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Standard Model Predictions for B Physics – A Statistical Analysis

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ABSTRACT

The standard model predictions for B_s^0-$\overline{B_s^0}$ mixing and CP violating hadronic decay asymmetries are presented in the form of probability distributions. From these distributions, it can be easily seen what the most likely values of these quantities are, which of the CP asymmetries is expected to have the largest signal, and which measurements would clearly be signs of new physics.

There has been a great deal of interest recently in the prospects for testing the Cabibbo-Kobayashi-Maskawa (CKM) matrix through B physics [1], particularly after the measurement by ARGUS [2] of unexpectedly large B_d^0-$\overline{B_d^0}$ mixing (x_d) and its subsequent confirmation by CLEO [3]. The two main possibilities which have been discussed are CP violation in the B system, and a measurement of B_s^0-$\overline{B_s^0}$ mixing, x_s.

There are several possibilities for CP violation in the B system [4]. The most promising of these is a certain class of hadronic decay asymmetries [5] which are directly calculable from the CKM matrix without additional (unreliable) hadronic information. These asymmetries involve the decays of B^0 and $\overline{B^0}$ mesons to a common final hadronic state f, which is a CP eigenstate ($f = \pm\overline{f}$). Because of B-$\overline{B}$ mixing, a state which starts out as a pure B^0 or $\overline{B^0}$ meson will evolve in time to a mixture of B^0 and $\overline{B^0}$. CP violation is then manifested by a nonzero value of the asymmetry

$$A_f = \frac{\Gamma(B^0(t) \to f) - \Gamma(\overline{B^0}(t) \to \overline{f})}{\Gamma(B^0(t) \to f) + \Gamma(\overline{B^0}(t) \to \overline{f})} \,. \tag{1}$$

For the above class of decays the time-integrated asymmetry is then given by

$$A_f = \frac{x_q}{1 + x_q^2}\,\mathrm{Im}\,\lambda_f \,, \tag{2}$$

where x_q is the B_q^0-$\overline{B_q^0}$ ($q = d, s$) mixing parameter. The form of $\mathrm{Im}\,\lambda_f$ depends on which final state is being considered, but in all cases can be related directly to CKM matrix parameters. This is most easily seen using the Wolfenstein [6] parametrization of the (three generation) CKM matrix:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\rho\lambda^3 e^{-i\delta} \\ -\lambda(1 + A^2\lambda^4\rho e^{i\delta}) & 1 - \frac{1}{2}\lambda^2 - A^2\rho\lambda^6 e^{i\delta} & A\lambda^2 \\ A\lambda^3(1 - \rho e^{i\delta}) & -A\lambda^2(1 + \lambda^2\rho e^{i\delta}) & 1 \end{pmatrix} . \tag{3}$$

Here, $A \simeq 1.05$, $\rho \leq 0.9$ [7], and $\lambda = 0.22$ is the Cabibbo angle. With this parametrization, one sees immediately that only V_{ub} and $V_{td} \equiv |V_{td}|e^{-i\phi}$ have potentially large phases. In Ref. 8 it was shown that there are four classes of model-independent CP asymmetries A_f, three of which can be sizeable. Depending on the decay mode, the quantity $\mathrm{Im}\,\lambda_f$ obtained from A_f measures one of the following combinations of phases in the CKM matrix:

$$\begin{aligned} &\text{Cabibbo allowed } B_d \text{ decays [e.g. } B_d^0(\overline{B_d^0}) \to \Psi K_s] : \mathrm{Im}\,\lambda_1 \simeq \sin 2\phi \\ &\text{Cabibbo suppressed } B_d \text{ decays [e.g. } B_d^0(\overline{B_d^0}) \to \pi^+\pi^-] : \mathrm{Im}\,\lambda_2 \simeq \sin(2\phi + 2\delta) \\ &\text{Cabibbo suppressed } B_s \text{ decays [e.g. } B_s^0(\overline{B_s^0}) \to \rho K_s] : \mathrm{Im}\,\lambda_3 \simeq \sin 2\delta \\ &\text{Cabibbo allowed } B_s \text{ decays [e.g. } B_s^0(\overline{B_s^0}) \to \Psi\phi] : \mathrm{Im}\,\lambda_4 \simeq 0 \,. \end{aligned} \tag{4}$$

In the standard model, $\sin 2\phi$ is expected to lie in the range between about 0.1 and 1, while the other phases can take any value, $-1 \leq \sin(2\phi + 2\delta)$, $\sin 2\delta \leq 1$ [8]. In this paper I will be concerned with the CP asymmetries, A_1, A_2, A_3, associated with the nonzero values of $\mathrm{Im}\,\lambda_f$ in eqn. 4.

The other promising possibility for obtaining information about the CKM matrix is the measurement of x_s (B_s^0-$\overline{B_s^0}$ mixing) [9]. In the standard model, x_d and x_s arise theoretically from identical box diagrams (except for the $s \leftrightarrow d$ interchange), and one has

$$x_s = x_d \left|\frac{V_{ts}}{V_{td}}\right|^2 \left[\frac{f_{B_s}^2 B_{B_s}}{f_{B_d}^2 B_{B_d}}\right], \tag{5}$$

where $M_{B_s} \simeq M_{B_d}$ and $\tau_{B_s} \simeq \tau_{B_d}$ have been assumed. The factor in brackets encompasses our lack of knowledge about the hadronic matrix elements, and differs from unity only through $SU(3)$ breaking effects. (It must be pointed out, however, that these effects are difficult to calculate, and preliminary lattice results [10] indicate corrections up to 50%.) Assuming that this factor is 1, the standard model prediction for B_s^0-$\overline{B_s^0}$ mixing is $2 \leq x_s \leq 25$ [7,8].

The analyses to date have dealt only with the standard model predictions for the ranges of x_s and the CP asymmetries, A_f (or the related quantities $\mathrm{Im}\,\lambda_f$). A perhaps more interesting question, however, is "what is the most likely value of x_s or A_f?". That is, can one do a statistical analysis of existing experimental data to obtain probability distributions for the standard model predictions? The knowledge of such distributions could aid in the planning of experiments, and will indicate which values of the above quantities most constrain the standard model. It is this analysis which is presented in this paper.

Although λ and A in the CKM matrix (eqn. 3) are known, ρ and δ are almost completely undetermined. The only information we have comes from CP violation in the Kaon system, ϵ, and the measurement of B_d^0-$\overline{B_d^0}$ mixing, x_d. (In principle, the measurement of ϵ'/ϵ [11] provides additional information. However the theoretical uncertainties are quite large, so that the constraints on ρ and δ are weaker than those provided by ϵ and x_d [12].)

The theoretical expression for $|\epsilon|$ is [13]

$$\begin{aligned}|\epsilon| = &\frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} B_K \left(A^2 \rho \lambda^6 \sin\delta\right) \left(y_c \left\{\eta_3 f_3(y_c, y_t) - \eta_1\right\}\right. \\ &\left. + \eta_2 y_t f_2(y_t) A^2 \lambda^4 (1 - \rho\cos\delta)\right) + \frac{1}{\sqrt{2}}\xi .\end{aligned} \tag{6}$$

In eqn. 6, the η_i are QCD correction factors, $\eta_1 \simeq 0.7$, $\eta_2 \simeq 0.6$, $\eta_3 \simeq 0.5$, $y_i = m_i^2/M_W^2$,

and the functions f_2 and f_3 are given by

$$\begin{aligned} f_2(y_t) &= \frac{1}{4} + \frac{9}{4(1-y_t)} - \frac{3}{2}\frac{1}{(1-y_t)^2} + \frac{3}{2}\frac{y_t^2 \ln y_t}{(1-y_t)^3} \\ f_3(y_c, y_t) &= \ln\frac{y_t}{y_c} - \frac{3y_t}{4(1-y_t)}\left(1 + \frac{y_t}{1-y_t}\ln y_t\right) . \end{aligned} \tag{7}$$

B_K represents our present ignorance of the matrix element of $\left(\bar{d}\gamma^\mu(1-\gamma_5)s\right)^2$ between K^0 and $\overline{K^0}$, with $B_K = 1$ corresponding to the vacuum insertion approximation. Finally, ξ is the phase parameter of the $(I = 0)$ $K \to 2\pi$ weak amplitudes ($\xi = \mathrm{Im}A_0/\mathrm{Re}A_0$). Although there are large uncertainties in its exact value, the ξ term is small ($O(0.1|\epsilon|)$), and for this reason it is neglected in the following analysis. (I have checked that the inclusion of this term, even using the largest allowed value, has a negligible effect on the results.) The experimental value of $|\epsilon|$ is [14]

$$|\epsilon| = (2.28 \pm 0.02) \times 10^{-3} . \tag{8}$$

The box diagram in the B system is expected to be dominated by t-quark exchange, which yields, for x_d [13,15]

$$x_d = \tau_B \frac{G_F^2}{6\pi^2} M_B M_W^2 \left(f_{B_d}^2 B_{B_d}\right) \eta_B y_t f_2(y_t) \left\{A^2 \lambda^6 \left(1 + \rho^2 - 2\rho\cos\delta\right)\right\} . \tag{9}$$

Here the hadronic uncertainty is given by $f_{B_d}^2 B_{B_d}$, whose meaning is analogous to that of the corresponding quantities in the Kaon system, except that here also f_B is not measured. There is some controversy concerning the QCD correction, η_B. In Refs 13,15 an estimate of $\eta_B \simeq 0.85$ was obtained. However, a recent calculation [16], obtains a lower value, $\eta_B \simeq 0.63$. (The difference is mostly due to the scale at which the QCD corrections are evaluated.) In this analysis the latter value will mainly be used, but in order to compare the effects of the different QCD corrections, the result for A_1 using the former value will also be presented. x_d has been found experimentally to be [17]

$$x_d = 0.70 \pm 0.13 . \tag{10}$$

The theoretical expressions for $|\epsilon|$ and x_d then depend on the 5 parameters: ρ, δ, m_t, B_K and $f_{B_d}^2 B_{B_d}$, denoted collectively as $\{z_i\}$, $i = 1, ..., 5$. The requirement that the experimental and theoretical values of $|\epsilon|$ and x_d agree to some accuracy (for a given set of the $\{z_i\}$) implies that the four quantities of interest, x_s and the 3 CP asymmetries, depend indirectly on all 5 parameters. These will be denoted $f_j(z_i)$, $j = 1, ..., 4$. In order to obtain the standard model predictions for the $f_j(z_i)$ as probability distributions, the following procedure is used. The $\{z_i\}$ are allowed to vary in the ranges

$$\begin{aligned} 0 \le \rho \le 0.9 \\ 0 \le \delta \le \pi \\ 40\ \text{GeV} \le m_t \le 200\ \text{GeV} \\ 1/3 \le B_K \le 1 \\ (100\ \text{MeV})^2 \le f_{B_d}^2 B_{B_d} \le (200\ \text{MeV})^2 . \end{aligned} \tag{11}$$

For each set of $\{z_i\}$, the $f_j(z_i)$ are calculated. However, not all sets of parameters are equally probable, that is, they do not all give values of x_d and $|\epsilon|$ in agreement with experiment. In order to assign a statistical weight to each set of the $\{z_i\}$, the theoretical values of $|\epsilon|$ and x_d are obtained using this set of parameters, and the χ^2 is calculated:

$$\chi^2(z_i) = \left(\frac{|\epsilon|_{th}(z_i) - |\epsilon|_{exp}}{\Delta|\epsilon|}\right)^2 + \left(\frac{(x_d)_{th}(z_i) - (x_d)_{exp}}{\Delta x_d}\right)^2 . \tag{12}$$

For each set of parameters $\{z_i\}$, the contribution to the $f_j(z_i)$ probability distributions is then weighted by a statistical factor $e^{-\chi^2(z_i)/2}$. That is, the probability of finding one of the $f_j(z_i)$ with the value Z is

$$P_j(Z) = \frac{\int dz_1 ... dz_5\, e^{-\chi^2(z_i)/2} \delta(f_j(z_i) - Z)}{\int dz_1 ... dz_5\, e^{-\chi^2(z_i)/2}} . \tag{13}$$

The standard model prediction for x_s is found in Fig. 1. x_s is expected to be larger than x_d by a factor of O(10) (eqn. 5), and this figure shows this to be so. As can be seen, if x_s were measured to be less than 2, this would be a clear sign of new physics. The central value of this distribution is at $x_s = 4.4$, with $3.2 \le x_s \le 16.8$ at 90% c.l.

I now turn to the CP asymmetries. Experimentally, the most promising mode involves the final state ΨK_s. This is because the branching ratio is (relatively) large, and there are clean signatures. The probability distribution for this CP asymmetry, A_1, is shown in Fig. 2, for two values of the x_d QCD correction. There is a difference between the curves for $\eta_B \simeq 0.63$ and $\eta_B \simeq 0.85$. The former gives a central value for A_1 of 0.105, with $0.05 \le A_1 \le 0.24$ at 90% c.l. The latter QCD correction favours somewhat larger values, with a central value of 0.13, and a 90% c.l. between 0.06 and 0.3. However, it is clear that regardless of the QCD correction, if this CP asymmetry were found to be either negative *or near its maximum value*, this could not be accounted for by the standard model. The prediction for A_2 (the CP asymmetry found in Cabibbo suppressed B_d decays) is shown in Fig. 3. Although all values are allowed, negative values are definitely favoured, with a

roughly flat distribution between 0 and −0.5. In other words, on average one expects bigger CP asymmetries in final states such as $\pi^+\pi^-$ than in ΨK_S. The probability distribution for the CP asymmetry in Cabibbo suppressed B_s decays, A_3, is shown in Fig. 4. In general, because of the $x_s/(1+x_s^2)$ suppression and the fact that x_s is large in the standard model, this asymmetry is expected to be much smaller than A_1 and A_2. Although this figure shows this to be true, the asymmetries are not so small as to be dismissed as unmeasurable. In fact, the (negative) central value of this distribution is −0.1, which is of the same order as the central value of the A_1 distribution! It is therefore quite possible that this CP asymmetry could be found to be substantial, even within the standard model. Of course, if this were very large, it would be an indication of physics beyond the standard model.

In conclusion, I have presented the probability distributions of the standard model predictions for x_s and the hadronic CP asymmetries A_1, A_2, A_3, obtained from a statistical fit of the parameters of the standard model to the measurements of $|\epsilon|$ and x_d. The prediction for x_s has a central value of $x_s = 4.4$, with a 90% c.l. range of $3.2 \le x_s \le 16.8$. The distribution for A_1 (the CP asymmetry in (e.g.) $B_d^0(\overline{B_d^0}) \to \Psi K_S$) is found to be peaked at $\simeq 0.1$, with $0.05 \le A_1 \le 0.3$ at 90% c.l. These numbers are somewhat dependent on what one takes for the QCD correction in B_d^0-$\overline{B_d^0}$ mixing. All values are allowed for A_2 (e.g. $B_d^0(\overline{B_d^0}) \to \pi^+\pi^-$), but negative values are strongly favoured. On average, one expects a larger CP asymmetry in this mode than in $B_d^0(\overline{B_d^0}) \to \Psi K_S$. The third class of CP asymmetries, A_3 (e.g. $B_s^0(\overline{B_s^0}) \to \rho K_S$) is indeed smaller, on average, than A_1 or A_2. However, measurable asymmetries (O(0.1)) are still possible in the standard model. From these distributions it is easy to see which values of x_s and the A_i require new physics explanations. A measurement of $x_s < 2$, $A_1 \le 0$, $A_1 > O(0.4)$, $A_3 < O(-0.3)$ or $A_3 > O(0.1)$ would all be clear indications of physics beyond the standard model.

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Figure Captions

Fig 1: The probability distribution for the standard model prediction of x_s (B_s^0-$\overline{B_s^0}$ mixing).

Fig 2: The probability distribution for the standard model prediction of A_1 (the CP asymmetry in Cabibbo allowed B_d decays, e.g. $B_d^0(\overline{B_d^0}) \to \Psi K_S$). The solid (dashed) line corresponds to an x_d QCD correction $\eta_B \simeq 0.63$ (0.85).

Fig 3: The probability distribution for the standard model prediction of A_2 (the CP asymmetry in Cabibbo suppressed B_d decays, e.g. $B_d^0(\overline{B_d^0}) \to \pi^+\pi^-$).

Fig 4: The probability distribution for the standard model prediction of A_3 (the CP asymmetry in Cabibbo suppressed B_s decays, e.g. $B_s^0(\overline{B_s^0}) \to \rho K_S$).

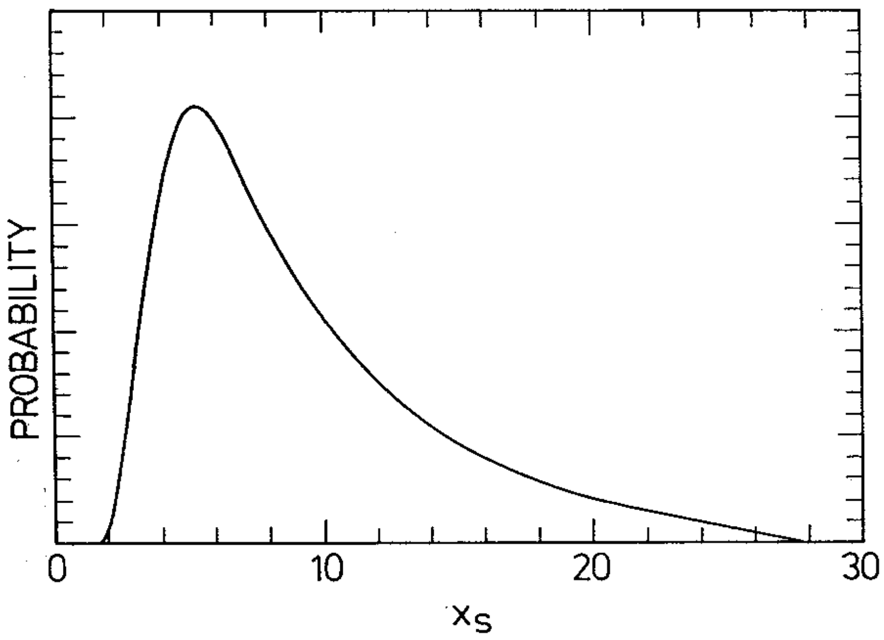

Fig. 1

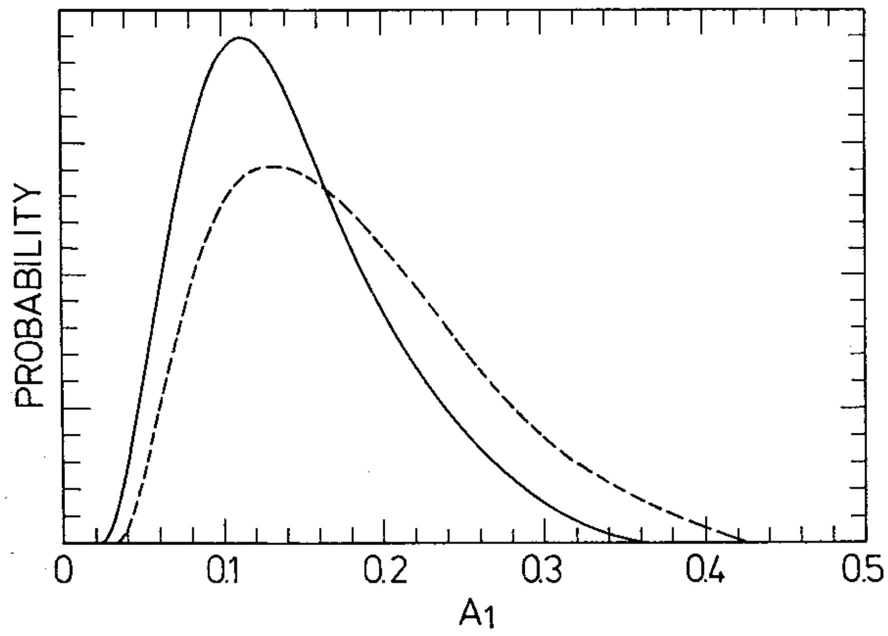

Fig. 2

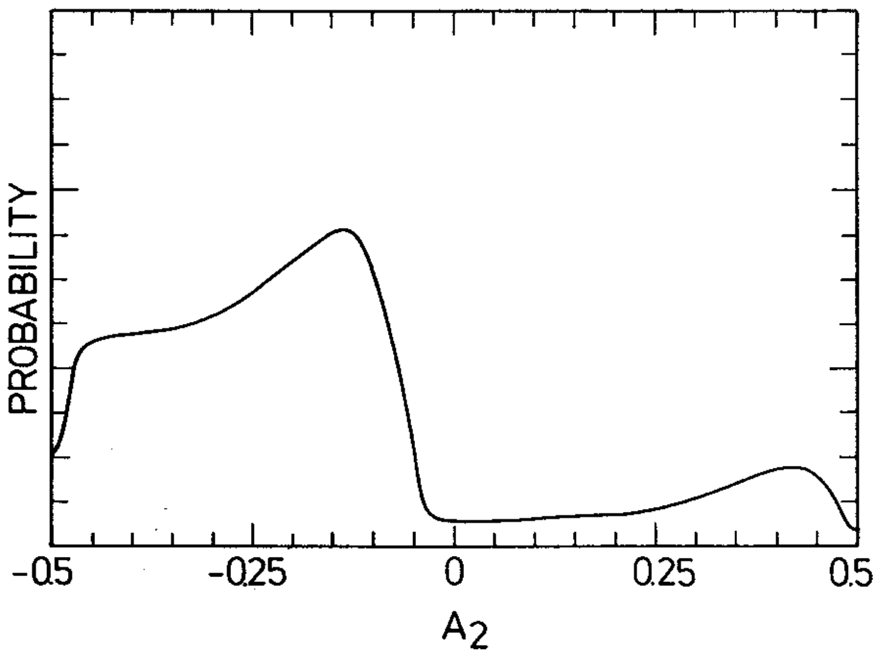

Fig. 3

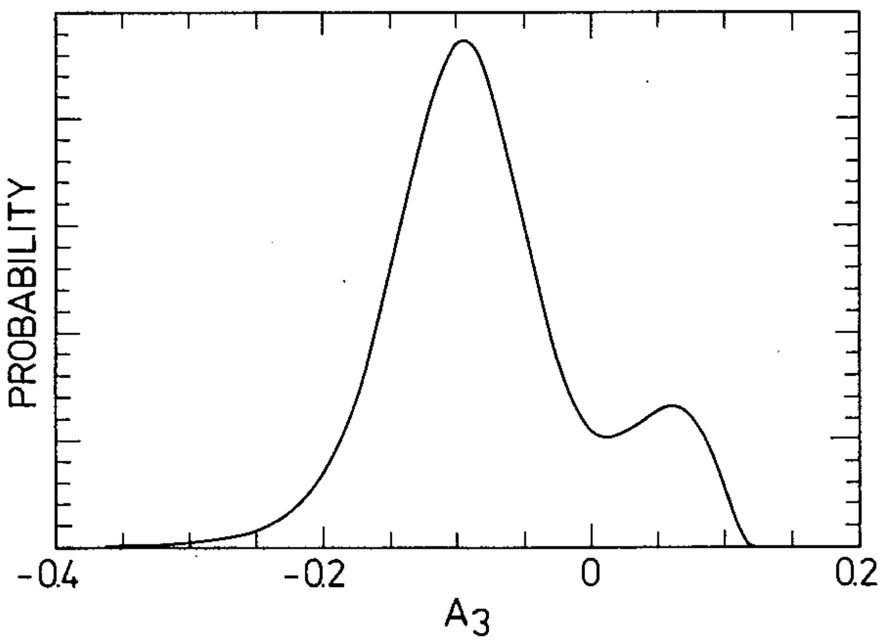

Fig. 4