DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 89-040
March 1989



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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY
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Notkestrasse 85
2 Hamburg 52
Germany

DESY 89-040
March 1989

ISSN 0418-9833

# THE POMERON IN QCD AND EXPERIMENT[1]

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**Abstract:**

An overview is given of the Pomeron in experiment and its interpretation within QCD. The discussion also includes the small-x limit in deep-inelastic scattering.

## I Introduction

In this talk I will attempt to give an overview of the present situation of the Pomeron. This includes the main experimental features and, in particular, the question how well the Pomeron is understood within QCD. As it is well-known, the theoretical explanation of the Pomeron is an old problem in particle physics, and despite numerous and elaborate efforts in the past, we have,in my view, not yet reached a satisfactory solution in the framework of the Standard Model. On the other hand, hadron colliders which are presently working or being planned for the future (CERN-collider, Tevatron, HERA, SSC, LHC, Eloisatron,...) are or will be full of events where the majority of particles is produced close to the forward direction. In would also be hard to argue that we can reach an understanding of the dynamics of quark and color confinement without eventually being able to explain diffraction scattering. Therefore, interest in the Pomeron problem remains, and an orientation of where we are standing now may be of some use.

First a few introductory remarks. The name "Pomeron Physics" refers to phenomena seen in the small-t region of hadron-hadron scattering at high energies. Quantities of primary interest are: the energy dependence of the total cross section, energy and t-dependence of the elastic cross section (in the range $-t << s$, the energy dependence of the real part of the elastic forward scattering amplitude F(s,t). Given a successful theory for these quantities, standard Regge theory then allows to describe also inelastic processes in multiregge limits and inclusive particle spectra. New and recent interest in the Pomeron is also connected with the small-x behavior of deep-inelastic structure functions. Here one also probes the Pomeron, although in a different kinematic limit. As I will discuss below, the advantage of this limit is that one is somewhat closer to the region of validity of perturbation theory. In the language of QCD, in both contexts one probes the content of very slow quarks and gluons inside the hadron, but in slightly different spatial regions. Hadron hadron scattering at small angles probes the slow partons at large impact parameters, whereas deep-inelastic scattering in looks at small transverse distances. Only if, at fixed $Q^2$, x is made very small, one also enters the region of larger and larger impact parameters. In the first case, one cannot ignore confinement (which makes the problem so hard), whereas in the second case one

[1]Invited Lectures given at the Eloisatron Workshop held in Erice, Sicily, Italy in June 1988.



at least begins with perturbation theory and then tries to move deeper and deeper into the nonperturbative regime. That is why it may be helpful to have both approaches described in one paper. To give them a name, I will call them the "soft" and the "hard" Pomeron, resp.

The talk will be organized in three parts. In the first part I summarize a few gross features of the experimental situation in hadron-hadron scattering ("soft" Pomeron). Then I discuss a few models and how they relate to experimental data. The main conclusions of this section are: (i) at present day energies the total cross section shows a rather clear $lns^2$-behavior, but there is little doubt that we are still far away from truly asymptotic energies. One should therefore not rule out the possibility that some new physics will leave its signature also in the total cross section (e.g.in the TeV-region).(ii) The recent UA4 measurement of the real part of the forward scattering amplitude is of extreme interest, since it contains information on the behavior of the total cross section at higher energies. It should, therefore, be confirmed by another independent measurement.

In the second part I will review what has been done in order to derive the Pomeron from QCD. Starting point are extensive perturbative calculations (most economically done with the help of dispersion relations). They seem to result in a complete (nonlocal) reggeon field theory. When trying to solve this field theory one faces a principal difficulty: in the infrared region the Feynman diagrams from which the reggeon field theory is derived become sensitive to confinement dynamics. Suggestions have been made how to overcome this difficulty and how to solve the reggeon field theory. The most promising candidate seems to be critical reggeon field theory.

The third part reviews the situation of the small-x limit of deep-inelastic structure functions. It is well-known that the Standard QCD-description comes into conflict with s-channel unitarity when $x^{-1} = s/Q^2$ is getting too large. By taking into account much larger classes of Feynman diagrams (so-called fan diagrams, the building units of which are the standard QCD-ladders) it is possible to extend the range of validity of perturbative QCD up to larger values of $1/x$. Existing data, however, do not yet allow to test this "improved" QCD description. HERA will be the first machine where measurements down to $x = 10^{-4}$ will be possible and where these theoretical ideas can be tested. It will also be discussed how this "hard" Pomeron appearing in the small-x region comes closer and closer to the reggeon field theory description of the "soft" Pomeron. Finally, a few words will be said about the concept of the recently proposed "Pomeron structure function".

## II The Experimental Situation

In this part I first review the most important experimental facts of the Pomeron in hadron-hadron scattering and confront them with a few theoretical models.

I begin with the total cross section in $p\bar{p}$-scattering (Fig.1a)[1]. The arrow should remind us that we are still waiting for the Tevatron measurement of the total cross section. The rise of $\sigma_{tot}$ is well- described by the classical fit of Amaldi et al.[2] (Fig.2a):

$$\sigma_{\pm} = C_1 E^{-\nu_1} \mp C_2 E^{-\nu_2} + B_1 + B_2 (ln\, s)^{\gamma} \tag{1}$$

with

$$C_1 = 41.9,\ C_2 = 29.2,\ B_1 = 27.0,\ B_2 = 0.17,\ \nu_1 = 0.37,\ \nu_2 = 0.55,\ \gamma = 2.1 \tag{2}$$

(all constants B and C are in mb, energy in Gev/c). A more recent fit is due to Bourrely and Martin [3], (Fig.3). For the scattering amplitude they make the ansatz:

$$F = is \frac{A + B[ln\, s/s_0 - i\pi/2]^2}{1 + C[ln\, s/s_0 - i\pi/2]^2}\ . \tag{3}$$

The constants A,B,C are fitted to the data, and two different scenarios are used: in (I) C=0 (which implies $\sigma_{tot} = const * lns^2$); in (II) C is allowed to be different from zero (which implies asymptotically that $\sigma_{tot}$ approaches a constant value). The results of the fits are:

| | I | II |
|---|---|---|
| A | 41.95 | 41.824 |
| B | 0.43 | 0.815 |
| C | 0 | 0.006 |
| $s_0$ | 243.6 | 277.7 |

(4)

One notices that in scenario II the value for C is very small: the plateau $\sigma_{tot}$ = constant is reached only for extremely high energies. Hence these fits support the conclusion that, for present energies, $\sigma_{tot}$ is best described by a $lns^2$-behavior, i.e.it goes as the Froissart bound (although it is far from saturating it).(See, however, also [4] where slightly different conclusions are drawn).

The next quantity of interest is the elastic cross section. As shown in Fig.1b [1], the ratio $\sigma_{el}/\sigma_{tot}$ rises from 0.175 at ISR to 0.215 at the collider. For the differential cross section $d\sigma_{el}/dt$ a compilation of data [5] is shown in Fig.4: in the $p\bar{p}$-data a shoulder is seen at $/t/$=0.9 $(GeV)^2$. With a little imagination one also sees some structure at $/t/=1.6(GeV)^2$. A very interesting topic is the measurement of $\rho = ReF(s,0)/ImF(s,0)$. The UA4 measurement [6] gives (Fig. 5)

$$\rho = 0.24 \pm 0.04 \quad . \tag{5}$$

This has to be confronted with the extrapolations from data at lower energies (Fig. 2b). They all lie in the range $\rho$=0.10 to 0.15 (e.g. $\rho$=0.119 for scenario II and $\rho$=0.154 for scenario I of Bourelly and Martin [3]). The measured value is thus off by almost a factor of two from the expected value. If it is correct, it implies that, via the use of dispersion relations, at $s = 0(TeV^2)$ the total cross section should rise faster than seen presently. Fig.6 shows a few "predictions", taken from [7]. Since such a growth cannot continue forever and eventually has to flatten off again, one expects to see some "structure", e.g. a broad enhancement. It is this "predictive" power of $\rho$ which makes it such an interesting quantity. It would, therefore, be very desirable to have an independent experimental confirmation of this unexpected high value of $\rho$.

In a more detailed survey one would now go on and discuss other quantities which allow more specific tests of theoretical ideas, for example: $\sigma_{inel}$, the rise of the plateau of inclusive cross sections, multiplicities, KNO-scaling,.... For this rather general review, however, I have to restrict myself to the very few quantities discussed above.

In the next step one might ask what kind of model agrees with these data. Here is a list of a few:

1) Critical reggeon field theory [8]
2) Flavoured reggeon field theory [9]
3) Eikonal Model [10]
4) Models containing an Odderon [11]
5) Model based upon the analogy Pomeron-photon[12]
6) QCD inspired three gloun exchange [13]
7) Dual Topological Unitarization [14]

Since I am unable to present a critical discussion of all these models, I make the subjective selection and say a few words only about 1) and 3). I do this for the following reason. Model 1) is, from the theoretical point of view, the most sophisticated one, and as I will argue below, the efforts of analyzing the high energy behavior of QCD point in the direction as if critical reggeon could be an effective field theory which described the high energy limit of QCD. On the other hand, the asymptotic predictions of critical reggeon field theory do not match with presently observed behavior. Some of the other models, on the other hand, do provide successful descriptions of present data, and model 3) seems to do particularly well. Moreover, it has some basis in quantum field theory (massive QED).

I begin with critical reggeon field theory. It has been set up as a solution to the t-channel partial wave unitarity equations and has its roots in general S-matrix theory rather than any specific quantum field theory model. It has also been shown to satisfy constraints that follow



from s-channel unitarity and that have been problematic for other models. All this makes it, from an esthetic point of view, a very attractive candidate for a self consistent theory for asymptotic hadron-hadron scattering. It contains, however,a large number of parameters (in particular, the couplings of reggeons to external hadrons) which have to be computed from the underlying theory, QCD. This defines the task of "deriving critical reggeon field theory from QCD", which I will discuss below in more detail. At this place let me list the most important asymptotic predictions which follow from critical reggeon field theory without any further reference to any underlying theory. For the total cross section one finds:

$$\sigma_{tot} = const(ln\, s/s_o)^{-\gamma}[1 + ln(s/s_0)]^{-\gamma}[c_0 + C_1 ln\, ln(s/s_0) + \ldots] \tag{6}$$

with the approximate numerical values $-\gamma = 0.2, \lambda = 1$. The elastic scattering amplitude obeys the scaling law:

$$F_{el}(s,t) = i\beta_A(t)\beta_B(t)ln(s/s_o)^{-\gamma} f[t(ln\, s/so)^z] \tag{7}$$

where f denotes a universal function which is independent of the scattering particles A and B, and z=1.1 approximately. Integrating $|F_{el}(s,t)|^2$ over t leads to:

$$\frac{\sigma_{el}}{\sigma_{tot}} = const \cdot (lns/s_0)^{-2\gamma - z} \quad , \tag{8}$$

$$-2\gamma - z \approx -3/4 < 0 \quad , \tag{9}$$

i.e., the ratio $\sigma_{el}/\sigma_{tot}$ is predicted to decrease with energy. All these formulae are asymptotic and have corrections which vanish as s goes to infinity. All predictions have been compared with present data: the total cross section in [15], the elastic cross section in [16]. None of them fits well, but the strongest argument against this theory is presently the prediction (8) which is clearly violated. The conclusion then is that critical reggeon field theory, at least in its asymptotic limit, is not applicable at present energies. However, for the obvious possibility that we may still be too far away from truly asymptotic energies, it seems to me that we cannot dismiss this theory altogether: it still may turn out to be the right theory at much higher energies.

Next we turn to a successful theory, model 3). As it was said before, this form of high-energy behavior has been derived from massive QED, by iterating in the s-channel the so-called tower diagrams [17]. In this picture the hadron looks like a black disc with the radius increasing as lns. I again list the main predictions. For the scattering amplitude one has:

$$F(s,t) = \frac{is}{2\pi} \int d^2b e^{iqb} [1 - e^{\Omega(s,b)}] \tag{10}$$

$$\Omega(s,b) = F(b^2) \left[ \frac{s^c}{(lns)^{c'}} + \frac{u^c}{(lnu)^{c'}} \right] + R_o(s,b) \tag{11}$$

where $q^2 = -t$, $c = 0.167$, and $c' = 0.748$, and $R_0(s,b)$ stands for secondary Regge pole exchange which becomes negligible when s goes to infinity. From these expressions on obtains the predictions:

$$\sigma_{tot} \sim const \cdot ln\, s^2 \tag{12}$$

$$\frac{\sigma_{el}}{\sigma_{tot}} \to 0.5 \tag{13}$$

A numerical evaluation of (10) for $d\sigma_{el}/dt$ gives impressive agreement for both pp-scattering at ISR and $p\bar{p}$-scattering at the collider. The authors also present their predictions for much higher energies. If one defines asymptopia to be at those energies where observables reach their limiting values (e.g. $\rho$ or $\sigma_{el}/\sigma_{tot}$), then one has to conclude that we are still far below [18] (Fig.7). Very similar conclusions could also be drawn from Model 4 [19].

Attempting to draw simple conclusions from this brief overview, I would like to stress the following three points: (i) Up to collider energies the behavior of $\sigma_{tot}$ shows $lns^2$ behavior ,and the eikonal model 3) gives a good description. The asymptotic formulae of critical reggeon field theory cannot be used. (ii) A crucial question then seems to be: do we observe already the truly asymptotic behavior? The simplest possibility is that the behavior seen now can smoothly be extrapolated up to infinite energies. But then still the comparison of present values for, e.g. $\rho$ and $\sigma_{el}/\sigma_{tot}$, with the limiting values predicted by the successful models indicates that we are far below asymtopia. From the theoretical point of view there remains the task of obtaining the $lns^2$ behavior from QCD. Alternatively, the behavior seen at present energies may not yet be the asymptotic behavior, and some changes will be observed at higher energies. Theoretical explanations for such a change could be: the increasing importance of hard QCD processes and their contribution to $\sigma_{tot}$, heavy flavour production thresholds, new physics in the TeV-region which not only affects the electroweak sector but also leaves some signature in the strong sector,.... In any of these cases, of course, the question of the asymptotic behavior of hadron-hadron scattering will remain open until we can reach much higher energies. (iii) An independent measurement of the $\rho$-value at the collider or at the Tevatron would be very important. Because of its predictive power it sheds some light on the behavior of $\sigma_{tot}$ at higher energies.

Let me conclude this section by repeating an argument [20] which, in my opinion, is important because it is related to one of the striking features of hadron-hadron scattering at present energies, the validity of the additive quark model. Within the QCD-inspired quark parton model, the validity of the quark counting rules is explained by assuming that inside the fast hadron with transverse radius $R_h$ each valence quark is surrounded by a cloud of (virtual) sea quarks and glouns. The radius of this cloud, $r_q$, is smaller than the hadron radius $R_h$, and most of the time the clouds of the two or three valence quarks do not overlap. Therefore the total cross section is to a good approximation given by the sum over valence quark - valence quark interaction cross sections. As a function of energy, $r_q^2$ grows as $4*\alpha'_{pomeron} * ln\, s/s_0$, and using the estimates of [20], $r_q$ is expected to reach $R_h$ at $\sqrt{s} = O(10^6 GeV)$. Once the clouds start to overlap, the additive quark model should break down and one expects to see a change in the hadronic cross sections. This argument supports the view that what we are seeing at present energies is not yet the true asymptotic behavior in soft hadron scattering.

# III The Soft Pomeron in QCD

In this section I will try to give an overview on our present understanding of the high energy behavior of QCD. There is no doubt that the Pomeron is an intrinsically nonperturbative object, i.e. it cannot be calculated from perturbation theory alone. Nevertheless, existing calculations start from the analysis of the perturbative (mainly gluonic) content of the Pomeron and then try to move deeper and deeper into the nonperturbative region. It will become clear that this program is not completed, and more theoretical efforts will be needed in the future.//

This overview can naturally be organized into three steps.In the first step I want to describe what may be called the "leading-lns" Pomeron in QCD [21]-[23]. It consists of the sum of ladder diagrams (Fig.8) with reggeized gluons along the side lines. The rungs of the ladders are due to gluons being produced multiperipherally. They can also be viewed as reggeon interaction vertices: two reggeons $\rightarrow$ two reggeons. They have a nontrivial momentum dependence which is related to the reggeization of the gluon. This set of QCD diagrams represents the leading-lns approximation to the scattering of colorless hadrons in the limit $s \rightarrow \infty$, t small and fixed: for each power of the strong coupling constant $\alpha_s$, these diagrams yield the highest power of lns. As an example, Fig.9 shows the scattering of two heavy flavor mesons [24]: the momentum scale of $\alpha_s$ is then given by the mass of the heavy quark. (These QCD ladder diagrams with reggeized gluons play a role analogous to that of the tower diagrams in QED [17]: there is, however, the fundamental difference between the photon and the nonabelian vector particles. The latter reggeize whereas the former does not. This makes the analysis of the high energy behavior of QED from the start different from that of nonabelian gauge theories.) The QCD ladder diagrams when coupled to colorless external bound states are all infrared finite and ultraviolet finite; the strong coupling constant remains fixed in the leading-lns approximation.

There are a few remarkable properties of these diagrams that should be mentioned. First of all one would like to know what kind of high energy behavior comes out if the ladder diagrams are summed up to all orders. The answer is [23]:

$$F(s,t) = i\, const \cdot s^{\alpha_c} \ , \quad \sigma_{tot} = const' \cdot s^{2\alpha_c - 1} \qquad (14)$$



with

$$\alpha_c = 1 + \frac{3g^2 ln2}{\pi^2} \quad (\alpha_s = \frac{g^2}{4\pi}) \qquad (15)$$

That is one finds a fixed-cut singularity in the angular momentum plane to the right of j=1, and the total cross section violates the Froissart bound. This clearly shows that these diagrams provide an invalid approximation if s becomes too large. In order to have a better understanding of what goes wrong it is useful to take a closer look of how this bad high energy behavior is generated. As it has been said before, at each order of $\alpha_s$ all internal momentum integrations (integration is always over two-dimensional transverse momentum.Its conjugate variables can be understood as the transverse distance between wee partons inside the scattering hadron) are perfectly convergent both in the infrared and ultraviolet region, and the average value of momentum is of the order of the scale in $\alpha_s$ (in our example, the mass of the heavy quark). However, if one starts to sum over all orders in $\alpha_s$ and includes higher and higher powers of the coupling constant, and if one asks which values of internal momentum are responsible for building up the leading singularity in the j-plane, then, according to the analysis of [23], one finds:

$$< ln^2 \frac{k_t^2}{m^2} > \sim const \cdot n \qquad (16)$$

where n is the order of $\alpha_s$. Hence the leading-s behavior depends largely upon the behavior of the diagrams in both the infrared and the ultraviolet regions. But there the leading-lns approximation is not quite correct: for large momenta, the dependence of the strong coupling $\alpha_s$ upon momentum ought to be taken into account, and in the infrared region where the strong coupling is becoming large perturbation theory is expected to break down. One therefore has to conclude that these ladder diagrams, as they come out from the leading-lns analysis, are valid only as long as the internal momenta are neither to small nor too large, i.e. one is not allowed to go to infinite order in $\alpha_s$, and s has to be such that $\alpha_s * lns < 1$.

It is also instructive to see to what extent a modification of the ladder diagrams in the ultraviolet and/or the infrared region is able to cure the bad high energy behavior. This has been studied by L.N.Lipatov in [25]. For large momentum he replaces the fixed coupling constant by the momentum dependent one, and in the low-momentum region where the coupling becomes strong he imposes as certain boundary condition onto the behavior of the perturbative amplitude which represents confinement dynamics. As a result of this, the leading fixed-cut singularity is resolved into a string of moving poles, but the right-most one is still to the right of j=1. (The slope of the moving Regge poles depends upon the assumed boundary condition, i.e upon the unknown strong interaction dynamics). Thus one of the undesirable features has disappeared, namely the fixed-cut nature of the leading j-plane singularity. The other one, however,the violation of the Froissart bound is still present. These modifications alone are therefore not yet sufficient to provide an acceptable high energy description.

In the next step one therefore has to ask which further modifications are necessary. The answer is: more diagrams have to be included. The reason is that the ladder diagrams with two reggeized gluons are lacking s-channel unitarity. An iterative way of restoring s-channel unitarity while preserving at each step partial wave unitarity in the t-channel has been outlined (and partly been carried out [26]) in [27]. The central idea to this program is the use of dispersion relations for multiparticle amplitudes rather than doing perturbation theory order by order. In order to apply this powerful technique to QCD, the gluons have first to made massive (by introducing Higg's scalars and invoking the Higg's mechanism). Then the gluon reggeizes like an ordinary massive vector particle, and extended use can be made of the known analytic structure of multiparticle amplitudes in multiregge limits. (Note that this technique could, most likely, not be used in massive QED: since the photon does not reggeize, scattering amplitudes have a much more complicated analytic structure than in nonabelian theories where the vector mesons do reggeize). Once all these calculations have been done, the gluon mass has to be removed (see below).

The outcome of this unitarization scheme (still with the gluon mass being nonzero) is a complete reggeon field theory: the reggeized gluons are the "elementary fields", all n-reggeon $\rightarrow$ m-reggeon couplings are present (obeying the rules of signature conservation), and all these vertices are momentum dependent, i.e. they are nonlocal. A few diagrams are shown in Fig.10. In elastic scattering amplitudes with color zero exchange channel (which is, of course, what we are interested in) only even numbers of gluon lines appear in the t-channel, and the Pomeron appears as a bound state (glueball). For full QCD this description is still incomplete: fermions have to be included by introducing flavor nonsinglet exchange channels (reggeization of fermions in nonabelian gauge theories has been studied in [28]).

As it has been said before, after all these diagrams have been included and the elements of the reggeon field theory have been defined, it still remains to remove the gluon mass. The whole program described above relies upon the assumption that QCD can be reached by taking a certain limit in the parameter space of a spontaneously broken SU(3) gauge theory. There are arguments which strongly support this assumption: (i) in the framework of perturbation theory, one can check whether the two approaches give the same answer: either calculating order by order in QCD or applying the dispersion relation technique to a spontaneously broken gauge theory and then removing the scalar fields. This has been done for the two-gluon ladder diagrams in SU(2) gauge theory. Further support comes from the study of the small-x limit of hadronic structure functions in QCD (see below): this analysis seems to lead to the same diagrams as the dispersion relation-based analysis of the massive theory.(ii) Beyond perturbation theory, an analysis of the phase structure of lattice gauge theories [29] shows that, as long as the Higg's scalars are in the fundamental representation of the color group, the Higg's phase and the confinement phase are analytically connected. There exists therefore a path in the parameter space which leads from the phase with massive reggeizing vector particles to that of confinement. (iii) Finally, one may ask directly how the massive reggeon diagrams behave when the mass of the vector particle is taken to zero. Most remarkably, one sees striking differences between color zero and color-nonzero exchange channels. In the former case, there is an enormous cancellation of infrared divergencies (which, however, is not expected to be complete in more complicated reggeon diagrams). In channels with color-nonzero exchange, on the other hand, the infrared logarithms always seem to pile up in such a way that these amplitudes are suppressed: this is, of course, consistent with the expectation that open color should not be produced at high energies. Taking together all these bits of evidence, it seems very probable that this scheme of isolating QCD diagrams which in the high energy limit satisfy both t-channel and s-channel unitarity might really work.

In the third step, which to me seems to be the most difficult one, on has to solve this reggeon field theory. In doing so, one has to keep in mind the restrictions which I have mentioned in the context of the two-gluon ladders. That means that, in addition to the difficulty that the obtained reggeon field theory is nonlocal, comes with arbitrary high order interaction vertices and that we are asking for bound states rather than corrections to the elementary reggeon field, both the infrared and the ultraviolet part of the momentum integrals of the underlying Feynman amplitudes must be handled with great care. In the ultraviolet part it may be enough to simply put in by hand the momentum dependence of the strong coupling, but in the infrared region confinement dynamics must be brought in. A signal for this are the infrared logarithms, which are expected to appear in more complicated reggeon diagrams when the gloun mass is removed. All experience from statistical mechanics and the use of the renormalization group suggests that when trying to solve this theory one should look for a phase transition: higher order interactions and nonlocalities in the interaction verticies could then be dismissed as irrelevant operators. This is why critical reggeon field theory appears to be the strongest candidate for a solution.

As the simplest attempt one could think of the following scheme of arranging the summation of all reggeon diagrams [25]. Define the two-gluon ladders - with ultraviolet and infrared modifications being built in following the ideas of [25] - to be the "bare Pomeron". It is a moving pole with intercept above one, and its slope depends upon the (unknown) confinement properties of the low-momentum region. The vertex: two reggeized gluons $\rightarrow$ four reggeized gluons then generates a triple Pomeron vertex which leads to self interactions of the bare Pomeron and brings the intercept down by some amount. By gradually including more and more of the higher order reggeon interaction vertices one then might hope to lower the (renormalized) intercept down to one (one could, of course, also end up below one). In carrying out these steps it might be helpful to make use of the conformal invariance of the interaction vertices [25]. A somewhat unsatisfactory aspect of this scheme is the lack of an ar-



gument why one should end up with the critical theory. In any case, before this program can be carried out, more information on the higher order reggeon interaction vertices is needed. Work on this line is in progress.

A much more sophisticated and ambitious program for deriving critical reggeon field theory from QCD has been formulated by A.White [30]. Strictly speaking, his ideas do not apply to QCD with the usual number of color triplet quarks, but they require the existence of two additional quarks in color sextet representations. They are expected to bind together into three color sextet pions which serve as the standard Higg's scalars in the electroweak sector. As an important consequence of this, the longitudinal components of the W's and the Z vector bosons couple to the glouns. At sufficiently high energies - most likely in the TeV region - the production of W's and Z's then start to contribute to the Pomeron (which at lower energies consisted mainly of gluons), and one expects to see some signal of this in hadron-hadron scattering. The true high energy behavior, therefore, can set in only at energies sufficiently above the onset of these effects. The main dynamical idea for obtaining for this theory the critical Pomeron is a subtle mapping of the phase of the underlying QCD onto different phases of reggeon field theory. Initially QCD (augmented by the additional quark sextets and provided with an ultraviolet cutoff) is taken to be in the spontaneously broken phase (Higg's phase), and its high energy behavior is described by the sort of reggeon field theory which I have described above. At this stage no critical behavior for the Pomeron is expected, and the total cross section might even be falling with energy. It is then only after a very careful removal of gluon mass and ultraviolet cutoff (which actually has to proceed in several steps) that the associated reggeon field theory becomes critical. It would be very helpful to have more of the details of this very interesting proposal written up, such that one could take a closer look.

What can be said to summarize this review on the theoretical work on the high energy behavior of QCD: on the whole it seems to me as if the perturbative content of the Pomeron is under control, although quite a few details of the reggeon field theory still have to be worked out. The solution of this reggeon field theory seems to me to be the major task which has to be worked on. Needless to say, that more understanding of the strong interaction confinement dynamics will be of extreme importance.

## IV The Hard Pomeron in QCD

In this section I want to review another kinematical limit which also probes the Pomeron, although from a slightly different direction. One of the motivations for including this into my review is the consistency which seems to emerge when both limits are studied within QCD: the Regge limit and the small-x limit. But there is also some interest in the small-x behavior of structure functions by itself, in particular with the HERA machine coming up soon.

I begin with a few basic features of the standard QCD description of deep-inelastic scattering [31]. The $Q^2$-evolution of a structure function $D_a^b(x,Q^2)$ is usually described in terms of the Altarelli-Parisi equations (the structure function $D_a^b(x,Q^2)$ expresses the probability of finding, at scale $Q^2$, parton b with momentum fraction x inside parton a. Parton stands for quark or gluon. The scaling variable x is defined as usual: $x = -q^2/2pq$. For small x the approximate relation holds: $1/x = s/Q^2$, where s is the energy of the system: photon + hadron). This description is completely equivalent to representing $D_a^b(x,Q^2)$ as the sum of flavor singlet QCD ladder diagrams (Fig.11). In an obvious manner, they describe the cascade of partonic decay processes, beginning with parton a at low momentum scale $Q_0^2$ and momentum fraction 1, and ending with the slower parton with momentum fraction x and scale $Q^2$. At their lower end, these diagrams have still to be linked to the valence quarks inside the hadron, and the x-distribution at the low-momentum scale $Q_0^2$ has to be put in by hand. The ladders then describe how this x-distribution changes as a function of $Q^2$. A characteristic feature of these diagrams is the ordering of the momenta:when moving along the ladder from the bottom to the top, the momentum fraction decreases from 1 down to x, whereas the virtuality of the parton (or, in the language of Sudakov variables, the square of its transverse momentum) increases from the initial low scale up to $Q^2$. In the region of small x, gluons generally dominate over quark contributions:in the following I will restrict myself to gluonic ladders (it should be clear that these ladders are not identical of those of the previous section).

These QCD ladders, together with a suitable ansatz for the x-distribution at some low-momentum scale, provide a good description for hadronic structure functions in deep-inelastic scattering and in other hard processes, provided the variable x is not taken to be too small (there are also words to be said about the limit $x \to 1$, but this is not my topic). In the limit of very small x the gluon ladders have the following behavior [32]:

$$F(\neq 6^2) \approx \frac{\exp[\sqrt{2(\xi - \xi_0)y}]}{\sqrt{2\pi}[2(\xi - \xi_0)]^{1/4}} \qquad (17)$$

where

$$\xi = \ln \ln Q^2/\Lambda^2 \; , \; y = \frac{8N}{\beta_0} \ln \frac{1}{x}$$
$$\beta_0 = \frac{11}{3} N - \frac{2}{3} n_F \; , \; N = 3 \text{ for } QCD \; . \qquad (18)$$

This growth is too strong and violates unitarity: if one uses the photon-hadron analogy (vector dominance), hadronic unitarity requires:

$$\sigma^{\gamma}_{tot} = \frac{4\pi^2 \alpha_{em}}{Q^2} F(x,Q^2) < 2\pi R_h^2(s) \; , \qquad (19)$$

with

$$R_h(s) \sim \ln s \qquad (20)$$

From (17) it follows, however, that F grows faster than any power of lns, and eqs.(19), (20) are violated. To put this differently [32],the QCD ladders when coupled to a fast constituent quark describe the cascade of decays into slower and slower partons. The photon then "sees" the quark as being surrounded by a cloud of gluons and sea quarks, and as long as the clouds of the different constituent quarks are not too dense, it interacts with one at a time. This situation changes if the density increases. When the probability of an interaction between partons of different clouds becomes of order unity, the single ladder approximation no longer holds, and the interaction of several ladders has to be taken into account. To make this more quantitative, interpret $F(x,Q^2)/R_h^2$ as the density of gluons with scale $Q^2$ and momentum fraction x, and take for the interaction cross section of partons at scale $Q^2$ $\alpha_s(Q^2)/Q^2$. Then the probability for partons to interact is:

$$P(x,Q^2) \approx \frac{\alpha_s(Q^2)}{Q^2} \cdot \frac{F(x,Q^2)}{R_h^2} \; , \qquad (21)$$

This probability function should be less than one in order to justify the one-ladder approximation. With (17), however, P reaches unity at some small value x, and the approximation breaks down.

It is therefore necessary to "improve" this QCD description in the small-x region by including more diagrams. This has been worked out by the Leningrad group [32]. As it has been said above, these new contributions represent interactions between ladders evolving from different constituent quarks at low momentum. After the interaction the ladders combine and continue their further evolution in one common ladder (Fig.12). These diagrams have suggestively been named "fan diagrams". Their sum is given as a solution of an integral equation and has been discussed in [32] (see also [33]). It allows, at fixed $Q^2$, to extend the QCD description of structure functions further down in x, but at low x there is again a limit to its validity. In order to cross this limit two modifications have to be made. The first one concerns the QCD ladder itself: so far the QCD gluon ladders of this section differ from those of the previous one in that the gluon has not been reggeized. These effects have now to be included, and from now on the QCD ladders consist of the same diagrams as those of the Regge limit. Because of the different kinematic limits and choices of variables (x and $Q^2$ versus s and t), however, the analytic expressions one is dealing with are different. (As a consequence of the reggeization of the gluon lines in the QCD ladders, the small-x behavior in (17) becomes even stronger [34], and the conflict with unitarity more acute). The second modification to be applied to the fan diagrams is the removal of a restriction:



when moving from the top to the bottom, in the fan diagrams the number of QCD ladders never decreases. This restriction is now lifted, as a result of which the set of diagrams to be summed takes the form of a field theory (Fig. 13). The propagator of the "bare field is derived from the QCD ladders [32]. Since the ladder diagrams are the same as those of the Regge limit, we clearly see a convergence appearing. It is only because of the different limits and variables, that the analytic expressions obtained from ladder graphs are not the same and the rules of the field theories are different. In fact, because of this difference it has not yet been possible to find a method of carrying out calculations in this new form of a "reggeon field theory" [32]. Nevertheless, the mere fact that the perturbative analysis of both the Regge limit and the small-x limit in deep-inelastic scattering in QCD lead to the same set of diagrams presents a very important consistency check and clearly supports that we are going in the right direction. Once it has been established that both limits require the same set of diagrams, it seems inevitable that the difficulties mentioned in the context of the Regge limit will eventually also show up in the small-x limit. In particular, at some very small value x the ordering of momenta inside the QCD ladders which so far has protected us from the infrared problems becomes invalid and the difficulties of dealing with this region will appear. As a consequence, even if one could handle the new "reggeon field theory" which appears in the small-x limit, also this description would not yet provide the ultimate theory for $x \to 0$.

Next one has to answer the important question for what numerical values of x and $Q^2$ these various versions of QCD approximations should be used. Let me first repeat the estimate given by the inventors of this theory [32] (Fig.14). Starting from the $\xi$-axis below and moving upwards in y, one first is in a region where the standard ladders of the beginning of this section provide a good description. The line "1" marks the boundary of this region: above this line the first step of improvement, the fan diagrams, have to be used. The second curve "2" shows where this step of approximation fails: above this line one would have to solve the new "reggeon field theory", and eventually the infrared problems mark the end of any perturbative treatment. Line "1" is tangent to curve "2" and the point $\xi_0$. A numerical evaluation of these curves is shown in Fig.15: the results depend upon the QCD scale $\Lambda$ and the momentum scale $Q_0^2$, where the QCD evolution is assumed to start. The results indicate a little inconsistency at the left end: for realistic values of $\Lambda$ and $Q_0^2$, the boundary curves "1" and "2" intersect rather than being tangent. This implies that the expressions for the two curves presumably have to be modified for low $\xi$-values. Leaving this question aside and taking the third figure to be valid for larger values in $Q^2$, the conclusion would be that as long as x is not smaller than $10^{-2}$ the effects of the fan diagrams will not be seen.

Another numerical estimate has been carried out by Kwiecinski [36]. He has investigated at which values x the first fan diagram becomes nonnegligeable compared to the standard ladders. For $Q^2$ between 10 and $10^4$ $GeV^2$, the first fan diagram is less than approximately 5% of the standard ladder, if x is bigger than $10^{-2}$. The 10% level is reached at about $x \approx 10^{-5}$. He has also calculated the probability function $P(x,Q^2)$ (eq.(4.5)): it is well below one in all these regions. This supports the conclusions stated before: in order to test the "improvement" due to fan diagrams, x has to be as small as $10^{-4}$. Experimental data so far lie in the region $x > 10^{-2}$. HERA will be the first machine which will allow to reach the interesting $x \approx 10^{-4}$.

A somewhat different analysis of Glueck et al.[37] leads to a similar conclusion. The standard QCD-ladder analysis, as it has been said before, depends upon the x-dependence which is used as an input at low momentum. In particular, the small-x behavior at higher $Q^2$ values will be sensitive to this input. The numerical analysis of [37] however shows that these effects will also not be seen unless x becomes smaller than $10^{-2}$ (see also [38]). All this makes it extremely desirable to have data for x-values as small as $10^{-4}$ or even less.

To conclude this part of the discussion, I would like to stress three points.(i) From the theoretical viewpoint I find it very satisfactory that both the Regge limit and the small-x limit consistently lead to the same sort of Feynman diagrams. In both cases, however, the extreme asymptotic limits ($s \to \infty$ and $x \to 0$) require nonperturbative components and are still waiting for a complete solution. (ii) The ideas of Gribov et al.[32] have not been tested yet, since existing data have not yet reached the interesting region of small x. For practical purposes it would be desirable to have manageable expressions for the sum of the fan diagrams (see, for example, [33]). (iii) HERA will be the first machine which probes the part of the small-x region where the new ideas can be tested. Theorists should be prepared when HERA starts to work.

In the last part of my talk I would like to say a few word on an issue which goes under the name "Pomeron structure function". Ingelman and Schlein [39] (see also Fritzsch and Streng [40]) proposed to measure the gluon content of the Pomeron in high-mass diffractive $p\bar{p}$-scattering: the antiproton scatters elastically and emits a Pomeron (with low momentum transfer t), which combines with the proton to produce two large-$p_T$ jets inside the high-mass Pomeron-proton system (Fig.16). Viewing the Pomeron as an "incoming particle", the two jets arise from the fusion of a gluon inside the Pomeron and a quark (or gluon) taken from the proton. The Pomeron structure function which enters the calculation of this process is, a priori, a new and unknown function: a first comparison with data [41] seems to favor a rather soft (near x=1) gluon distribution, but there is some uncertainty about the normalization. In fact, it seems to me, that the situation may be more favorable and the Pomeron structure function may be calculable within perturbative QCD (provided we stay away from any extreme kinematical limit). In the above picture it has been assumed [42],[43] that the Pomeron which fuses with the proton and then gives rise to hard processes is really the same as in hadron hadron scattering: factorization holds, and the energy dependence is the same as in the triple-Regge limit of inclusive scattering:

$$\frac{d^2\sigma_{jet\,jet}}{dtdM_x^2} = \frac{d^2\sigma_{SD}}{dtdM_x^2} \cdot \frac{\sigma_{P\,Pom\to jet\,jet}}{\sigma_{P\,Pom\to X}} \tag{22}$$

$$\frac{d^2\sigma_{SD}}{dt\,dM_x^2} = \frac{1}{16\pi}\beta_{\bar{p}}(t)^2 G_{triple\,Pom}(t)\beta_P(0) \; s^{2\alpha_p(t)-1}(M_x^2)^{\alpha(0)-2\alpha_p(t)} \tag{23}$$

$$\sigma_{P\,Pom\to X} \approx const \approx 1\,mb$$

$$\sigma_{P\,Pom\to jet\,jet} = \int dx_1 dx_2 \sum_i G(x_1,Q^2) f_i(x_2,Q^2)\frac{d\hat{\sigma}_i}{d\hat{t}}$$

$$G = \text{Pomeron structure function}$$

$$f_i = \text{proton structure function}$$

This implies that the Pomeron is still soft when it meets the proton, and only from then on it starts to develop hard constituents. Based upon the QCD picture it seems more probable that the evolution of hard constituents inside the Pomeron starts much earlier, namely as soon as the Pomeron is emitted from the antiproton. To illustrate this in some more detail, it is convenient to consider this "Pomeron structure function" in deep-inelastic ep-scattering (Fig.17). The kinematical configuration analogous to that in $p\bar{p}$-scattering outlined above would be the following: in the subprocess: photon+proton $\to$ anything take those final states where a high-mass cluster is produced along the photon direction and, in rapidity, is well-separated from the proton. In other words, there is a "hole" in the final state between this cluster and the proton which goes through elastically. Such a process obviously is just the diffractive cut through the first fan diagram (Fig.17) which I have described above. The two ladders above and below the cutting line then represent the Pomeron. The assumption made in [42] that the Pomeron stays "soft" until it starts to produce the high-mass final state, translated to this QCD picture, then means that the internal momenta inside the Pomeron ladder remain small all the way up to the "triple vertex" and then start to grow in the usual fashion. It seems much more natural that the main contribution should be given by the configuration of internal momenta which I have described in the beginning of this section: the growth of transverse momenta and the decrease of momentum fraction starts right at the end of the diagrams when the Pomeron takes off from the proton. If this expectation is correct, the Pomeron structure function could be calculated (as long as we stay away from



too small values of x) in QCD. In fact, all this has been discussed already in one of the later sections of [32], and it follows from this that eqs. (22),(23) have to be changed slightly. A numerical study is presently underway [44].

## V Conclusion

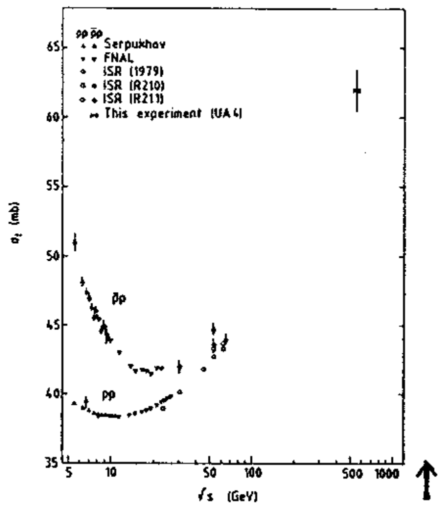
The aim of this talk has been to give an overview of the present situation of Pomeron-related topics in the framework of QCD. As to the purely hadronic part of this, it seems to me that there the total cross section may very well exhibit some changes when higher energies are reached. The question of the true asymptotic behavior in hadron hadron scattering is, therefore,not settled. The theoretical status of the Pomeron in QCD is, in my opinion, not satisfactory. The Pomeron has its roots both in the perturbative and the nonperturbative parts of QCD, and a clear separation between both seems not possible.It is, therefore, conceivable that we first have to have a better understanding of the confinement dynamics before the Pomeron problem can be solved. In hard scattering the situation is somewhat better: the extreme small-x limit where perturbation theory breaks down is far away. Before experiments will reach this "dangerous" region, we first should test the idea of "improvement" due to the fan diagrams which will give a useful hint whether we are going in the right direction.

**Acknowledgements:** I would like to thank A. Ali and A. Zichichi for inviting me to this stimulating and very pleasant meeting. I also would like to express my gratitude to the Institute for Nonlinear Science (INLS) at the University of California, San Diego where most of this written version has been prepared.

### References

1. M.Bozzo et al.,Phys. Lett. 147B (1984) 392.
2. U. Amaldi et al.,Phys. Lett. 66B(1977) 390.
3. A. Martin in: Proc. Fourth Topical Workshop on pp Collider Physics, Bern, 1984, p.308; C. Bourrely, A. Martin in: Proc.of the ECFA-CERN Workshop on Large Hadron Collider in the LEP Tunnel, held at Lausanne and Geneva, 1984, p.323. M.Haguenauer, G.Matthiae, ibid. p.303.
4. M.M. Block and R.N. Cahn, in "Strong Interactions and Gauge Theories," Proceedings of 21st Recontre de Moriond, Les Ares, France, 1986, edited by Tran Thanh Van (Editions Frontiére, Gif-sur-Yvette, 1986), Vol. 2, p.679.
5. D.Bernard et al.,Phys.Lett. 171B (1986) 142.
6. D.Bernard et al.,Phys.Lett. 198B (1987) 583.
7. P.M.Kluit and J.Timmermans, Phys.Lett. 202B (1988) 458.
8. For general reviews see: H.D.I. Àbarbanel, J.B.Bronzan, R.L.Sugar, A.R.White, Phys.Rep. 21C (1975) 119; M.Baker, K.A.Ter-Martirosyan, Phys.Rep. 28C (1976) 3; M.Moshe, Phys. Rep 37C (1978) 255.
9. S.T.Jones, J.W.Dash, Phys.Rev. D37 (1988) 583.
10. C. Bourrely, J. Soffer and T.T.Wu, Nucl.Phys. B247 (1984), 15; Phys.Rev. Lett. 54 (1984) 757; Z. Phys. C37 (1988) 369.
11. P.Gauron, E.Leader, B.Nicolescu, Phys Rev.Lett. 54 (1985), 2656; Phys. Rev. Lett. 55 (1985) 639; preprint IPNO/TH 86-55 (1986).
12. A.Donnachie, P.V. Landshoff, Nucl. Phys. B244 (1984) 322.
13. A.Donnachie, P.V. Landshoff, Phys. Lett. 123B (1983) 345; Nucl. Phys. B231 (1983) 189;P.V.Landshoff, O.Nachtmann, Z. Phys. C 35 (1987) 405.
14. A.Capella, J.Tran Thanh Van, Z. Phys. C38 (1988) 177 and references therein.
15. J.Baumel, M. Feingold, M.Moshe, Nucl. Phys. B198 (1982) 13.
16. M.Baig, J. Bartels, J.W.Dash, Nucl. Phys. B237 (1984) 502.
17. H.Cheng and T.T. Wu, Phys. Rev. Lett. 22 (1969) 666; Phys. Rev. Lett. 23 (1969) 1311; Phys. Rev. Lett. 24 (1970) 759 and 1456; Phys. Rev. 182 (1969) 1852; Phys. Rev. D1 (1970) 2775.
18. Second reference of [10].
19. D. Bernard, P. Gauron and B. Nicolescu, Phys. Lett. 199B (1987) 125.

20. V.M. Schekhter, Sov. Journ. Nucl. Phys. 33 (1981) 426.
21. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP 44(1976) 443.
22. E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. TETP 45(1977) 199.
23. Ya.Ya. Balitzky, L.N. Lipatov, Sov. Journ. Nucl. Phys. 28(1978) 822.
24. Ya.Ya. Balitzky, L.N. Lipatov, JETP Lett.30 (1979) 355.
25. L.N. Lipatov, Sov. Phys. JETP 63 (1986) 904.
26. J. Bartels, Nucl. Phys. B151 (1979) 293; Nucl. Phys. B175(1980) 365.
27. J. Bartels, Acta Phys. Polonica B11(1980) 281; unpublished.
28. R. Kirschner and L.N. Lipatov, Sov. Phys. JETP 56(1982;) Phys. Rev. D26 (1982) 1202.
29. E. Fradkin and S.H. Shenker, Phys. Rev. D26 (1982) 3682.
30. A.R. White, talk given at the IInd International Conference on Elastic and Diffractive Scattering, Rockefeller University, New York, October 1987; preprint ANL-HEP-CP-87-120; in "Hadron Multiparticle Production," edited by P. Carruthers (World Scientific 1988), and references therein.
31. For general reviews see: C.H.Llewellyn Smith, Schladming Lectures 1978, Acta Physica Austr., Suppl. XIX (1978) 331; Yu.Dokshitzer, D.I.Dyakonov, S.I.Troyan, Phys. Rep. 58 (1980) 269;E. Reya, Phys. Rep. 69 (1981) 195; A.H. Mueller, Phys. Rep.73 (1981) 237.
32. L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Rep. 100 (1983) 1.
33. J.P. Ralston, D.W. McKay, in 'Physics Simulations at High Energy,' Madison, Wisconsin, 1986,p.30.
34. T. Jaroscewicz, Phys. Lett. 116B (1982) 291.
35. G. Schuler, unpublished.
36. J. Kwiecinski, Z. Phys. C29 (1985) 561.
37. M. Glueck, R.M. Goodbole, E. Reya, Dortmund preprint DO-TH 88/9.
38. E. Eichten, I. Hinchliffe, K. Lane, C. Quigg, Rev. Mod. Phys. 56 (1984) 579.
39. G. Ingelman, P. Schlein, Phys. Lett. 152B (1985) 256.
40. H. Fritzsch, H.H. Streng, Phys. Lett.164B (1985) 391.
41. R. Buino et al., CERN-EP/88-60.
42. E.L. Berger, J.C. Collins, D.E.Soper, G.. Sterman, Nucl.Phys. B286 (1987) 704.
43. A. Donnachie, P.V. Landshoff, CERN-TH.5020/88.
44. J. Bartels, G. Ingelman, in preparation.

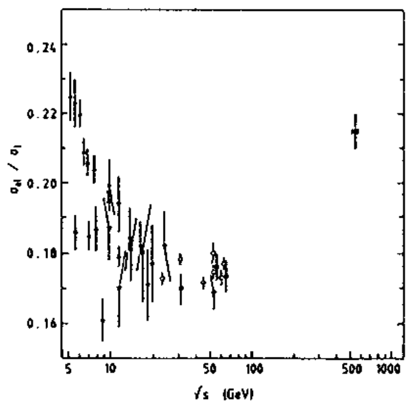
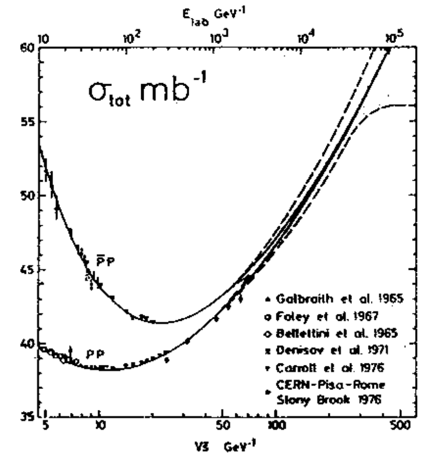




Figure 1a. The total cross section for $pp$ and for $p\bar{p}$-scattering (from [1]). The arrow marks the Tevatron point which has not been published yet.

Figure 1b. The ratio $\sigma_{el}/\sigma_{tot}$ (from [1]).



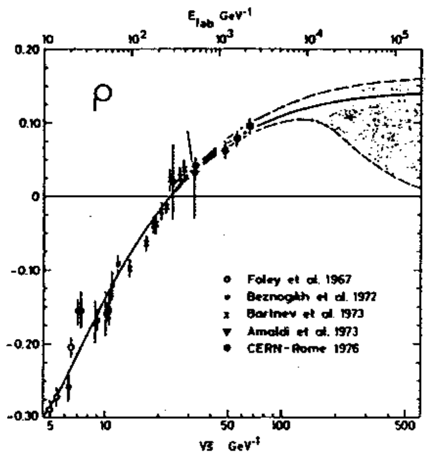




Figure 2a. Amaldi's fit to $\sigma_{tot}$ for $pp$ and $p\bar{p}$-scattering (from [2]).

Figure 2b. Amaldi's fit to $\rho = ReF(s,0)/ImF(s,0)$ (from [2]).

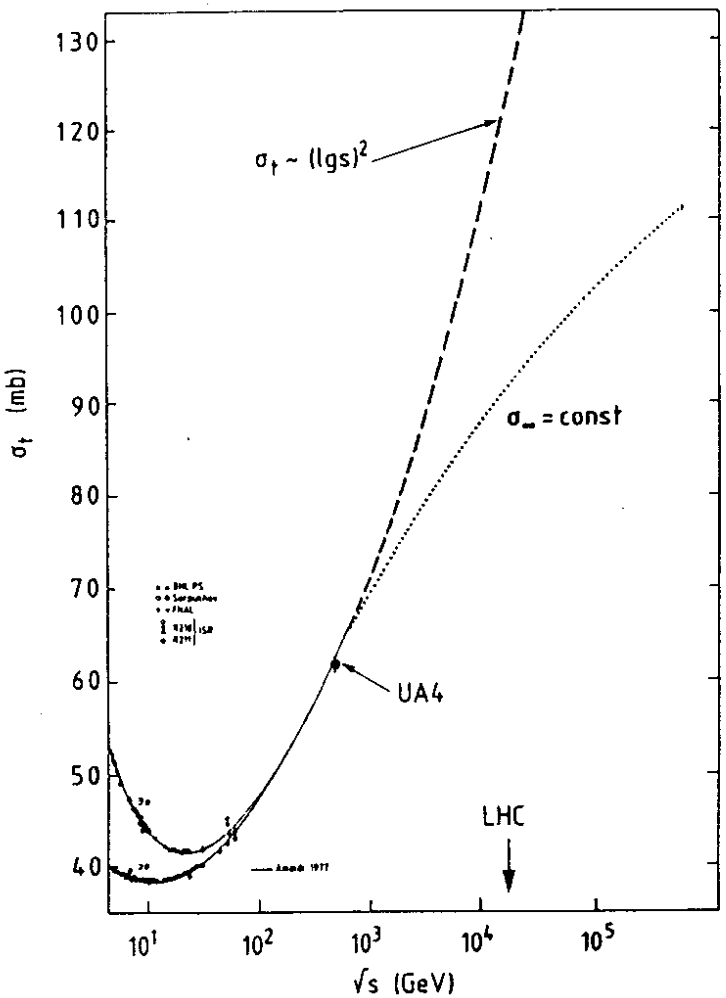



Figure 3. Fit of Bourrely and Martin (taken from [3]).

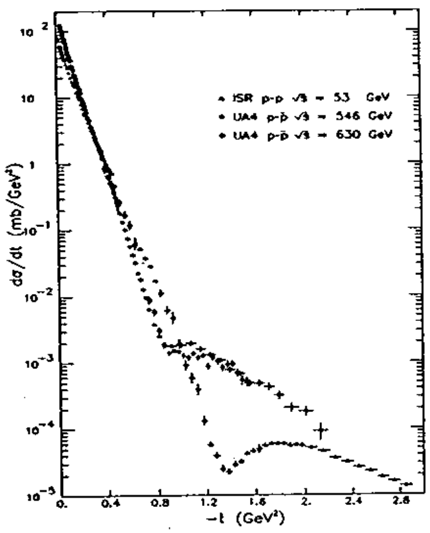



Figure 4. Compilation of $d\sigma_{el}/dt$ for pp at ISR and for $p\bar{p}$ at the collider (from [5]).

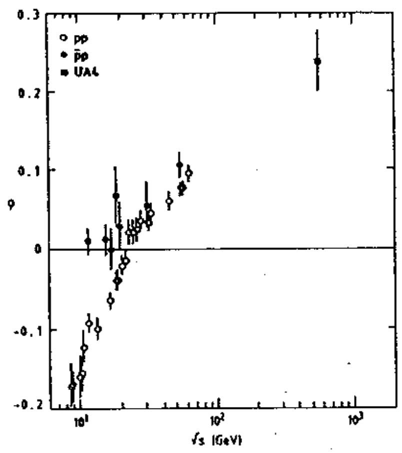



Figure 5. Data points for the $\rho$-parameter (from [6]).

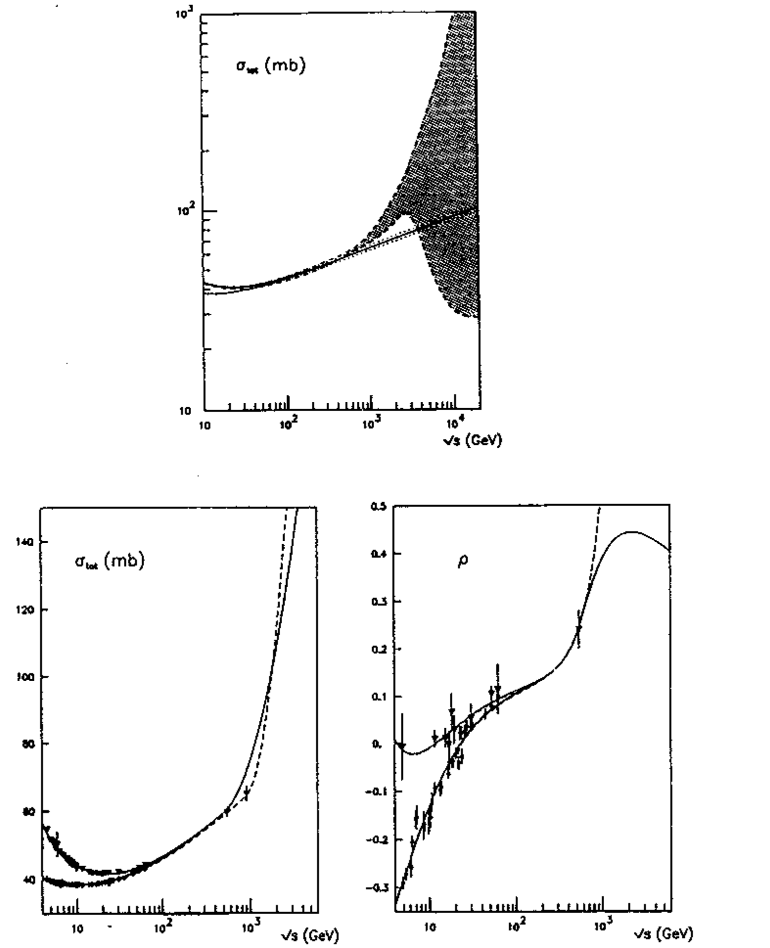



Figure 6. "Predictions" for $\sigma_{tot}$ and $\rho$ (using the UA4 measurement of $\rho$ and dispersion relations). The models are described in [7], and the curves are taken from the same reference. (a) No energy threshold is assumed, and the dashed area shows the allowed variation of the constant in front of the $ln^2 s$ term. (b) An energy threshold is assumed at $\sqrt{s_0} = 500GeV$.

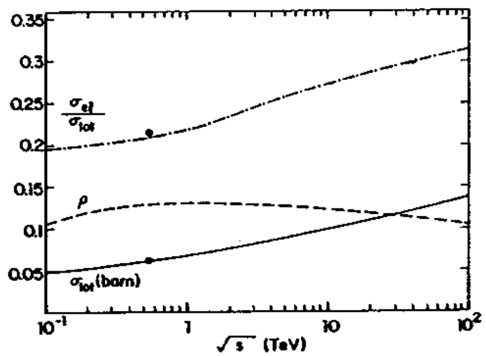



Figure 7. Extrapolations for $\sigma_{el}/\sigma_{tot}$, $\rho$, and $\sigma_{tot}$ to higher energies (from [18]).

Figure 8. QCD ladder diagrams in the Regge limit (leading -lns approximation): The wavy lines denote reggeized gluons, and the open circles stand for momentum dependent reggeon interaction vertices.

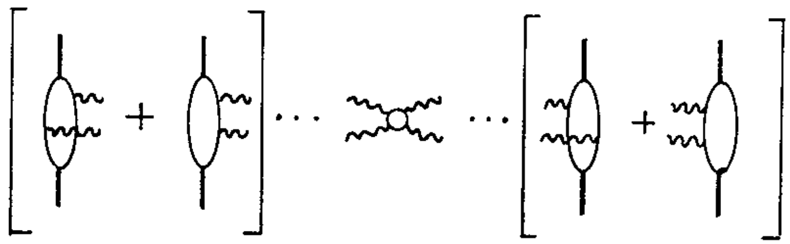


Fig.9 : Meson-meson scattering in the leading-lns approximation (with heavy quarks).

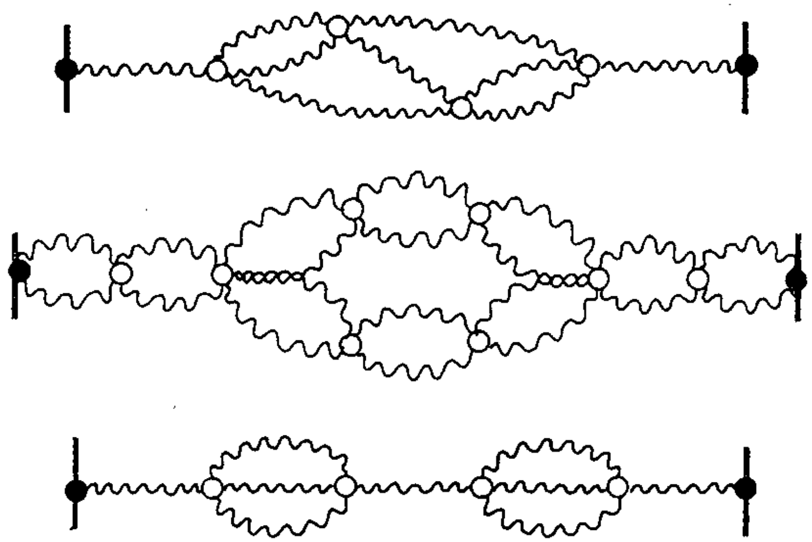


Fig.10: A few reggeon diagrams which emerge from the s-channel unitarization descibed in [27].The notation is the same as in Fig. 8.

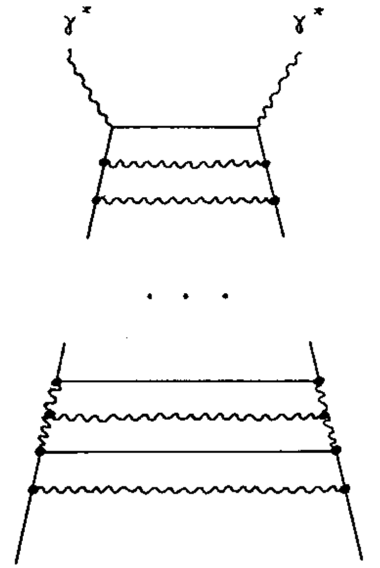



Fig.11: QCD diagrams in the small-x limit. Straight lines and wavy lines are quarks and gluons, resp.

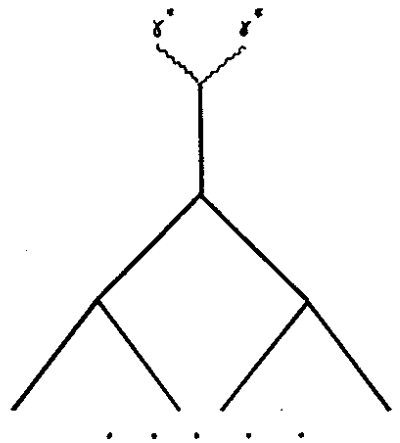



Fig.12: A few "fan diagrams". The black lines denote ladders as shown in Fig.11 (but with gluons only).

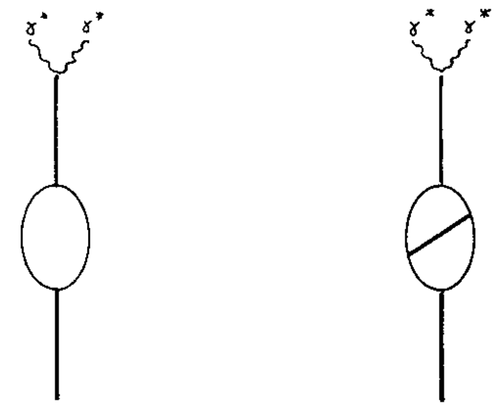



Fig.13: More diagrams have to be included if one goes to even smaller x-values.The ladders are now the same as in Fig. 8.

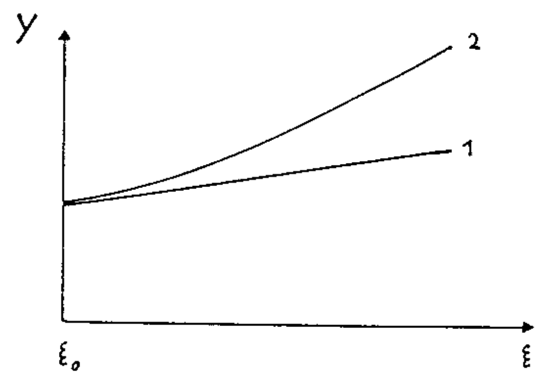



Fig.14: Boarder lines in the $\xi - y$ plane (for explanations see text).

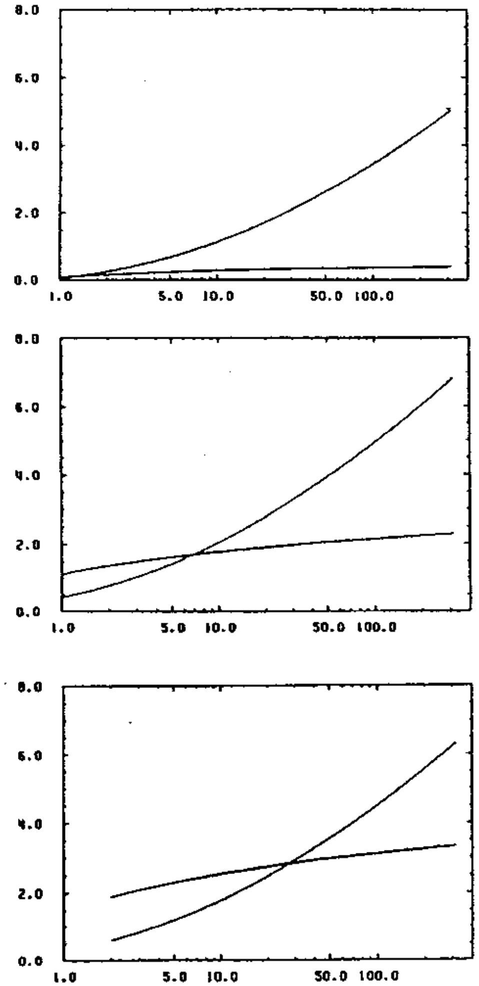



Fig.15: Numerical examples for the curves 1 and 2 of Fig.14 for different values $Q_0^2$ and $\xi_0$. (from [35]). We plot $n = -\log_{10} x$ versus $\sqrt{Q^2}$ (in GeV). The values for $Q_0$ and $\Lambda$ are: 1 GeV and 440 MeV, 1 GeV and 200 MeV, 2 GeV and 200 MeV, resp.

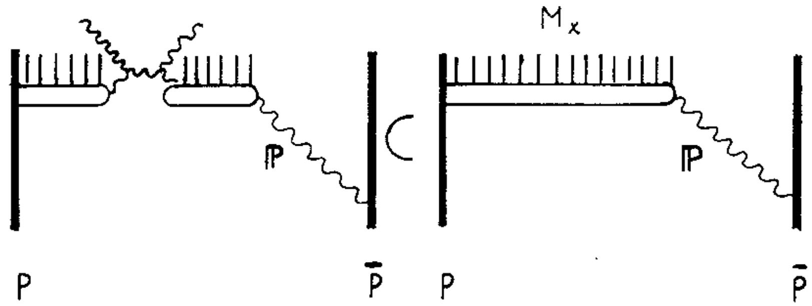



Fig.16: The Ingelman-Schlein final state configuration for extracting the Pomeron structure function.

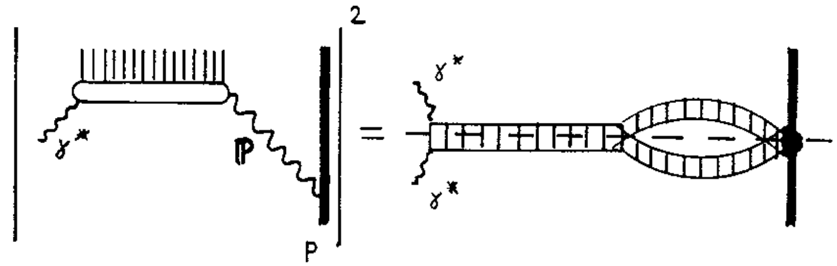



Fig.17: The Pomeron structure function in deep-inelastic scattering.