure 15: Ratio of the decay rates of $b \to s\gamma$ to $b \to ce\bar{\nu}_e$ with (solid line) and without (dotted line) QCD corrections, plotted as a function of m_t. From [32].

$B \to K^*\gamma$ and various estimates [31] suggest[12]

$$\Gamma(B \to K^*\gamma) \simeq (4.5 - 7) \times 10^{-2}\Gamma(B \to X_s\gamma) \ . \qquad (22)$$

Taking the above at face value, the present CLEO limit [34]

$$B(B \to K^*\gamma) < 1.7 \times 10^{-4} \qquad (23)$$

places a bound on the rate $b \to s\gamma$ which is about an order of magnitude away from the prediction of the standard model. This is already beginning to be interesting, since the expectations for $b \to s\gamma$ can be substantially modified, if there exists physics beyond the CKM paradigm. We shall discuss three different possibilities which were examined during the Snowmass Summer Study, involving charged Higgses (CG,JNN), fourth generation (JH), and supersymmetry (GB,AIS).

3.1.1 Charged Higgs

The minimal charged Higgs model arises when one introduces two doublet fields Φ_1 and Φ_2, coupled to right-handed charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks, respectively, rather than just Φ and its charge conjugate $\tilde{\Phi}$. The model has two additional parameters: the ratio of the two Higgs vacuum expectation values $\xi = v_2/v_1$, and the charged Higgs boson mass. Some restrictions on these parameters are imposed by B_d-$\overline{B_d}$ mixing, since now diagrams involving charged Higgses also contribute. Typically [35] ξ must be greater than unity and it grows as M_{H^+} grows, but decreases as m_t increases. However, the detailed value for ξ depends on assumptions one makes about V_{td}, and is influenced by the uncertainties on the hadronic matrix elements. The value of V_{td} used in our analysis ($V_{td} = .01$) is, quite naturally, smaller

[12] However, there exist in the literature [33] other estimates in which $B \to K^*\gamma$ makes up as much as 30% of all $b \to s\gamma$ decays!

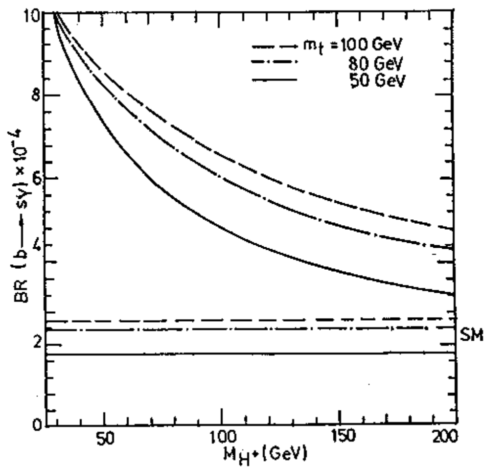

Figure 16: Branching ratio for the process $b \to s\gamma$ in a model with charged Higgses, plotted as a function of M_{H^+}, for various values of m_t. The standard model predictions are also included.

than that allowed by a global fit using the standard model only, as charged Higgses provide additional contributions to x_d.

The decay rate for $b \to s\gamma$ is always increased by the presence of charged Higgses. This is easy to understand since this rate is not influenced by V_{td}. Furthermore, corresponding to each graph in Fig. 14, there is now also one in which $W \leftrightarrow H$. Also for the charged Higgs diagrams QCD corrections, which have been computed by Grinstein and Wise [36], turn out to be important and they have been included in our analysis. In Fig. 16 we plot the branching ratio for the process $b \to s\gamma$, as a function of the charged Higgs mass, for various values of m_t, taking into account the restriction on ξ imposed by B_d-$\overline{B_d}$ mixing. Clearly a large enhancement over the standard model is obtained for small values of m_t and M_{H^+}. Indeed, recalling that the present CLEO limit [34] on $B \to K^*\gamma$ corresponds to, roughly, $B(b \to s\gamma) \simeq$ (2-4) $\times 10^{-3}$, a modest improvement in the experimental limit could be important for ruling out low mass charged Higgses.

3.1.2 4th Generation

The situation is not as clear if one imagines that there are four and not three generations. In this case, since unitarity does not restrict V_{ts} to be approximately equal to V_{cb}, the predictions for $b \to s\gamma$ can vary considerably from those of the standard model [37]. Performing the same type of analysis as that discussed in Sec. 2.2, one finds a rather large range allowed for the branching ratio $b \to s\gamma$, particularly for small values of m_t. The results are displayed in Fig. 17 and one notes that, in contrast to the charged Higgs case, here it is also possible to have a branching ratio below that predicted by the standard model. The even rarer decay $b \to se^+e^-$, which has a standard model branching ratio of the order of (4-10) $\times 10^{-6}$, depending on m_t, is perhaps an even better place to look for 4th generation effects [38]. In fact, typically, for the whole range of m_t, we find that with 4 generations this branching ratio

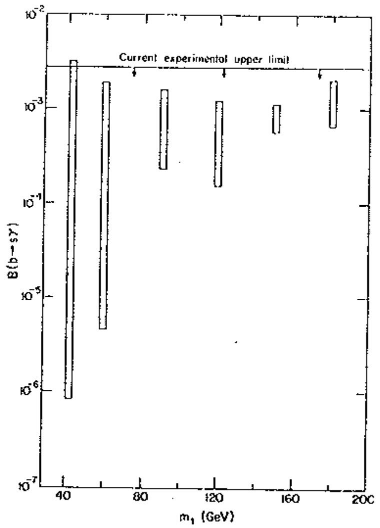

Figure 17: Predictions for $b \to s\gamma$ in a 4 generation CKM model.

can be as large as (2-4) $\times$ 10^{-4}.

3.1.3 SUSY

A supersymmetric extension of the standard model produces a situation which is akin to that of the case of 4 generations. Here also there are many unknown parameters and, in certain circumstances, one can expect a sizable enhancement over the standard model rate. A general discussion, which is valid for all parameter space, is difficult, and one is forced to make simplifying assumptions. In supersymmetric models the flavor changing $b \to s$ transition can occur through gluino exchange, since the quark-squark-gluino coupling is not flavor diagonal [39]. In the simplest scenario [40], this mixing is characterized by a parameter, A, detailing the amount of squark chiral mixing. Furthermore, the supersymmetric contributions to the rate for $b \to s\gamma$ vanish in the limit of degenerate squark masses. Hence the rate is sensitive to $\Delta\tilde{m}^2$. In a minimal model, one can approximate $\Delta\tilde{m}^2 \simeq Cm_t^2$, with C being a parameter of O(1). In addition to A, C, and m_t, the rate depends on the actual values one takes for the gluino mass and the central mass of the squarks.

Given this plethora of parameters, it is difficult to make strong assertions. Nevertheless, as Bertolini et al [40] have emphasized, light SUSY particles ($\tilde{m}$, $m_{\tilde{g}} \sim$ 40-60 GeV) can give rather substantial effects, particularly if m_t is large[13]. This is illustrated in Fig. 18, which shows the excluded region in $\tilde{m}$-$m_{\tilde{g}}$ space for $B(b \to s\gamma) = 2 \times 10^{-3}$, for various values of m_t. If a signal for $b \to s\gamma$ is seen around this level, which is an order of magnitude above the standard model predictions, a supersymmetric explanation may well be in order. However, the reverse assertion is probably not warranted. Given the many unknowns, one should not use present experimental limits on $b \to s\gamma$ to argue for excluding certain regions in the squark-gluino mass parameter space.

[13]This is perhaps not surprising, since $\Gamma_{SUSY}(b \to s\gamma) \sim (\Delta\tilde{m}^2)^2$, and so grows like m_t^4.

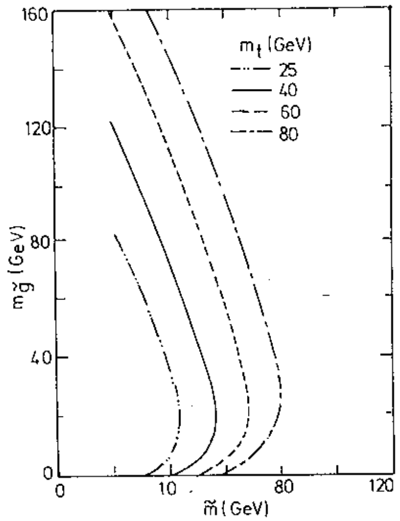

Figure 18: Excluded region (to the left) of values for $\tilde{m}$ and $m_{\tilde{g}}$, for different m_t, assuming $B(b \to s\gamma) = 2 \times 10^{-3}$ and $A = 3$, $C = 0.5$. From [40].

3.2 B_s-$\overline{B}_s$ Mixing

A second topic in B physics which was studied at Snowmass was the prospects for determining the mixing parameter $x_s = \frac{(\Delta m)_{B_s}}{\Gamma_{B_s}}$ for the B_s system. An accurate determination of x_s is of considerable interest, as it would severely constrain the allowed values for ρ and δ. The mixing parameters x_s and x_d arise theoretically from identical box graphs, save for the $s \leftrightarrow d$ interchange. Hence one has, simply

$$x_s = x_d \left|\frac{V_{td}}{V_{ts}}\right|^2 \left[\frac{\tau_{B_s} f_{B_s}^2 B_{B_s}}{\tau_{B_d} f_{B_d}^2 B_{B_d}}\right]. \qquad (24)$$

The factor in the square brackets in (24) is expected to be close to one, since $\tau_{B_s} \simeq \tau_{B_d}$[14] and the ratio of hadronic matrix elements can differ from unity only through $SU(3)$ breaking effects. Actually these latter effects are difficult to compute and some caution is in order, since some preliminary attempts to calculate this ratio on the lattice [41] indicate corrections as large as 50%. At any rate, we shall assume in what follows that the square bracket in Eq. (24) is unity and hope that, by the time x_s is measured, one will have a more reliable theoretical estimate for this factor. Thus

$$x_s \simeq \frac{x_d}{\lambda^2|1 - \rho e^{i\delta}|^2} \simeq \frac{14.4}{|1 - \rho e^{i\delta}|^2}, \qquad (25)$$

where for the numerical value above we have used $x_d = 0.7$, which is the central value of the combined ARGUS [42] and CLEO [43] measurements.

It is clear from Eq. (25) and from the allowed range for ρ and δ given in Fig. 4, that one expects rather large values for x_s in the CKM model ($x_s \gtrsim 4$). This, unfortunately, makes life more difficult rather than easier! In a time integrated measurement, like that performed

[14]For inclusive decays of B mesons the spectator model, which implies the equality of the B_s and B_d lifetimes, is supposed to work very well.

for B_d at the $\Upsilon(4S)$, what is measured is not the mixing parameter but the ratio r_s of the rates for the B_s mesons to decay to leptons of opposite charge. Neglecting the contribution due to a possible lifetime difference between the two neutral B_s eigenstates, one has that

$$r_s = \frac{\Gamma(B_s \to l^- X)}{\Gamma(B_s \to l^+ X)} \simeq \frac{x_s^2}{2 + x_s^2} . \tag{26}$$

Thus if indeed $x_s \geq 4$, then $r_s \geq 0.89$ and one needs a measurement which is extremely accurate to extract x_s. Furthermore, at the $\Upsilon(5S)$ the situation is much more complicated, since the produced leptons can arise not only from B_s (or $\overline{B_s}$) decays, but also from produced B_d's, B_d^*'s and charged B's. For an accurate measurement, therefore, one needs to be able to tag, somehow, the B_s meson sample.

Two ideas have been studied for measuring x_s. In the first method, one works at the $\Upsilon(5S)$ using a time integrated method, but enriches the B_s signal by tagging on D_s mesons. In the second approach, one tries to take advantage of the fast time oscillations of the B_s signal to extract x_s. Both these techniques are examined in detail in [44] and they have also been amply discussed by some of our experimental colleagues [45,46]. Thus we shall concentrate here only on some highlights of both methods.

3.2.1 Time Integrated Techniques

The idea here is to work at the $\Upsilon(5S)$ and hope that the expectations of potential models [47] that the $B_s\overline{B_s}$ pairs are dominantly produced in a given angular momentum state are correct[15]. This is necessary since, if both angular momentum states are produced, what is measured is not r_s but the incoherent ratio [44]

$$R_s^{\rm inc} \simeq \frac{2r_s}{1+r_s^2} \overset{x_s \text{ large}}{\simeq} 1 - \frac{2}{x_s^4} , \tag{27}$$

which is even more insensitive to x_s. To proceed at all the B_s sample needs to be enriched with respect to the B_d sample. This is most efficiently done by requiring an additional D_s to be present (arising from the decay $B_s \to D_s l X_s$) so that the signal one is looking for is $\Upsilon(5S) \to D_s l^+ l^- X$. The small contamination of B_d remaining, arising from the decay $B_d \to D_s l X_d$, can be further purified by an invariant mass cut [48]. The state X_d necessarily must involve a Kaon and a neutrino, while X_s is dominantly just a neutrino. Thus requiring that $m_X^2 = 0$ is a useful cut to make.

Typically [47] the ratio of B_s to B_d production at the $\Upsilon(5S)$ is around 10-30%. According to the analysis of [44], requiring a D_s in the final state increases this ratio by a factor of about 35, while the neutrino mass cut provides another factor of 5 enhancement. Even with this purified sample one can reduce the systematic uncertainty produced in r_s by misidentified B_d's only to about 6%[16]. A matching statistical error, so that $\delta r_s \sim 8\%$, calls for 800 $\Upsilon(5S) \to l^+ l^- D_s X$ decays. This in turn, according to the estimates of Krawczyk et al [44], requires at least 2×10^8 $b\bar{b}$ pairs to be produced. Thus this method, although interesting, does not look too promising, unless one can somehow or other reduce the statistical onus.

[15] It may be necessary to operate a little below the $\Upsilon(5S)$ in energy, if one wants to eliminate kinematically $B_s^*\overline{B_s^*}$ production

[16] Some of the factors entering in this estimate also take into account uncertainties in the expected initial ratio of B_s to B_d and the production of charged B's.

3.2.2 Time Dependent Methods

The situation looks quite a bit better if one can study the time development of the produced leptons from B_s decays. The number of l^+'s produced by a decaying sample of B_s at rest is a modulated exponential

$$N_s^+(t) = N^+(0) e^{-\Gamma t} \left[1 + \cos \frac{(\Delta m)_s t}{2} \right] . \tag{28}$$

For large x_s, the oscillating factor above will vary numerous times in an e-fold. Since $x_d \simeq 0.7$, on the other hand, a decaying sample of B_d's will have a much milder undulation. Hence, as far as the time development goes, the presence of some B_d contamination is not so damaging, since it will only change the magnitude of the oscillating B_s signal.

The time development of B_s's can be studied in a hadronic collider or a Z factory. However, the necessity of tagging to determine whether one is dealing with an initial B_s or $\overline{B_s}$, combined with the needed precision in energy and position measurements, require again more than 10^8 $b\bar{b}$ pairs. However, with this method values of x_s up to 15 are accessible [44]. A much more promising avenue is afforded by having an asymmetric e^+e^- collider operating at the $\Upsilon(5S)$. In this case, one can measure the distribution in the proper time difference between the leptons produced by the decay of the two B mesons by a distance measurement. These distributions, assuming again that the $B_s\overline{B_s}$ mesons are produced in an even angular momentum state $[B_s\overline{B_s^*}]$, depend directly on x_s. One has [44]

$$N(l^+l^-;\Delta t) = \frac{N_0}{2(1+x_s^2)} e^{-\frac{\Delta t}{\tau}} \left\{ 2\cos^2(\frac{x_s \Delta t}{2\tau}) + x_s^2 - x_s \sin(\frac{x_s \Delta t}{\tau}) \right\} , \tag{29}$$

$$N(l^+l^-;\Delta t) = \frac{N_0}{2(1+x_s^2)} e^{-\frac{\Delta t}{\tau}} \left\{ 2\sin^2(\frac{x_s \Delta t}{2\tau}) + x_s^2 + x_s \sin(\frac{x_s \Delta t}{\tau}) \right\} . \tag{30}$$

Although both the same sign and opposite sign dilepton distributions have the same type of oscillatory behaviour, experimentally the same sign distribution suffers much less from backgrounds produced by $B^\pm$ and B_d decays. Fig. 19 from [44] shows the expected distribution of same sign dileptons as a function of the proper time difference for different values of x_s. In the figure the ratio of B_s to B_d produced at the $\Upsilon(5S)$ was that of [47]. Furthermore a mistagging probability of 5% and a vertex resolution of 25μm [corresponding to $\Delta t/\tau = 0.1$] were assumed. As can be seen the different x_s patterns are quite visible. This is to be contrasted with the opposite sign pattern shown in Fig. 20, computed with the same parameters. Krawczyk et al [44] conclude that a measurement of x_s up to about 10 is feasible, under the above assumed conditions. Since the detection of a D_s meson is not necessary, this measurement can be done with considerably fewer $b\bar{b}$ pairs, approximately 5×10^6. However, one should emphasize that the range of x_s accessible is quite dependent on the assumed production rates for B_s versus B_d. Furthermore, a factor of two worsening of the vertex resolution washes out the signal. Nevertheless, this appears the most promising way to try to measure x_s.

4 CP Violation in the B System

Observing CP violation in B decays has become the holy grail of the subject. This is understandable since, within the CKM paradigm, there exist quite definite predictions which

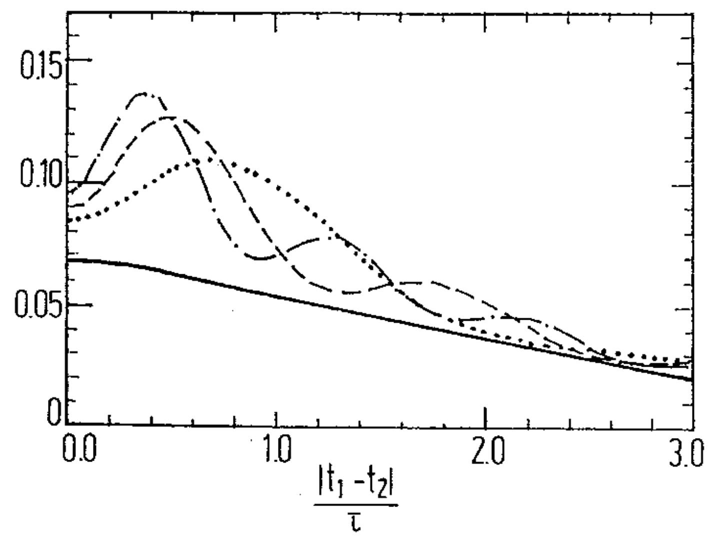

Figure 19: Expected distribution of same sign dileptons as a function of the proper time difference, under the conditions discussed in the text, for $x_s = 0$ (solid); 3 (dotted); 5 (dashed) and 7 (dot-dashed).

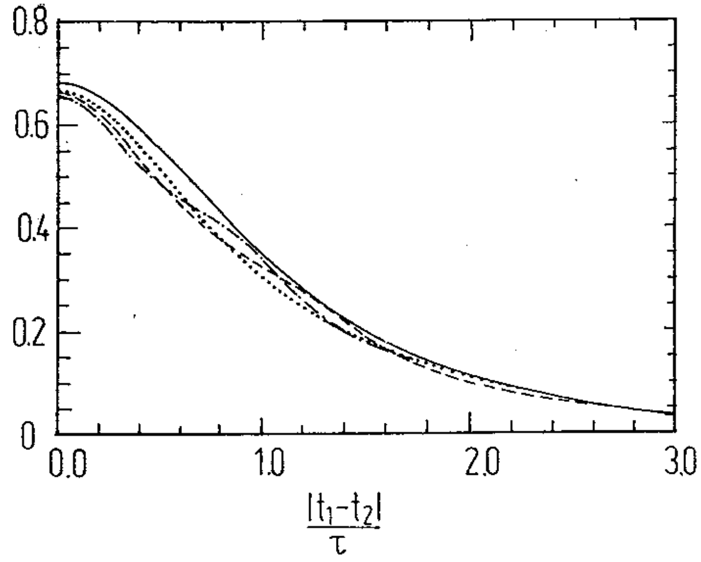

Figure 20: As in Fig. 19, except for opposite sign dileptons.

should - hopefully in the 1990's - either be proven or falsified. Unfortunately, this is a non-trivial endeavour, since it will require studying of the order of 10^8 $b\bar{b}$ pairs. This is a subject which has also been discussed rather extensively in the past [49]. Thus our discussion here, rather than being encyclopedic, will focus on a few points of interest which arose during the summer study.

Roughly speaking, there exist three categories of CP violating processes in the B system.

- Processes where, in the standard model, one expects rather small CP violating effects;
- Processes where the CP violating effects are intrinsically intertwined with dynamics;
- Processes where there may well be rather large, and substantially model independent, CP violating signals.

Each of these classes of CP violation is interesting and worth commenting upon, although, obviously, the last class is the one which holds the most promise.

An example of the first kind of CP violation is provided by the lepton charge asymmetry [50], which measures the difference in the number of same sign dileptons, arising from the production of $B\bar{B}$ pairs:

$$a = \frac{N(l^+l^+) - N(l^-l^-)}{N(l^+l^+) + N(l^-l^-)} = \frac{r - \bar{r}}{r + \bar{r}} . \tag{31}$$

In the standard model r and $\bar{r}$, and therefore the number of same sign dileptons, can be large (e.g. $r_s \simeq 1$). However, unfortunately, the difference between r and $\bar{r}$ is very small, leading to predictions for a_d and a_s in the range 10^{-3}-10^{-4} [51]. Obviously, such an asymmetry requires an enormous number of $B\bar{B}$ pairs (of order 10^9-10^{10}), for a statistically significant measurement. Thus these asymmetries, at least for the 1990's, are experimentally completely out of reach. Nevertheless, they should be looked for, since the observation of any signal in the foreseeable future will mean that $a \gg 10^{-3}$. Although this would constitute a prima facie case for new physics, the overwhelming probability is that nothing will be found at all!

Processes where the observable CP violation might be large, but where the signal depends in detail on the hadronic dynamics, are less useful for testing the CKM paradigm. Nevertheless, these processes may very well be interesting to study for their own sake, because of the intricate interplay between the strong and weak dynamics. An example is provided by the decays of charged B's into CP conjugate states [52]. The ratio $\Gamma(B^- \to f)$ and $\Gamma(B^+ \to \bar{f})$ can differ from each other if there is an interference between two amplitudes which have different CP violating phases and are subject to different strong final state interactions. Then

$$\Gamma(B^- \to f) - \Gamma(B^+ \to \bar{f}) \sim \sin(\delta_1 - \delta_2)_{\rm weak} \sin(\delta_1 - \delta_2)_{\rm strong} . \tag{32}$$

In Snowmass some of us (BK,MK,RDP,AIS) studied a different example of this sort, involving the measurement of some final state triple correlation. Strictly speaking, the effect studied is really a T violating effect, rather than a CP violating effect, but these are equivalent through the CPT theorem.

For B decay processes, where the weak decay rate is well approximated by its lowest order expression, it is easy to show [53] using a combination of CP and T invariance that

$$\Gamma[B \to a_1(\vec{p}_1, \vec{s}_1) + ... + a_n(\vec{p}_n, \vec{s}_n)] = \Gamma[B \to a_1(-\vec{p}_1, -\vec{s}_1) + ... + a_n(-\vec{p}_n, -\vec{s}_n)] , \tag{33}$$

where $a_i(\vec{p}_i, \vec{s}_i)$ denotes a daughter particle of given momentum $\vec{p}_i$ and spin $\vec{s}_i$. From this form, we see that the observation in a given B decay, of any triple correlation of the type $\vec{a} \cdot \vec{b} \times \vec{c}$, where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are each the momentum or spin of some daughter particle, would be a signal of CP violation. We should caution, however, that this conclusion ceases to be true in the presence of strong interaction rescattering phases. So in practice, it is necessary to be able to estimate the strong interaction contribution, so that it can be subtracted from an observed signal to reveal the interesting CP violating part.

There are many candidates for decay modes which involve kinematically nontrivial CP violation in the B system - including examples in which one compares particle-antiparticle rates, like $B^{\pm} \to K^{*\pm}\Psi$, in different helicity states [53]. Here we want to illustrate some of the challenges involved in analyzing these effects by discussing two examples briefly: $\Lambda_b \to \Lambda D^0, \Lambda\overline{D^0}$, with $D^0, \overline{D^0} \to K_s\pi^0$, and $\Lambda_b \to \Lambda\pi^0$. Clearly, in both cases the presence of a term like $\vec{s}_{\Lambda_b} \cdot \vec{s}_\Lambda \times \vec{p}_\Lambda$ in the rate would be a possible signal for CP violation effects. Naively, in each case, it appears that this may be possible theoretically, since two, relatively complex, amplitudes contribute. [For the first process $b \to c(\overline{u}s) \sim \lambda^3$ and $b \to u(\overline{c}s) \sim \lambda^3 e^{-i\delta}$; for the second process $b \to u(\overline{u}s) \sim \lambda^4 e^{-i\delta}$ and the Penguin contribution $b \to s(\overline{u}u) \sim \lambda^2\alpha_s$.] Nevertheless, only in the second example does a nontrivial triple correlation appear. In the first case, at least in the simple dynamical approximation considered, although one has two terms with different phases, they contribute equally to the parity conserving and parity violating amplitudes. Thus, the amplitudes for the two subprocesses are kinematically identical and no **kinematically nontrivial** effect, such as the triple correlation, can arise from this interference. It appears that it is always necessary (but not always sufficient) to have two kinematically distinct amplitudes at the quark level for a triple correlation to be nonvanishing.

4.1 Hadronic Asymmetries and CP Violation

To observe CP violating processes requires that there should be interference between two amplitudes with different phases. This can come about in the B system through B-$\overline{B}$ mixing. Because of this mixing, a state which at $t = 0$ was a pure B meson can evolve in time into a mixture of B and $\overline{B}$ mesons. The physical state at time t is easily seen to be [54]

$$|B_{\rm phys}(t)\rangle = f_+(t)|B\rangle + f_-(t)\eta_M e^{i\phi_M}|\overline{B}\rangle \ , \tag{34}$$

where

$$\begin{aligned} f_+(t) &= e^{-imt}e^{-\frac{\Gamma}{2}t}\cos\frac{\Delta mt}{2} \\ f_-(t) &= ie^{-imt}e^{-\frac{\Gamma}{2}t}\sin\frac{\Delta mt}{2} \ . \end{aligned} \tag{35}$$

In the above $\eta_M e^{i\phi_M}$ is related to the ratio of the off diagonal mass and decay width matrix elements of the B-$\overline{B}$ system

$$\eta_M e^{i\phi_M} = \left[\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}\right]^{1/2} \ . \tag{36}$$

Since $\Gamma_{12} \ll M_{12}$, the magnitude η_M is essentially unity. The phase ϕ_M is convention dependent. However, in the quark phase convention we shall adopt, it is just minus twice the phase

of the $\Delta B = 2$ mass matrix, calculated in terms of the usual quark box graphs. Since these graphs are dominated by the t-quark contribution [55] one has simply

$$\phi_M \simeq \begin{cases} -2\phi & B = B_d, \\ 0 & B = B_s. \end{cases} \tag{37}$$

In the above $-\phi$ is the phase of V_{td} [see Fig. 3], and we have used the fact that V_{ts} is approximately real.

A CP violating asymmetry can be measured in decays of B and $\overline{B}$ mesons to a common hadronic final state f. If CP is not conserved, the decay probability of a state $B_{\rm phys}$, which at $t = 0$ was a pure B meson, into f need not agree with the decay probability of $\overline{B}_{\rm phys}$, a state which at $t = 0$ was a pure $\overline{B}$ meson, into $\overline{f}$, the CP conjugate state of f. The interesting asymmetry

$$A_f(t) = \frac{\Gamma(B_{\rm phys}(t) \to f) - \Gamma(\overline{B}_{\rm phys}(t) \to \overline{f})}{\Gamma(B_{\rm phys}(t) \to f) + \Gamma(\overline{B}_{\rm phys}(t) \to \overline{f})} \tag{38}$$

depends both on $\eta_M e^{i\phi_M} \simeq e^{i\phi_M}$ and on the dynamics contained in the ratio

$$\rho_f = \frac{A(\overline{B} \to f)}{A(B \to f)} \ . \tag{39}$$

A very interesting circumstance occurs [54] if f is a **CP eigenstate** [$f = \pm\overline{f}$] and the decays $B \to f$, $\overline{B} \to f$ are dominated by **one** amplitude. In this case $|\rho_f| \simeq 1$ and all the remaining dynamical information is contained in its phase[17]

$$\rho_f \simeq e^{i\phi_D} \ . \tag{40}$$

Since $V_{ub} = |V_{ub}|e^{-i\delta}$ and V_{cb} is real, neglecting Penguin effects to which we shall return later, one sees that, in these circumstances,

$$\phi_D \simeq \begin{cases} 0 & \text{Cabibbo allowed decays } (b \to c), \\ -2\delta & \text{Cabbibo suppressed decays } (b \to u). \end{cases} \tag{41}$$

The rate for B decay to charge conjugate states, which obey the above mentioned dynamical circumstances, read simply [54]:

$$\begin{aligned} \Gamma(B_{\rm phys}(t) \to f) &= e^{-\Gamma t}[1 + \alpha_f \sin\Delta mt] \\ \Gamma(\overline{B}_{\rm phys}(t) \to f) &= e^{-\Gamma t}[1 - \alpha_f \sin\Delta mt] \ , \end{aligned} \tag{42}$$

where

$$\alpha_f = -{\rm Im}\rho_f\eta_M e^{i\phi_M} \simeq -\sin(\phi_M + \phi_D) \ . \tag{43}$$

Using Eqs (37) and (41), we see that there are four classes of decays [15,56] which measure different combinations of the phases entering in the CKM matrix and in the unitarity triangle of Fig. 3:

$$\begin{aligned} \text{Cabibbo allowed } B_d \text{ decays [e.g. } B_d \to \Psi K_s] &: \quad \alpha_f \simeq \sin 2\phi \\ \text{Cabibbo allowed } B_s \text{ decays [e.g. } B_s \to \Psi\varphi] &: \quad \alpha_f \simeq 0 \\ \text{Cabibbo suppressed } B_d \text{ decays [e.g. } B_d \to \pi^+\pi^-] &: \quad \alpha_f \simeq \sin(2\phi + 2\delta) \\ \text{Cabibbo suppressed } B_s \text{ decays [e.g. } B_s \to K_s\pi^0] &: \quad \alpha_f \simeq \sin 2\delta \ . \end{aligned} \tag{44}$$

[17] There is in general a $\pm$ sign, associated with the CP properties of f, which we have dropped in Eq. (40).

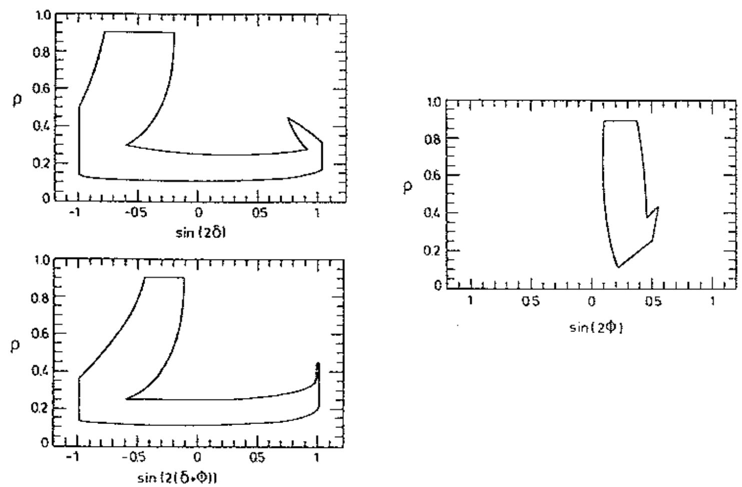

Figure 21: Allowed region for $\sin 2\phi$, $\sin 2(\phi+\delta)$ and $\sin 2\delta$, as a function of ρ, corresponding to the allowed region in ρ-δ space given in Fig. 4. From [15].

The measurement of the various types of CP violating asymmetries of Eq. (38) would provide the most stringent test of the CKM paradigm. Ideally, one could check the unitarity of the CKM matrix experimentally by proving that the sum of the angles in the unitarity triange – related to the different α_f's – indeed add up to 180°! Lacking such experimental input at the moment, one can at least see what values for the different α_f's are allowed by our present knowledge of ρ and δ. Fig. 21, taken from [15], shows the different allowed regions for $\sin 2\phi$, $\sin 2\delta$ and $\sin(2\phi + 2\delta)$, as a function of ρ, corresponding to the allowed region in ρ-δ space of Fig. 4. One sees that for $\rho > 0.3$, the values permitted for the various α_f's are largely ρ independent and can be quite large in magnitude.

Although the hadronic asymmetries in B decay can be substantial, one needs to check theoretically that the assumption of having a dominant weak amplitude, and thus a well defined weak phase for ρ_f, really holds for the decays of interest.. Furthermore, it is important to try to devise some sensible experimental strategy for measuring these asymmetries. Concerning the first point, two of us (DL,RDP) have looked at the possible influences that Penguin diagrams can have on the predictions of the hadronic asymmetries, Eq. (44). Roughly speaking, the situation can be summarized as follows [57]. Since the momentum transfer in B decays is typically greater than, or of order of, the charm mass, effectively, after making use of the CKM unitarity condition, only the t-quark loop contributes to the Penguin amplitude. Hence the $b \to s$ Penguin is approximately real, while the $b \to d$ Penguin has the phase of V_{td}^*. These simple observations lead to a number of consequences:

1. The predictions for the decay process $B_d \to \Psi K_s$, which is particularly interesting experimentally, are quite safe. The $b \to s$ Penguin contribution is real, as is the main $b \to c\bar{c}s$ quark decay amplitude. Thus the asymmetry associated with this decay should measure purely the mixing phase ϕ_M and $\alpha_f = \sin 2\phi$, without any corrections.

2. In decays like $B_d \to \pi^+\pi^-$, or $B_s \to K_s\pi^0$, where the $b \to d$ Penguins may be important, the predictions for the corresponding α_f's do change. However, the corrections to Eq. (44) are proportional to the relative strength of the Penguin amplitude to the pure quark decay amplitude, which is typically of $O(\frac{\alpha_s}{12\pi}\ln\frac{m_t^2}{m_c^2})$, and may well be small. In decays like $B_d \to K_s\pi^0$ or $B_s \to K^+K^-$, which are very Cabibbo suppressed at the quark level, the Penguin effects, which are Cabibbo enhanced [58], may change the answer for α_f totally.

3. There are decays which are purely Penguin dominated, where one could also expect interesting hadronic asymmetries. An interesting example is provided by $B_d \to \phi K_s$, which proceeds through the $b \to s$ Penguin. Since this graph is approximately real, one expects that also $\alpha_{\phi K_s} = \sin 2\phi$, just as was the case for the decay $B_d \to \Psi K_s$.

We shall only make two general remarks on the experimental strategy to measure CP violation in the B system, as this subject is discussed in much more detail in the reports of our experimental colleagues in these Proceedings [46,59]. One of these comments concerns the question of time integrated versus time dependent asymmetry measurements. Time integrated measurements are only sensible for B_d decays, since one is not punished unduly by the mixing parameter x. Integrating Eq. (38) over time, using (42), gives for the time integrated asymmetry

$$\langle A_f \rangle = \frac{x}{1+x^2}\alpha_f \,. \qquad (45)$$

For the B_d system, the mixing parameter x_d is such that the mixing factor in front of α_f above is essentially maximal, $\frac{x_d}{1+x_d^2} \simeq 0.47$. For the B_s system, on the other hand, since x_s is large in the CKM model, the relevant time integrated asymmetry can be much reduced. In this case, it is probably better to try to follow the time development of the decay amplitude. Indeed, the considerations needed for measuring the CP asymmetry are quite similar to those we discussed in the case of B_s-$\overline{B}_s$ mixing.

Our second comment concerns the question of tagging. To measure the hadronic asymmetry (38) one needs to know how a decaying B meson was born. That is, was the state at $t = 0$ of this meson a B^0 or a $\overline{B^0}$? There is no universal answer on how to tag. Rather, this is a machine dependent question. For instance, a leptonic tag – with a concomitant signal dilution – may be sensible at a hadronic collider. In the case of an asymmetric e^+e^- machine, operating at the $\Upsilon(4S)$, because of the p-wave wave function of the produced B_d's one really needs no tag [60]. Here the first decay, because of Bose statistics, essentially serves to tag the second. Finally, for a linearly polarized collider operating at the Z^0, one can tag the initial B by a forward-backward tag [61], since particles originating from a b quark will preferentially go along the direction of the incoming electron.

Some of us in the summer study (BK,SM,AIS) have investigated some aspects of this last proposed method for tagging. In particular, there was some concern on whether hadronization corrections could affect the quark level formulas for the asymmetry and whether the formula for the asymmetry in [61]

$$A^f_{\rm meas} = \frac{N(f,\cos\theta) - N(\bar{f},-\cos\theta)}{N(f,\cos\theta) + N(\bar{f},-\cos\theta)} = A_f A_{FB} \qquad (46)$$

itself had any corrections. Here $N(f,\cos\theta)$ is the number of decays into the final state f in the forward direction, while $N(\bar{f},-\cos\theta)$ is the number of decays to the charge conjugate state, $\bar{f}$, in the backward direction. Finally, A_{FB} is the quark level forward-backward asymmetry.

Essentially, the answer to the first question is no, even though in the case of some two body decays – like $Z^0 \to B\overline{B}$ – there is no memory of the quark parent. The point is that the decays of $Z^0 \to$ jet + jet predominate, and for these decays the correlation with the quark parent exists, as the successful measurement of the forward-backward asymmetry for b-quarks at PETRA [62] has demonstrated. As far as the second question goes, it turns out that there are corrections to Eq. (46). However, these corrections vanish if $|\rho_f| = 1$. Hence for the cases of most interest, these corrections are also not important.

5 Concluding Remarks

Our conclusions are quite simple. In weak decays, the task of the 1990's is to go and test the CKM paradigm. Here we will need the help of our experimental friends. However, there is plenty of work to do for theorists also, who should continue to refine the phenomenology and work hard to reduce the uncertainties associated with hadronic matrix elements.

It may be that, on the way to test the CKM model, we shall come upon new physics in the form of violent disagreements of experiment with standard model predictions and/or the discovery of flavor changing neutral currents processes. Although everybody should keep their eyes open for this kind of opportunity – and indeed it is useful to quantify the possible signals and predictions of theories beyond the standard model – we should admit that, in this realm, present theories are incapable of trustworthy predictions. Our hope is also that, in the 1990's, we shall be granted some illumination in this area!

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