DEUTSCHES ELEKTRONEN – SYNCHROTRON DESY

DESY 89-024
February 1989

How to avoid imperfection spin resonances in a proton ring with snakes

K. Steffen
Deutsches Elektronen-Synchrotron DESY, Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 89-024
February 1989

ISSN 0418-9833

How to avoid imperfection spin resonances in a proton ring with snakes

K. Steffen
Deutsches Elektronen-Synchrotron DESY, Hamburg

Abstract

The coupling of spin motion to the corrected closed orbit in a model ring with snakes is investigated as a confined walk problem, and a closed orbit relaxation scheme is proposed for avoiding imperfection resonances even at SSC energies. Specific orbit corrections for suppressing specific coupling modes are also suggested.

Contents

1	Introduction	p. 2
2	Coupling of the spin to the vertical closed orbit	p. 2
3	Effect of the snake configuration on spin orbit coupling	p. 3
4	Spin coupling to the corrected closed orbit as a confined walk problem	p. 4
5	Principle of closed orbit relaxation	p. 6
6	Closed orbit relaxation scheme	p. 7
7	Effect of dipole magnet misalignment	p. 9
8	Compensation of spin orbit coupling modes	p. 10
9	Conclusions for the SSC	p. 12

1. Introduction

The question of whether polarized proton beams can be accelerated to very high energies, say 20 TeV in the SSC, has recently become the focus of more intense studies. In a recent workshop on Siberian snakes for the SSC collider ring[1], it appeared that the intrinsic spin resonances caused by betatron oscillations can be avoided by inserting 15-23 snake pairs in a strongly spin matched pattern[2,3], but that, on the other hand, the imperfection resonances due to the errors remaining in a corrected vertical closed orbit are a subject of great concern. K. Yokoya[4] estimated that, to avoid these resonances, the remaining closed orbit amplitudes must be kept below 80 μm, a value almost impossible to achieve in practice.

Will this prevent the acceleration of polarized beams to 20 TeV? I believe it will not, and will try to display in this note the reasons for being optimistic. Employing a simplified ring model for the sake of clarity, I will calculate the coupling of the spin to a corrected closed orbit that is bound between limits, using a confined walk analysis, and will then show how this coupling can be significantly reduced by further straightening the orbit in a simple relaxation procedure. With realistic alignment tolerances, assuming careful engineering, the coupling in the SSC will then be well below the threshold for imperfection spin resonances. For added safety, I will recommend specific orbit corrections as a cure for those specific coupling modes that might still be strong after orbit relaxation.

2. Coupling of the spin to the vertical closed orbit

In linear approximation the rotation of the spin away from the periodic $\vec{n}$-axis due to a vertical closed orbit $z(s)$ is given by the spin orbit coupling integral

$$I(\sigma) \approx \gamma a \cdot \int_0^\sigma e^{i\psi(s)} \cdot z''(s) ds \tag{1}$$

where the real and imaginary parts represent the two orthogonal phases $\psi(s)$ of the spin precession, with

$$\psi(s) = \gamma a \cdot \int_0^s \frac{1}{\rho(s)} ds \quad \text{and} \quad \gamma a = \frac{E[GeV]}{0.52335} \tag{2}$$

$1/\rho$ is the strength of the main bending magnets in the ring.

Let us take a ring where all bending magnets are perfectly aligned in the horizontal plane and where a vertical correcting magnet is placed at each quadrupole. Then, due to the correctors and vertical quadrupole displacements, the vertical closed orbit will be kicked at the quadrupoles only and will be straight in between, and the coupling integrals (1) may be written

$$\begin{aligned} I_c &= \gamma a \textstyle\sum_{i=1}^{n} \cos\psi_i \cdot \Delta z_i' \\ I_s &= \gamma a \textstyle\sum_{i=1}^{n} \sin\psi_i \cdot \Delta z_i' \end{aligned} \qquad (3)$$

where the index i refers to the i^{th} quadrupole. Assuming a periodic ring structure with evenly spaced quadrupoles, we have between kicks a constant spin precession phase advance $\delta\psi$ which is proportional to energy and which, during acceleration in the SSC for example, rises from $\delta\psi = \approx 3 \cdot 2\pi$ at 1 TeV to $\delta\psi \approx 60 \cdot 2\pi$ at 20 TeV.

In this case the coupling integrals become:

$$\begin{aligned} I_c &= \gamma a \textstyle\sum_{i=1}^{n} \cos(i-1)\delta\psi \cdot \Delta z_i' \\ I_s &= \gamma a \textstyle\sum_{i=1}^{n} \sin(i-1)\delta\psi \cdot \Delta z_i' \end{aligned} \qquad (4)$$

To evaluate them, let us focus on a set of particular precession frequencies labeled by the index m, where the same fractional phase repeats after a sequence of m kicks, i.e.

$$\delta\psi = 2\pi \cdot \frac{p}{m}, \quad \text{p integer} \qquad (5)$$

Then, it immediately appears that for spin frequencies with m = 1 the coupling integrals (4) will vanish over one revolution since there is an integer number of spin precessions between kicks, and the kicks will add up coherently to zero due to the fact that the orbit is closed. This is, however, only the case if the ring does not contain more than one phase jump in spin precession, as induced by the Siberian snakes. The influence of the snake configuration will be reviewed in the next section.

For m = 2 and odd p, on the other hand, the integral will be the alternating sum of the kicks and we have the case of strongest coupling. In section 4, we shall therefore try to carefully evaluate this alternating sum by using an orbit model that takes fully into account the confined nature of the closed orbit after correction.

3. Effect of the snake configuration on spin orbit coupling

The general properties of snakes and snake configurations are given in ref. [2]. Each snake rotates the spin by 180° about the transverse horizontal axis, and subsequently by an angle α about the vertical axis. α is called the precession angle of the snake. Snakes composed of transverse bending magnets can be built for any value of α. Useful species, as chosen e.g. for the SSC snake configurations of ref. [3], are snakes with

$\alpha = \pm 180^\circ$ and $\alpha = \pm 90^\circ$, in contrast to snakes with $\alpha = 0^\circ$ which would be less compact. If, while approaching a snake in the ring, the spin precession angle increases, it will, behind the snake, decrease after having encountered a phase jump α in the snake.

Remembering now that there are two orthogonal coupling integrals I_C and I_S (eq. 3) which refer to orthogonal precession phases and both must be kept small, we can distribute the effect of all kicks in the ring equally onto I_C and I_S by designing the precession phase in half of the ring to be at 90° with respect to the precession phase in the other half. By choosing proper snake precession angles α, it is possible to do this, either in complete half rings, or alternating in smaller subsections of equal length. The sum $I_C^2 + I_S^2$ will then be minimal, and in case of m = 2, i.e. with $\delta\psi = \pi$ between quadrupoles, each integral will see only half of the kicks, thus being kept a factor of $1/\sqrt{2}$ below the maximum. When designing the SSC sample configurations[3], I was not yet aware of this aspect, and they might now call for a small revision, which should be easy to make.

4. Spin coupling to the corrected closed orbit as a confined walk problem

After the vertical closed orbit has been subjected to a thorough correction routine, we assume that its remaining deviation from the median horizontal plane will nowhere exceed the amplitude $\pm\hat{a}$. If, for example, the quadrupoles would deviate from this plane by less than a_q, and the beam monitors from the quadrupoles by less than a_m, being mechanically attached to them, and if all monitor readings were reduced to zero by the correction routine, we would have $\hat{a} = a_q + a_m$. With a practical limit of, say $a_q \approx a_m \approx 0.2\ mm$, we then have $\hat{a} = 0.4\ mm$ for example. The maximum orbit kick that may occur within the band of width $2\hat{a}$ is

$$\varepsilon = 2 \cdot \frac{2\hat{a}}{\ell} \tag{6}$$

where ℓ is the distance between quadrupoles.

Let us assume that in each quadrupole the closed orbit can assume any one out of q discrete amplitudes which are equally spaced over the band width $2\hat{a}$ and that, starting from the previous quadrupole, the orbit can go to any one of these amplitudes with equal probability. With n quadrupoles in the ring, there are then q^n possible closed orbits, all equally probable. Examples of these discretized closed orbits are shown in Fig. 1 for q = 2, 3, and 4. The orbit kick at the i^{th} quadrupole is then proportional to

$$(a_{i+1} - a_i) - (a_i - a_{i-1})$$

with a_i being the discrete normalized amplitude at position i, and the alternating sum (section 2) of all kicks in the ring is, except for a factor $\varepsilon = 4\hat{a}/\ell$, given by

$$S = \begin{cases} 2 \sum_{i=1}^{n} (-1)^i \ a_i & \text{for even n} \\ a_1 + a_n + 2 \sum_{i=1}^{n} (-1)^i \ a_i & \text{for odd n ,} \end{cases} \tag{7}$$

where a_i can assume any one of the values

$$a_i = \frac{r-1}{q-1} - \frac{1}{2} \ , \quad \text{with} \quad r = 1,\ 2,\ ...,\ q\ . \tag{8}$$

With n quadrupoles in the ring, the alternating sum (7) can now be evaluated for each of the q^n possible closed orbits, and we find the distribution of these sums in a sample set of q^n machines.

The result is composed in Tab. 1 for

$$S = a_1 - (-1)^n a_n + 2 \sum_{i=1}^{n} (-1)^i \ a_i \tag{9}$$

For each q, we can calculate the numbers N(S) of sample machines that have a particular value of the alternating sum, S. We can then arrange the N(S) in a pyramid in which each line gives the distribution of S in a ring with n kicks. and n increases toward the base. The generation of these pyramids is beautifully simple: Each number is the sum of those q interleaved numbers in the line above which are located symmetrically. This is indicated in Tab. 1, where the square is the sum of the q circles above. For each line, i.e. for each distribution of S, Tab. 1 gives the mean square value of S

$$\sigma^2 = \frac{1}{q^n} \sum S^2 \cdot N(S) \tag{10}$$

and we see that, for q = 2, we have $\sigma^2 = n - 3/2 \approx n$ for large n, and that for higher q the σ^2-values are the same except for a common, q-dependent reduction factor. For n = 2 we have

$$\sigma_2^2(q) = \frac{2}{q^2(q-1)^2} \sum_{i=1}^{q-1} i^2(q-i) = \frac{1}{6} + \frac{1}{3(q-1)} \tag{11}$$

which, for large q, goes to 1/3 of its value at q = 2, and we may therefore, in large rings, generally assume

$$\sigma^2(n) = \frac{n}{3} \tag{12}$$

With this result, we can now evaluate the spin orbit coupling integral (4) for the worst case m = 2, when the kicks add up with alternating signs. Using eqs. (6), (7):

$$I_{rms}(\mathrm{m{=}2}) = \gamma a \cdot \frac{4\hat{a}}{\ell} \cdot \sigma = \gamma a \cdot \varepsilon \cdot \sqrt{\frac{n}{3}} \tag{13}$$

where n is half the number of quadrupoles in the ring if I_C and I_S are made equal by the snake configuration, and $\varepsilon = 4\hat{a}/\ell$ the maximum kick. For the SSC with $\hat{a} = 0.4\ mm$, $\ell = 120\ m$ and $n = \frac{1}{2} \cdot 700$, for example, at 20 TeV the model yields the spin orbit coupling integral

$$I_{rms}(\mathrm{m}{=}2) = 5.5 \text{ rad} = 315^{\circ}$$

which is larger than can be tolerated. K. Yokoya[4] has calculated the spin tune shift due to the orbit errors to be

$$\Delta\nu_{rms} \approx \frac{1}{4\pi}\left[(\gamma a)^2\ \phi^2_{rms}\ n\right] \tag{14}$$

which, after replacing $\phi^2_{rms} \cdot n$ by $\varepsilon^2 \cdot n/3$, gives

$$\Delta\nu_{rms}(\mathrm{m}{=}2) = 2.4$$

with the parameters of our example. If, in the ring without errors, the fractional spin tune is set to 0.5 by the snakes, the spin tune shift due to errors must remain below this value in order to avoid resonance excitation. To keep the spin tune shift below, say, 0.3 in our example, we would have to reduce the maximum kick by a factor of 3 and could then expect that more than 80 % of the sample machines would not suffer resonance depolarization caused by quadrupole errors. Fortunately, it is possible to obtain an even bigger reduction factor by applying the closed orbit relaxation technique proposed in the next section. On top of that, spin orbit coupling can even be further reduced by devising specific orbit corrections for specific precession frequencies, as outlined in section 8.

5. Principle of closed orbit relaxation

Instead of accepting, in the quadrupoles of our orbit-corrected model ring, the maximum kick $\varepsilon = 4\hat{a}/\ell$ as given in eq. (6), it is helpful to observe that the actual kick ε_i in each quadrupole is known by measurement with an accuracy

$$\delta = \frac{a_m}{f} \tag{15}$$

where a_m is again the offset of the monitor from the quadrupole and f is the modulus of the focal length, alike in all quadrupoles. Denoting the monitor reading in the ith quadrupole by d_i and the deflecting angle of the corresponding correction magnet by α_i, the actual kick is

$$\varepsilon_i = \begin{cases} \alpha_i - \frac{d_i}{f}[\pm\delta] & \text{in the F-quads} \\ \alpha_i + \frac{d_i}{f}[\pm\delta] & \text{in the D-quads} \end{cases} \tag{16}$$

For a betatron phase advance ϕ per cell, we have

$$f \approx \frac{\ell}{2 sin\phi/2} \tag{17}$$

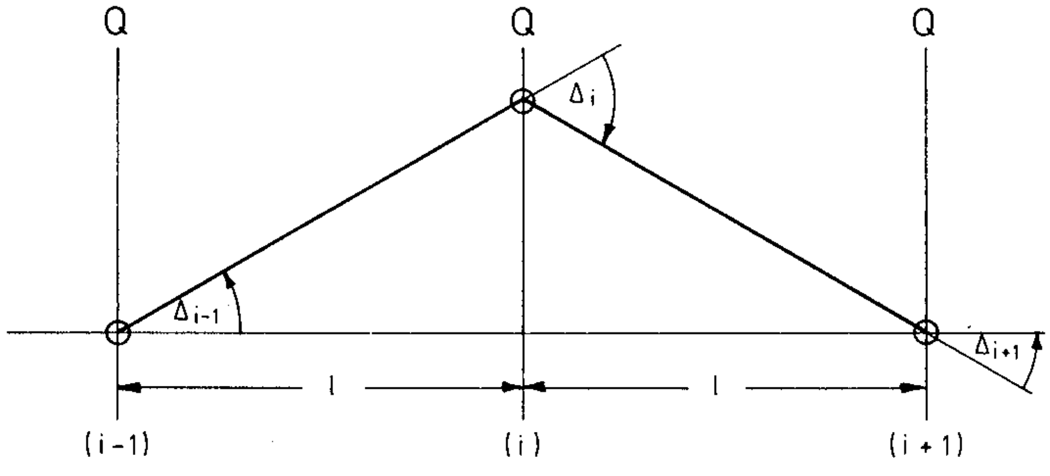
i.e. $f = \ell$ for $\varphi = 60^o$ and $f = \ell/\sqrt{2}$ for $\phi = 90^o$. With $a_m = \hat{a}/2$ and $\phi = 90^o$, as in our SSC model for example, we have the relative measuring accuracy

$$\frac{\delta}{\varepsilon} = \frac{\sqrt{2}}{8} = \frac{1}{5.66} \tag{18}$$

and thus we suggest to decrease the vertical kicks by this factor by reducing the measured orbit angles ε_i to zero reading in a way similar to that which we had formerly used to reduce the measured orbit amplitudes d_i when applying the standard orbit correction techniques. To do this, the following closed orbit relaxation scheme is proposed.

6. Closed orbit relaxation scheme

What we suggest is, in principle, a local superposition of a closed beam bump that involves 3 consecutive correction magnets and, at the inner corrector (i), shifts the orbit half-way toward its average vertical position between the two outer correctors (i−1) and (i+1):



If, before relaxation, we have measured the kicks ε_i, the angles Δ_i in the bump are chosen to be

$$\begin{aligned} \Delta_i &= -\frac{1}{2}\,\varepsilon_i \\ \Delta_{i-1} = \Delta_{i+1} &= \frac{1}{4}\,\varepsilon_i \end{aligned} \tag{19}$$

Instead of applying the bumps one after another. we suggest to relax the closed orbit as a whole by simultaneously applying the appropriate beam bump at every quadrupole, going in one step from the measured kicks $\varepsilon_i^{(0)}$ to a new set of kicks $\varepsilon_i^{(1)}$ as given by

$$\varepsilon_i^{(1)} = \frac{1}{4}\,(\varepsilon_{i-1}^{(0)} + 2\varepsilon_i^{(0)} + \varepsilon_{i+1}^{(0)}) \tag{20}$$

This relaxation scheme converges very quickly and would, after a few steps, reduce all kicks to almost zero and make the closed orbit approach an almost straight line almost parallel to the median plane if the monitors were perfectly centered on the quadrupoles. In that case, even the first relaxation step alone would reduce the maximum deflection by a factor of 4. This is seen as follows: Denoting the initial orbit amplitudes by $a_i^{(0)}$, we have e.g. the initial kick

$$\begin{aligned}\varepsilon_3^{(0)} &= \frac{1}{\ell}[(a_4^{(0)} - a_3^{(0)}) - (a_3^{(0)} - a_2^{(0)})] \\ &= \frac{1}{\ell}(a_2^{(0)} - 2a_3^{(0)} + a_4^{(0)}) \;; \qquad \mid \varepsilon^{(0)} \mid \leq \frac{4}{\ell}\,\hat{a}\end{aligned} \tag{21}$$

and after the first relaxation step

$$\begin{aligned}a_3^{(1)} &= \frac{1}{4}(a_2^{(0)} + 2a_3^{(0)} + a_4^{(0)}) \\ \varepsilon_3^{(1)} &= \frac{1}{\ell}(a_2^{(1)} - 2a_3^{(1)} + a_4^{(1)}) \\ &= \frac{1}{4\ell}(a_1^{(0)} - 2a_3^{(0)} + a_5^{(0)}) \;; \qquad \mid \varepsilon^{(1)} \mid \leq \frac{1}{\ell}\,\hat{a}\end{aligned} \tag{22}$$

Since the sum of a quadrupole and a dipole field may be viewed as a translated quadrupole, the relaxation procedure may be viewed as a vertical repositioning of the initially misaligned quadrupole fields onto a smooth beam line by superposing in situ the required corrector dipole fields.

In the real world, the monitors are not perfectly centered; they have displacements of up to a_m and thus, for zero reading, permit kicks of up to $\delta = a_m/f$. After a few relaxation steps, we will therefore essentially be left with an upper limit $\hat{\varepsilon} = \delta$ for the orbit kicks:

$$\hat{\varepsilon} = \frac{a_m}{f} \tag{23}$$

Note that $\hat{\varepsilon}$ does not depend on the alignment accuracy a_q of the quadrupoles any more, but only on the centering of the monitors; a remarkable, very useful result!

Then, in our SSC example after orbit relaxation, with f = 85 m and $\hat{\varepsilon}$ = 0.2 mm/85 m = 2.4 μrad, we have the spin orbit coupling integral (eq. 13)

$$I_{rms}(\mathrm{m}=2) = 0.97 \text{ rad} = 56^{\circ}$$

and the spin tune shift due to orbit errors (eq. 14)

$$\Delta\nu_{rms}(\mathrm{m}=2) = 0.075$$

These confortably small values indicate that closed orbit relaxation, applied after standard closed orbit correction, may be a sufficient tool to avoid depolarization by orbit imperfections even up to SSC energies. In addition, however, there remains the possibility of devising specific orbit correction schemes which decouple the spin motion from the orbit at specific precession frequencies, as described in section 8.

The relaxation scheme, slightly modified and somewhat less effective, can also be applied in a ring where a beam position monitor and a vertical correction magnet are only available at every vertically focusing quadrupole, i.e. at every second quadrupole along the circumference[5]. In that case, the upper limit for the kicks will be instead

$$\hat{\varepsilon} = \frac{a_m}{f} + \frac{2f}{2f+\ell} \cdot \frac{a_q}{f} \tag{24}$$

where a_q is again the maximum offset of the quadrupoles from the median horizontal plane. Here, $\hat{\varepsilon}$ depends on a_q since the offset of the D-quads cannot be compensated by a correction magnet in situ.

We have here discussed closed orbit relaxation only for the vertical motion. It should, of course, also be applied in the horizontal plane in order to reduce the variations in spin precession phase that are generated by horizontal closed orbit kicks.

7. Effect of dipole magnet misalignment

In our ring model, we have so far ignored the additional vertical kicks due to the bending magnets between quadrupoles. By carefully building these magnets, their roll angle, i.e. the variation of field direction over the length, can be kept below, say, ± 1 *mrad*. It can be measured as a function of s, and the *average* orientation of the magnet can then be levelled in the tunnel to an accuracy of, say, ± 0.2 *mrad*. It is essential to note here that, due to the stiffness of coil and magnet structures and due to the smoothness of stacking fixtures, the roll of the magnet will not be a rapidly varying function of s but will, most probably, resemble a sine wave of not more than about one full oscillation per magnet length. On the other hand, there will be many spin precessions per magnet at very high energy, and the effect of the roll on the spin will thus partially cancel.

In the SSC, for example, there are 10 spin precessions per magnet at 20 TeV, and we see from

$$\left| \int \cos 10x \ \cos x dx \right| = \left| \frac{1}{2} \left(\frac{1}{11} \ sin 11x + \frac{1}{9} \ sin 9x \right) \right| \leq \frac{10}{99}$$

that the effective kick ε_B given to the spin by each magnet will be about an order of magnitude smaller than the maximum roll angle suggests. Thus, with a bending angle of 1.6 mrad in the SSC magnet, and 6 magnets between quadrupoles

$$\varepsilon_{B,rms} \approx 0.2 \cdot 10^{-3} \cdot 1.6 \cdot 10^{-3} \cdot \sqrt{6} = 0.8 \ \mu\text{rad}$$

which is reasonably small as compared to the maximum kick $\hat{\varepsilon} = 2.4$ μrad that remains in the SSC quadrupoles after orbit relaxation. The proposed closed orbit relaxation scheme will therefore work with similar effectiveness when the roll of bending magnets is included.

8. Compensation of spin orbit coupling modes

With the spin precession phase advance $\delta\psi = 2\pi p/m$ between successive quadrupoles, the strength of the spin orbit coupling mode (m, p) can be determined in a way similar to that used for the worst case m = 2 in section 4. We have done this here only for a few more examples with p = 1

m	2	3	4	6
σ^2	5.5	2	1.5	1

where the mean square value of the orbit-induced spin rotation per revolution happens to go down approximately as 1/m. For a given snake configuration the calculation of coupling mode strengths should include the jump and sign inversion of the spin precession phase in each snake (section 3). One can then select the modes (m, p) with the strongest coupling and devise for each of them a specific orbit correction pattern. This pattern can be applied after acceleration to the corresponding energy by empirically adjusting its strength for maintaining maximum polarization. Once compensation has been achieved for a certain mode (m, p), it will be effective at a large number of equidistantly spaced energies, e.g. in the SSC at ca. 57 different energy levels, at least.

How, in principle, the correction can be individually addressed to a certain m-value is explained in Fig. 2 for the examples of m = 2, 3, ..., 9 in a ring with a betatron phase advance of 45° per half cell. Over a certain part of the ring, a kick of same sign is applied at every m^{th} quadrupole, and the orbit distortion generated by each kick is cancelled again by another one of these kicks which is (4 + 8n) half cells apart, since then the betatron phase advance is an odd multiple of 180° between them. For some values of m, the orbit-cancelling chain of kicks gets rather long and might not fit any more into one interval between snakes. In these cases, the chain might be subdivided, or extended over one or two snakes, and if these methods do not work, a more complex scheme of kicks with orbit cancellation must be devised.

Compensation of spin orbit coupling is rather difficult for m = 8 because orbit cancellation would demand the kicks to *alternate* between every 8^{th} half cell, while for the compensation they are required to be *equal*. A solution is provided by centering the kicks about a 180° snake which inverts the spin and adds a 180° spin precession, because that particular spin oscillation which has its maximum amplitude in the snake will there be subjected to a 180° phase jump, and the subsequent kicks on the other side of the snake must then be inverted.

So far, we have only addressed the compensation of spin orbit coupling for that particular spin phase which has its maximum amplitude at the quadrupoles of our selected chain of kicks, spaced m half cells apart. Obviously, another compensation must be provided for the orthogonal spin phase which is not coupled to these kicks. This is somewhat complicated by the fact that, at the positions of the m quadrupoles within a period of m half cells, $\cos 2\pi\ p/m$ and $\sin 2\pi\ p/m$ assume different amplitude patterns depending on the value of p. However, the number of these different patterns is $\leq m$ since, with increasing p, the patterns repeat themselves periodically. If, for the k^{th} amplitude pattern, the precession amplitudes in the m quadrupoles are composed into the vector $\vec{a}_k$ and if the i^{th} correction mode provides the kick δ_i at only the i^{th} quadrupole, we need an independent correction sum $b_k = \sum_i a_{ki}\ \delta_i$ for each pattern and thus have the set of linear equations

$$\begin{aligned} \vec{a}_1 \cdot \vec{\delta} &= b_1 \\ \vec{a}_2 \cdot \vec{\delta} &= b_2 \\ &\cdot \\ &\cdot \\ &\cdot \\ \vec{a}_m \cdot \vec{\delta} &= b_m \end{aligned} \tag{25}$$

which can be solved for the m components δ_i of the vector $\vec{\delta}$.

Admittedly, more work is needed to better understand the offerings and shortcomings of such mode correction [6], but I believe that schemes can be found to make it a practicable concept for curing the small number of modes that might still be dangerous after orbit relaxation.

9. Conclusions for the SSC

The workshop report ref.[1] concludes that "... the key to the feasibility of polarized beams in the SSC will be how to overcome the tolerance problem". With alignment tolerances of ± 0.2 mm for the quads and for the position monitors with respect to the quads, and with an average roll of bending magnets not exceeding ± 0.2 mm, I believe that the proposed method of closed orbit relaxation suffices to bring the spin orbit coupling well below the threshold for imperfection resonances. This assumes that the SSC will be equipped with a vertical position monitor and correction magnet at every quadrupole. As an added safety against coupling modes that might still be strong after orbit relaxation, the recommended patterns for specific orbit corrections may be applied.

Acknowledgement

I wish to thank A. Chao and K. Yokoya for discussions and I am grateful to D. Barber for carefully reading the manuscript and giving helpful comments.

References

[1] **A.W. Chao, E. Courant, S.Y. Lee, K. Steffen, L.C. Teng, K. Yokoya**: "Workshop on Siberian snakes for the SSC collider ring", workshop report by K. Yokoya, SSC-SR-1036 , Sept. 1988.

[2] **K. Steffen**: "Strong spin matching with and without snakes, a scheme for preserving polarization in large ring accelerators", Part. Acc. Vol. 24, p. 45, 1988, and DESY 88-068, May 1988.

[3] **K. Steffen**: "An example of strong spin matching: Sample lattices for polarized proton acceleration in the Supercollider (SSC)", DESY HERA 89-06, Feb. 1989, and also a part of ref.[1].

[4] **K. Yokoya**: "On multiple Siberian snakes", SSC-189, Sept. 1988.

[5] **K. Steffen**: "Closed orbit relaxation for polarized beam operation in HERA and LEP", DESY HERA 89-08, Feb. 1989.

[6] **K. Yokoya**: private communication 1989.

q = 2

n												σ^2
2					1	2	1					0.5 = σ_2^2
3				1	2	2	2	1				1.5
4			1	2	3	4	3	2	1			2.5
5		1	2	4	6	6	(6)	4	(2)	1		3.5
6	1	2	5	8	10	12	10	[8]	5	2	1	4.5
S:	-5	-4	-3	-2	-1	0	1	2	3	4	5	

q = 3

n																σ^2
2					1	2	3	2	1							0.33 = $0.66 \cdot \sigma_2^2$
3			1	2	4	4	5	4	4	2	1					1
4	1	2	5	6	10	10	13	(10)	10	(6)	5	(2)	1			1.66
5			16	18	28	26	33	26	28	[18]	16	8	6	2	1	2.33
S:					-1	-.5	0	.5	1	1.5	2	2.5	3	3.5	4	

q = 4

n																	σ^2
2				1	2	3	4	3	2	1							0.277 = $0.55 \cdot \sigma_2^2$
3	1	2	4	6	7	(8)	8	(8)	7	(6)	4	(2)	1				0.833
4			16	20	24	26	28	26	[24]	20	16	12	8	5	2	1	1.388
S:					-.66	-.33	0	.33	.66	1	1.33	1.66	2	2.33	2.66	3	

q = 5

n																		σ^2
2	1	2	3	4	5	4	3	2	1									0.25 = $0.5 \cdot \sigma_2^2$
3			12	(12)	13	(12)	12	(10)	9	(6)	4	(2)	1					0.75
4			50	50	55	50	50	[42]	39	30	26	18	14	8	5	2	1	1.25
S:			-.5	-.25	0	.25	.5	.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	

q = 6

n																						σ^2
2	1	2	3	4	5	6	5	4	3	2	1											0.233 = $0.466 \cdot \sigma_2^2$
3				17	(18)	18	(18)	17	(16)	14	(12)	9	(6)	4	(2)	1						0.7
4				86	89	92	89	86	79	[72]	63	54	45	36	28	20	14	8	5	2	1	1.166
S:				-.4	-.2	0	.2	.4	.6	.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3	

$$S = a_1 - (-1)^n a_n + 2 \sum_{i=1}^{n} (-1)^i a_i$$

Tab. 1 Distributions of the alternating sum S of kicks for a confined closed orbit with n kicks, approximated by q discrete amplitude levels. For each distribution, i.e. each line, the mean square deviation $\sigma^2 = 1/q^n \sum S^2 N(S)$ is given. Each element N(S) in the pyramids (e.g. in the square) is the sum of the q symmetrically located interspaced elements in the line above (e.g. in the circles).

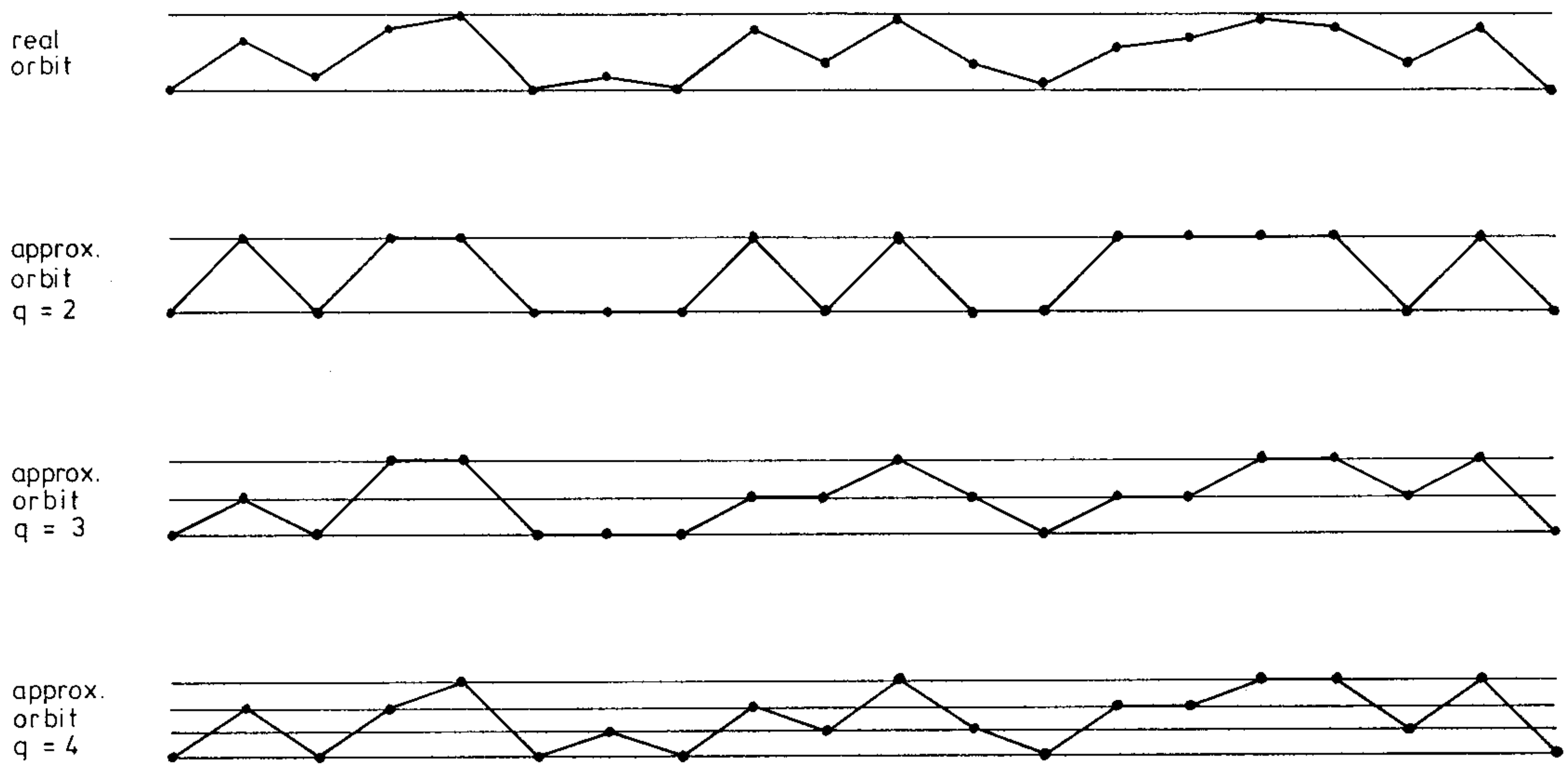


Fig. 1 Example of confined closed orbit and its approximations with $q = 2, 3$ and 4 discrete amplitude levels.

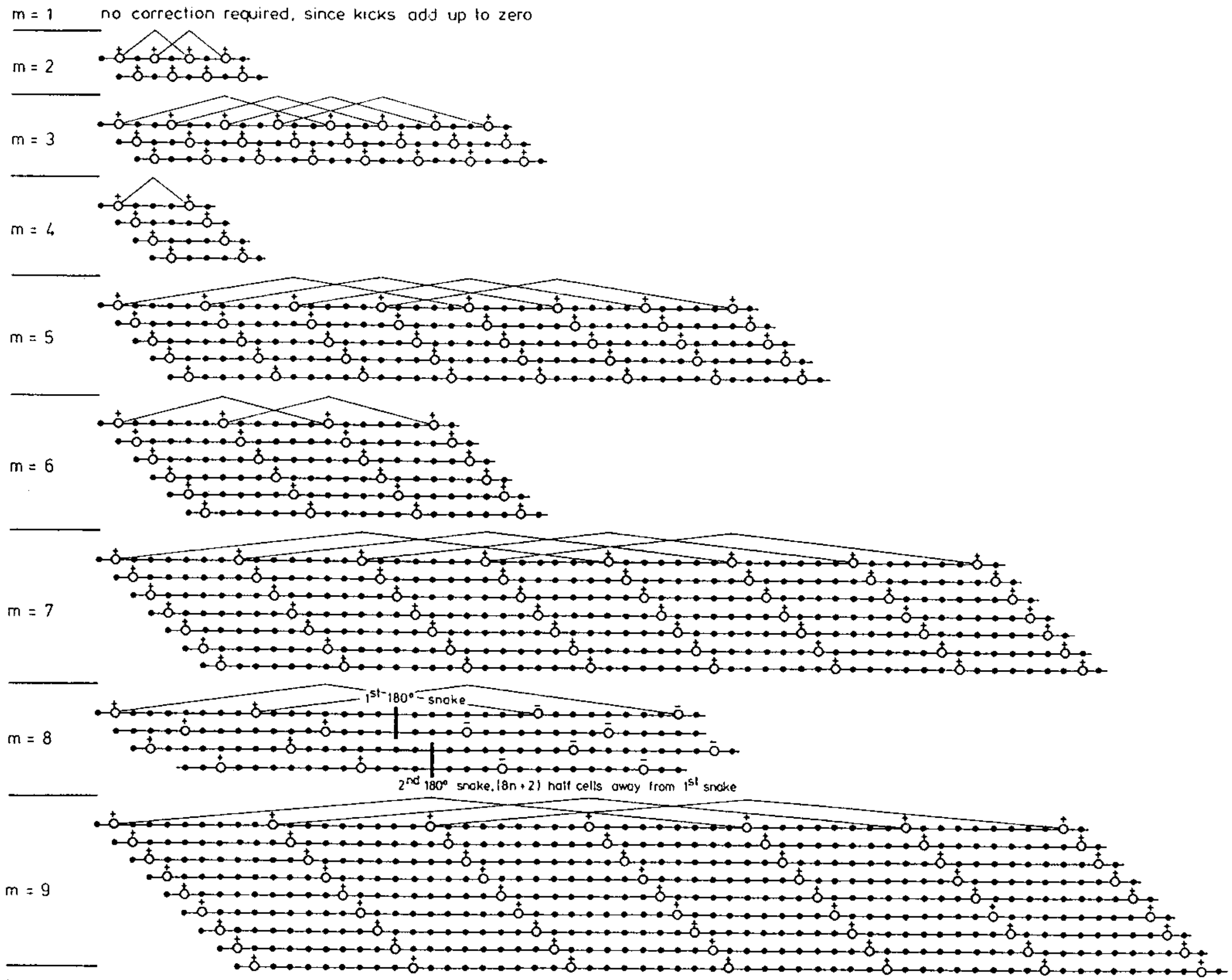


Fig. 2 Choice of equal strength correction coils to compensate spin orbit coupling for spin precession frequencies characterised by $m = 2, 3, \ldots, 9$. One bin denotes one half cell with a betatron phase advance of 45°. For each m, generally m groups of correction coils must be provided which are shifted toward each other by one half cell, respectively.